

SUPPLEMENTARY - NOTE 1

Hardware

We use a camera, Hercules HD Twist (max. resolution 1024x768px) for our projection mapping, and a pico projector, iCODIS LED mini projector (dimension 149x79x18mm, max. resolution 854x480px) for graphical projection.

Software

We wrote computer programs to computationally control our transducers phase and the mist, calibrate camera and projector, and produce image plane for our laminar mist. The programs was written in C++ language. The graphics is handled by OpenGL (version 4.6.0) and the calibration process is handled by OpenCV (version 2.4.13.6).

To perform the camera-projector calibration, we refer to a work by [3, 2, 1], which mainly incorporate pinhole model in their calibration algorithm. First, we form a general equation of the pinhole model (see Equation (1)) where m' is image plane coordinates, A is a camera intrinsic parameters, $[R|t]$ is camera extrinsic parameters and M' is the world coordinates. The equation is further expressed as Equation (2). In the equation, (X, Y, Z) are the 3D point coordinates in the world, (u, v) are image plane coordinate in pixels, matrix, (C_x, C_y) is a central point of the image plane and f_x, f_y are the focal lengths in pixel units. Then, we captured 10 images of 10x7 chessboard and 10 projected image of that chessboard (640x480px). We summarised a complete procedure for the calibration¹ system in two main steps: camera calibration, and projector calibration. In these steps, we denote subscript 'c' for camera parameters and subscript 'p' for projector parameters. It is worth to note here that we used the same equations (Equation (1) and (2)) for our projector calibration, but with the idea that projector is an inverse camera.

$$m' = A[R|t]M' \quad (1)$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 1 \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (2)$$

Camera Calibration

1. Capture a printed chessboard and detect corner points of the chessboard and define them as the image point coordinates, m'_c .
2. Define the known point coordinates of the printed chessboard as the world coordinates, M'_c .
3. Using the camera intrinsic matrix A_c and information in (1) and (2), compute the camera extrinsic matrix $[R|t]_c$.

Projector Calibration

1. Place printed chessboard in the camera view, capture its photo with the same camera that we have calibrated.

¹We refer to calibration OpenCV code at URL: https://docs.opencv.org/2.4/modules/calib3d/doc/camera_calibration_and_3d_reconstruction.html

2. Identify the chessboard corners and using equation(1), compute the extrinsic parameters.
3. Using a projector, project chessboard on the same position and orientation as chessboard in (1).
4. Using the camera intrinsic in camera calibration (1) and extrinsic value in (2), compute the M'_p .
5. Define the coordinates of known image plane chessboard that has been used in (3) as m'_p .
6. With the pieces informations above, compute the projector intrinsic parameters A_p .
7. By using the informations above and scale factor, the image plane on the projector can then be used to determine the image plane needed for the real world point coordinates, in our case the laminar mist.

After we calibrated the camera and projector, we mapped the graphical content with the laminar mist. We simplify the process of our reconfigurable mid-air display system with a flowchart in figure 1. The flowchart consists of three main modules: user, laminar mist reconfiguration, and projection mapping. The user will input any type of transformation for laminar mist control point such as to define a movement of a butterfly in a circle. The information then will be sent to both mist reconfiguration and projection mapping. Mist reconfiguration module receives the changes of the coordinate in meter and computes the phase delays required for the PAT (0 to 2π). Meanwhile, the projection mapping will receives the data in meter and transform it into pixels coordinate by using equation (1) and (2).

REFERENCES

- [1] I. Din, H. Anwar, I. Syed, H. Zafar, and L. Hasan. 2014. Projector Calibration for Pattern Projection Systems. *Journal of Applied Research and Technology* 12, 1 (2014), 80 – 86. DOI:[http://dx.doi.org/https://doi.org/10.1016/S1665-6423\(14\)71608-6](http://dx.doi.org/https://doi.org/10.1016/S1665-6423(14)71608-6)
- [2] D. Moreno and G. Taubin. 2012. Simple, Accurate, and Robust Projector-Camera Calibration. In *2012 Second International Conference on 3D Imaging, Modeling, Processing, Visualization Transmission*. 464–471. DOI:<http://dx.doi.org/10.1109/3DIMPVT.2012.77>
- [3] Z. Zhang. 2000. A flexible new technique for camera calibration. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 22, 11 (Nov 2000), 1330–1334. DOI:<http://dx.doi.org/10.1109/34.888718>

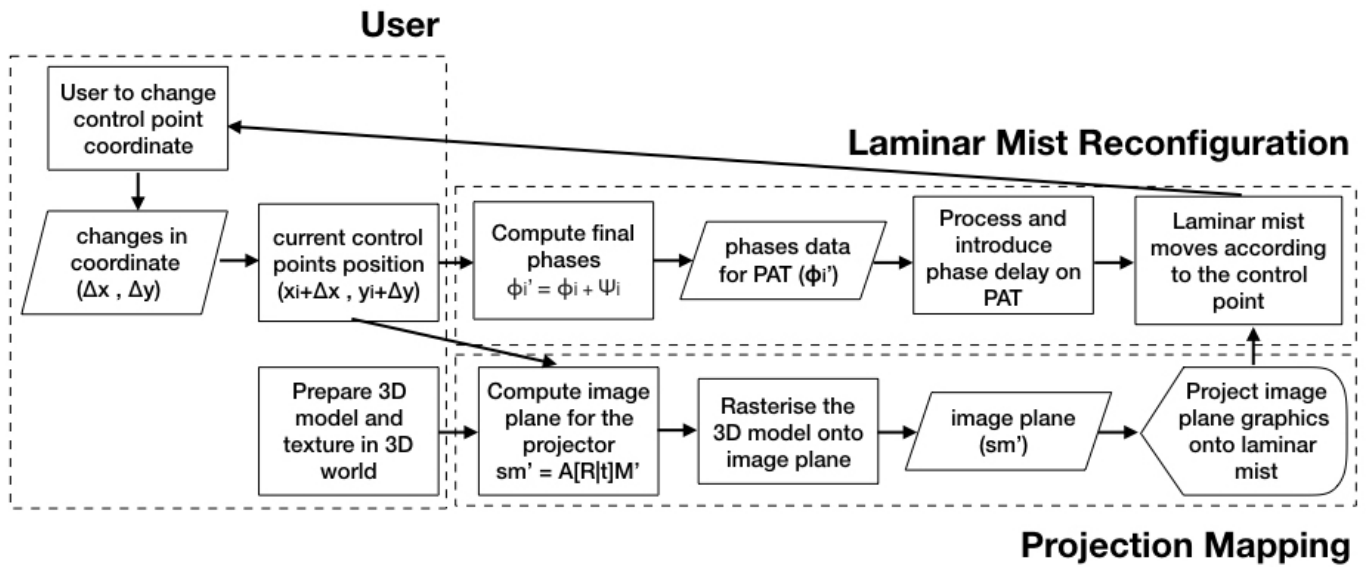


Figure 1: Flowchart for SonicSpray contains three main modules which are user, laminar mist reconfiguration and projection mapping.