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The state of the art development of AHP (1979-2017): A literature review with a social network analysis

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Abstract

Although many papers describe the evolution of the Analytic Hierarchy Process (AHP), most adopt a subjective approach. This paper examines the pattern of development of the AHP research field using Social Network Analysis (SNA) and scientometrics, and identifies its intellectual structure. The objectives are: (i) to trace the pattern of development of AHP research; (ii) to identify the patterns of collaboration among authors; (iii) to identify the most important papers underpinning the development of AHP; and (iv) to discover recent areas of interest.

We analyse two types of networks: social networks, that is, co-authorship networks, and cognitive mapping or the network of disciplines affected by AHP. Our analyses are based on 8,441 papers published between 1979 and 2017, retrieved from the ISI Web of Science (WoS) database. To provide a longitudinal perspective on the pattern of evolution of AHP, we analyse these two types of networks during the three periods 1979-1990, 1991-2001 and 2002-2017. We provide some basic statistics on AHP journals and researchers, review the main topics and applications of integrated AHPs and provide direction for future research by highlighting some open questions.

Keywords: AHP development, review, Analytic Hierarchy Process, Matrix Consistency, Pairwise Comparisons Matrix (PCM)

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1. Introduction

Analytic Hierarchy Process (AHP) is a problem solving framework (Saaty, 1986) and a theory of measurement (Saaty, 1990a). It has been proposed as a decision analysis technique to evaluate complex multi-attribute alternatives among one or more decision makers. Since it allows the inclusion of subjective factors, it is considered an advancement compared to other decision-making methods. AHP has been applied extensively, especially to large-scale problems involving multiple criteria, and where
the evaluation of alternatives is mostly subjective. This paper describes how applications of stand-alone and integrated AHPs evolved and discusses the development over time of the main contributions in this field, to provide an original historical perspective on AHP. The aim is to identify seminal studies that have played a major role in the development of AHP and, also, to identify areas of its adoption. The study uses quantitative methods to identify the set of papers that have contributed most to AHP development and to discover recent major AHP activities.

The literature contains several important surveys (Chai et al., 2013; Ho, 2008; Ishizaka and Labib, 2011; Sipahi and Timor, 2010), but the present paper is the first to investigate AHP adopting a longitudinal perspective on both its methodological development and applications, based on quantitative analysis. Our aim is to provide an in-depth understanding of the scientific communities working on specific applications of AHP and to analyse the on-going debate on the different AHP approaches proposed over recent decades. This study method can be described as quantitative, qualitative and citations network based. The need for a quantitative analysis of this work emerged as the result of the growing number of publications that no longer allow comprehensive qualitative analysis.

This paper contributes to our understanding of the patterns of development to date, of the AHP. It traces the evolution of the method within the communities of authors interested in application of the AHP to problem-solving in different contexts, and in the methodological advancements to overcome the shortcomings of the method identified over years. We explain how the weaknesses of the AHP and shortcomings related to an individual approach have been addressed over time and discuss the advantages of using AHP based methods for decision making.

2. Methodology

Studying paper citations networks using a scientometric approach and SNA has become popular in recent years and provides an understanding of various dynamics
such as collaboration among researchers (Lee et al., 2014) and emerging knowledge trends within disciplines (Emrouznejad and Marra, 2014; Lampe and Hilgers, 2014). In this paper, we combine insights from a scientometric mapping technique and SNA to study collaboration networks. We apply the scientometric mapping technique *overlay mapping* to obtain a cognitive map of the AHP field, and use SNA to study co-authorship networks.

Overlay mapping is a recently developed scientometric technique, which has become a ‘strategic intelligence’ tool, which is able to detect the evolution and emergence of innovations in patent citation networks (Rotolo et al., 2013). We chose this approach for a number of reasons. First, it has been proven to be helpful to benchmark and to track temporal changes and to analyse the growing numbers of scientific developments within a discipline (Rafols et al., 2010). Second, the mapping captures and displays the variety of disciplines by depicting them as nodes. Another key aspect is that scholars have invested effort in making these tools available to researchers interested in exploring the evolution of science and knowledge. There is a range of on-line tools available to conduct such analyses†.

Figure 1 provides a depiction of the idea underpinning overlay mapping, which is to use data representing an *entity*, a focal subject area, to construct an *overlay*. This is projected over a *basemap*, which represents the totality of the contemporary research areas which are grouped into 19 categories covering social studies, to mathematical methods and computer science. Each node in the map represents one of 19 factors that proxies for a scientific discipline.‡ These areas are identified using the 225 Web of Science (WoS) subject categories, which classify journals included in the Science Citation Index (SCI) into disciplinary and sub-disciplinary structures. This allows a

† In developing the cognitive maps using data retrieved from the ISI WoS academic database, we followed instructions provided by Loet Leydesdorff on his website, accessible at http://www.leydesdorff.net/software.htm. Freeware-based toolkit available here http://www.leydesdorff.net/overlaytoolkit/

‡ The 19 factors are: mathematical methods; computer science; physics; mechanical engineering; chemistry; environmental science and technology; materials science; geoscience; ecology; agriculture; biomed science; infectious diseases; psychological science; health and social issues; clinical psychology; clinical medicine; social studies; business and management; economics politics and geography.
visualization of how the publications in a certain field (in our case AHP) relate to different scientific disciplines. The term *cognitive map* refers to the projection on an *overlay* of data on published works, showing the cognitive space which is the contemporary universe of research areas. In the resulting cognitive map, the node size is proportional to the number of publications related to a given topic, were published in the given discipline represented by the node (Leydesdorff et al., 2013; Rotolo et al., 2013). Different colours are used to represent the 19 factors and to enable an immediate visual understanding.

![Figure 1. Overlay mappings](image)

This technique allows the mapping of three spatial dimensions - the cognitive, social and geographical spaces. For the purposes of this paper, we analyse only the cognitive and social spaces since the geographical space is more relevant for analysing patent publications because it identifies companies located in different countries.

The cognitive map provides a classification of the publications into research areas (Waltman and van Eck, 2012). Rotolo et al. (2013) highlight that mapping emergence in the cognitive space can reveal a number of features. These include the direction of diffusion of a given topic across the key knowledge areas involved in its emergence,
how these areas interact, and in which domain the actors’ knowledge production processes are located. Overall, it provides an immediate snapshot of the disciplinary evolution of a field or a topic, in our case AHP. The cognitive maps resemble a group of poles arranged roughly in a circle, whose thickness varies and which is different from a regular ring. It has been suggested that this shape is in line with the concept of scientific enterprise where no discipline dominates by occupying the centre (Knorr-Cetina, 1999).

We chose this approach for the present study in order to highlight the attention being given to AHP by scholars working in various fields, to show how AHP is influencing multiple disciplinary contexts and to demonstrate the utility and power of AHP as a method for assessing the decision-makers in disparate fields.

The approach can be applied to longitudinal studies to show the evolution of areas of interest along time. For the purposes of the present study, we divide the period 1979 to 2017 into three sub-periods (1979-1990, 1991-2001 and 2002-2017). For each period we provide the corresponding cognitive map, that is, the network of disciplines to which AHP is applied, and the corresponding social map, that is, the network of authors working in those disciplines. These are integrated with detailed information derived from an analysis of co-authorship networks. In these co-authorship networks, the connections among authors are the channels through which they gain access to knowledge and generate research outcomes. We show the evolution of collaboration networks over time and how they reflect the evolution of the topics within the field. Several studies demonstrate the utility of SNA approaches, such as citation and co-citation networks, to identify clusters of knowledge within a discipline (Lampe and Hilgers, 2014; Liu et al., 2013). When considering the specific field of AHP, which we show is characterized by efforts from many researchers to deal with complex issues in multiple research contexts, the need for collaborative activities among scholars is expected to play a central role in providing impactful contributions. Thus, we aim to map the social space underpinning the evolution of the AHP field, that is, the collaboration networks
among co-authors. We use the emergence of the main collaboration networks during the three periods to shed light on the most important themes to which AHP has been applied, and to identify the most important papers and their interrelations, the topic trends over time, and the major authors and their evolving co-authorship networks. Data are analysed using Pajek, Sci² software (Sci² Team, 2009) and HistCite. Specifically, Pajek was used to create the cognitive maps. We exploit the procedures and tools made available on line by Rafols et al. (2010) for mapping publications in relation to WoS categories. We used SCI² for the analysis of co-author networks emerging in each of the three sub-periods. We selected SCI² because the visual output produced allows us to work on it in an effective way to improve its readability. HistCite was used to compute the basic statistics presented in the next section.

Figure 2 provides a visualization of the methodology by depicting the flowchart for the research process. We extracted data from the ISI WoS academic database. AHP papers were searched for and retrieved using the keywords “analytic hierarchy process”; “AHP”; “comparison matrix”; “pairwise comparison matrix” and “PCM”; “matrix consistency”. The data cover the period 1979 to 2017. We obtained an initial 8,814 results, 373 of which were not imported since they were considered not relevant despite containing a keyword in the text. This generally referred to the references; the topic of the 373 papers was not AHP. We reviewed the content of these papers to ensure their inconsistency with the overall sample. Among the remaining papers, we analysed the abstracts to ensure each paper related to the field of AHP. We downloaded our initial results as a text file and imported it into SCI², which allowed us to visualize and organize the abstracts systematically. The whole sample has split by author to make the analysis more manageable.

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Cleaning of the dataset, both off line as the text file, and on-line by excluding the papers from the list provided by WoS provided the final dataset.
We exported the WoS data into an analyze.txt file, which was further transformed by a freely available mini programme. This resulted in a file that could be analysed using Pajek.

Figure 2. Flowchart for the research process

We analysed the dataset using SCI² to obtain the co-author networks. The data were split into three sub-periods. For each sub-periods the software identified the nodes (authors) and linked them if they had co-authored one or more papers, that is, if they
represented an edge”. Given the large number of papers in our network and the even larger numbers of authors involved, we focus only on the top edge (co-author relations) and top nodes (authors) in each period; in other words, the resulting co-authorship networks are the most representative of the number of co-authored works and the number of citations received, but they are not the only networks.

The final panel of papers includes 8,441 published works: 4,721 papers, 3,362 conference proceedings, 211 articles and proceedings papers, 19 editorial pieces and 128 other document types.

In this study, we combine analysis of co-authorship networks with cognitive mapping related to AHP and, to provide a longitudinal perspective on the evolution of the field, we split the period under investigation (1979-2017) into the three sub-periods 1979-1990, 1991-2001 and 2002-2017.

3. Data and basic statistics

Figure 3 shows that the number of publications related to the topic of AHP has increased over the last 10 years, with the highest numbers - more than 800 published works - in 2013 and 2015. The total sample includes papers published up to January 2017.

We rank journals (Table 1) according to the number of papers published. We provide Total Local Citation Score (TLCS) and Total Global Citation Score (TGCS). The former refers to how many times the journal’s papers included in this collection were cited by other papers in the collection; the latter refers to how many times the papers in the journals included in this collection were cited in the WoS database.

This score is calculated based on the Times Cited score retrieved from the WoS.

** An edge in a network can be defined as an undirected link between two nodes. Thus, links do not show directionality. Co-author networks are an example of undirected networks where the links are reciprocal. This is due to the ‘mutual consent’ characteristic of this type of network (Jackson, 2010).
Among the most active journals, we find *European Journal of Operational Research* with 214 papers, followed by *Expert Systems with Application* with 211 published papers. The third most active journal is *International Journal of Production Research* with 94 papers, followed by *Mathematical and Computer Modelling* with 73 papers and *International Journal of Production Economics* with 72 papers.

Table 1. Top 10 most active journals in AHP

<table>
<thead>
<tr>
<th>Rating</th>
<th>Journals</th>
<th>Amount</th>
<th>TLC</th>
<th>TGCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>European Journal of Operational Research</td>
<td>214</td>
<td>1630</td>
<td>3012</td>
</tr>
<tr>
<td>2</td>
<td>Expert Systems with Application</td>
<td>211</td>
<td>4387</td>
<td>6720</td>
</tr>
<tr>
<td>3</td>
<td>International Journal of Production Research</td>
<td>94</td>
<td>448</td>
<td>1163</td>
</tr>
<tr>
<td>4</td>
<td>Mathematical and Computer Modelling</td>
<td>73</td>
<td>508</td>
<td>819</td>
</tr>
<tr>
<td>5</td>
<td>International Journal of Production Economics</td>
<td>72</td>
<td>1243</td>
<td>2496</td>
</tr>
<tr>
<td>6</td>
<td>International Journal of Advanced Manufacturing Technology</td>
<td>66</td>
<td>260</td>
<td>591</td>
</tr>
<tr>
<td>7</td>
<td>Computers &amp; industrial Engineering</td>
<td>59</td>
<td>305</td>
<td>614</td>
</tr>
<tr>
<td>8</td>
<td>Environmental Earth Science</td>
<td>54</td>
<td>63</td>
<td>93</td>
</tr>
<tr>
<td>9</td>
<td>Journal of Environmental Management</td>
<td>52</td>
<td>401</td>
<td>866</td>
</tr>
<tr>
<td>10</td>
<td>Journal of Operational Research Society</td>
<td>50</td>
<td>462</td>
<td>677</td>
</tr>
</tbody>
</table>
Table 2 presents the 10 most influential papers ranked by TLCS. Note that we provide also the TGCS, which accounts for the impact of the paper within the entire ISI database. For this reason, a paper can be highly cited within the entire ISI collection and slightly less cited in the selected sample, or vice versa.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Title</th>
<th>Journal</th>
<th>TLCS</th>
<th>TGCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Saaty (1990b)</td>
<td>How to make a decision – The analytic hierarchy process</td>
<td>European Journal of Operational Research</td>
<td>642</td>
<td>836</td>
</tr>
<tr>
<td>2 Saaty (1986)</td>
<td>Axiomatic foundation of the analytic hierarchy process</td>
<td>Management Science</td>
<td>257</td>
<td>332</td>
</tr>
<tr>
<td>3 Dyer (1990a)</td>
<td>Remarks on the analytic hierarchy process</td>
<td>Management Science</td>
<td>257</td>
<td>319</td>
</tr>
<tr>
<td>4 Saaty (1994)</td>
<td>How to make a decision – The analytic hierarchy process</td>
<td>Interfaces</td>
<td>201</td>
<td>277</td>
</tr>
<tr>
<td>7 Saaty (1990a)</td>
<td>An exposition of the AHP in reply to the paper remarks on the analytic hierarchy process</td>
<td>Management Science</td>
<td>172</td>
<td>190</td>
</tr>
<tr>
<td>8 Crawford and Williams (1985)</td>
<td>A note on the analysis of subjective judgment matrices</td>
<td>Journal of Mathematical Psychology</td>
<td>165</td>
<td>256</td>
</tr>
</tbody>
</table>
4. Results and evolution of AHP

4.1. First period (1979-1990)

Figure 4 shows the cognitive map for each sub-period identified. AHP relies on the area of mathematics, which is represented by the largest nodes (coloured grey). We observe also that AHP is an attractive application in other disciplines, for example, business and management, followed by economics and to a lesser extent health. As expected, in this first period, we observe that the number of disciplines in which AHP is applied is smaller than in the second and third periods.
The first period includes 86 papers and is characterized by few groups of authors (Figure 5) that actively participate to the initial debate on AHP. The thickness of links and the dimensions of the nodes refer to the weight of the relation measured as citations received and number of papers co-authored. The largest group consists of authors proposing AHP first formulations; this includes to Saaty, Vargas and Harker.
and their co-authors, which show the strongest relationships. They were the first to propose the mathematical AHP formulation and conceptualizations of various aspects, such as the measurement of judgments (Saaty and Vargas, 1987), and proposed the theoretical foundations for the method (Crawford and Williams, 1985; Harker, 1987; Harker and Vargas, 1990, 1987). This first period is characterized not only by the inception of AHP but also by some works highlighting some limitations such as operational difficulties. This applies to Dyer (1990a, 1990b) and Vargas (Harker and Vargas, 1990).

Dyer highlights two of the most controversial issues in the original AHP conceptualization: the phenomenon of rank reversal and the fact that axioms are "flawed".

![Co-authorships network (1979-1990)](image)

There is a small network which includes these three authors and some others. This network refers to co-authors giving examples of the usefulness of AHP for different objectives, such as faculty promotions decisions (Saaty & Ramanujam, 1983) and marketing applications (Wind & Saaty, 1979), or proposing improvements to the classical approach (Millet and Harker, 1990). The main co-authorship networks which emerged during the first period demonstrate that AHP was used in different
contexts to support decision-making in relation to consumers' bank selections (Javalgi et al., 1989), bond ratings (Johnson et al., 1990; Srinivasan and Bolster, 1990) and important medical and health care decisions. We show how applications of AHP to health care and the medical sector increase in the next two periods.


The cognitive map for the period 1991-2001 (Figure 6) shows the growing number of publications and the contamination from other research areas in this second period. Figure 5 shows the increased incidences of co-authorship in the same period. As collaboration among authors increases, the number of papers and the disciplines affected by AHP applications also increase. The second period includes 716 papers. Compared to the first period analysed, we see growing attention from such research areas as mathematical methods, computer science and business and management studies, and its introduction in new research areas. The most active among these are environmental science and technology, followed by mechanical engineering, ecology, social studies and materials science. These are macro areas which include multiple similar disciplines. This period is characterized by studies addressing emerging new concerns, such as attention to environmental issues, which are studied more extensively in the 2002-2017 papers. To better discriminate among topics, we analyse the contributions from the most representative author collaborations.

Mathematical advancements 1

The debate around improvements to the AHP method has been the motivation for several different studies. Saaty and Vargas are among top nodes in both this and the first period. Vargas worked with Arbel to explore new approaches to priority derivation when preferences are expressed as interval judgments (Arbel and Vargas, 1993), and he worked with Saaty to propose application of AHP to support medical decisions (Saaty and Vargas, 1998).
Note that the debate on the drawbacks of the AHP formulation encompasses all three periods. We find several contributions to improve AHP in the other periods. Scholars focus mostly on the inconsistency of PCM.

**Higher education sector**

If we look at the co-authorship networks for the period 1991-2001 (Figure 5), first, we observe a strong relationship between Liberatore, Nydick, Stylianou and Sanchez. Liberatore and Nydick (1999) study application of AHP to the higher education sector and benefits such as improvements to the quality of masters courses and student satisfaction. They also investigate the benefits related to the evaluation of research papers (Liberatore et al., 1992) and improvements to universities’ decision-making processes (Liberatore and Nydick, 1997). The work co-authored by these researchers deals with applications of AHP combined with other methods. Liberatore and Stylianou (1995, 1994, 1993) integrate knowledge-based systems with scoring models, logic tables and AHP for strategic market assessment.

**Health sector**

During this period, AHP was applied widely in the health sector and studied by many including scholars such as Saaty and Vargas (1998) who proposed AHP to determine which tests should be performed given certain symptoms, to scholars with a background in medical studies using AHP as a method to evaluate different medical treatment strategies (Carter et al., 1999; Castro et al., 1996).

**Supply chain management and logistics**

There is a strong co-authorship relation between Korpela, Tuonimen and Lehmusvaara, who apply AHP to supply chain management (Korpela et al., 2001; Korpela and Lehmusvaara, 1999) and logistics (Korpela and Tuominen, 1997, 1996a, 1996b, 1996c). In work on AHP applied to supply chain management, an important contribution is the paper co-authored by Ghodsypour and O’Brien (1998). This deals
with the supplier selection problem involving quantitative and qualitative factors. The advancements proposed in this paper consist of combining AHP with linear programming to consider tangible and intangible factors, which, at the time it was written, were among the main limitations of existing methods.

**Computer science applied to chemical engineering**

The collaboration among Dudukovic, Joseph and Hanratty is an example of the usefulness of applying AHP to the chemical sector for laboratory reactor selection, for instance (Hanratty et al., 1992; Hanratty and Joseph, 1992).

**Energy sector and manufacturing**

Ramanathan authored 13 of the papers in the whole sample, and with Ganesh co-authored 3 papers in the second period. Two of these co-authored papers consist of applications of AHP to the energy sector (Ramanathan and Ganesh, 1994a) and energy allocation problems (Ramanathan and Ganesh, 1995a). Ramanathan also worked, on his own, on the application of AHP to environmental management (Ramanathan, 2001) and proposed a version of AHP, the multiplicative version, to support group decisions in climate change negotiations (Ramanathan, 1998). A third co-authored paper proposes an advancement to AHP, providing, for the first time, a formal evaluation of the group preference aggregation method using an eigenvector based method (EM) to determine intrinsically the weights for group members, using members' subjective opinions (Ramanathan and Ganesh, 1994b). Ganesh worked with Rajendran and Gajpal (1994) to propose AHP to evaluate the criticality of spares in manufacturing organizations.
Among studies applying AHP to ecology we find a network around Kangas, who applies AHP combined with SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis to support forest management planning and decision making (Kangas and Kuusipalo, 1993; Kurttila et al., 2000).
4.3. Third period (2002-2017)

The third period analysed is the largest in terms of number of papers published (7,639). The third cognitive map (Figure 7) shows clearly that this period is dominated by research in mathematical methods, computer science and management studies. With respect to the previous period we observe a growing numbers of contributions in the area of mechanical engineering and environmental science and technology, followed by contributions in geoscience. Two aspects characterize the third period: it is dominated (i) by a fuzzy-based approach; and (ii) by the so-called integrated AHP. The increasing complexity of the knowledge related to more sophisticated methods proposed to improve AHP, and the demand for ways to deal with new complex decision-making problems, led scholars to propose AHP in combination with other multi-criteria decision-making methods. An in-depth analysis of co-authorship networks sheds more light on these aspects.

**Mathematical advancements II**

AHP conceptualization has received considerable attention in recent years and Table 3 summarizes the main issues discussed during the period analysed in relation to advancements and drawbacks to AHP, and highlights the different approaches proposed in different disciplinary domains. Table 3 displays recent papers only, published during the last 5 years.

<table>
<thead>
<tr>
<th>Complex issues</th>
<th>Proposed AHP approach</th>
<th>Authors/research</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Treating judgements qualitatively</strong></td>
<td>DEAHP</td>
<td>Ramanathan and Ramanathan (2010)</td>
</tr>
<tr>
<td><strong>Sorting problems</strong></td>
<td>AHP-Sort</td>
<td>Ishizaka et al. (2012);</td>
</tr>
<tr>
<td></td>
<td>AHP-K-means algorithm - Veto</td>
<td>Lolli et al. (2014)</td>
</tr>
<tr>
<td><strong>PCM</strong></td>
<td>Eigenvector method and mean relative error</td>
<td>Tomashevskii (2015);</td>
</tr>
<tr>
<td></td>
<td>(i) A scheme which yields an estimate for the probability of rank reversal and test the applicability of this scheme under various conditions; (ii) a theoretical</td>
<td>Dede et al. (2016, 2015)</td>
</tr>
</tbody>
</table>
model for estimating the probability of the consequent rank reversal using the multivariate normal cumulative distribution function.

Criteria for determining when the COP is met. This supplement the two conditions formulate dby Bana e Costa and Vansnick (2008).

Kulakowski (2015)

Judgement scales

To model the expert estimation process.

Tsyganok et al. (2012)

Consistency in PCM

Sub-optimal heuristic algorithm.

Siraj et al. (2012a)

A method for calculating the missing elements of an incomplete matrix of PCM, by minimizing a measure of global inconsistency.

Fedrizzi and Giove (2007)

Hadamard product induced bias matrix model

Kou et al. (2014)

Consistency through optimization.

Benitez et al. (2012)

Principal eigenvector approach

Saaty (2013)

New definition of interval multiplicative comparison matrices (IMCMs) incorporating consistency and indeterminacy levels of interval judgments

Li et al. (2016)

New simulation algorithm designed for the AHP

Kazibudzki (2016)

Two step logarithmic goal programming and lexicographic goal programming

Bozorgi-Amiri and Asvadi (2015)

Hesitant AHP

Zhu and Xu (2014)

Indirect judgments

Siraj et al. (2012b)

New method for deriving priority vectors that although based on the eigenvalue method is optimization-based

Grzybowski (2013)

Bayesian Priorization Procedure (BPP) and Systemic Decisionn Making in AHP

Salvador et al. (2014); Altuzarra et al. (2007); Moreno-Jimenez et al. (2016) Altuzarra et al. (2013)

Precise consistency consensus matrix

Escobar et al. (2015); Dong et al. (2015); Grzybowski (2016)

Triangular FAHP to combine a triangular fuzzy weighted power geometric operator the recovery methods and extent analysis method effectively

Dong et al. (2015); Dong and Cooper (2016)

AHP-group decision making model in a local context (a unique criterion) based on the individual selection of the numerical scale and prioritization method and a new individual consistency index

Dong and Cooper (2016)

Two phase algorithm (1) Two-dimensional Sammon’s mapping; (2) consensus convergence model

Srdjevic et al. (2013)

Group Euclidean distance, group minimum violations, and distance between weights for the purpose of evaluation.

Groselj et al. (2015)

New inconsistency index

Grzybowski (2016)

Two new measures, termed congruence and consistency deadlock

Siraj et al. (2015)
To investigate the link between consensus and consistency; and between group decision and consistency, by defining general boundary properties for the inconsistency. To identify axiomatic properties of inconsistency indices.  

Brunelli and Fedrizzi (2013; 2015, 2014);  

**Fuzzy approach to AHP (FAHP)**

Kahraman is one of the most influential authors within this literature stream. He co-authored many papers proposing FHAP. One of the most important proposes hierarchical Fuzzy Axiomatic Design (FAD) which contributed positively to classical FAD by selecting problems through a hierarchical structure (Kahraman and Çebi, 2009). Kahraman and Çebi co-authored 7 of the papers in our sample which deal with this approach to support decisions (Cebi and Kahraman, 2010; Kahraman and Çebi, 2009).

Kahraman and Kaya (2010) proposed a method based on AHP with fuzziness to select among energy alternatives.

Within this area of research, one of the most significant authors is Mikhailov L. He contributed to the debate on deriving priorities from fuzzy pairwise comparison judgments (Mikhailov, 2003). In his conceptualization, assessment of the priorities from pairwise comparison intervals is formulated as an optimization problem, maximizing the decision-maker’s satisfaction with a specific crisp priority vector.

Chan F.T.S. contributed to the development and application of fuzzy AHP to problems such as global supplier development (Chan et al., 2008; Chan and Kumar, 2007).

Che (2010) proposes FSHP to analyse defective supply chain system.
**AHP and Technique for order preference by similarity to ideal solution (TOPSIS)**

Büyüközkan seems to bridge two co-authorship networks. The first (Figure 6 – left side) refers to the fuzzy approach to AHP, the second to the integration of AHP with TOPSIS. Regarding this second application, the authors propose this integrated approach as effective to evaluate e-logistics-based strategic alliance partners, using a fuzzy logic approach (Büyüközkan et al., 2008). Yurdaku and İÇ (2007) propose a Performance Measurement Model (PMM) which can be used to obtain an overall performance score by measuring the success of a manufacturing company’s operational activities. AHP is used to weight (the relative importance of) the dimensions and their sub-components; then weights and performance scores are combined using TOPSIS. This integration has been applied successfully to the complex problem of the vague and imprecise nature of linguistic assessments in the case of facility location selection (Ertuğrul and Karakaşoğlu, 2007). Similarly, Kaya and Karman (2014) apply AHP and TOPSIS to the assessment of intelligent buildings in a fuzzy environment to deal dealing with the uncertainty and imprecision of evaluations, in which the expert's comparisons are represented as fuzzy numbers.

On the right side of the top of Figure 6 there are smaller networks relating the application of AHP and TOPSIS to evaluation of supply chain performance (Eraslan and Atalay, 2014). In Eraslan and Atalay’s paper, the authors first apply the fuzzy extension of AHP and TOPSIS to overcome problems related to linguistic assessments of expert decision-makers, and the propose a ranking to support the decisions.

**AHP and Data Envelopment Analysis (DEAHP)**

This branch of the literature includes a group of works combining insights from DEA with AHP (Ramanathan, 2006), in some cases with fuzzy AHP (FAHP) (Che et al., 2010). Following Ramanathan (2006), Sevkli et al. (2007) apply this hybrid approach to a real industry case and show that DEAHP outperforms AHP method.
for supplier selection. On another hand, this paper has been criticised by Wang et al. (2009) that show the weaknesses of the DEAHP.

The main advantage highlighted by the authors in this field is the chance to use DEA quantitative criteria to evaluate a decision problem, and to apply AHP to collect qualitative data (Ertay et al., 2006). The usefulness of this method has been proved in the case of solving practical design problems (Yang and Kuo, 2003) by combining the subjective opinions of decision makers with objective data on the relevant factors in the case of vendor selection (Zhang et al., 2006). Advances were proposed by Wang and Chin (2009) for priority determination in AHP, that is, to derive the best local priorities from a pairwise comparison matrix or a group of pairwise comparison matrices. Lozano and Villa (2009) propose a new target for DEA approaches, which consists of an interactive multi-objective method where, in each step of the process, the decision maker is asked which inputs and outputs he/she wants to improve, using a method that employs a lexicographic multi-objective approach in which the decision maker specifies a priori a set of priority levels and, using AHP, the relative importance given to the improvements of the inputs and outputs at each priority level. Sueyoshi et al. (2009) propose the combined method to support companies’ internal auditing in order to better identify the most critical businesses units within a corporation. Lin et al. (2011) apply the integrated method to evaluate the economic performance of local governments in China. Focusing on the most recent advancements we observe Anvari et al. (2014) working on the integrated method with desirable and undesirable variables, to assess the relative efficiency of lean manufacturing tools and techniques. The main advantage of the proposed method is the chance to consider desirable and undesirable variables in the production process.

**AHP-DEA and TOPSIS**

The integration of AHP, DEA and TOPSIS characterizes mostly mainly the last five years of research. In this stream of research we find the approach applied in the
automotive sector to supplier selection and evaluation quality (Zeydan et al., 2011). Zeydan et al. (2011) demonstrate the superiority of this approach for making decisions in an automotive company. Similarly, Yousefi and Hadi-Venchez (2010) propose an integrated model based on AHP and TOPSIS to evaluate improvements in Iran's automobile industry and, more specifically, to rank automobile problems. They also propose an AT index, to combine the two rankings obtained, and suggest use of a DEA model to evaluate the efficiency of the alternatives as a basis for comparing three multi-criteria decision-making techniques. Their main finding is that the AT index outperforms AHP and TOPSIS. Recently, stochastic DEA has been shown to be useful in the optimisation of facility layout design problems (Azadeha et al., 2015). The proposed method deals with multiple inputs and stochastic outputs, and uses mathematical programming for optimum layout alternatives. Kumar and Singh (2012) demonstrate that fuzzy AHP and TOPSIS is useful in evaluating the performance of global third party logistics service providers for effective supply chain management.

**AHP and SWOT analysis**

There is a long tradition of studies which employ AHP integrated with SWOT analysis. The most representative collaboration network consists of authors working with Zavadskas and applying these two methods to the construction industry. Zavadskas and co-authors propose a methodology based on AHP and SWOT analysis to determine management strategies in construction enterprises (Zavadskas et al., 2011). In this paper AHP is applied along with expert judgment and a permutation method to deduce feasible alternatives. The first stage of the analysis is aimed at selecting the most preferred strategy; the second relies on SWOT analysis of the current state and the feasible future alternatives for construction enterprises. Recently, Tavana et al. (2016) applied Intuitionistic Fuzzy AHP and SWOT analysis
to evaluate the relative importance of the weights of the criteria and the corresponding sub-criteria in a Reverse Logistic.

_AHP and Quality Function Deployment (QFD)_

The co-authorship networks among Bhattacharya, Sarkar and Mukherjee and among Bhattacharya, Gerarghty and Young, are the most representative of integration of AHP with QFD. In the first case, the authors demonstrate that a combined AHP/QFD model allows determination of whether deployment of robots in industry enhances performance from a requirements perspective (Bhattacharya et al., 2005); in the second, AHP and QFD are combined with cost factor measures (CFM) to rank and select suppliers (Bhattacharya et al., 2010).
AHP and sensitivity analysis

The integration of AHP with sensitivity analysis was very popular in this period. We observe a large co-authorship network (Figure 6) with several authors working on applications of this integrated AHP to different cases. Wu et al. (2007) apply AHP,
sensitivity analysis and a modified version of the Delphi method, to the selection of an optimal location for a hospital. AHP sensitivity analysis has been further integrated with the balance scorecard approach to measure financial services (Wu et al., 2011). Chatzimouratidis and Pivalachi (2009a, 2008a) apply the approach based on AHP and sensitivity analysis to evaluate the technological and economic sustainability of power plants in Greece and show that giving priority to the technology and sustainability criteria favours renewable energy power plants, while prioritizing economic criteria favours nuclear power plants at the expense of four types of fossil fuel power plants. Pilavachi and Chatzimouratidis are also involved in other papers in our dataset dealing with analysis of the energy sector, combining the two methods and also using AHP on its own (Chatzimouratidis and Pilavachi, 2009b, 2008b; Papalexandrou et al., 2008).

5. Discussion – the development of AHP and open problems

*Advantages of using AHP for decision-making*

AHP is a multi-criteria decision making method that is easy to use and flexible. It allows complex problems with multiple and sometimes conflicting criteria to be addressed. It is suited to a number of domains and to different problems since relies on the innate human propensity to conduct comparison. Among the advantages of using AHP for decision making is that it offers the opportunity to consider the different importance of criteria and, consequently, to assign different weights so that some criteria dominate the decision.

Research on AHP developments is organized in two strands. We can trace the ongoing and lively debate on improving the fundamentals of AHP, which proposes various advancements to overcome the shortcomings of existing conceptualizations. The definition of criteria and the calculation of their weights are central to this method used to assess the alternatives and derive weights from PCMs.
The main advantages of the most advanced conceptualizations of AHP are that it allows hierarchical modelling of the problem, and the possibility to make verbal judgements and to confirm consistency (Ishizaka and Labib, 2011). As summarized in Table 3, a number of complex issues have been identified in recent years, and addressed by researchers proposing different AHP approaches. Among these crucial open problems, we would highlight: (i) issues connected to the mathematical theory of PCMs such as the consistency of decision-maker’s judgments (Kazibudzki, 2016; Kazibudzki and Grzybowski, 2013); (ii) the introduction of new prioritization methods to deal with nonreciprocal PCMs (Grzybowski, 2013); (iii) problems related to group decisions (Dong and Cooper, 2016; Grošelj et al., 2015; Srdjevic et al., 2013); problems of consistency indices (Li et al., 2016) and inconsistency indices in PCM (Brunelli and Fedrizzi, 2015, 2014; Siraj et al., 2015). Some of these issues are closely intertwined and lead on from one another.

In the next section, we discuss how recent contributions address these crucial and complex issues, which, in some cases, require more research in the future.

At the same time, AHP has proven effective for dealing with problems in various disciplinary domains, as shown in the previous sections. A number of applications have been proposed to show the usefulness of AHP for decision making and its wide applicability in several sectors, not necessarily related to those in which it originated, such as the health sector (Cheever et al., 2009; Liberatore, 1987; Liberatore et al., 2003) and education (Liberatore and Nydick, 1999). AHP flexibility has been shown to be useful for the supplier selection process (c.f. Chan et al., 2008; Chan and Kumar, 2007; Che, 2010; Labib, 2011; Tsai and Hung, 2009; Vahdani and Zandieh, 2010). Within this line of research, Handfield et al. (2002) show that AHP can help managers in assessing suppliers by taking into account important aspects related to environmental issues; Şen et al. (2008) presents a framework for defining the supplier selection criteria by considering quantitative and qualitative criteria; Levary (2008) applies AHP for ranking and evaluating potential suppliers; Liao and Kao (2010) integrates the Taguchi loss function, AHP and multi-choice goal
programming (MCGP) model to select the best supplier; Labib (2011) demonstrate that AHP addresses the issue of the subjectivity inherent in human assessments and for this reason can be useful in the selection process of the most appropriate supplier.

AHP has been applied also for the analysis of outsourcing (Wang et al., 2010), for supply chain quality management (Kuei et al., 2011; Murata and Katayama, 2013), customer satisfaction (Li et al., 2014; Medjoudj et al., 2015) and manufacturability evaluation (Nagahanumaiah et al., 2007). For example, Ic et al. (2012) propose a component based AHP model to improve the use of technical specifications provided by machining-centre manufacturers; Sarfaraz et al. (2012) apply FAHP to improve customization of an enterprise resource planning system. A number of successful applications have been described in the context of the management of limited resources (Ramanathan and Ganesh, 1995b), computer science applications, the transportation sector (Caliskan, 2006; Ferrari, 2003) strategic planning (Cengiz Toklu et al., 2016) and in the area of logistics (c.f. Chan et al., 2008; Fung et al., 1998; Jain et al., 2006; Lu et al., 2007; Singh et al., 2007; Tiwari, 2010; Zhang et al., 2012).

For example, Agarwal et al. (2002) use the Analytic Network Process (ANP) approach to prioritise the performance improvement of a supply chain; Sarkis and Talluri (2004) show that AHP can help supply chain directors in selecting the most suitable electronic commerce technology media and software for the supply chain; Chen and Wu (2010) combine AHP, ANP and Interpretive Structure Modeling (ISM) as a tool to evaluate the automobile-distributor partnership within the automobile industry.

Falsini et al. (2012) show how AHP combined with DEA and linear programming can effectively support the multi-criteria evaluation of third party logistics service providers. Singh and Singh (2011) develop a three-level AHP-based heuristic approach for solving multi-objective facility layout problem which characterises the manufacturing system. Larrodé et al. (2012) proposed an AHP-based methodological framework to analyse the process of technological differentiation in the automotive
industry. Cannavacioulo et al. (2012) research deals with another aspect related to effective value chain management, that is the assessment of the impact of individual competencies on value creation. Salgado et al. (2012) demonstrate the applicability of AHP to prioritise activities of the new product development process. Bhagwat and Sharma (2013) develop and AHP based model and integrate this with pre-emptive goal programming (PGP) for supply chain performance evaluation. Rezaei and Ortt (2013) analyse the supplier segmentation problem by applying FAHP and prove that this approach can incorporate the uncertainty of human judgment.

Recently, Muerza et al. (2014) propose AHP to deal with a general technological diversification process in the automotive industry. Along a similar line of research, Kengpol and Tuammee (2015) combine AHP with DEA, failure mode and effects analysis, risk contour plot and quantitative risk assessment to assess quantitative risk in multimodal green logistics. Liao and Kao (2014) develop a new method based on the integration of fuzzy extended AHP, QFD and multi-segment programming for designing the logistics system. Adebanjo et al. (2016) show the advantages of using FAHP to understand the perceptions of experts about the prioritisation of healthcare performance measures and their relationship with lean supply chain management. Dey et al. (2016) use AHP to overcome the potential bias when dealing with the heterogeneous degree of expertise in group decision making. The example of a warehouse location selection in a supply chain is used to demonstrate the usefulness of the proposed method.

Razi and Karatas (2016) use AHP to generate rankings and assign weights to different incident types in the context of an Incident Based-Boat Allocation Model used to decide the location of search and rescue boats. In this case, AHP represents the first stage in a three-stage methodology.

*How AHP weaknesses have been addressed over time?*
Since Saaty’s (1990b) conceptualization, the AHP method has attracted attention and also criticism. One of the main criticisms relates to PCMs and their principal right eigenvector ability to generate true rankings (Kazibudzki, 2012). This central argument in AHP development, the PCM and its limitations, have been the topic of lively debate and insightful elaboration. Within this line of research, Tomashevskii (2015) points to the problem of rank reversal, that is, the change in the ranking of alternatives when a non-optimal alternative is introduced, and shows that the problem leading to unreliable EM rankings can be overcome by taking account of the numerical values of the EM errors. In Tomashevskii’s formulation, the decision support tool consists of pairwise comparisons, EM as a data processor, and the formula obtained for EM errors as an indicator.

Closely linked to the previous issue, is another heavily debated problem: AHP relies on decision makers’ pairwise comparisons, and problems can arise if some of these comparisons are not performed well. For example, the decision-maker’s arbitrary judgement can lead to some inconsistency. It is assumed that the reliability of the decision taken depends on the consistency of the decision-maker’s pairwise judgment. This has led to work which provides tools to detect the degree of inconsistency of pairwise comparisons.

Brunelli and Fedrizzi (Brunelli et al., 2013; Brunelli and Fedrizzi, 2015, 2014) have contributed by identifying the axiomatic properties of inconsistency indices. They demonstrate that previous inconsistency indices ignore their general definition and do not provide accurate inconsistency indices.

The most appropriate prioritization method is the open problem. Within this line of research, efforts have been dedicated to proposing new solutions to real-world problems. For example, Zhu and Xu (2014) consider a situation where the decision-maker’s judgments can be considered hesitant, that is, they cannot be aggregated and revised. To overcome this, they developed a hesitant multiplicative programming method as a new prioritization method to derive ratio-scale priorities from hesitant judgments. Another advancement was proposed by Grzybowski
and constitutes a new prioritization method based on the original eigenvalue method, but optimization-based. The new method provides a tool to deal with nonreciprocal PCMs. This represents an advancement on the traditional AHP since the original formulation excludes application to nonreciprocal PCMs. Another prioritization procedure, that is the Bayesian Priorization Procedure (BPP), was proposed by Altuzarra et al. (2007). This enrich the two conventional procedures used, the aggregation individual judgements (AIJ) and the aggregation of individual priorities (AIP). This approach has a number of advantages, as it is flexible, realistic and practical. For example, it does not require intermediate filters for the initial judgements of the actors and allows for the inclusion of the uncertainty associated with the priorities estimation process in the analysis of individual preference structures (c.f. Altuzarra et al., 2013; Moreno-Jimenez et al., 2016; Salvador et al., 2014).

Group decision making processes pose the problem of reaching consensus. Dong and Cooper’s (2016) model overcomes the need for a moderator within a group. Their model provides an automatic feedback mechanism and ensures consistency preservation, democracy within the group and adaptive judgment revision. Srdjevic et al. (2013) deal with another relevant issue in group decision making - consensus building to derive the final group decision. They propose a two-dimensional Sammon’s mapping and a convergence consensus model. This combination overcomes the problems inherent in the heterogeneous composition of groups.

*How integration of AHP with other methods has helped to overcome the shortcoming of individual approaches*

As discussed above, scholars have proposed advancements to the AHP formulation, and the integration of this with other methods to address the weaknesses identified. Some problematic aspects of AHP have been highlighted, such as the use of an exact value to express the decision-maker’s opinion in a comparison of alternatives, while,
in reality, the preference model can be uncertain (Wang and Chen, 2007). A contribution in this direction is Mikhailov’s (2003, 2002) work which uses interval values to express comparisons and develops the Fuzzy Preference Programming (FPP) method to calculate the weight of every level which can then be applied to the AHP method to determine the global priorities, by aggregating the local priorities (Emrouznejad and HO, 2017).

Another limitation of the standalone AHP is the potential arbitrary judgment of the decision-maker, which can lead to inconsistency. To overcome this, combined AHP and QFD have been proposed (Bhattacharya et al., 2010, 2005).

Classical AHP was extended by use of the D-AHP to model various types of uncertainty, and represents an extension of the Dempster–Shafer theory (Deng et al., 2014; Fan et al., 2016). The D-AHP allows determination of the weights of the alternatives and has proved effective to address the supplier selection problem, to represent the decision matrix of pairwise comparisons given by experts (Deng et al., 2014) and to deal with problems of grouting efficiency evaluation (Fan et al., 2016).

In some cases, methods such as TOPSIS and mathematical programming, can compensate for the AHP by considering not only qualitative and quantitative factors, but also information about real-world resources limitations.

The integration of DEA with AHP (DEAHP) was proposed to generate local weights of alternatives from pair-wise comparison judgment matrices. Further advancements in this direction are provided in Wang et al. (2008), which proposes a DEA model with an assurance region for priority derivation in the AHP. The authors demonstrate that this model provides better priority estimates and better decision conclusions than the DEAHP. Kuo et al. (2010) combine FAHP and DEA to develop a new performance evaluation method for improving the supplier selection decision.

The combination of AHP, Analytic Network Process and the balanced scorecard has been proposed to help group decision making for helping managers to improving action plans (Poveda-Bautista et al., 2012). Cabral et al. (2012) apply ANP to select the best lean, agile, resilient and green supply chain management practices. The
integration of AHP, ANP and the failure, mode, effects, and criticality analysis (FMECA) has been presented by Silvestri et al. (2012). This led to the development of the safety improve risk assessment (SIRA) for risk assessment.

Open questions in AHP need further attention. For example, Conditions for Order Preservation (COP) is highlighted by Bana e Costa and Vansnick (2008) and further conceptualized by Kulakowski (2015) who proposed precise criteria for determining when the COP are met. A second and lively debated issue refers to avoiding rank reversal (Dede et al., 2015; Wang and Luo, 2009; Wang and Elhag, 2006); although this is a key issue since AHP was first proposed and has attracted several contributions, further research could shed light on its use in different contexts. Similarly, research on nonreciprocal PCMs and implementation of additional conditions imposed on the priority weights would be helpful.

6. Conclusions and direction for future research

This paper reviewed the growing body of work on AHP published between 1979 and 2017. Given the large number of works in the field (8,441 published pieces), we opted for quantitative analysis, based on scientometric mapping and SNA. Compared to other reviews of AHP, this study deals with both its theoretical bases and its application methods. It also covers and a longer time span than other reviews. We show that AHP has attracted the attention of scholars in various fields because of its ability to provide support to different decision-makers, in areas ranging from medical issues to computer science and environmental studies.

The identification of areas of research expertise highlights several clusters including theoretical AHP developments, fuzzy approaches to decision-making and specific applications of AHP to support supply chain management activities including selecting the most efficient suppliers, environmental planning, and expert systems. We described the evolution of AHP along the three periods selected, both in terms of growing areas of application of AHP method and evolution of the debate on
drawbacks of AHP formulation. This reflects the development of the AHP debate and the contributions to AHP from its theoretical foundations to the proposed integration with other multi-criteria methodologies to support traditional and more contemporary decision-making problems.

This study has some limitations. First, the sample was taken from the ISI WoS, which is recognized as the largest citations-based academic database. However, some published works on AHP might not be included in the WoS. Second, we used keywords to retrieve the papers, which might have led to the inclusion of papers not strictly related to the AHP field. Finally, citation practices involve some 'noise' problems. Citations to some authors and works could be due to opportunistic behaviour and not just to thematic connections. For these reasons, the results should be interpreted bearing in mind these caveats.

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