First measurement of neutrino oscillation parameters using neutrinos and antineutrinos by NOvA

Article  (Published Version)


This version is available from Sussex Research Online: http://sro.sussex.ac.uk/id/eprint/88137/

This document is made available in accordance with publisher policies and may differ from the published version or from the version of record. If you wish to cite this item you are advised to consult the publisher's version. Please see the URL above for details on accessing the published version.

Copyright and reuse:
Sussex Research Online is a digital repository of the research output of the University.

Copyright and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable, the material made available in SRO has been checked for eligibility before being made available.

Copies of full text items generally can be reproduced, displayed or performed and given to third parties in any format or medium for personal research or study, educational, or not-for-profit purposes without prior permission or charge, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

http://sro.sussex.ac.uk
First measurement of neutrino oscillation parameters using neutrinos and antineutrinos by NOvA


(NOvA Collaboration)

1Argonne National Laboratory, Argonne, Illinois 60439, USA

2Universidad del Atlantico, Km. 7 antigua via a Puerto Colombia, Barranquilla, Colombia

3Department of Physics, Institute of Science, Banaras Hindu University, Varanasi, 221 005, India

4California Institute of Technology, Pasadena, California 91125, USA

5Charles University, Faculty of Mathematics and Physics, Institute of Particle and Nuclear Physics, Prague 116 36, Czech Republic

6Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, USA

7Department of Physics, Cochin University of Science and Technology, Kochi 682 022, India

8Department of Physics, Colorado State University, Fort Collins, Colorado 80523-1875, USA

9Czech Technical University in Prague, Brehova 7, 115 19 Prague 1, Czech Republic

10University of Dallas, 1845 E Northgate Drive, Irving, Texas 75062 USA

11Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India

12Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

13Instituto de Física, Universidade Federal de Goiás, Goiânia, Goiás 74690-900, Brazil

14Department of Physics, IIT Guwahati, Guwahati 781 039, India

15Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

16Department of Physics, University of Houston, Houston, Texas 77204, USA

17School of Physics, University of Hyderabad, Hyderabad 500 046, India

18Department of Physics, IIT Hyderabad, Hyderabad 502 205, India

19Indiana University, Bloomington, Indiana 47405, USA

20Institute of Computer Science, The Czech Academy of Sciences, 182 07 Prague, Czech Republic

21Department of Physics, Illinois Institute of Technology, Chicago, Illinois 60616, USA

22Institute for Nuclear Research of Russia, Academy of Sciences 7a, 60th October Anniversary prospect, Moscow 117312, Russia

23Institute of Physics, The Czech Academy of Sciences, 182 21 Prague, Czech Republic

Published by the American Physical Society
The NOvA experiment has seen a 4.4σ signal of $\bar{\nu}_e$ appearance in a 2 GeV $\bar{\nu}_e$ beam at a distance of 810 km. Using $12.33 \times 10^{20}$ protons on target delivered to the Fermilab NuMI neutrino beamline, the experiment recorded 27 $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ candidates with a background of 10.3 and 102 $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ candidates. This new antineutrino data are combined with neutrino data to measure the parameters $|\Delta m^2_{32}| = 2.48^{+0.11}_{-0.06} \times 10^{-3} \text{ eV}^2/c^4$ and $\sin^2 \theta_{23}$ in the ranges from $(0.53-0.60)$ and $(0.45-0.48)$ in the normal neutrino mass hierarchy. The data exclude most values near $\delta_{CP} = \pi/2$ for the inverted mass hierarchy by more than $3\sigma$ and favor the normal neutrino mass hierarchy by $1.9\sigma$ and $\theta_{23}$ values in the upper octant by $1.6\sigma$. DOI: 10.1103/PhysRevLett.123.151803

The observations of neutrino oscillations by many experiments [1–9] are well described by the mixing of three neutrino mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$ with the flavor eigenstates $\nu_e$, $\nu_\mu$, and $\nu_\tau$. The mixing is parametrized by a unitary matrix which depends on three angles and a phase, $\delta_{CP}$, that may break charge-parity ($CP$) symmetry. The oscillation frequencies are proportional to the neutrino mass splittings, $|\Delta m^2_{21}| \equiv m_2^2 - m_1^2 \approx 7.5 \times 10^{-5} \text{ eV}^2/c^4$ for $\Delta m^2_{32} \approx 2.5 \times 10^{-3} \text{ eV}^2/c^4$, and the angles are known to be large: $\theta_{12} \approx 34^\circ$, $\theta_{13} \approx 8^\circ$, $\theta_{23} \approx 45^\circ$ [10]; $\delta_{CP}$, however, is largely unknown.

Within this framework, several questions remain unanswered. The angle $\theta_{23}$ produces nearly maximal mixing but has large uncertainties. If maximal, it would introduce an unexplained $\mu - \tau$ symmetry; should it differ from $45^\circ$, its octant would determine whether $\nu_3$ or $\nu_\mu$ couples more strongly to $\nu_2$. Furthermore, while it is known that the two independent mass splittings differ by a factor of 30, the sign of the larger splitting is unknown. The $\nu_1$ and $\nu_2$ states that contribute most to the $\nu_e$ state could be lighter ["normal hierarchy" (NH)] or heavier ["inverted hierarchy" (IH)] than the $\nu_3$ state. This question has important implications for models of neutrino mass [11–15] and for the study of the Dirac vs Majorana nature of the neutrino [16,17]. Additionally, neutrino mixing may be a source of $CP$ violation if $\sin \delta_{CP}$ is nonzero.

These questions can be addressed by the measurement of $\nu_\mu \rightarrow \nu_\mu$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$, $\nu_\mu \rightarrow \nu_e$, and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, oscillations in matter over baselines $L$ of order (100–1000) km, with neutrino energies $E[\text{GeV}] \approx L[\text{km}] \times |\Delta m^2_{32}[\text{eV}^2/c^4]|$. Several long-baseline experiments have reported observations of $\nu_\mu \rightarrow \nu_\mu$ [18–21], $\nu_\mu \rightarrow \nu_e$ [19–21], and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ [19,20], but a statistically significant observation of...
$\bar{\nu}_\mu \to \bar{\nu}_e$ has not previously been made. This report combines the first antineutrino measurements by NOvA with the neutrino data reported in Ref. [21] in a reoptimized analysis yielding a new determination of the oscillation parameters $|\Delta m^2_{32}|$, $\sin^2 \theta_{23}$, $\delta_{CP}$, and the neutrino mass hierarchy.

The NOvA experiment measures oscillations by comparing the energy spectra of neutrino interactions in two detectors placed in the Fermilab NuMI beam [22] at distances of 1 km [near detector (ND)] and 810 km [far detector (FD)] from the production target. The 14 kton FD and 290 ton ND are sampling calorimeters constructed from PVC and liquid scintillator [23,24]. The ND is located 100 m underground. The FD operates on the surface with modest shielding resulting in 130 kHz of cosmic-ray activity. The detectors are located 14.6 mrad off the beam axis where the neutrino energy spectrum peaks at 2 GeV. Magnetic focusing horns in the beamline charge-select neutrino parents giving 96% (83%) pure $\nu_\mu$ ($\bar{\nu}_\mu$) event samples between 1 and 5 GeV. Most contamination is “wrong sign” ($\bar{\nu}$ in the $\nu$ beam, or vice versa) with $<1\% \nu_e + \bar{\nu}_e$.

This Letter reports data from an antineutrino run spanning from June 29, 2016, to February 26, 2019, with an exposure of $12.33 \times 10^{20}$ protons on target (POT) delivered during 317.0 s of beam-on time, combined with the previously reported [21] neutrino beam exposure of $8.85 \times 10^{20}$ POT and 438.2 s. During these periods, the proton source achieved a peak hourly averaged power of 742 kW.

The flux of neutrinos delivered to the detectors is calculated using a simulation of the production and transport of particles through the beamline components [22,25] reweighted to incorporate external measurements [26–45]. Neutrino interactions in the detector are simulated using GENIE [46] tuned to improve agreement with external measurements and ND data, reducing uncertainties in the extrapolation of uncertainties in the ND to the FD. As in Ref. [21], we set $M_A$ in the quasielastic dipole form factor to 1.04 GeV/$c^2$ [47] and use corrections to the chargeder current (CC) quasielastic cross section derived from the random phase approximation [48,49]. In this analysis, we also apply this effect to baryon resonances as a placeholder for the unknown nuclear effect that suppresses rates at a low three-momentum transfer in our and other measurements [50–53]. Additionally, we increase the rate of deep-inelastic scattering with hadronic mass $W > 1.7$ GeV/$c^2$ by 10% to match our observed counts of short track-length $\nu_\mu$ CC events. We model multinucleon ejection interactions following Ref. [54] and adjust the rates in bins of energy transfer, $q_0$, and three-momentum transfer, $|\vec{q}|$, for $\nu_\mu$ and $\bar{\nu}_\mu$ separately to maximize agreement in the ND. The calculation of the $\nu_e$ and $\bar{\nu}_e$ rates uses these same models.

The energy depositions of final-state particles are simulated with GEANT4 [25] and input to a custom simulation of the detector response [55]. The absolute energy scale of the detectors is calibrated to within $\pm5\%$ using the minimum ionizing portion of cosmic-ray muon tracks that stop in the detectors.

Cells with activity above threshold (hits) are grouped based on their proximity in space and time to produce candidate neutrino events. Events are assigned a vertex, and clusters are formed from hits likely to be associated with particles produced there [56]. These clusters are categorized as electromagnetic or hadronic in origin using a convolutional neural network (CNN) [57]. Hits forming tracks are identified as muons by combining information on the track length, $dE/dx$, vertex activity, and scattering into a single particle identification (PID) score [58]. The same reconstruction algorithms are applied to events from data and simulation in both detectors.

The $\nu_\mu$ and $\bar{\nu}_\mu$ candidates are required to have a vertex inside the fiducial volume and no evidence of particles exiting the detector. Following Ref. [21], the $\nu_e$ and $\bar{\nu}_e$ candidates are divided into a “core” sample which satisfies these containment requirements, and a “peripheral” sample which loosens these requirements for the most signal-like event topologies. A second CNN [59] serves as the primary PID, classifying events as $\nu_e$ CC, $\nu_\mu$ CC, $\nu_\tau$ CC, neutral current (NC), or cosmic ray. The network is trained on simulated neutrino and antineutrino beam conditions and cosmic-ray data. It has an improved architecture and higher rate of cosmic ray rejection over the previous network [21]. Events identified as $\nu_\mu$ CC are required to contain at least one track classified as a muon.

Several requirements target cosmic-ray backgrounds. For the $\nu_\mu$ CC sample, a boosted decision tree (BDT) algorithm based on vertex position and muonlike track properties is used. Events in the core $\nu_e$ sample not aligned with the beam direction and that are near the top of the detector are rejected. Events are removed whose topology is consistent with detached bremsstrahlung showers from cosmic tracks and with photons entering from the detector’s north side where there is less shielding. Cosmic-ray backgrounds in the $\nu_e$ peripheral sample are removed with a BDT based on position and direction information.

The selection of $\nu_\mu$ and $\bar{\nu}_\mu$ CC events is 31.2% (33.9%) efficient for true interactions in the fiducial volume, resulting in 98.6% (98.8%) pure samples at the FD during neutrino (antineutrino) beam operation. Both $\nu_\mu$ and $\bar{\nu}_\mu$ are counted as a signal for the disappearance measurements. Selections against exiting particle tracks are the largest source of inefficiency. The efficiency for selecting signal $\nu_\mu$ CC ($\bar{\nu}_\mu$ CC) events is 62% (67%). Purities for the signal $\nu_\mu$ ($\bar{\nu}_\mu$) samples fall in the range 57–78% (55–77%) depending on the impact of oscillations on the signal and wrong-sign background levels. These efficiencies and purities differ from those quoted in Ref. [21] due to a reoptimization of the selection algorithms [60]. The wrong-sign component of the selected $\nu_\mu$ sample in the ND is calculated to be
2.8 ± 0.3% and 10.6 ± 1.1% for the neutrino and antineutrino beams. These fractions are consistent with a data-driven estimate based on the rate of νµ CC and NC interactions with associated detector activity indicative of neutron capture.

The incident neutrino energy is reconstructed from the measured energies of the final-state lepton and recoil hadronic system. The lepton energy is estimated from track length for muon candidates and from calorimetric energy for electron candidates. The hadronic energy is estimated from the sum of the calibrated hits not associated with the primary lepton. The neutrino energy resolution at the FD is 9.1% (8.1%) for νµ CC (ν̄µ CC) events and 10.7% (8.8%) for νe CC (ν̄e CC) events. We analyze the νµ and ν̄µ events in quartiles of hadronic energy fraction as events with less hadronic energy have the best energy resolution and lowest backgrounds [21].

The energy spectra of the selected νµ CC and νe CC interactions in the ND during neutrino and antineutrino beam operations are shown in Fig. 1. The selected ND νe sample consists entirely of background sources for the νe appearance measurement, predominantly the intrinsic beam νe component, along with misidentified νµ CC and NC interactions. We analyze the νe candidate energy spectra in two bins of νe PID (“low” and “high”) to isolate a highly pure sample of νµ → νe and ν̄µ → ν̄e at the FD. In the ND, the high-PID sample is dominated by intrinsic beam νe. A third bin containing the “peripheral” events is added for the FD.

The νµ and ν̄µ signal spectra at the FD are predicted for the neutrino and antineutrino beams separately using the observed spectra of νµ candidate events in the ND. The true neutrino energy spectrum at the ND is estimated using the measured event rates in bins of reconstructed energy and the energy distributions of simulated events found to populate those bins. This true spectrum is corrected for differences in flux and acceptance between the ND and FD, as well as differences in the νµ and νe cross sections;
osculations are applied to yield predictions for the true $\nu_\mu$ and $\bar{\nu}_e$ spectra at the FD. These spectra are then transformed into reconstructed energy using the underlying energy distributions from simulated neutrino interactions in the FD.

The predicted background spectra at the FD are also primarily data driven. Data collected out of time with the NuMI beam provide a measurement of the rate of cosmic-ray backgrounds in the $\nu_\mu$ and $\nu_e$ samples. Neutrino backgrounds calculated to populate the FD $\nu_e$ spectra are corrected based on the reconstructed $\nu_e$ candidates at the ND. The procedure from Ref. [21] is followed to determine corrections for each background component in the neutrino-mode beam, while for the antineutrino-mode beam a single scale factor is used. The remaining backgrounds, which include any misidentified neutrino events in the $\nu_\mu$ samples and misidentified $\nu_e$ interactions in the $\nu_e$ samples, make up less than 2% of the FD candidates and are taken directly from simulation.

To evaluate the impact of systematic uncertainties we recompute the extrapolation from the ND to the FD varying the parameters used to model the neutrino fluxes, neutrino cross sections, and detector response. The procedure accounts for changes in the composition of the $\nu_e$ background, and for impacts on the transformation to and from true and reconstructed energies due to variations in the model parameters. We parametrize each systematic variation and compute its effect in each analysis bin. These parameters are included in the oscillation fit, constrained within their estimated uncertainties by penalty terms in the likelihood function.

The oscillation parameters that best fit the FD data are determined through minimization of a Poisson negative log-likelihood, $-2 \ln L$, varying three unconstrained parameters, $\Delta m^2$, $\sin^2 \theta_{23}$, and $\delta_{CP}$, as well as 53 constrained parameters covering the other oscillation parameters and the sources of systematic uncertainty summarized in Tables I and II. The two-detector design and extrapolation procedure significantly reduce the effect of the $\approx 10\text{--}-20\% \ a \ priori$ uncertainties on the beam flux and cross sections. The principal remaining uncertainties are neutrino cross sections, the energy scale calibration, the detector response to neutrinos, and differences between the ND and FD that cannot be corrected by extrapolation.

The selection criteria and techniques used in the analysis were developed on simulated data prior to inspection of the FD data distributions. Figure 1 shows the energy spectra of the $\nu_\mu$ CC, $\bar{\nu}_\mu$ CC, $\nu_e$ CC, and $\bar{\nu}_e$ CC candidates recorded at the FD overlaid on their oscillated best-fit expectations. Table III summarizes the total event counts and estimated compositions of the selected samples. We recorded 102 $\bar{\nu}_\mu$ TABLE III. Event counts at the FD, both observed and predicted at the best fit point (see Table IV).

<table>
<thead>
<tr>
<th>Neutrino beam</th>
<th>Antineutrino beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\mu$ CC</td>
<td>$\bar{\nu}_\mu$ CC</td>
</tr>
<tr>
<td>$\bar{\nu}_\mu$</td>
<td>112.5</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>7.2</td>
</tr>
<tr>
<td>$\nu_\mu \rightarrow \nu_e$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$</td>
<td>0.0</td>
</tr>
<tr>
<td>Beam $\nu_e + \bar{\nu}_e$</td>
<td>0.0</td>
</tr>
<tr>
<td>NC</td>
<td>1.3</td>
</tr>
<tr>
<td>Cosmic</td>
<td>2.1</td>
</tr>
<tr>
<td>Others</td>
<td>0.7</td>
</tr>
<tr>
<td>Signal</td>
<td>$^{+0.2}_{-0.3}$</td>
</tr>
<tr>
<td>Background</td>
<td>$^{+0.5}_{-0.4}$</td>
</tr>
<tr>
<td>Best fit</td>
<td>119.7</td>
</tr>
<tr>
<td>Observed</td>
<td>113</td>
</tr>
</tbody>
</table>

TABLE IV. Summary of oscillation parameters. The top three are inputs to this analysis [10], while the rest are the best fits for different choices of the mass hierarchy (NH, IH) and $\theta_{23}$ octant (UO, LO), along with the significance (in units of $\sigma$) at which those combinations are disfavored. In addition to the region indicated, for NH, LO a small range of $\sin^2 \theta_{23}$ 0.45--0.48 is allowed at 1$\sigma$ [61].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NH, UO</th>
<th>NH, LO</th>
<th>IH, UO</th>
<th>IH, LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{21}/(10^{-5} \text{ eV}^2/c^4)$</td>
<td>7.53</td>
<td>$^{+0.18}_{-0.18}$</td>
<td>$^{+0.18}_{-0.18}$</td>
<td>$^{+0.18}_{-0.18}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$^{+0.04}_{-0.04}$</td>
<td>$^{+0.04}_{-0.04}$</td>
<td>$^{+0.04}_{-0.04}$</td>
<td>$^{+0.04}_{-0.04}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$^{+0.01}_{-0.01}$</td>
<td>$^{+0.01}_{-0.01}$</td>
<td>$^{+0.01}_{-0.01}$</td>
<td>$^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$\delta_{CP}/\pi$</td>
<td>$^{+1.3}_{-1.3}$</td>
<td>$^{+1.3}_{-1.3}$</td>
<td>$^{+1.3}_{-1.3}$</td>
<td>$^{+1.3}_{-1.3}$</td>
</tr>
</tbody>
</table>

$^{+1.6\sigma}_{-1.6\sigma}$ | $^{+1.8\sigma}_{-1.8\sigma}$ | $^{+2.0\sigma}_{-2.0\sigma}$ | $^{+2.0\sigma}_{-2.0\sigma}$ |
FIG. 2. The 90% confidence level region for $\Delta m^2_{23}$ and $\sin^2 \theta_{23}$, with best-fit point shown as a black marker [61], overlaid on contours from other experiments [19,20,64,65].

candidate events at the FD, reflecting a significant suppression from the unoscillated expectation of 476. We find 27 $\bar{\nu}_\mu \to \bar{\nu}_e$ candidate events with an estimated background of $10.3^{+0.6}_{-0.5}$, a 4.4$\sigma$ excess over the predicted background. This observation is the first evidence of $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam over a long baseline. These new antineutrino data are analyzed together with 113 $\nu_\mu$ and 58 $\nu_\mu \to \nu_e$ candidates from the previous data set.

Table IV shows the overall best-fit parameters and the best fits for each choice of $\theta_{23}$ octant and hierarchy. The best-fit point is found for the normal hierarchy with $\theta_{23}$ in the upper octant where $-2 \ln L = 157.1$ for 175 degrees of freedom (goodness of fit $p = 0.91$ from simulated experiments). The measured values of $\theta_{23}$ and $\Delta m^2_{23}$ are consistent with the previous NOvA measurement [21] that used only neutrino data, and are consistent with maximal mixing within 1.2$\sigma$.

Confidence intervals for the oscillation parameters are determined using the unified approach [62,63]. Figure 2 compares the 90% confidence level contours in $\Delta m^2_{23}$ and $\sin^2 \theta_{23}$ with those of other experiments [19,20,64,65]. Figure 3 shows the allowed regions in $\sin^2 \theta_{23}$ and $\delta_{CP}$. These results exclude $\delta_{CP}$ values in the inverted mass hierarchy from $-0.04$ to $0.97\pi$ in the lower $\theta_{23}$ octant and $0.04$ to $0.91\pi$ in the upper octant by more than 3$\sigma$. The data prefer the normal hierarchy with a significance of 1.9$\sigma$ ($p = 0.057$, CL$_{ex} = 0.091$ [66]) and the upper $\theta_{23}$ octant with a significance of 1.6$\sigma$ ($p = 0.11$) [67].

We are grateful to Stephen Parke (FNAL) for useful discussions. This document was prepared by the NOvA collaboration using the resources of the Fermi National Accelerator Laboratory (Fermilab), a U.S. Department of Energy, Office of Science, HEP User Facility. Fermilab is managed by Fermi Research Alliance, LLC (FRA), acting under Contract No. DE-AC02-07CH11359. This work was supported by the U.S. Department of Energy; the U.S. National Science Foundation; the Department of Science and Technology, India; the European Research Council; the MSMT CR, GA UK, Czech Republic; the RAS, RFBR, RMES, RSF, and BASIS Foundation, Russia; CNPq and FAPEG, Brazil; STFC, and the Royal Society, United Kingdom; and the state and University of Minnesota. This work used resources of the National Energy Research Scientific Computing Center (NERSC), a U.S. Department of Energy Office of Science User Facility operated under Contract No. DE-AC02-05CH11231. We are grateful for the contributions of the staffs of the University of Minnesota at the Ash River Laboratory and of Fermilab.

FIG. 3. The 1$\sigma$, 2$\sigma$, and 3$\sigma$ contours in $\sin^2 \theta_{23}$ vs $\delta_{CP}$ in the normal hierarchy (NH, top panel) and inverted hierarchy (IH, bottom panel) [61]. The best-fit point is shown by a black marker.


[38] K. Abe et al. (T2K Collaboration), Phys. Rev. D 87, 012001 (2013); 87, 019902(A) (2013).
[60] T. Blackburn, Measurement of $\Delta m^2_{31}$ and $\sin^2 \theta_{23}$ using muon neutrino and antineutrino beams in the NOvA experiment, Ph.D. thesis, Sussex University, 2019.
See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.123.151803 for the muon neutrino distributions in each quartile of hadronic energy fraction, for the profiles of significance surfaces on the $\Delta m^2_{32}$, $\sin^2 \theta_{23}$, $\delta_{CP}$ axes as well as the surfaces computed for the inverted hierarchy.


This preference for the upper octant is stronger than the significance of the best fit in the lower octant because binary questions, like the octant and hierarchy, have fewer effective degrees of freedom, analogous to profiling a two-dimensional to one-dimensional confidence interval.