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A Novel Iterative Learning Approach for Tracking Control of High-Speed Trains subject to Unknown Time-Varying Delay

Yong Chen, Deqing Huang, Member, IEEE, Yanan Li, Member, IEEE, and Xiaoyun Feng

Abstract—In this paper, a novel iterative learning control scheme is proposed for high-speed trains, aiming to track the desired reference displacement and velocity, where the Krassovskii function is constructed to compensate for the negative influence of unknown time-varying speed delays. The main feature of proposed approach is that the hyperbolic tangent function and the command filter are integrated into the learning controller to overcome the singularity problem that may occur during the control process, and relax the requirement for the derivability of desired velocity. The stability of control system is strictly proved through establishing the composite energy function, and the effectiveness is confirmed via numerical simulations. Compared with the existing works, the merits of proposed control scheme lie in that more general nonlinear uncertainties are imposed on the dynamic model of train instead of the Lipschitz condition, and the reference acceleration assigned by the railway department is not required.

Note to Practitioners—High-speed train always runs periodically on the same railway, e.g., the same tunnels, slopes, bridges, according to the scheduling plans developed by the railway department. Owing to the repetitive operation pattern, the iterative learning control has the prospect of becoming an inherent method for devising the tracking controller of trains. Nevertheless, the unknown speed delays, which are inevitable due to the damping effect of wheel-rails, couplers, etc., as well as the disturbance of external environments, may degrade the performance of control system and even cause instability in severe cases. As a result, this paper exploits a compensation method to eliminate the effects of unknown delay under the iterative learning control framework, thus guaranteeing the safety of train operation and the comfort of passengers. To enhance the practicability, the hyperbolic tangent function is introduced to keep the continuity of control signal, and the command filter is synthesized to reduce the complexity of controller implementation. Although the stability analysis and numerical simulations have confirmed the feasibility and effectiveness of proposed scheme, it is still expected to be verified by experiments in the future.

Index Terms—Iterative learning control, high-speed trains, tracking control, time-varying delay, command filter.

I. INTRODUCTION

Iterative learning control is very suitable for devising the tracking controller of high-speed trains that run periodically on the same railway, e.g., the same tunnels, slopes, bridges [1], [2], [3], [4], [5], [6], [7], [8], [9]. However, on account of the damping effect of wheel-rails, couplers, etc., as well as the disturbance of external environments, the speed delay occurs frequently during the operation of train, thus attenuating the control performance of trains. Hence, this paper proposes a novel iterative learning control approach for the high-speed trains with time-varying delay to track the given reference displacement and velocity.

Tracking control of displacement and velocity for high-speed trains has been intensively studied with fruitful results [10], [11], [12], [13]. The linearization technique was first applied to deal with the cruising control problem for high-speed trains. Some representative publications can be found in [14], [15]. In detail, Yang et al. [14] proposed a mixed $H_2/H_\infty$ cruise controller to achieve the goals of cruising speed tracking and gust attenuation for different traction types of trains. Tang et al. [15] presented a hierarchical multi-mode speed controller based on $H_\infty$ to enable train to track the reference position and speed while possessing a capacity to pass the steep uphill sections. Nevertheless, the linearized model ignores the complex dynamics of trains, thus greatly limiting the potentials of control systems. Therefore, it is preferable to devise the tracking controller of high-speed trains using the nonlinear control methods. Song et al. [16] developed a robust and adaptive algorithm to solve the automatic control problem of high-speed train under immeasurable aerodynamic drag. Faieghi et al. [17] utilized the same technique to deal with the cruise control problem in the presence of unknown parameters and external disturbances. Li et al. [18] investigated the robust output feedback cruise control of trains. Hou et al. [19], [20], [21], [22] demonstrated a series of iterative learning tracking control schemes. Besides, more achievements with respect to the nonlinear tracking control method for high-speed trains are available in [23], [24], [25].

Unknown speed delay introduces challenges to the design of tracking controller for high-speed trains, that motivate researchers to carry out more study. Kaviarasan et al. [26] proposed a dissipativity-based reliable sampled-data controller to ensure that the high-speed trains with probabilistic time-varying delays can well track the desired velocity and the relative displacement between the two adjacent cars is asym-
concludes this work. The effectiveness of the proposed scheme. Finally, Section V concludes this work. In Section IV, numerical simulations are executed to validate the proposed scheme. 5) The unknown time-varying delays are considered to be more general nonlinear terms instead of the Lipschitz functions. 3) By virtue of the backstepping technique, the proposed control strategy can construct the learning control scheme to address the speed tracking problem of trains with speed delays and input saturations. However, the proposed control methodology in [29], [30] requires that the nonlinear term of dynamic model satisfies the Lipschitz condition which is a strong assumption.

In this section, a novel adaptive iterative learning controller is proposed to address the tracking control problem of high-speed trains, where the unknown speed delay is compensated by choosing the appropriate Krasovskii function. Different from [29], [30], the command filtered backstepping technique is adopted to assist the design of learning controller. Particularly, the hyperbolic tangent function is employed to deal with the controller singularity problem that may arise from compensating for delays. Compared with the existing works, the main contributions of this paper include the following. 1) The iterative learning control scheme fully takes advantage of the repetitive operation pattern of train, in which the estimations of unknown time-varying parameters can be adjusted online in a very simple way. 2) The lumped uncertainties of train dynamics are considered to be more general nonlinear terms instead of the Lipschitz functions. 3) By virtue of the backstepping technique, the proposed control strategy can construct the controller to realize the tracking control of displacement and velocity by directly utilizing the train dynamics that is modeled as a class of second-order nonlinear system, without any auxiliary system. 4) Since the introduction of command filter could avoid taking continuously the derivative of reference trajectories, the train control system no longer requires the reference acceleration assigned by the railway department to be available, which can improve the practicality of the proposed scheme. 5) The unknown time-varying delays are compensated through constructing the Lyapunov-Krasovskii function, where the hyperbolic tangent function is integrated to ensure the continuity of control signal.

This paper is organized as follows. In Section II, the tracking control problem of high-speed trains is described, and some necessary assumptions and mathematical tools are reviewed. In Section III, an adaptive iterative learning control scheme is proposed to achieve the tracking control of the desired displacement and velocity, where the stability of control system is strictly proved by establishing the composite energy function. In Section IV, numerical simulations are executed to validate the effectiveness of the proposed scheme. Finally, Section V concludes this work.

II. PRELIMINARIES

In this section, the following dynamic model of high-speed trains over the interval \([0, T]\) is employed to formalize the tracking control problem of displacement and velocity [30],

\[
\begin{align*}
\dot{s}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= u_i(t) + c_0(t) + c_v(t)v_i(t - d(t)) \\
&\quad + c_a(t)v_i^2(t - d(t)) + f(v_i(t - d(t))),
\end{align*}
\]

where \(t \in [0, T]\), \(s_i(t)\) and \(v_i(t)\) denote respectively the displacement and velocity at the \(i\)th operation (iteration), \(u_i(t)\) is the control input signal, i.e., traction force or braking force, at the \(i\)th iteration, \(m(t)\) is the mass of train, \(c_0(t)\), \(c_v(t)\) and \(c_a(t)\) are the unknown state-independent time-varying parameters representing the damping coefficients of train, \(f(\cdot)\) is a known state-dependent continuous function that denotes the additional resistance suffered by trains during the operational process, \(d(t)\) is the unknown time-varying speed delay.

The iterative learning controller to be devised is expected to drive the displacement \(s_i(t)\) and velocity \(v_i(t)\) of trains to track the desired reference displacement \(s_r(t)\) and velocity \(v_r(t)\), where \(\dot{s}_i(t) = v_i(t)\).

Some requisite assumptions are specified for sake of the controller design and the stability analysis.

**Assumption 1:** The initial states of train at each operation are consistent with the desired reference trajectories, i.e., \(s_i(t) = s_r(t)\) and \(v_i(t) = v_r(t)\) for \(t \leq 0\).

**Assumption 2:** The derivative of time-varying delay satisfies \(\dot{d}(t) \leq \sigma < 1\), where \(\sigma\) is the known constant.

**Assumption 3:** The following inequality with respect to the unknown function \(f(\cdot)\) holds,

\[
|f(v_i(t - d(t)))| \leq \chi(t)\psi(v_i(t - d(t))),
\]

where \(\chi(t)\) is the unknown state-independent time-varying parameter, \(\psi(\cdot)\) is the known state-dependent continuous function.

**Remark 1:** It is well known that **Assumption 1** can be easily satisfied through preassigning the reference trajectories since the train always starts at the prescriptive station [1]. **Assumption 2** is a common constraint condition for the time-delay systems, and is reasonable in the practical physical systems. Moreover, **Assumption 3** implicates that \(f(\cdot)\) is bounded in the operational interval of train [31], [32], which is quite weak.

During the controller design, the hyperbolic tangent function is used to dispose the possible controller singularity problem, so some of its basic properties are reviewed here.

**Lemma 1 ([32]):** For any positive constant \(\epsilon > 0\) and any variable \(z \in \mathbb{R}\), it has \(\lim_{z \to 0} \frac{1}{2} \tanh^2 \left( \frac{z}{\epsilon} \right) = 0\).

**Lemma 2 ([32]):** For any positive constant \(\epsilon > 0\) and the set \(\Omega_z = \{z||z| < 0.8814\epsilon\}\), if variable \(z \in \mathbb{R}\) satisfies \(z \notin \Omega_z\), it has \(1 - 2\tanh^2 \left( \frac{z}{\epsilon} \right) \leq 0\).

The railway department usually does not assign the specific desired acceleration for train, namely, the derivative of reference velocity \(\dot{v}_r\) is not available to the designed controller. Hence, the following command filter is conducive to the controller design,

\[
\dot{\beta}_i = -\omega (\beta_i - \alpha_i) ,
\]

where \(\omega\) is the positive constant, \(\alpha_i\) is the virtual controller to be devised, \(\beta_i\) is the output of command filter with the initial...
condition \( \beta_i(0) = \alpha_i(0) \). To overcome the influence caused by the command filter, the compensation mechanism of error signal is defined as follows,

\[
\begin{align*}
\dot{\xi}_{1,i} &= -q_1 \xi_{1,i} + \xi_{2,i} + (\beta_i - \alpha_i), \\
\dot{\xi}_{2,i} &= -q_2 \xi_{2,i} - \xi_{1,i},
\end{align*}
\]

(4) - (5)

where \( q_1 > 0 \) and \( q_2 > 0 \) are the controller parameters, and the initial values of the error compensation signals are set as \( \xi_{1,i}(0) = 0 \) and \( \xi_{2,i}(0) = 0 \). Therewith, the following lemma in regard to the error compensation mechanism is necessary for the stability analysis.

**Lemma 3 ([33]):** For any constant \( \mu > 0 \), as well as the command filter (3) and the error compensation mechanism (4)-(5), it can always find the appropriate parameter \( \omega \) to make the following inequalities true,

\[
|\beta_i - \alpha_i| < \mu \quad \text{and} \quad \|\xi_i\| \leq \frac{\mu}{2q_0},
\]

(6)

where \( \xi_i = [\xi_{1,i}, \xi_{2,i}]^T \) and \( q_0 = \frac{1}{2} \min (q_1, q_2) \).

**Remark 2:** The command filter can yield the approximate signals of virtual controller, which obviates to calculate continuously analytic derivatives in the standard backstepping technique and reduces the complexity of implementing the controller. Particularly, the effect of command filter on the stability of closed-loop control system has been rigorously analyzed in [33], [34].

Moreover, since the projection mechanism will be applied to the parameter learning rules, a well-known lemma is given as follows.

**Lemma 4 ([35]):** For the projection operator defined as

\[
P(\theta) = \begin{cases} 
\bar{\theta}, & \theta > \bar{\theta}, \\
\theta, & \theta \leq \theta \leq \bar{\theta}, \\
\bar{\theta}, & \theta < \bar{\theta},
\end{cases}
\]

(7)

it has the following property,

\[
[(\gamma + 1) \beta - (\gamma \theta + P(\theta))] [\theta - P(\theta)] \leq 0,
\]

(8)

where \( \theta, \bar{\theta} \in \mathbb{R}, \theta \leq \beta \leq \bar{\theta} \) and \( \gamma \geq 0 \).

In this paper, for the convenience of presentation, some arguments of parameters and functions will be ignored without causing confusion, e.g., \( s_i, c_0 \) and \( \psi(v_i) \).

**Remark 3:** The dynamic model of trains is considered as a class of second-order nonlinear system, rather than the first-order form used in [29], [30], which is convenient for analyzing the tracking control of displacement. From the perspective of train group, the displacement regulation of single train can be easily extended to the coordinated control of multiple trains to ensure the safety distance of two adjacent trains, that will be investigated in future work. Moreover, the unknown nonlinear term in the train model is not required to satisfy the Lipschitz condition, so it is more relaxed than the assumptions in [29], [30], and has stronger universality.

### III. Controller Design and Analysis

In this section, a novel iterative learning controller of high-speed trains is presented for tracking the desired displacement and velocity, and the stability of proposed closed-loop control system is strictly proved via constructing the appropriate composite energy function.

Following the procedure of backstepping technique, the coordinate transformation of closed-loop system is established as follows,

\[
\begin{align*}
e_{1,i} &= s_i - s_r, \\
e_{2,i} &= v_i - \beta_i,
\end{align*}
\]

(9) - (10)

where \( \beta_i \) is the filtered version of virtual controller \( \alpha_i \). Through integrating the error compensation mechanism of command filter, the compensated error signals are defined as

\[
\begin{align*}
z_{1,i} &= e_{1,i} - \xi_{1,i}, \\
z_{2,i} &= e_{2,i} - \xi_{2,i}.
\end{align*}
\]

(11) - (12)

The construction of control system is accomplished in two steps, where the virtual controller \( \alpha_i \) and the real controller \( u_i \) are devised in turn.

**Step 1.** Taking the derivative of \( e_{1,i} \) and combining (9) and (10), it yields

\[
\dot{e}_{1,i} = v_i - v_r = \alpha_i + e_{2,i} - v_r + (\beta_i - \alpha_i).
\]

(13)

Along (11) and (13), as well as the error compensation mechanism (4), it could get the derivative of \( z_{1,i} \) as

\[
\dot{z}_{1,i} = \alpha_i + z_{2,i} - v_r + q_1 \xi_{1,i}.
\]

(14)

The Lyapunov function is defined as \( V_{1,i} = \frac{1}{2} z_{1,i}^2 \), and its derivative could be calculated through invoking (14),

\[
\dot{V}_{1,i} = z_{1,i}(\alpha_i + z_{2,i} - v_r + q_1 \xi_{1,i})
\]

(15)

It is feasible to construct the following virtual controller as

\[
\alpha_i = -q_1 e_{1,i} + v_r.
\]

(16)

Substituting (16) into (15), it could finally get the derivative of \( V_{1,i} \) as

\[
\dot{V}_{1,i} = -q_1 z_{1,i}^2 + z_{1,i} z_{2,i}.
\]

(17)

**Step 2.** According to (1) and (10), the derivative of \( e_{2,i} \) could be obtained as follows,

\[
\dot{e}_{2,i} = u_i + c_0 + c_v v_i(t - d) + c_a v_i^2(t - d) + f(v_i(t - d)) - m \dot{v}_i + \dot{v}_i - \dot{\beta}_i.
\]

(18)

Integrating (5), (12) and (18), it could derive that

\[
\dot{z}_{2,i} = u_i + c_0 + c_v v_i(t - d) + c_a v_i^2(t - d) + f(v_i(t - d)) - m \dot{v}_i + \dot{v}_i - \dot{\beta}_i
\]

(19)

+ q_2 \xi_{2,i} + \xi_{1,i}.

To compensate the unknown time-varying delay, an appropriate Lyapunov-Krasovskii function is considered as follows,

\[
V_{2,i} = \frac{1}{2} z_{2,i}^2 + \frac{1}{2(1 - \sigma)} \int_{t \sigma}^{t} \varphi(v_i(\tau)) d\tau,
\]

(20)
where \( \varphi(\cdot) \) will be defined later. In terms of (19), the taking the derivative of \( V_{2,i} \) yields

\[
\dot{V}_{2,i} \leq z_{2,i} \left( u_i + c_0 - m \dot{v}_i + \dot{v}_i - \beta_i + q_2 \xi_{2,i} + \xi_{1,i} \right) + c_a z_{2,i} v_i (t - d) + c_a z_{2,i} v_i^2 (t - d) + |z_{2,i}| |f (v_i (t - d))| + \frac{1}{2(1 - \sigma)} \varphi (v_i) - \frac{1}{2(1 - \sigma)} \varphi (v_i (t - d)).
\]

Applying Assumption 3 and the Young’s inequality, the fourth item on the right hand side of (21) could be handled as

\[
|z_{2,i}| |f (v_i (t - d))| \leq |z_{2,i}| \chi \psi (v_i (t - d)) \leq \frac{1}{2} \chi^2 z_{2,i}^2 + \frac{1}{2} \psi^2 (v_i (t - d)).
\]

Again, by the Young’s inequality, it could deduce the following results regarding the second and third items on the right hand side of (21),

\[
c_a z_{2,i} v_i (t - d) \leq \frac{1}{2} c_a^2 z_{2,i}^2 + \frac{1}{2} v_i^2 (t - d),
\]

\[
c_a z_{2,i} v_i^2 (t - d) \leq \frac{1}{2} c_a^2 z_{2,i}^2 + \frac{1}{2} v_i^2 (t - d).
\]

Substituting (22)-(24) into (21) and combining the fact \( \frac{1 - d}{1 - \sigma} \geq 1 \) indicated by Assumption 2, it has

\[
\dot{V}_{2,i} \leq z_{2,i} \left( u_i + \theta^T \phi - m \dot{v}_i + \dot{v}_i - \beta_i + q_2 \xi_{2,i} + \xi_{1,i} \right) + \frac{1}{2} (1 - \sigma) \varphi (v_i) + \frac{1}{2} \left( 1 - \frac{1 - d}{1 - \sigma} \right) \varphi (v_i (t - d)) \leq z_{2,i} \left( u_i + \theta^T \phi - m \dot{v}_i + \dot{v}_i - \beta_i + q_2 \xi_{2,i} + \xi_{1,i} \right) + \frac{1}{2} (1 - \sigma) \varphi (v_i),
\]

where \( \theta = \left[ c_0, c_1^2 + c_2^2 + \chi^2 \right]^T \in \mathbb{R}^{2 \times 1} \) is the unknown state-independent time-varying parameter vector, \( \phi = \left[ 1, \frac{1}{2} z_{2,i} \right]^T \in \mathbb{R}^{2 \times 1} \) and \( \varphi (v_i) = v_i^2 + v_i^4 + \psi^2 (v_i) \) denote the known state-dependent continuous function vector and function respectively.

It may lead to the singular problem if \( \frac{1}{2(1 - \sigma)} \varphi (v_i) \) is used to devise controller directly due to the case of \( z_{2,i} = 0 \). Hence, one introduces the hyperbolic tangent function to dispose the last item on the right hand side of (25), and get the following equality immediately,

\[
\frac{1}{2(1 - \sigma)} \varphi (v_i) = \frac{1}{2(1 - \sigma)} \left[ 1 - 2 \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \right] \varphi (v_i) + \frac{1}{1 - \sigma} \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i),
\]

where \( \epsilon > 0 \). Using (26) to replace \( \frac{1}{2(1 - \sigma)} \varphi (v_i) \) on the right hand side of (25), it follows that

\[
\dot{V}_{2,i} \leq z_{2,i} \left[ u_i + \theta^T \phi - m \dot{v}_i + \dot{v}_i - \beta_i + q_2 \xi_{2,i} + \xi_{1,i} + \frac{1}{(1 - \sigma) z_{2,i}} \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i) \right] + \frac{1}{2(1 - \sigma)} \left[ 1 - 2 \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \right] \varphi (v_i).
\]

On account of the convergence property indicated by Lemma 1, it can be found that \( \frac{1}{(1 - \sigma) z_{2,i}} \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i) \) is well defined at \( z_{2,i} = 0 \).

To satisfy the stability criteria, the iterative learning control law at the \( i \)th is established as follows,

\[
u_i = -qz_{2,i} - c_i - \beta_i + \theta_i^T \phi + \tilde{m}_i \dot{v}_i - \dot{v}_i + \beta_i \]

\[
- \frac{1}{(1 - \sigma) z_{2,i}} \tanh \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i),
\]

where \( \theta_i \) is the estimation of \( \theta \), \( \tilde{m}_i \) is the estimation of \( m \), at the \( i \)th iteration. Imposing the controller \( \nu_i \) on (27), it leads to that

\[
\dot{V}_{2,i} \leq -qz_{2,i}^2 - qz_{2,i} - \beta_i^T \phi + qz_{2,i} \tilde{m}_i \dot{v}_i + \frac{1}{(1 - \sigma) z_{2,i}} \tanh \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i),
\]

where \( \theta_i = \theta - \theta \) and \( \tilde{m}_i = \tilde{m}_i - m \) denote the estimation errors at the \( i \)th iteration.

Considering \( z_{1,i} \) and \( z_{2,i} \) simultaneously, the Lyapunov-Krasovskii function for the closed-loop control system of high-speed trains is defined as,

\[
V_i = V_{1,i} + V_{2,i}.
\]

Taking the derivative of \( V_i \) and invoking (17) and (29), it has

\[
\dot{V}_i \leq -qz_{1,i}^2 - qz_{2,i}^2 - z_{2,i} \theta_i^T \phi + z_{2,i} \tilde{m}_i \dot{v}_i + \frac{1}{(1 - \sigma) z_{2,i}} \tanh \left( \frac{z_{2,i}}{\epsilon} \right) \varphi (v_i) \]

where \( \gamma_1 > 0 \) and \( \gamma_2 > 0 \) are the parameter learning gains, \( m > 0 \) is a known constant representing the self-weight of train. It should be pointed out that the projection mechanism is applied to (32) and (33) to prevent the learning divergency.

**Theorem 1:** Under the premise that Assumptions 1-3 hold, the closed-loop control system of high-speed trains consisting of the dynamic model (1), iterative learning control laws (16) and (28), as well as the parameter learning rules (32) and (33), can guarantee that

1. All variables are bounded;
2. The tracking errors \( e_{1,i} \) and \( e_{2,i} \) will converge along the iteration axis and remain in a compact set throughout the entire operational interval \([0, T] \), i.e., \( \lim_{i \to \infty} e_{1,i} \leq \frac{\mu}{2 \delta_0} \) and \( \lim_{i \to \infty} e_{2,i} \leq 0.8814 \mu + \frac{\mu}{2 \delta_0} \).

**Proof:** The convergence of errors and the boundedness of variables are proved through constructing the appropriate composite energy function. Considering the case of \( z_{2,i} \notin \Omega_z \) and recalling Lemma 2, it gets that

\[
\frac{1}{2(1 - \sigma)} \left[ 1 - 2 \tanh^2 \left( \frac{z_{2,i}}{\epsilon} \right) \right] \varphi (v_i) \leq 0.
\]

So, (31) could be rewritten as

\[
\dot{V}_i \leq -qz_{1,i}^2 - qz_{2,i}^2 - z_{2,i} \theta_i^T \phi + z_{2,i} \tilde{m}_i \dot{v}_i.
\]
Formally, one defines the composite energy function as
\[
E_i = V_i + \frac{1}{2\gamma_1} \int_0^t \left\| \dot{\theta}_i \right\|^2 d\tau + \frac{1}{2\gamma_2} \int_0^t \dot{m}_i^2 d\tau.
\] (36)
The proof process is composed of three parts. In the first part, the non-increasing of \(E_i\) along the iteration axis will be investigated. In the second part, the boundedness of composite energy function at the 1st iteration will be validated. Meanwhile, combining the non-increasing and non-negativity of \(E_i\), the boundedness of \(E_i\) in each iteration can be ensured. By virtue of the limit theorem, the convergence of errors can be finally confirmed. On the basis of the first and second parts, the third part will analyze the boundedness of variables. The details are given as follows.

Part 1. To discuss the variation of \(E_i\) along the iteration axis, the difference of \(E_i\) is calculated as
\[
\Delta E_i = E_i - E_{i-1} = V_i - V_{i-1} + \frac{1}{2\gamma_1} \int_0^t \left( \left\| \dot{\theta}_i \right\|^2 - \left\| \dot{\theta}_{i-1} \right\|^2 \right) d\tau + \frac{1}{2\gamma_2} \int_0^t (\dot{m}_i^2 - \dot{m}_{i-1}^2) d\tau.
\] (37)
Let us first analyze the third item on the right hand side of (37). Considering its integral term, it could derive through invoking the parameter learning rule (32) that
\[
\frac{1}{2\gamma_1} \left( \left\| \dot{\theta}_i \right\|^2 - \left\| \dot{\theta}_{i-1} \right\|^2 \right) = \frac{1}{\gamma_1} \left( \dot{\theta}_i^T \dot{\theta}_i - \dot{\theta}_{i-1}^T \dot{\theta}_{i-1} \right) - \frac{1}{2\gamma_1} \left\| \dot{\theta}_i - \dot{\theta}_{i-1} \right\|^2 \\
\leq \frac{1}{\gamma_1} \left( \dot{\theta}_i^T - \dot{\theta}_{i-1}^T \right) \left( \theta - \theta_i \right) \\
= \frac{1}{\gamma_1} \left[ \dot{\theta}_i^T + \gamma_1 z_{2,i} \phi^T - P_1 \left( \dot{\theta}_{i-1} + \gamma_1 z_{2,i} \phi \right)^T \right] \cdot \left[ \theta - P_1 \left( \dot{\theta}_{i-1} + \gamma_1 z_{2,i} \phi \right) \right] + z_{2,i} \dot{\theta}_i^T \phi.
\] (38)
Recalling Lemma 4 with \(\gamma = 0\), \(\beta = \theta\) and \(\theta = \dot{\theta}_{i-1} + \gamma_1 z_{2,i} \phi\), it has
\[
\left[ \dot{\theta}_i^T + \gamma_1 z_{2,i} \phi^T - P_1 \left( \dot{\theta}_{i-1} + \gamma_1 z_{2,i} \phi \right)^T \right] \cdot \left[ \theta - P_1 \left( \dot{\theta}_{i-1} + \gamma_1 z_{2,i} \phi \right) \right] \leq 0.
\] (39)
Substituting (39) into (38), it leads to that
\[
\frac{1}{2\gamma_1} \left( \left\| \dot{\theta}_i \right\|^2 - \left\| \dot{\theta}_{i-1} \right\|^2 \right) \leq z_{2,i} \dot{\theta}_i^T \phi.
\] (40)
Following the analogous process of (38)-(40), it gets that
\[
\frac{1}{2\gamma_2} (\dot{m}_i^2 - \dot{m}_{i-1}^2) \leq -\dot{z}_{2,i} \dot{v}_i \dot{m}_i.
\] (41)
According to Assumption 1, it could obtain from (35) that
\[
V_i - V_{i-1} \leq -q_1 \int_0^t \dot{z}_{1,i}^2 d\tau - q_2 \int_0^t \dot{z}_{2,i}^2 d\tau + \int_0^t \dot{z}_{2,i} \dot{\theta}_i^T \phi d\tau + \int_0^t \dot{z}_{2,i} \dot{m}_i \dot{v}_i d\tau.
\] (42)
Integrating (40)-(42) into (37), the following inequality can be deduced immediately,
\[
\Delta E_i \leq -q_1 \int_0^t \dot{z}_{1,i}^2 d\tau - q_2 \int_0^t \dot{z}_{2,i}^2 d\tau \leq 0.
\] (43)
So, the non-increasing of \(E_i\) along the iteration axis is validated via (43).

Part 2. Convergence of errors
In terms of (35) and (36) with \(i = 1\), as well as the results of (40) and (41) with \(i = 1\), i.e.,
\[
\frac{1}{2\gamma_1} \left( \left\| \dot{\theta}_1 \right\|^2 - \left\| \dot{\theta}_0 \right\|^2 \right) \leq z_{2,1} \dot{\theta}_1^T \phi \quad \text{and} \quad \frac{1}{2\gamma_2} (\dot{m}_1^2 - \dot{m}_0^2) \leq -z_{2,1} \dot{v}_1 \dot{m}_1,
\] it could derive the derivative of \(E_1\) as
\[
\dot{E}_1 \leq -q_1 z_{1,1}^2 - q_2 z_{2,1}^2 - z_{2,1} \dot{\theta}_1^T \phi + z_{2,1} \dot{m}_1 \dot{v}_1 \\
- \frac{1}{2\gamma_1} \left( \left\| \dot{\theta}_1 \right\|^2 + \frac{1}{2\gamma_2} \dot{m}_1^2 \right) + \frac{1}{2\gamma_2} (\dot{m}_1^2 - \dot{m}_0^2) \leq -\frac{1}{2\gamma_1} \left( \left\| \dot{\theta}_1 \right\|^2 + \frac{1}{2\gamma_2} (m - m_0)^2 \right).
\] (44)
Considering the fact that \(\theta\) and \(m\) are the finite and continuous signal, it could draw the conclusion that \(\dot{E}_1\) is bounded in the interval \([0, T]\). Owing to the integral theorem, \(E_1\) is also bounded in the interval \([0, T]\). Finally, since non-increasing of \(E_i\) has been proved, it could confirm the boundedness of \(E_i\) in the interval \([0, T]\) on account of the non-negativity of \(E_i\).

Considering (43) again, there exists the following inequality,
\[
E_i \leq E_1 - q_1 \sum_{k=2}^i \int_0^t \dot{z}_{1,k}^2 d\tau - q_2 \sum_{k=2}^i \int_0^t \dot{z}_{2,k}^2 d\tau.
\] (45)
Taking the limit of both sides of (45), it has
\[
\lim_{i \to \infty} E_i \leq E_1 - q_1 \lim_{i \to \infty} \sum_{k=2}^i \int_0^t \dot{z}_{1,k}^2 d\tau - q_2 \lim_{i \to \infty} \sum_{k=2}^i \int_0^t \dot{z}_{2,k}^2 d\tau.
\] (46)
According to the limit theorem, as well as the boundedness of \(E_i\) and \(E_1\), the necessary conditions for (46) are given as follows,
\[
\lim_{i \to \infty} \int_0^t \dot{z}_{1,i}^2 d\tau = 0 \quad \text{and} \quad \lim_{i \to \infty} \int_0^t \dot{z}_{2,i}^2 d\tau = 0.
\] (47)
It should be noted that the results of (47) is obtained with \(z_{2,i} \notin \Omega_z\). Hence, the convergence of \(z_{2,i}\) should be rewritten as \(\lim_{i \to \infty} \left| z_{2,i} \right| < \infty\). Finally, combining Lemma 3, as well as the facts of \(\left| e_{1,i} \right| \leq \left| z_{1,i} \right| + \left| \xi_i \right|\) and \(\left| e_{2,i} \right| \leq \left| z_{2,i} \right| + \left| \xi_i \right|\), one could derive the convergence of tracking errors, i.e.,
\[
\lim_{i \to \infty} \left| e_{1,i} \right| \leq \frac{\mu_0}{2\gamma_1} \quad \text{and} \quad \lim_{i \to \infty} \left| e_{2,i} \right| \leq 0.8814 \epsilon + \frac{\mu_2}{2\gamma_2}.
\]

Part 3. Boundedness of variables
Since the boundedness of \(E_i\) has been confirmed, it can conclude from the definition of \(E_i\) that \(\dot{\theta}_i, \dot{m}_i, e_{1,i}, e_{2,i}, z_{1,i}, z_{2,i}\) are bounded. Combining the finiteness of \(s_r\) and \(v_r\), the state signals of high-speed train, i.e., \(s_i\) and \(v_i\) are bounded.
Moreover, according to (16) and (28), the boundedness of control signals $\alpha_i$ and $u_i$ can also be guaranteed.

**Remark 4:** The proposed iterative learning controller integrates two components, where the feedback control in the time domain can effectively maintain the comfort of passengers during the train operation, and the parameter learning in the iteration domain can ensure the pointwise convergence of tracking errors. In addition, it could be found that the design of controller can still follow the process of standard backstepping method although the command filter is assembled in our work. Specifically, since the inverse of compensated error signal may cause the controller singularity problem, the hyperbolic tangent function is introduced to prevent the divergence of control signal. Moreover, according to (16) and (28), the boundedness of control signals $\alpha_i$ and $u_i$ can also be guaranteed.

### IV. SIMULATION AND DISCUSSION

Numerical simulations are executed to validate the performance of iterative learning control scheme, where the parameter settings are given in TABLE I. The reference trajectory of trains is mainly composed of five stages, involving the acceleration phase, the cruising phase, and the braking phase, which basically cover all the operational states. The maximum cruising velocity is set as 83.5 m/s (approximately 300 km/h). The more details could be found in the simulation results.

**TABLE I** 
**SIMULATION PARAMETERS [30].**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$3.45 \times 10^5$</td>
<td>kg</td>
</tr>
<tr>
<td>$c_d$</td>
<td>$-2977 + 275 \sin(0.00376)$</td>
<td>N</td>
</tr>
<tr>
<td>$c_u$</td>
<td>$-25.17 + 2.5 \sin(0.00376)$</td>
<td>N · s/m</td>
</tr>
<tr>
<td>$c_m$</td>
<td>${0.3864 + 0.04 \sin(0.00376)}$</td>
<td>N · s²/m²</td>
</tr>
</tbody>
</table>

The following continuous piecewise function is adopted to simulate the time-varying delay,

$$d(t) = \begin{cases} 
0, & 0 \leq t \leq 100, \\
20 \sin^2(0.005\pi(t - 100)), & 100 < t \leq 200, \\
20, & 200 < t \leq 300, \\
10 \cos^2(0.005\pi(t - 300)) + 10, & 300 < t \leq 400, \\
10 \cos^2(0.0025\pi(t - 400)), & 400 < t \leq 600.
\end{cases}$$

It could be found that the derivative of $d(t)$ complies with the condition specified by Assumption 2, i.e., $\dot{d}(t) < 0.8 < 1$. So, $\sigma = 0.8$ is considered as the upper bound of $\dot{d}(t)$ to implement the learning controller.

**A. Performance of Adaptive Iterative Learning Control**

According to the stability requirements of control system, the controller parameters and the learning gains are set as $q_1 = 0.01$, $q_2 = 0.1$, $\gamma_1 = 20$, and $\gamma_2 = 100$. Meanwhile, $\epsilon = 100$ and $\omega = 100$ are chosen as the parameters of the hyperbolic tangent function and the command filter respectively. Overall, 100 iterations are performed to verify the effectiveness of proposed scheme.

**Fig. 1** demonstrates the profiles of displacement and velocity at the 1st and 100th iterations respectively. It could be seen that there are some drastic deviations between the measured velocity profile at the 1st iteration and the desired reference velocity. Nevertheless, after 100 iterations, the errors of displacement and velocity have been basically eliminated. Hence, it can draw the conclusion that the tracking performance of high-speed train on the reference trajectories has been significantly enhanced via the iterative learning.

**Fig. 2.** Error profiles of displacement and velocity.

Furthermore, the tracking error profiles of displacement and velocity at the 1st and 100th iterations are given in Fig. 2. The results show that the errors at the 1st iteration are significantly smaller than the errors at the 100th iteration during the entire operational period. Moreover, Fig. 3 analyzes the variation of the maximum absolute errors of displacement and velocity at each iteration. It could be found that the maximum absolute errors decrease monotonically along the iteration axis. Overall, Figs. 2 and 3 confirm the convergence of tracking errors.

**Fig. 3.** Displacement errors at the 1st and 100th iterations.

Finally, Fig. 4 exhibits the control input signal profiles of high-speed trains at the 1st and 100th iterations. Combining...
In summary, the simulation results in Figs. 1-4 validate the effectiveness and robustness of the iterative learning control scheme.

**B. Comparison with Robust Adaptive Control**

To further demonstrate the advantages of the proposed learning controller, a robust adaptive controller is applied to the train systems for comparison [16], [36], where the tracking profiles are shown in Fig. 5. Since the singularity problem in the robust adaptive control scheme is handled via the piecewise function rather than the hyperbolic tangent function, it could be observed from Fig. 5 that the velocity profile includes obvious oscillation, which will seriously influence the comfort of passengers. Moreover, due to the shortage of learning ability, there are dramatic deviations between the measured displacement and velocity profiles of trains and the desired trajectories although the controller parameters have been carefully selected.

**V. CONCLUSION**

In this paper, an adaptive iterative learning controller of high-speed trains for the tracking control of displacement and velocity is devised via the command filtered backstepping technique, where the unknown time-varying speed delay is compensated through choosing the appropriate Lyapunov-Krasovskii function. During the design of controller, the hyperbolic tangent function is utilized to deal with the singularity problem to ensure the continuity of control signal. It should be emphasized that our approach does not require the nonlinear uncertainties to satisfy the Lipschitz condition. By constructing the composite energy function, the stability of train control system has been proved strictly. Finally, numerical simulation confirmed the effectiveness of the proposed control scheme. One of our future works is to investigate the tracking control problem of high-speed trains with more nonlinear physical constraints, e.g., input hysteresis, actuator failure, and state saturation.

**REFERENCES**


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