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**Bond Portfolio Management under Solvency II Regulation**

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**Abstract**

We develop a novel approach to the bond portfolio optimization for insurance companies that are subject to the new Solvency II regulation. The regulatory efficient portfolios are determined using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The characteristics of the estimated efficient portfolios are examined in different market regimes. Our findings suggest low cardinality of all estimated efficient portfolios despite explicit regulatory penalties for highly concentrated portfolios. The efficient portfolios are dominated by short term and BBB rated bonds. The lack of diversification and over-exposure to bonds with higher credit risk in different market regimes represents a weakness of the Solvency II regulation with unintended consequences for management of insurance companies.

**Key words:** Portfolio management; Insurance; Solvency II; Market risk; NSGA II.

**JEL:** C61, G11, G12, G18, G22.
1. Introduction
The Solvency II regulatory framework for the European insurance industry came into effect on 1\textsuperscript{st} January 2016. Solvency II is also relevant for US insurers and has already been used as a model of “best practice” by insurance companies in the Standard & Poor’s 500 Index. Solvency II framework dramatically changed the way regulators quantify risk. Key principles in Solvency II are the introduction of the risk-based approach to prudential capital requirements and the mark-to-market approach for balance sheet items. The framework establishes the Solvency Capital Requirements (SCR) for the exposure of insurance companies to market, underwriting, counterparty default, and operational risks (see EC, 2010 and EU, 2015). Insurance companies are required to report the amount of capital they hold as a percentage of the SCR. Furthermore, the recently introduced International Financial Reporting Standard (IFRS 17) requires clear distinction between insurers’ investment portfolio returns and underwriting profits, starting from 2021 (IASB, 2017). Investment portfolio management in the new regulatory environment thus represents an important challenge for the insurance industry.\textsuperscript{1}

Previous studies examine Solvency II’s impact on investment strategies and allocation of equities (Fischer and Schluetter, 2015; Filipovic et al., 2014; Kerkhof et al., 2010; Gatzert and Martin, 2012) and bonds (Arias et al., 2012). For example, Arias et al., (2012) examine adequacy of measuring bond risk using the standard SCR formula and suggest that SCR is an adequate measure. More recently, some studies examine efficient allocation problems with several asset classes (including corporate bonds) where risk is determined by the Solvency II standard formula (Kouwenberg, 2018a; b; Braun et al. 2017; 2018). Common feature of these studies is consideration of both aggregated assets and aggregated liabilities, where asset positions are proxied by respective market indices. There is, however, a paucity of literature on bond portfolio management by insurance companies strictly complying with the new Solvency II framework.

In this paper, we focus on a bond portfolio optimization by insurers strictly adopting the SCR formula for market risk.\textsuperscript{2} In order to obtain regulatory efficient portfolios we study the optimal

\textsuperscript{1} European insurance companies, with around €6.7 trillion of assets under management, are among the largest institutional investors in the world (Braun et al., 2017).

\textsuperscript{2} Market risk subsumes around 69% of insurers’ assets and is thus the most significant part of Solvency II (Hoering, 2013).
trade-off between the yield to maturity (YTM) and the SCR of bond portfolios. SCR calculations are performed in accordance with Solvency II “look through” approach.\(^3\) According to this approach, SCR of insurers must be calculated on the basis of each of the underlying assets of collective investment undertakings and/or other investments packaged as funds. The purpose of the “look-through” approach is to ensure insurers understand the underlying risk of the financial instruments in which they have invested. We base our SCR calculations on market risk regulatory sub-modules for interest rate, spread, and concentration risk. We then compare capital charges corresponding to different market conditions and among portfolios with different levels of concentration. Consideration of concentration risk makes the optimization problem complex since the objective function is non-differentiable and non-convex. In order to solve it, we develop software in which we implement the NSGA-II algorithm from Deb et al. (2002). In contrast to traditional optimization methods (such as linear programming), NSGA-II (and other multi-objective evolutionary algorithms) are population-based and, hence, have the ability to generate the entire Pareto-optimal front in a single run (see Deb et al., 2002; and Coello et al., 2007). Our choice of the algorithm was motivated by the strengths of NSGA-II in addressing complex portfolio optimization problems (see Deb et al., 2000 and Ponsich et al., 2013). We test our approach on a sample of Euro denominated financial and industrial investment grade bonds. These bonds constitute the majority of the outstanding European corporate bond universe and they are the most important asset class for European insurance companies (EIOPA, 2018).\(^4\)

Our study differs significantly from previous related studies (e.g. Kouwenberg (2018a; b; Braun et al. 2017; 2018). First, we make every effort not to make assumptions and approximations which are not strictly in the spirit and letter of Solvency II. Most importantly, by adopting Solvency II “look through” approach we base our portfolio SCR calculation on characteristics of individual bonds rather than on aggregated positions proxied by market indices. Focusing on individual bonds we are able to: i) estimate weights for each of individual bonds in order to reach optimal solutions; ii) examine composition, characteristics (across ratings, maturities, and industries) and cardinality of regulatory efficient portfolios; iii) apply exact Solvency II procedure for calculation of interest rate risk component of the SCR formula

\(^3\)This approach is intended to capture the risk which arises by virtue of the assets and/or investments held by a fund in which a (re)insurer itself has invested. See Article 84 of the 2015/35 Delegated Regulation (EU) (EU, 2015). See also EC (2010) (parts SCR 5.4.5.14) and Bank of England (2016).

\(^4\) European insurance industry’s overall corporate bond holdings were €1.2 trillion in 2017 (EC, 2017, p.33). This was twice as much compared to their equity holdings in the same year.
for market risk.; and iv) consider SCR’s concentration risk sub-module aimed at penalizing excessive exposures to single issuer. Second, contrary to studies considering both investments and liabilities, we consider only investments (i.e. bond portfolios). Consequently, we are able to estimate the SCR for the entire efficient frontier of performance seeking portfolios, independent of any given liability structure (i.e. asset-liability duration gap). Our approach is also in line with IFRS 17, requiring a separate disclosure of the investment portfolio returns. Finally, unlike Arias at al. (2012) who compare ex-post (i.e. historical) returns of individual bonds with corresponding SCR, we examine the trade-offs between YTM (i.e. forward-looking performance measure) and SCR for bond portfolios. YTM is most widely used bond portfolio performance measure by practitioners and depends on the same parameters as SCR for the bond portfolios: risk-free yield curve, credit rating, credit spreads, and duration. From insurers’ perspective, regulatory capital (i.e. SCR) involves an opportunity cost since it is required to be available at all times. With the adoption of Solvency II, therefore, investors have to consider both, bond yields and SCR. Thus, insurance companies face a trade-off between yields and SCR which provides economic rational to maximize YTM and minimize SCR. Our optimization algorithm is designed precisely to address optimal YTM-SCR trade-offs of bond portfolios. The rational for the regulatory efficient portfolios is similar to one used in an extensive banking literature, in the context of Basel II. Our paper is one of the first attempts to examine regulatory efficient portfolios in insurance industry, strictly in line with Solvency II.

Our results suggest that all efficient portfolios involve investing in highly concentrated (i.e. under-diversified) portfolios despite regulatory penalties for excessive concentration risk. Furthermore, the optimal YTM-SCR trade-offs, implied by the standardized approach, lead to portfolios with high exposure to investment grade bonds with the lowest credit rating. There is also a notable absence of long and medium-term bonds from the efficient portfolios. We therefore identify a direct impact of the SCR regulation on insurance firms’ demand for bonds of various maturities and ratings.

5 Our focus on assets (without liabilities) is also in line with core-satellite approach widely used by portfolio managers of insurance companies and pension funds (see Amenc et al., 2006; van Bragt and Kort, 2011; and Deguest et al., 2015).

6 It is worth mentioning that Solvency II allows asset-liabilities matching approach provided that certain conditions are met. “Currently, the required yield that insurers need to match their liabilities is not offered on the market due to the low yield environment, therefore making this provision often ineffective.” (EC 2017; p.37).

7 Notably, Solvency II requires that assessment of the overall solvency needs is forward-looking (see EIOPA, 2013). Previous evidence also suggests that insurance companies do pay attention to YTM and tend to favor bonds with a higher YTM, within the same rating category (e.g. Becker and Ivashina, 2015).
The above results are important for insurers who are required to disclose amount of capital as a percentage figure relative to SCR. Our results provide valuable insights about the standardized SCR formula which may impact management strategies and decisions whether to adopt an internally developed instead of the standardized SCR model. Our results also expose several potential weaknesses of the current Solvency II regulation. Specifically, we highlight need to recalculate the concentration submodule aiming to penalize more severely portfolios with low cardinality. Furthermore, documented regulatory advantage of short-term bonds may reduce aggregate demand for long term bonds thus creating a negative effect on corporations that rely on long term issues. Our results therefore, lend support to recommendations that the capital requirements for all investment grade rating bonds with longer maturities (in particular beyond 10 years) should be reduced (see EC, 2017). Finally, we show that the EU’s recently imposed 1% floor for the spot curve upward stressing (EU, 2015) does not significantly alter the composition of the regulatory efficient portfolios and appears to be overly simplistic relative to the initial intentions of the regulator.

The rest of the paper is organized as follows. In Section 2 we present related literature and further highlight our contributions. In Section 3, we introduce three market sub-risks and adapt the market SCR formula for corporate bond portfolios. Section 4 presents the YTM-SCR optimization problem and explains the optimization method. Section 5 provides sample descriptive statistics. Section 6 presents YTM-SCR regulatory efficient portfolios and assesses their characteristics. In Section 7 we test the robustness of our results. Section 8 concludes.

2. Related literature

Previous literature examined effects of various aspects of the Solvency II regulation on asset-liability (AL) management of insurance companies (Amenc et al., 2006; van Bragt et al., 2010). Amenc et al. (2006), for example, examine impact of IFRS and Solvency II on AL and asset management in insurance industry. Authors recommend a core-satellite approach, with a dedicated liability hedging portfolio (i.e. core portfolio) to fully immunize liabilities. Once liabilities are fully hedged with the core asset portfolio, Amenc et al. show that the remaining surplus assets (i.e. performance seeking portfolio) can be managed with traditional asset

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8 See Article 51, 256 and 256a of 2009/138/EC Directive (EU, 2009), and Articles 290 to 298 and 359 to 364 of 2015/35 Delegated Regulation (EU, 2015).
allocation techniques. Van Bragt et al. (2010) suggest that insurance companies should pursue asset allocation policies that consider both short term Solvency II capital requirement and the long-run risk-return profile of asset allocation.

Literature on Solvency II’s impact on bond related investment strategies is more recent. For example, Arias et al. (2012), examine the SCR for individual European corporate bonds using the standardized Solvency II approach. Authors report that the SCR is overall an appropriate measure of risk but that it does not fully reflect the risk associated with long maturity investment grade bonds, high yield and unrated bonds. In particular, the authors suggest that BBB, or lower rated bonds, could be neglected by investors due to the excessive additional marginal cost in proportion to the return generated. In terms of risk-taking efficiency (measured by historical returns-SCR ratio) the authors report that Solvency II favours short duration (three years or less) irrespective of credit rating.

Contrary to Arias et al. (2012) several studies examine both aggregated investment positions (proxied by indices) and aggregated liabilities. For example, Kouwenberg (2018a;b) analytically solve the investment problem with three asset classes (stocks, corporate and government bonds, and listed property) where risk is determined by the Solvency II standard SCR formula (assuming zero concentration risk and parallel downward and upward interest rate shocks). Aggregate asset positions were proxied with corresponding market indices. Author argue that the SCR formula tends to lead to relatively under-diversified portfolios with large concentrations of government bonds. Houweling and Swinkels (2019) examine low-risk investment strategies in corporate bonds and equities in the context of solvency capital requirements across Nordic countries and Netherlands. Similar to Kouwenberg (2018a;b), authors do not consider concentration risk and examine aggregated positions proxied by indices. Authors report that solvency regulation does not seem to discourage allocations to low-risk corporate bonds.

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9 It is important to note that sample Nordic countries (and Netherlands) do not follow same regulation and are not required to strictly follow Solvency II. Other regulatory frameworks were also examined in the literature. For example, Market Consistent Embedded Value (MCEV) framework for consolidated value of shareholders interest in insurance companies, developed in parallel to the Solvency II and IFRS principles, was examined in Gambaro et al. (2018; 2019). For papers examining effects of US rating-based capital requirements on investment in corporate bonds, see Becker and Ivashina (2015), Ellul et al. (2011) and Murray and Nikolova (2017).
Braun et al. (2017; 2018) also consider both aggregated assets and liabilities. Braun et al. (2018) derive a European life insurer’s return on risk-adjusted capital (RORAC) under the Solvency II capital requirements. Authors construct a large number of hypothetical asset allocation scenarios for German life insurance industry. They show that under Solvency II, a RORAC-based performance measurement may have detrimental effects for a life insurer’s stakeholders. Braun et al. (2017) ran a mean-variance quadratic optimization of a life insurance company’s exposure to six asset classes (including corporate bonds). The authors then calculate the corresponding market risk capital charges under the standard Solvency II formula, as well as the proposed internal model, to identify those mean-variance efficient portfolios that are attainable for an exogenously given amount of equity. The results suggest that the standard Solvency II formula, in some instances, favours suboptimal mean-variance portfolios.

3. Solvency II market risk capital requirements-Standard approach

3.1. Standardized framework for SCR calculation

Solvency II represents a departure from the previous regulatory approach in which the regulator specified investment asset classes for insurance companies and set constraints on their positions. With the new regulation, insurers have much more freedom in terms of their investments but, on the other hand, have the responsibility to measure and manage their risks explicitly. A standardized framework is developed and companies have to follow it unless they can measure risks more precisely using internally developed models. The underlying risk concept for estimation of capital requirements is VaR. Solvency II requires insurers to report risks (by components and in aggregate) at a 99.5% level of confidence and for one-year horizon (see EC, 2010; and EU, 2015). All parameters and assumptions used for the SCR calculation reflect this calibration objective.

According to Solvency II, SCR is divided into market, health, counterparty default, life, non-life and intangibles modules (see Figure 1). The market risk module deals with the impact of the level and volatility of market prices of various financial instruments. Life, health and non-life are underwriting risk modules.
Source: Adapted from EC (2010). For brevity, Figure 1 presents sub-risks only for Market module.

3.2. Market risk sub-modules for European corporate bonds

We consider market risk module related to investments in Euro denominated bonds that are constituents of Markit iBoxx Corporates Composite bond index. Our focus on Euro denominated bonds makes the currency sub-module redundant. We therefore specifically consider: interest rate risk (i.e. change in asset value due to changes in interest rates), spread risk (i.e. changes in the level or volatility of credit spreads over the risk-free term structure of interest rates), and concentration risk (i.e. excess exposure to a single bond/issuer), marked in bold in Figure 1. We denote the corresponding capital requirements as: $\text{Mkt}_{\text{int}}$ (capital requirement for interest rate risk); $\text{Mkt}_{\text{sp}}$ (capital requirement for spread risk); and $\text{Mkt}_{\text{Conc}}$ (capital requirement for concentration risk).10

We follow the latest regulation and, for a given bond portfolio, calculate interest rate component of the market SCR as follows:

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10 A detailed description of interest rate risk, spread risk, and concentration market risk sub-modules is provided in Appendix.
\[ Mkt_{int} = \sum_i w_i \cdot \%\Delta NAV_i \]  
(1)

\[ \%\Delta NAV_i = \frac{P_{bond_i} - stressedP_{bond_i}}{P_{bond_i}} \]  
(2)

\( \%\Delta NAV_i \) is the relative price change of bond \( i \). In all our calculations we use dirty bond prices \( P_{bond_i} \) from Markit database. The term \( stressedP_{bond_i} \) is the stressed price of bond \( i \) which corresponds to the shock that gives rise to the highest capital requirement for a given bond portfolio. \( w_i \) is the fraction of the bond portfolio’s value invested in each individual bond \( i \), calculated using dirty prices. \( Mkt_{int} \) is, therefore, the relative change of the bond portfolio value due to a shift in interest rates.

An important element in determining \( Mkt_{int} \) is the estimation of the term structure of interest rates and its stressing. For this purpose, various versions of the Nelson and Siegel (1987) model have often been used both due to model parsimony and the fact that estimated parameters have economic interpretation. For example, European Insurance and Occupation Pension Supervisors (EIOPS) derive term structure of interest rates using the Nelson and Siegel (1987) model with a single convexity parameter (see CEIPOS, 2010). We follow the ECB (2017) procedure which utilizes the Svensson (1994) extension of that model:

\[ R^c(0, \theta) = \beta_0 + \beta_1 \left[ \frac{1 - e^{-\theta/\tau_1}}{\theta/\tau_1} \right] + \beta_2 \left[ \frac{1 - e^{-\theta/\tau_2}}{\theta/\tau_2} \right] + \beta_3 \left[ \frac{1 - e^{-\theta/\tau_3}}{\theta/\tau_3} \right] \]  
(3)

Here, \( R^c(0, \theta) \) is a continuously compounded zero-coupon rate at time zero with maturity \( \theta \). Parameter \( \beta_0 = \lim_{\theta \to \infty} R^c(0, \theta) \), is the long term level of interest rates while \( \beta_2 = \lim_{\theta \to 0} \left( R^c(0, \theta) - \beta_0 \right) \) is the long-to-short-term spread (in fact the slope factor). The sum of these two parameters represents the vertical intercept that can be interpreted as a short rate. Parameter \( \beta_3 \) is a curvature factor, determining the magnitude and the direction of the interest rate “hump”. Parameter \( \tau_1 \) is a rate of decay for the second (short-term) and the third term

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11 With respect to the original Nelson and Siegel (1987) model, the Svensson model adds an additional convexity parameter. The estimation utilizes the root mean squared error (RMSE) method (See ECB, 2017).
(medium-term) components. Parameter $\beta_3$ is the second curvature factor which brings additional flexibility to short-term segment modelling, and $\tau_2$ is the rate of decay for the forth component.

For the stressing of the yield curve we use the following procedure: a) the Svensson curve based on AAA spot rates, provided by the ECB, is used as the reference curve; b) given the benchmark curve the Z-spread is calculated for each bond in the opportunity set; c) the Svensson curve is stressed according to the Table A.1, in Appendix; d) each bond is repriced using the stressed curve and the Z-spread previously calculated.

The stressed bond price is therefore calculated as follows:

$$stressedP_{bond} = \sum_{c=1}^{n} e^{-(R^c_{stressed}(0,t_c) + Z\text{-spread}) \cdot \tau_2} \cdot CF_c$$  \hspace{1cm} (4)

Here, $CF_c$ is the expected cash flow at time $t_c$ and $R^c_{stressed}(0,t_c)$ is the stressed zero-coupon rate with maturity $t_c$. Since we do not consider bonds with embedded options, the bonds’ Z-spread (relative to the ECB spot rate curve) embodies only credit and illiquidity risk.

Investment bond portfolio spread risk capital requirement is calculated using the following formula (see EC, 2010; p.122):

$$Mkt_{sp}^{bonds} = \sum_r \%MV_r^{bonds} \cdot F^{up}(\text{rating}_r) \cdot duration_r$$  \hspace{1cm} (5)

Here, $\%MV_r^{bonds} = \sum_{j=1} w_j$  \hspace{1cm} (6)

$I = \{\text{bonds of rating } r\}$ is fraction of the bond portfolio invested in bonds of rating $r$. Factor $F^{up}(\text{rating}_r)$ is a function of the rating class of the credit risk exposure which is calibrated to deliver a shock consistent with 99.5% VaR following a widening of credit spreads. $duration_r$ is average duration of bond portfolio at rating $r$, weighted with the market value of the bonds.

---

12 Since all bonds in our sample are issued in Euros, it suffices to consider just one benchmark spot curve (at any point in time) with respect to priced bonds (see Fabozzi, 2006; p.139).
13 The Z-spread, or zero-volatility spread, is a measure of the spread that would be achieved over the entire benchmark spot rate curve, when the bond is held to maturity (see Fabozzi, 2006; p.467).
The calculation of market SCR regarding concentration risk is performed in three steps: i) determination of excess exposure per ‘name’ \( n \) \( (XS_n) \); ii) risk concentration capital requirement per ‘name’ \( n \) \( (Conc_n) \); iii) aggregation across single names. For a given bond portfolio market SCR regarding concentration risk is calculated using the following formula (EC, 2010; p.130):

\[
Mkt_{Conc} = \sqrt{\sum_n (Conc_n^2)}
\]  

(7)

where \( n \) refers to single name (issuer) and assuming zero correlation among the requirements for each counterparty \( n \).

### 3.3. Market risk SCR formula for corporate bond portfolios

The market sub-risks should be combined into an overall capital requirement \( SCR_{mkt} \) for market risk utilizing the correlation matrices presented in Table 1. The correlation coefficients intend to reflect potential dependencies in the tail of the distributions, as well as the stability of any correlation assumptions under stress conditions (EC, 2010; p.92).

#### Table 1: Market risk correlation matrices in different market scenarios

**Panel A: Correlation matrix for market (i.e. interest rate) down**

<table>
<thead>
<tr>
<th></th>
<th>Interest</th>
<th>Equities</th>
<th>Real</th>
<th>Spread</th>
<th>Currency</th>
<th>Concentration</th>
<th>Illiquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real estate</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>0.5</td>
<td>0.75</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentration</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Illiquidity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Panel B: Correlation matrix for market (i.e. interest rate) up**

<table>
<thead>
<tr>
<th></th>
<th>Interest</th>
<th>Equities</th>
<th>Real</th>
<th>Spread</th>
<th>Currency</th>
<th>Concentration</th>
<th>Illiquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real estate</td>
<td>0</td>
<td>0.75</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>0</td>
<td>0.75</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Currency</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentration</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Illiquidity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The general formula for the total market SCR is: \(^{14}\)

\[
SCR_{mkt} = \max \left( \sum_{r \leq c} CorrMktUp_{r,c} \cdot Mkt_{up,r} \cdot Mkt_{up,c}, \sum_{r \leq c} CorrMktDown_{r,c} \cdot Mkt_{down,r} \cdot Mkt_{down,c} \right)
\]

Here, \(CorrMktUp_{r,c}\) are entries of the correlation matrix presented in Table 1 (Panel A); \(Mkt_{up,r}\), \(Mkt_{up,c}\) are capital requirements for the individual market risks under the interest rate up stress movement according to the coefficients of the correlation matrix \(CorrMktUp\); \(CorrMktDown_{r,c}\) are entries of the correlation matrix presented in Table 1 (Panel B); \(Mkt_{down,r}\), \(Mkt_{down,c}\) are capital requirements for the individual market risks under the interest rate down stress movement according to the coefficients of the correlation matrix \(CorrMktDown\).

The formula for the total SCR of a bond portfolio facing interest rate risk, spread risk, and concentration risk is:

\[
SCR_{mkt} = \max \left( \sqrt{Mkt_{int}^{up} + \left(Mkt_{sp}^{bonds}\right)^2 + \left(Mkt_{Conc}\right)^2 + 0.5Mkt_{int}^{up}Mkt_{sp}^{bonds}}, \sqrt{Mkt_{int}^{down} + \left(Mkt_{sp}^{bonds}\right)^2 + \left(Mkt_{Conc}\right)^2} \right)
\]

Since we consider only asset management (without liabilities), the market SCR formula can be further simplified to allow only stress scenario with an upward movement of spot curves. Namely, downward movement would increase the value of a long position in bonds implying zero interest rate capital requirements. Therefore, we calculate total market SCR as follows:

\[
SCR_{mkt} = \sqrt{Mkt_{int}^{up} + \left(Mkt_{sp}^{bonds}\right)^2 + \left(Mkt_{Conc}\right)^2 + 0.5Mkt_{int}^{up}Mkt_{sp}^{bonds}}
\]

4. YTM-SCR optimization problem and the solution method

4.1. Optimization problem

We perform optimization with two objectives, to minimize market risk regulatory capital requirements SCR and, at the same time, to maximize YTM. A set of solutions for such a problem is referred to as an efficient frontier. Formally, we solve the following problem:

\[
\begin{align*}
\min_w & \quad SCR_{mkt} \\
\max_w & \quad YTM \\
\text{subject to} & \quad \sum_{i=1}^{N} w_i = 1 \\
& \quad 0 \leq w_i \leq 1, \quad i = 1, \ldots, N
\end{align*}
\]

Here, \(SCR_{mkt}\) are the Solvency II capital requirements for market risk expressed as a fraction of the bond portfolio value (equation 10). A portfolio YTM is determined as the weighted average of YTM's for the bonds entering that portfolio.\(^{15}\) YTM is the internal rate of return (annual yield to maturity), calculated using bond price formula, with cash flows that would be received if a bond is held until the maturity date and all coupons reinvested at the rate YTM:

\[
P_{bond}(YTM) = PV \left( \sum_{c=1}^{n} CF_c \right) = \sum_{c=1}^{n} \frac{CF_c}{(1 + YTM)^{t_c}}.
\]

Here, \(P_{bond}(YTM)\) is the bond price as a function of YTM, \(PV\) denotes present value, \(CF_c\) is promised \(c\)-th cash flow, \(n\) is the number of promised cash flows, \(t_c\) is time in years between the current date and the \(c\)-th cash flow \(CF_c\).

Decision variables are represented by the vector of portfolio weights \(w\), and \(N\) is the number of assets in the opportunity set. The first constraint ensures that the weights sum up to one.

\(^{15}\) Our approach for calculation of bond portfolios’ YTM’s is adopted from Luenberger, (1998; p.62). The same approach is also widely used by practitioners.
(equation 13). The second constraint (equation 14) ensures the non-negativity of each investment, consistent with the absence of short sales.

4.2. Optimization method

The concentration risk part of the market SCR (equation 7) is a non-differentiable and non-convex function of portfolio weights. This makes the YTM-SCR optimization problem highly complex and not solvable using traditional analytical techniques. In order to solve it we employ a multi-objective evolutionary algorithm (MOEA), specifically, NSGA-II. This is the most widely used MOEA for solving complex portfolio optimization problems non-solvable by standard techniques.\textsuperscript{16} It has been successfully applied to mean-variance equity portfolio optimization problems where more complex constraints are imposed (Branke et al., 2009, and Deb et al., 2011) and when complex risk measures are employed (Ranković et al., 2016, Anagnostopoulos and Mamanis, 2011), including mean-capital requirements optimization in Basel 2.5 framework (Drenovak et al., 2017). Several advantages of NSGA-II over the Pareto Archived Evolutionary Strategy (PAES) and Strength Pareto Evolutionary Algorithm (SPEA) were reported in the literature (e.g. Deb et al. 2000). Similarly, Anagnostopoulos and Mamanis, (2010) showed NSGA-II’s good approximation of risk-return trade-offs in a 3-objectives optimization problem, independent of the risk measure used.

All evolutionary algorithms search for optimal solutions to a given problem through an emulation of Darwin’s principle of natural selection. The algorithm begins with a set of randomly generated candidate solutions (portfolios), referred to as a population. In each iteration (generation) the following nature-inspired steps are performed: i) selection of good solutions (portfolios with the lowest SCR for a corresponding YTM) from the current population which form a set of potential parent solutions (mating pool);\textsuperscript{17} ii) crossover (combination) of randomly selected pairs of parent solutions which results in new solutions (offspring solutions); and iii) mutation (slight modification) of some randomly selected offspring solutions.\textsuperscript{18} The obtained offspring solutions form the population of the next

\textsuperscript{16} See Ponsich et al. (2013) for a comprehensive survey of MOEA applications in portfolio optimization. According to the survey, NSGA-II was used in twice as many studies compared to second most popular method.

\textsuperscript{17} As in the natural evolution, only good individuals (in our case portfolios) survive and are able to produce offspring.

\textsuperscript{18} Here, crossover refers to combination of weights of the two selected parent portfolios. Mutation refers to slight modification of weights of the selected offspring portfolios.
generation. As these processes are repeated over a number of generations, solutions evolve and improve in terms of the chosen objectives.

Implementation of NSGA-II implies adopting settings for the solution representation, the population size and parameters of the crossover and mutation operators. For the stated optimization problem, the solution is defined as a non-negative real-valued vector of portfolio weights that satisfy the appropriate constraints. Population size $S$ is set to 5 times the number of constituents in a given opportunity set.

For the breeding of an offspring population we apply a uniform crossover. Execution of crossover operator consists of two steps: random selection of two solutions (portfolios) from mating pool and their recombination with a predefined crossover probability. Two parent solutions produce two offspring solutions. Thus, crossover operator is performed $S/2$ times. We use crossover probability equal to 1 which means that recombination is performed to each pair of solutions. In uniform crossover the recombination implies that every allele (i.e., individual weight) is exchanged between the pair of parent solutions with a certain probability (swapping probability). In accordance with the previous literature, we set the swapping probability to 0.5, which means that approximately 50% of weights are exchanged (Sastry et al., 2005; Drenovak et al., 2017).

For the mutation process, we apply a uniform mutation operator. In general, mutation operator implies that each allele is selected with a predefined mutation probability and replaced with replacement value. We set the mutation probability to 0.05 which implies that approximately 5% of weights are replaced. In uniform mutation, replacement value is a realization of a random variable, uniformly distributed in the range defined by the lower and upper domain bounds (here 0 and 1).

Uniform crossover and uniform mutation operators satisfy the constraint defined by equation 14, for each offspring. However, for the most of generated offspring budget constraint (equation 13) is not satisfied. Thus, we normalize each of the offspring portfolios by dividing each weight by sum of all weights.

Note that, due to concentration risk formula, our objective function (SCR) (equations 10 and 11) is non-convex, but the space of decision variables (i.e. weights) is convex. Hence, crossover operator which combines two vectors of decision variables (i.e. portfolios) results always in a feasible solution.
5. Sample description

5.2.1. Sample selection and descriptive statistics

Traditionally, government bonds have been the most important investment class for insurance and pension industries. However, more recently yields on European government bonds have entered negative territory. At the same time corporate bonds were included in the Asset-backed Securities Purchase Programme (ABSPP) (ECB, 2014). Both events have contributed to unprecedented demand for corporate bonds by insurance companies.\textsuperscript{20}

We simulate the outcomes of YTM-SCR optimization on 16\textsuperscript{th} September 2008 and 21\textsuperscript{st} November 2014. The first date is associated with the Lehman Brothers’ bankruptcy filing and the peak of the recent financial crisis.\textsuperscript{21} The second date (21\textsuperscript{st} November 2014) is associated with the beginning of the European Central Bank’s (ECB) quantitative easing program.\textsuperscript{22} The above two events could therefore be described as a “black-swan” and a unique market-timing event respectively. The dates exhibit very different benchmark interest rate curves and therefore allow us to compare the optimization results under different market scenarios.

Our sample bonds are constituents of the Markit iBoxx Corporates Composite bond index. The index encompasses Euro-denominated bonds from non-financials such as industrials, automobiles, chemicals, food and beverages, utilities, health care, oil and gas, retail, and telecommunications, as well as financial sectors such as banks and other financial services.\textsuperscript{23} Consideration of bonds from different industrial sectors is important due to both differences among the sectors (supply side) and among insurers (demand side). Bond issuers from financial and industrial (non-financial) industries, for example, tend to be affected differently by changing market conditions. On the demand side, non-life-only insurers prefer to invest in industrial bonds while life-only insurers prefer financial bonds. For example, European non-life-only insurers hold 27% of their invested portfolios in corporate bonds with 80% in industrial bonds (EC, 2017, p.33).

\begin{itemize}
\item \textsuperscript{20} For example, European insurance companies have increased their holdings of corporate bonds from €1.1 trillion (in 2013) to €1.2 trillion (in 2017) (EC, 2017, p.33).
\item \textsuperscript{21} On 16\textsuperscript{th} September 2008, the asset swap spread for the iBoxx Corporate Composite bond index recorded its highest single day jump of 17.4 points (see Aussenegg et al., 2016).
\item \textsuperscript{22} ABSPP was announced on 4\textsuperscript{th} September 2014. This was followed by the release of relevant amendments and technical annexes during October and its start in November 2014 (e.g. ECB, 2014).
\item \textsuperscript{23} All bonds included in the iBoxx Corporates Composite bond index are investment grade with fixed coupons and a minimum amount outstanding of at least €500 million (excluding bonds with embedded options and collateralized debt obligations). The bonds are also required to have a minimum time to maturity of at least one year and could be senior or subordinated (see Market Group Limited, 2010).
\end{itemize}
Our main source of data is Markit, a leading global bond information provider. From the Markit database, we collect data required for calculation of capital charges which correspond to market risk submodules (e.g. duration, rating, coupon payments, notional amounts, dirty prices, accrued interest, etc.). We begin with 1,051 and 1,529 constituents of iBoxx Composite corporate bond index, on 16th September 2008, and 21st November 2014 respectively. We then adopt the following sample selection procedure. First, we select only bonds that have maturities from one to fifteen years categorized as senior debt. For uniformity, we keep only bonds that pay coupons once per year. Notably, the number of AAA rated corporate bonds is very small in the European market. This feature of the European corporate bond market has been documented in Aussenegg et al. (2015) and Biais et al. (2006), among others. For example, on 16th September 2008 (Panel A of Table 2), there were 38 AAA rated bonds issued by 5 different companies. However, none of these companies (i.e. bonds) are from Industrial sector. On 21st November 2014 (Panel B of Table 2) there were only 6 AAA rated bonds of 3 different companies. Again, none of these companies (i.e. bonds) were from Industrial sector. We were concerned that AAA bonds could potentially only affect opportunity sets for Financials bonds and decided not to consider AAA bonds. We, therefore, excluded AAA bonds keeping the investment grade bonds with ratings BBB to AA. We divide the sample bonds by the industry type of the issuer, into Financials and Industrials. The Financials sample contains bonds of financial companies, mostly banks. The Industrials sample includes bonds issued by companies from the following industrial subsectors: goods and services, construction, and materials. On 16th September 2008, our total sample consisted of 305 bonds, 232 Financials and 73 Industrials. On 21st November 2014, our sample consisted of 586 bonds, 439 Financials and 147 Industrials. For both Financials and Industrials, we consider the Total (containing all maturity brackets: 1-5Y, 5-10Y and 10-15Y) and subsamples containing bonds within the three maturity brackets.

The above procedure results in 4 subsamples by industry (2 dates x 2 industries) and 12 subsamples by bond maturity (2 dates x 2 industries x 3 maturities). We derive YTM-SCR efficient frontiers for each of the 16 opportunity sets. The number of constituents in each of the 16 opportunity sets (i.e. total sample and three maturity subsamples for 2 dates x 2 industries), is marked in bold in Table 2.
Table 2: Sample descriptive statistics

Panel A: 16th September 2008

<table>
<thead>
<tr>
<th></th>
<th>By credit rating</th>
<th>By maturity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AAA</td>
<td>AA</td>
<td>A</td>
</tr>
<tr>
<td>iBoxx Corporates</td>
<td>38</td>
<td>288</td>
<td>495</td>
</tr>
<tr>
<td>Our sample</td>
<td>0</td>
<td>156</td>
<td>97</td>
</tr>
<tr>
<td>- Financials</td>
<td>0</td>
<td>154</td>
<td>70</td>
</tr>
<tr>
<td>- Industrials</td>
<td>0</td>
<td>2</td>
<td>27</td>
</tr>
</tbody>
</table>

Panel B: 21st November 2014

<table>
<thead>
<tr>
<th></th>
<th>By credit rating</th>
<th>By maturity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AAA</td>
<td>AA</td>
<td>A</td>
</tr>
<tr>
<td>iBoxx Corporates</td>
<td>6</td>
<td>152</td>
<td>661</td>
</tr>
<tr>
<td>Our sample</td>
<td>0</td>
<td>85</td>
<td>293</td>
</tr>
<tr>
<td>- Financials</td>
<td>0</td>
<td>84</td>
<td>250</td>
</tr>
<tr>
<td>- Industrials</td>
<td>0</td>
<td>1</td>
<td>43</td>
</tr>
</tbody>
</table>

5.2.2. Benchmark and stressed spot curves

Figure 2 represents AAA spot curves (benchmark curves), and the corresponding stressed spot curves for the two dates.

**Figure 2: Benchmark and stressed spot curves for sample dates**

AAA spot curves (benchmark and stressed) for 16th September 2008 in black color (solid and dashed line correspondingly). AAA spot curves (benchmark and stressed) for 21st November 2014 in grey color (solid and dashed line correspondingly).

The shape of the 21st November 2014 spot curve reflects the fact that in the post-crisis environment short term benchmark rates dropped to zero and even became slightly negative while the entire spot curve lies in a low yield environment (below 2%). The original EC (2010) Solvency II formula for an upward stress scenario of the yield curve states that $r_{stressed} = r (1+s_{up})$. We follow the amended EU (2015) stressing rule, thus applying a 1% floor for an increase in interest rates. Also, we ignore the downward stress scenario since it would result in an increase of bond prices and therefore imply zero value of interest rate risk SCR for portfolios without liabilities (see Section A.1. of the Appendix for more details).

6. Results of the YTM-SCR bond portfolio optimization

6.1. YTM-SCR efficient portfolios

Figures 3 and 4 depict optimal YTM-SCR trade-offs (i.e. efficient frontiers) for the Total and for the three maturity subsamples (1-5y, 5-10y, 10-15y) for Financials and Industrials on the two dates (16th September 2008 and 21st November 2014). Both YTM and SCR values are expressed in percentage points. In the case of SCR, the percentage value represents the fraction of the portfolio value.24

As expected, efficient portfolios with higher yield have higher SCR levels under both market scenarios (Figures 3 and 4). The efficient frontiers for the samples of financial and industrial companies are substantially different. For example, Figure 3 reveals that the efficient frontier for the Total sample of Financials is almost entirely determined by the shorter term (1-5Y) bonds both in 2008 and 2014. In 2008, long-term bonds (segment 10-15Y on Figure 3) of financial companies provided the worst YTM-SCR trade-offs.

---

24 The presented efficient frontiers do not include outliers with unusually high YTM values relative to their sample peers. For example, from the 16th September 2008 subsample, we excluded Icelandic bank Glitnir Banki hf exhibiting a yield of 26.3% while from the 21st November 2014 subsample, we excluded Sberbank of Russia exhibiting a yield of 5.41%, significantly higher than the rest of the bonds. The inclusion of the above outliers would dramatically impact the shape of the efficient frontiers.
Figure 3: Efficient frontiers for Financials
Panel A: 16th September 2008

Panel B: 21st November 2014
Short-term (1-5Y maturity) efficient frontier for industrial bonds dominates the 5-10Y frontier for 2008. The opposite is true in 2014. In contrast to Financials, both 1-5Y and 5-10Y industrial bonds tend to contribute to the Total frontier in both 2008 and 2014 (Figure 4). In 2008, for example, 1-5Y and 5-10Y Industrials efficient portfolios bearing the same YTM of 8% exhibit SCR levels of 15.2% and 17.8% respectively. The Total Industrials portfolio frontier,
comprised of a combination of 1-5Y and 5-10Y bonds, exhibits a lower SCR at 13.2% for the same YTM.

Short term (1-5Y) efficient frontiers dominate frontiers of medium (5-10Y) and long term (10-15Y) tenors for Financials both in 2008 and 2014. For Industrials, evidence is mixed. Short term Industrial efficient frontiers dominate in 2008 but are taken over by medium term tenor in 2014. The reported absence of long-term bonds (10-15Y) in efficient Industrials and Financials portfolios, both before and after the crisis, is surprising given the business model of insurers and the results of previous studies documenting insurers’ preference for long term bonds (see Boermans and Vermaulen, 2016). Our results however are in line with the recent drop in insurers’ activity in the market for very long-term corporate bonds (15+Y) (EC, 2017). Our results indicate that even 10-15Y bonds may be disfavoured by the current regulation. This is in line with the results in Arias et al. (2012) who report that Solvency II favours bonds with short duration irrespective of credit rating.

6.2. YTM-SCR ratios
We compare the benefits of holding a particular bond portfolio to its regulatory cost by calculating its YTM-SCR ratio. This measure, appropriate for bonds and bond portfolios, is analogous to the Sharpe ratio for equity investments. Figure 5 presents the YTM-SCR ratio as a function of SCR for the efficient portfolios corresponding to the Total efficient frontiers for financial and industrial companies in the 2008 and 2014 samples.

Notably the YTM-SCR ratio is decreasing in SCR. Investors could have expected higher yield per unit of SCR in 2008 than in 2014. This is especially true for investors that were selecting portfolios with low regulatory cost and, in particular, for Financials portfolios with an SCR smaller than 20%. Since the efficient portfolios with low SCR are, also, the most diversified portfolios on the efficient frontier, they would arguably be the most interesting ones for insurance companies.

25 The insurance industry argues that a 30% regulatory charge for a 20-year BBB bond is too high given their actual credit loss rates tend to be below 1% (see EC, 2017, p.35-36).

26 A variant of this ratio was used in Arias et al. (2012) for individual corporate bonds.
In line with “flight to liquidity” phenomenon, in the aftermath of the global financial crisis investors changed their risk perceptions and increased the demand for corporate bonds (see Longstaff et al., 2004). The higher overall demand for corporate bonds inevitably resulted in higher prices and, consequently, lower yields. The above is evident in our results for 2014. For example, YTM-SCR ratios dropped precipitously from their 2008 values primarily as a result of the sharp decrease in yields. In addition, the YTM-SCR curves have flattened. As a result, it would still pay to select portfolios with low level of SCR in 2014. In terms of YTM - regulatory cost trade-off, the impact of this decision is much smaller than in 2008. The difference in ratios for financials and non-financials has effectively disappeared after the recent financial crisis.

6.3. SCR components in different market scenarios

Figure 5 shows that the range of SCR values did not change very much in 2014 compared to 2008. On the other hand, the structure of overall market risk SCR values has changed substantially. In order to demonstrate this, we present three market risk submodules, $Mkt_{int}$, $Mkt_{sp}^{bonds}$, and $Mkt_{Conc}$ as the functions of YTM in 2008 and in 2014 (Total) samples (see Figures 6 and 7, respectively).
We observe a significant decrease in the interest rate risk charge in 2014 compared to 2008, especially for the Financials. This is the result of significantly lower interest rates in 2014, and the fact that the Solvency II stress scenarios lead to relative changes of interest rates (see Section A.1. of the Appendix for more details). For example, the interest rate risk charge was reduced while the spread risk charge (i.e. changes in the level or volatility of credit spreads over the risk-free term structure of interest rates) increased in 2014. This is particularly pronounced for Industrials. Interestingly, these two effects tend to offset each other leading only to marginal change in the overall SCR.

The concentration risk profile is similar on both dates (except that similar values of $MktConc$ now correspond to lower yields). Concentration risk increases steeply with an increase in yields since portfolios with high yields tend to be concentrated in very few bonds. In particular, by construction, the portfolio with the highest yield on an efficient frontier contains only the bond with the highest yield. For this portfolio (as well as other portfolios with low cardinality and high yield), the dominant component of SCR is the concentration charge (see Figures 6 and 7).

Figure 6: SCR components for Financials and Industrials, 16th September 2008

Panel A: Financials

Panel B: Industrials
6.4. Cardinality and composition of the YTM-SCR efficient portfolios

We measure the cardinality of an efficient portfolio as a number of bonds included in the efficient portfolios. Here we consider only bonds with weights greater than 1%. The results are presented in Table 3. Importantly, all efficient portfolios have low cardinality, i.e. they are not well diversified. For example, the maximum number of bonds included in the Total Financials efficient portfolios (Panels A and B) is 22 and 42 respectively. This represents less than 10% of the number of bonds in the respective 16th September 2008 and 21st November 2014 opportunity sets. Unreported results show that the most diversified efficient portfolios are those at the left of the efficient frontier characterized by low levels of SCR and YTM.

Table 3: Cardinality of the efficient portfolios
Panel A: 16th September 2008

<table>
<thead>
<tr>
<th>Samples</th>
<th>Financials:</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>1-5Y</td>
<td>5-10Y</td>
<td>10-15Y</td>
</tr>
<tr>
<td>Number of bonds</td>
<td>232</td>
<td>158</td>
<td>69</td>
<td>5</td>
</tr>
<tr>
<td>Max cardinality</td>
<td>22</td>
<td>28</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Industrials:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bonds</td>
<td>73</td>
<td>25</td>
<td>39</td>
<td>8</td>
</tr>
<tr>
<td>Max cardinality</td>
<td>19</td>
<td>16</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>
Panel B: 21st November 2014

<table>
<thead>
<tr>
<th></th>
<th>Financials:</th>
<th>Industrials:</th>
<th>Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>1-5Y</td>
<td>5-10Y</td>
</tr>
<tr>
<td>Number of bonds in opportunity set</td>
<td>439</td>
<td>279</td>
<td>147</td>
</tr>
<tr>
<td>-Max cardinality</td>
<td>42</td>
<td>42</td>
<td>30</td>
</tr>
<tr>
<td>Number of bonds in opportunity set</td>
<td>147</td>
<td>73</td>
<td>67</td>
</tr>
<tr>
<td>-Max cardinality</td>
<td>34</td>
<td>34</td>
<td>17</td>
</tr>
</tbody>
</table>

Notes: Number of bonds in opportunity set refers to number of bonds in opportunity sets for the corresponding samples. Max cardinality refers to the cardinality of efficient portfolio(s) with maximum number of bonds.

Low cardinality of efficient portfolios, present in different market regimes and industry sectors, is unexpected given that Solvency II aims to penalize concentration risk. Importantly, the very low number of bonds constituting insurers’ efficient portfolios may potentially reduce demand for corporate bonds outside the efficient portfolios.

Composition of efficient portfolios by rating of constituent bonds is presented in Figures 8 (Financials) and 9 (Industrials). As expected, the proportion of BBB bonds is increasing in the portfolios with higher SCR. Opposite holds true for AA bonds. Across the entire SCR range, BBB bonds dominate (except perhaps for Financials featuring low SCR in 2008). The results for Financials are particularly interesting due to the large share of BBB bonds in efficient portfolios in 2014 (see Figure 8). Even portfolios with 10% SCR are constructed predominantly from BBB bonds.

Figure 8: Composition of efficient portfolios for Financials
It is evident that AA and A rated bonds gave way to BBB bonds, despite BBB’s lower concentration risk threshold and relatively small representation in the total sample (see Table 2). Absence of AA Industrial bonds from efficient portfolios in both 2008 and 2014 is striking. BBB bonds clearly dominate Industrial efficient portfolios, across all SCR levels, both in 2008 and 2014 (see Figure 9).

Figure 9: Composition of efficient portfolios for Industrials

The dominance of BBB bonds in our efficient portfolios contradicts Arias et al. (2012) who, based on the analysis of individual bonds, report that BBB bonds are likely to be neglected by investors. Our results are in line with reports that European insurers increased their allocation to lower rated bonds within the investment grade range during the post crisis period. For example, the average allocation to BBB-rated bonds increased from 10% to approximately 25% during 2011-16 (EC, 2017, p.33).

7. Robustness checks

7.1. YTM-SCR efficient portfolios on 2nd September 2011

We repeat the analysis for 2nd September 2011, which is a date in the middle between our two sample dates. There were no particularly important market announcements on that day although investors were concerned about the Eurozone (i.e. Greek government bond) crisis. Following the same procedure as before (see Section 5.2.1.) we identify 371 Financials and 113 Industrial
bonds. Unreported results are in line with the results reported in Section 6.\textsuperscript{27} For Financials, the efficient frontier is again dominated by short term bonds in a majority of the efficient portfolios. In contrast, for Industrials, the effect of medium-term bonds is much more prominent and they tend to dominate their short-term counterparts. SCR values range between 0\% and 35\% thus remaining similar to those reported in 2008 and 2014. YTM-SCR ratios’ range is also similar to one reported for 2008 and 2014. The slope of the curve depicting the YTM-SCR ratio across different SCRs is lower in 2011 than in 2008. As investors were gradually moving away from volatile equity markets, bond prices were increasing and yields decreasing. As the SCR remained in the same range, the ratios were gradually going down.

The composition of the efficient portfolios is presented in Figure 10. Looking across entire SCR range, financial efficient portfolios are dominated by A rated bonds. Exposure to AA rated financial bonds clearly shrank in favour of A rated bonds in 2011. The above results demonstrate a gradual increase in number of lower rated bonds in Financial efficient portfolios culminating in overall dominance of BBB bonds in 2014. It is worth noting that BBB financial bonds dominate efficient portfolios in spite of the fact that A rated bonds dominate opportunity set in 2014. For Industrials, structure of the efficient portfolios is similar to one reported for 2014. For example, all efficient portfolios with SCR beyond 20\% are constructed entirely from BBB bonds.

Figure 10: Composition of efficient portfolios in 2011

\textsuperscript{27} Unreported results are available upon request from the authors.
7.2. The effect of changes in spot curve stressing

The EU (2015) introduced a 1% floor for the spot curve upward stressing in order to accommodate for historically low level of interest rates. Here, we describe differences in stressed benchmark spot curves regarding the two versions of the regulation (EC, 2010 and EU, 2015) and present the implication of the 1% floor for the optimal YTM-SCR trade-offs (with differences in the structure of component SCR values).

We first note that the stressed spot curve for 16th September 2008 is the same regardless of the stressing scenario. In contrast, for 21st November 2014, EU 2015 rule implies more significant changes in interest rates (Figure 11). Effectively, the EU 2015 stress scenario (dashed gray line) turns out to be a 1% upward parallel shift in the benchmark AAA spot curve (solid gray line).

Figure 11: Benchmark and stressed spot curves on 21st November 2014: EC (2010) vs. EU (2015)

AAA spot curve (solid gray line). EC (2010) stressed curve is black line. EU (2015) stressed curve is dashed gray line.

An important consideration is how different stress scenarios affect YTM-SCR efficient frontiers in a low interest rate environment (i.e., for 21st November, 2014). For the sake of brevity, we summarize the unreported results here.28 The qualitative relationship between the efficient frontiers for different tenors remains roughly the same. Frontiers exhibit an almost parallel shift to the right under the new stress scenario. An increase in the SCR, for the same

28 Unreported results are available upon request from the authors.
level of YTM, is most pronounced in the long-term segment (10-15Y). This is expected given that the sensitivity of bonds to interest rate increases with bond tenor.

As expected, the interest rate SCR value increased significantly under the new (EU, 2015) rule. As regulators intended, the new upward stressing rule has led to an increase in the SCR values for interest rate risk in the low interest rate environment. The change in stress scenarios however did not seem to appreciatively changed the composition of the efficient portfolios. Moreover, the patterns for the concentration and spread risk components under the two scenarios are very similar.

7.3. The effect of AAA rated financial bonds

Due to very small overall number of European AAA rated bonds, and their complete absence from our Industrial subsample, we did not consider AAA rated (financial) bonds. To check for sensitivity of our results to exclusion of AAA rated financial bonds, we repeated estimates for the enlarged Total sample, including the AAA bonds. Unreported results suggest that 16 AAA financial bonds entered at least one of the efficient portfolios, in 2008. As expected, the shape of the efficient frontier did not change. The inclusion of the AAA rated bonds only provided a portfolio with slightly lower minimum SCR. In 2014, the shape of the efficient frontier remained the same, with only 2 AAA rated financial bonds entering the efficient portfolios. Maximum number of bonds in the efficient portfolios increased only marginally in 2008 and remained the same in 2014. Our results reported in Figure 3 and Table 3, therefore remained robust to the inclusion of AAA rated financial bonds.

8. Conclusion

As a response to the 2008 financial crisis financial regulators tightened the prudential rules by imposing explicit (additional) capital requirements to serve as a buffer for unexpected events. These made the objective of achieving regulatory efficient portfolios highly computationally challenging. The paper studies the asset allocation problem facing managers in insurance companies using the SCR formula. Our novel approach for bond portfolio optimization enables us to identify several important implications of the new regulation for portfolio management.

29 Detailed results available from authors upon request.
in insurance industry. For example, starting from a large universe of bonds, only a few enter regulatory efficient portfolios. Moreover, the efficient portfolios are dominated by short term, lower rated bonds. The dominance of short-term risky bonds in the efficient portfolios lend support to managers’ concerns that Solvency II overstates capital requirements for long term bonds and shed more light on the increasing presence of lower rated bonds in insurers’ portfolios. The lower demand for long term bonds by insurers may potentially create financing problems for companies issuing such bonds. Our results also lend support to recalibration of the concentration submodule aiming to penalize more severely portfolios with low cardinality. Finally, we highlight need for further changes in the EU stress scenarios for the benchmark AAA spot curve.

Appendix: Standard SCR – Market risk sub-modules
A.1. Interest rate risk sub-module for corporate bonds
For interest rate sub-module, the standard approach of Solvency II requires scenario-based calculation of the capital requirement. We therefore stress a given portfolio of bonds using up \((Mkt_{int}^{Up} = \Delta NAV|up)\) and down \((Mkt_{int}^{Down} = \Delta NAV|down)\) pre-specified moves of the spot curves. Here, \(\Delta NAV|up\) and \(\Delta NAV|down\) are the changes in bond prices due to upward and downward altered term structures. Where the scenario results in an increase of NAV (i.e. does not reflect a risk for the bonds) this should not lead to a "negative capital requirement". In this case, the corresponding capital requirement is zero (EC, 2010; p.92). \(Mkt_{int}^{Up}\) is calculated using the upward stressed term structure of interest rates. \(Mkt_{int}^{Down}\) is calculated using the downward stressed term structure. The capital requirement for interest rate risk is derived from the type of shock that gives rise to the highest capital requirement (EC, 2010; p.112):

\[
\text{If } Mkt_{int}^{Up} > Mkt_{int}^{Down} \text{ then } Mkt_{int} = \max (Mkt_{int}^{Up}, 0) \tag{A.1}
\]

\[
\text{If } Mkt_{int}^{Down} > Mkt_{int}^{Up} \text{ then } Mkt_{int} = \max (Mkt_{int}^{Down}, 0) \tag{A.2}
\]

The altered term structures are derived by multiplying the current interest rate curve by \((1+s_{up})\) or \((1+s_{down})\), where both the upward stress \(s_{up}(t)\) and the downward stress \(s_{down}(t)\) relative changes of interest rate for individual maturities \(t\) are specified in Table A.1. For maturities greater than 30 years, stress values of 25% and -30% should be applied. Importantly, the higher current interest rate, the greater is absolute change of rate under stress scenario and consequently the greater is its impact on discounting which affects bond price.
To accommodate the recent historically low interest rate environment, EU (2015) added a requirement that an increase of basic-risk-free interest rates under stress, at any maturity should, always, be at least 1%. Without these EU (2015) changes, negative benchmark interest rates would be stressed upward to become even more negative rates for short maturities. Specifically, stressing a negative interest rate by multiplying it by \((1+s_{up})\) factor (where \(s_{up}>0\)), as proposed by Solvency II (see Table A.1.), results in even more negative interest rate.

Table A.1: Upward and downward stress for different maturities \((t)\)

<table>
<thead>
<tr>
<th>Maturity ((t))</th>
<th>Relative change (s_{up}(t))</th>
<th>Relative change (s_{down})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>70</td>
<td>-75</td>
</tr>
<tr>
<td>0.5</td>
<td>70</td>
<td>-75</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>-75</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>-65</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>-56</td>
</tr>
<tr>
<td>4</td>
<td>59</td>
<td>-50</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>-46</td>
</tr>
<tr>
<td>6</td>
<td>52</td>
<td>-42</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>-39</td>
</tr>
<tr>
<td>8</td>
<td>47</td>
<td>-36</td>
</tr>
<tr>
<td>9</td>
<td>44</td>
<td>-33</td>
</tr>
<tr>
<td>10</td>
<td>42</td>
<td>-31</td>
</tr>
<tr>
<td>11</td>
<td>39</td>
<td>-30</td>
</tr>
<tr>
<td>12</td>
<td>37</td>
<td>-29</td>
</tr>
<tr>
<td>13</td>
<td>35</td>
<td>-28</td>
</tr>
<tr>
<td>14</td>
<td>34</td>
<td>-28</td>
</tr>
<tr>
<td>15</td>
<td>33</td>
<td>-27</td>
</tr>
<tr>
<td>16</td>
<td>31</td>
<td>-28</td>
</tr>
<tr>
<td>17</td>
<td>30</td>
<td>-28</td>
</tr>
<tr>
<td>18</td>
<td>29</td>
<td>-28</td>
</tr>
<tr>
<td>19</td>
<td>27</td>
<td>-29</td>
</tr>
<tr>
<td>20</td>
<td>26</td>
<td>-29</td>
</tr>
<tr>
<td>21</td>
<td>26</td>
<td>-29</td>
</tr>
<tr>
<td>22</td>
<td>26</td>
<td>-30</td>
</tr>
<tr>
<td>23</td>
<td>26</td>
<td>-30</td>
</tr>
<tr>
<td>24</td>
<td>26</td>
<td>-30</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>-30</td>
</tr>
<tr>
<td>30</td>
<td>25</td>
<td>-30</td>
</tr>
</tbody>
</table>

Source: EC (2010).

Thus, upward stressing in a negative interest rate environment would lead to virtually no change in bond prices (or even a slight increase in stressed bond prices) for shorter maturity bonds. For such bonds all (or most of) coupons would be discounted utilizing negative short-term benchmark interest rates, implying \(Mkt_{int} = 0\) for an upward stress movement. Within the downward stress scenario, the absolute change of interest rates should at least be 1%. If the unstressed rate is below 1%, the stressed rate of 0% should be used. (EC, 2010; p.112). Similarly, in cases of negative basic risk-free rates, the decrease of 0% should be used (EU, 2015; p.108). We follow the latest regulation and, for a given bond portfolio, calculate interest rate component of the market SCR as follows:
\[ Mkt_{\text{int}} = \sum_i w_i \cdot \%\Delta NAV_i \]  

(A.3)

\[ \%\Delta NAV_i = \frac{P_{\text{bond},i} - stressedP_{\text{bond},i}}{P_{\text{bond},i}} \]  

(A.4)

\%\Delta NAV\_i is the relative price change of bond \( i \). In all our calculations we use dirty bond prices \( P_{\text{bond},i} \) from Markit database. The term \( stressedP_{\text{bond},i} \) is the stressed price of bond \( i \) which corresponds to the shock that gives rise to the highest capital requirement for a given bond portfolio.\(^{30}\) \( w_i \) is the fraction of the bond portfolio’s value invested in each individual bond \( i \), calculated using dirty prices. \( Mkt_{\text{int}} \) is, therefore, the relative change of the bond portfolio value due to a shift in interest rates. The formula for \( Mkt_{\text{int}} \) (equation A.3) is intuitive and suggests that a portfolio interest rate risk is the weighted average of interest rate risk assessments of each its constituents.

**A.2. Spread risk sub-module for corporate bonds**

Spread risk submodule measures sensitivity of insurance companies’ assets and liabilities with respect to the changes in level or volatility of credit spreads over the risk-free term structure of interest rates. In our case, the (investment) bond portfolio spread risk capital requirement is calculated using the following formula (see EC, 2010; p.122):

\[ Mkt_{sp}^{\text{bonds}} = \sum_r \%MV_r^{\text{bonds}} \cdot F^{\text{up}}(\text{rating}_r) \cdot \text{duration}_r \]  

(A.5)

Here, \( \%MV_r^{\text{bonds}} = \sum_{j \in I} w_j \) \hspace{1cm} (A.6)

\( I = \{ \text{bonds of rating } r \} \) is fraction of the bond portfolio invested in bonds of rating \( r \). Factor \( F^{\text{up}}(\text{rating}_r) \) is a function of the rating class of the credit risk exposure which is calibrated to deliver a shock consistent with 99.5% VaR following a widening of credit spreads. Finally, \( \text{duration}_r \) is average duration of bond portfolio at rating \( r \), weighted with the market value of the bonds:

\(^{30}\) The superscript “bonds” highlights the fact that regulation differentiates spread for bonds, structured credit products, and credit derivatives (see EC, 2010; p.121).
duration_j = \sum_{j \in J} \sum_{j \neq i} \frac{w_j}{w_{ij}} \cdot duration_j  
\quad (A.7)

Here we use “street modified duration” from Markit database (see Markit, 2010). F^{up} specified for different bond ratings are reported in Table A.2 (EC 2010). Notably, duration floors and caps required by EC (2010) are not required by EU (2015) (see EC, 2010; p.122; and EU, 2015/35, Article 104, p.69).

<table>
<thead>
<tr>
<th>Bond ratings</th>
<th>F^{up} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.9</td>
</tr>
<tr>
<td>AA</td>
<td>1.1</td>
</tr>
<tr>
<td>A</td>
<td>1.4</td>
</tr>
<tr>
<td>BBB</td>
<td>2.5</td>
</tr>
<tr>
<td>BB</td>
<td>4.5</td>
</tr>
<tr>
<td>B or lower</td>
<td>7.5</td>
</tr>
<tr>
<td>Unrated</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Source: EC (2010)

To summarize, Mkt_{\delta}^{bonds} represents relative (percentage) change of portfolio value due to changes in credit spreads. It is a function of rating and duration which implies increase in SCR with a decrease of rating, while duration is primarily affected by bond residual maturity in any interest rate environment.

A.3. Concentration risk sub-module for corporate bonds

For a given bond portfolio, the calculation of market SCR regarding concentration risk is performed in three steps: i) determination of excess exposure per ‘name’ j (XS_{n}); ii) risk concentration capital requirement per ‘name’ j (Conc_{n}); iii) aggregation across single names.

All sample bonds with the same ticker symbol are issued by the same company. We therefore identify names using ticker symbols. The excess exposure is calculated as:

\begin{equation}
XS_{n} = \max \left(0; \frac{E_{n}}{Assets_{d}} - CT \right)
\end{equation}

\quad (A.8)

Where, \( \frac{E_{n}}{Assets_{d}} = \sum_{j \in J} w_{j} \)  
\quad (A.9)

Here, J is the set of bonds of the single name (i.e. all bonds with the same ticker symbol). E_{n} is the dollar exposure at default to counterparty n (i.e. exposure to single name) I. Assets_{d} is total amount of assets considered in this sub-module. In our case, the total assets is total market value of considered bond portfolio. Thus, the ratio of exposure and assets measures the
exposure to a particular name, relative to the value of the portfolio as a whole. Depending on rating of a name, the regulator sets different concentration threshold (CT). The values for CT are presented in Table A.3. (see EC, 2010, p.129; and EU, 2015, Article 185, p.121).

<table>
<thead>
<tr>
<th>Issuers rating</th>
<th>CT (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA-AAA</td>
<td>3</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>BBB</td>
<td>1.5</td>
</tr>
<tr>
<td>BB or lower</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Source: EC (2010).

Given the thresholds of 1.5% (rating BBB) and/or 3% (rating A and above), more diversified portfolios are favored by the regulation. For example, in order to construct a portfolio with 0 concentration charges one would need an opportunity set of at least 34 highly rated bonds (or 67 BBB rated bonds). If the ratio of exposure and assets is higher than the corresponding threshold CT, the risk concentration capital requirement per ‘name’ is calculated as (EC, 2010; p.129):

\[
Conc_n = XS_n \cdot g_n
\]

The parameter \( g_n \) depends on the credit rating of the bond issuer (i.e. name). The values for the rating parameter \( g_n \) are presented in Table A.4.

<table>
<thead>
<tr>
<th>Issuers rating</th>
<th>( g_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.12</td>
</tr>
<tr>
<td>AA</td>
<td>0.12</td>
</tr>
<tr>
<td>A</td>
<td>0.21</td>
</tr>
<tr>
<td>BBB</td>
<td>0.27</td>
</tr>
<tr>
<td>BB or lower</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Source: EC (2010).

Some adjustments to the calculation of the risk concentration capital per name are required in cases where some bonds of the same issuer (name) were given different ratings. Usually, this is done in so-called credit quality step where individual bond ratings for each bond of a single name were replaced with suitably averaged ratings for a given name. Since there are very few cases of such bonds in our sample, we simplify by assigning the first available rating for all bonds of the same issuer. This does not impact our results in any important way but simplifies the calculation. \( MktConc \) always increases with an increase in weights (above the threshold). Therefore, within a single efficient frontier, \( MktConc \) value is always expected to be greater for high-yield-low-cardinality portfolios (i.e. more likely at the right end of the efficient frontier).
Finally, for a given bond portfolio market SCR regarding concentration risk is calculated using the following formula (EC, 2010; p.130):

\[ Mkt_{Conc} = \sqrt{\sum_n (Conc_n^2)} \quad (A.11) \]

where \( n \) refers to single name (issuer) and assuming zero correlation among the requirements for each counterparty \( n \).
List of references


European Insurance and Occupational Pensions Authority (EIOPA), (2013). Guidelines on forward looking assessment of own risks (based on the ORSA principles).


