Relativizing identity

Article (Accepted Version)

Molto, Daniel (2019) Relativizing identity. Thought: A Journal of Philosophy, 8 (4). pp. 260-269. ISSN 2161-2234

Abstract: In this paper, I defend Peter Geach's theory of Relative Identity against the charge that it cannot make sense of basic semantic notions.

Discipline: Philosophy of Logic

Relativizing Identity

A minority view holds that numerical identity is relative. It is often claimed by its opponents that this theory of Relative Identity is incoherent because it cannot make sense of basic semantic notions like singular reference, quantification, or a domain of discourse (Quine 1964, Alston and Bennett 1984, Dummett 1996, Hawthorne 2003, Le Poidevin 2009, Bueno 2014, 2015). In this paper, I defend relative identity against the semantic objections.

Section I Relative Identity

The version of Relative Identity at which the semantic arguments are aimed, and which I shall defend, is what I will call 'Strong Relative Identity'. The characteristic components of this view of identity are:

> **Non-existence of Absolute Identity** (henceforth, NAI): There exists no relation which is universally reflexive, symmetric, transitive and satisfies the principle of the Indiscernability of Identicals (a relation fitting this description will henceforth be called 'absolute identity').
>
> **The Sortal Relativity of Identity**: All relations of numerical identity involve the specification, implicit or explicit, of some sortal term.
>
> **The Cross-cutting of Identity**: It is possible, for some x and y, for x to be the same F as y but a different G, where 'F' and 'G' are being used as sortal terms.

Strong Relative Identity must be distinguished from views on which cross-cutting relations of identity are possible, but there exists an absolute identity relation, for example Richard Griffin (1977), Eddy Zemach (1974, 1982), and Pawel Garbacz (2002). The Non-existence of Absolute Identity (henceforth, NAI) is the thesis which is targeted by the semantic objections and the only one with which I will be concerned in this paper. I think that if NAI can be defended the other theses become highly plausible. In this paper, I maintain that NAI is coherent, and that the semantic arguments presented by Quine, Michael Dummett, Otavio Bueno, and others fail to show otherwise.

Section II The Semantic Arguments

The various semantic arguments against NAI may be reduced to three:

> **Objection 1**: An interpreted language requires a domain and a domain must be composed of discrete entities. This requires absolute identity (Bueno 2014, see also *Internationales Zentrum für Philosophie NRW* 2017).

Objection 2: Any interpretation of a singular term must be distinct absolutely from any other possible interpretation. This requires absolute identity (Alston and Bennett 1984, van Inwagen 1988: 259-260, Le Poidevin 2009: 181-182).

Objection 3: Assigning values to variables depends on any given assignment being distinct absolutely from any other possible assignment. This requires absolute identity (Quine 1964, Dummett 1996, Hawthorne 2003: 119-120, Bueno 2014, 2015).

In this paper, I will assume that if Objection 2 can be answered then so can Objection 3, and with this assumption in mind, I will consider only languages with singular terms and no quantifiers for the sake of brevity. However, before I can do this, I need to make some further preliminary remarks on what I would take to be a successful defence of the coherence of Strong Relative Identity. As I say, I will be concerned only with NAI, on the assumption that if this is coherent, so is the theory as a whole. NAI is intended by its central defender, Peter Geach, to hold true for all languages. This is because Geach (1972) thinks that absolute identity is incoherent. Geach's own argument for this position has been universally rejected, and I will not defend it. In contrast to Geach, I maintain only that NAI is true of some languages. More particularly, I maintain that there are languages, interpreted except for a single dyadic predicate and where none of the already-interpreted predicates express identity, which are such that interpreting the uninterpreted predicate as an absolute identity relation leads to incoherence. In other words, I maintain that there are languages which *cannot* involve predicates expressing absolute identity. I will defend this claim by arguing that absolute identity does not play the crucial metalinguistic role that Objections 1-3 imply it does. In trying to demonstrate this, I will need to appeal to the coherence of a non-classical semantics, which I will sketch in Section IV. First, though, we must look a bit more closely at the nature of identity relations.

Section III Characterizations of Identity

Let us begin with a point made by Michael Dummett. Dummett argues (1996: 312-316) that any way we might have of specifying a domain of discourse for a particular interpretation will, in so doing, specify a maximally-fine-grained cutting-up of that domain. In other words, for any domain, although the contents might be described in various overlapping ways, there must be some ultimately-precise description which exhausts all the contents of the domain: even if we allow ambiguous referring expressions, these must in theory be capable of having their reference(s) more finely distinguished up to a maximal point at which ambiguity is no longer possible.[1]

Geach would disagree with Dummett here, but I intend to grant this point to Dummett. Given this assumption and assuming the language which we are currently talking is sufficiently precise to identify ultimately-fine-grained entities, we may characterize a domain as a set whose elements are all the ultimately-fine-grained objects that a language is equipped to refer to or quantify over

[1]Dummett's point is a semantic one, not a metaphysical one. This is not intended to be a statement of mereological atomism.

under a particular interpretation. Nevertheless, this assumption is compatible with the truth of the version of NAI that I am interested in, or so I shall argue.

Turning to characterizations of the absolute identity relation, we may note two traditional versions. One versions characterizes identity as the relation that satisfies the following two axioms:

(REF): $\forall x\ x = x$

and

(IND): $\forall x \forall y\ x = y \rightarrow \forall F(F(x) \leftrightarrow F(y))$.

Another characterizes it set-theoretically as the relation with the extension:

$<< X, X >: X \in D >$

These characterizations are usually thought to pin down one and the same relation. A relation with four central features: universal reflexivity, satisfaction of the Indiscernbility of Identicals, symmetry, and transitivity.

It is important to note that the first of the above characterizations involves a second-order quantifier which is intended to be read as ranging unrestrictedly over all properties of objects. Reading the second-order quantifier as restricted would have the consequence that some a can be "absolutely identical" with b in one interpreted language, but not "absolutely identical" to b in an extension of that interpreted language. But this is simply not absolute identity! If this is identity at all, it is language-relative identity. I think that virtually all critics of Strong Relative Identity, like Bueno, who explicitly rules out language-relative identities (2015: 258-259), would agree that a characterization of absolute identity in terms of indiscernibility principles require that the indiscernibility be absolute rather than language-relative. Therefore, I will henceforth distinguish these different relations with the following names: '$=_A$' shall stand for the stronger relation, characterized by (REF) and (IND) as those are intended to be read, and '$=_L$' for the weaker, language-relative, relation or rather relations since, when read restrictedly, there is a different relation characterizable in this way for every possible range of $\forall F$. $=_A$ is the relation whose existence is denied by NAI. More accurately, NAI is true of some object language if, when that language is interpreted but for a single dyadic predicate, then interpreting that predicate as the relation $=_A$ of our maximally-precise metalanguage will lead to incoherence. On the other hand, it is possible for NAI to be true of an object language even when some dyadic predicate of the object language is interpreted as the relation $=_L$ of the metalanguage, indeed the compatibility of NAI and $=_L$-relations will play an important role in my argument in favour of NAI, as we shall see.

I now turn to the attempted set-theoretic characterization of the absolute identity relation. I shall call this '$=_D$'. We have just seen that $=_L$ is a different relation from $=_A$. I anticipate this will be uncontroversial. By contrast, the characterizations by which we introduced $=_A$ and $=_D$ are supposed to pin down one and the same relation. In what follows, I shall argue that these two characterizations can, in fact, come apart. More specifically, I shall argue that some languages interpreted according to a proposed non-classical semantics are such that the relation $=_A$ cannot be introduced without contradiction, while, in these same languages, some dyadic relation of the object language might be interpreted as the relation $=_D$ of the metalanguage, but this relation can be shown not to be the relation of absolute identity. If this is right and if the proposed semantics is coherent, this would show that NAI is true of some interpreted languages. In the next section, I sketch this non-classical semantics.

Section IV A Non-Classical Semantics

In classical model-theoretic semantics, an interpretation involves a function, I, taking as input constants and delivering as output elements of a domain, D.[2] Hence, classical semantics has the following as a condition on the interpretation of a language, where a is any proper name:

(1) $I(a)$ is some $X \in D$.

It is a consequence of the nature of a function that the 'is' here is an 'is' of identity. A function can be defined by a one-one or a many-one mapping, but not by a one-many or many-many mapping. In the case of the interpretation function of classical semantics, it is defined by a many-one mapping, the mapping of names onto the elements of the domain for which they stand. The interpretation of a *just is* the object onto which a has been mapped, and this 'just is' must be interpreted as identity if interpretation is to count as a function at all.

This is not compatible with Strong Relative Identity, at least not in conjunction with the assumption that the elements of D are ultimately fine-grained in the sense that they are free from the possibility of subsequent disambiguation. Given this assumption, if interpreting any singular term involves mapping the singular term onto a single element of the domain, where the referent of the singular term is taken to be that single element, call the referent of the singular term 'X', then the relation between X and X will satisfy (IND) on pain of contradiction (if it fails to satisfy it, then X and X have different properties). Given that the set of referents of singular terms exhausts the objects capable of being referred to in the language (recall, we are looking only at languages with singular terms and no quantifiers), the relation between anything X and X will also satisfy (REF). In short, for any interpreted language where the semantics takes interpretation to be a function onto elements which are not open to more fine-grained discrimination, the relation $=_A$ holds between any X and X, so NAI is false for that language. In short, classical semantics is incompatible with Strong Relative Identity.

If NAI is true of any interpreted language, it is because the language is interpreted according to a semantics which allows a singular term, a to be interpreted by multiple elements in the domain simultaneously. In other words, NAI entails some kind of lexical ambiguity. Interpretation then is not a many-one mapping and therefore not a function according to a Strong Relative Identity semantics.

We are not, therefore, able to introduce a functional expression for interpretation into the metalanguage in this semantics, but we can still shed light on the nature of interpretation indirectly as follows. We eliminate the functional expression 'I' from the formal vocabulary and replace it with a term for a different

[2] I will restrict myself to looking at the language of first-order logic without identity for this section of the paper, but it would be easy to adjust what I say here to model-theoretic approaches to the semantics of natural languages.

function, which I shall name 'J'. Intuitively, J is defined by the mapping between any proper name and the set of objects which it ambiguously names (and the mapping between any monadic predicate and some set of sets, the mapping between a dyadic predicate and a set of pairs of sets, and so on for every use to which the function I is put in classical semantics). If a is unambiguous, $J(a)$ is a singleton. Call $J(a)$ the 'J-value for a'. We now introduce the following constraints on the interpretation[3] of a language:

(2) $J(a) \subseteq D$

(3) $J(P) \subseteq \wp(D)$, where P is any monadic predicate.

These tell us that names are mapped onto sets of elements and monadic predicates are mapped onto sets of sets of elements (i.e. a subset of the power set of the domain).[4] Although (2) tells us that names are mapped onto sets of objects, it is important to keep in mind that the function J is not itself an interpretation function, for example, the J-value of a is not the referent of a. A J-value is always a set (even when its argument is an unambiguous proper name), while the referent of a is very unlikely to be a set (how often do we name sets?). Crucially, the predicates that are satisfied by a are determined not by the properties of $J(a)$, but rather by the properties that are held in common by all of the members of $J(a)$.[5]

We are confronted with two questions. First, what are the consequences for identity relations of interpreting a language according to the proposed semantics? Second, is the semantics coherent? I take these in turn.

The consequences of the new semantics for identity relations are, first, that, for domain D of any object language, a relation can be defined in the metalanguage by $<< X, X >: X \in D >$, just as in classical semantics. That is to say, in any suitably expressive metalanguage, we can always define the relation $=_D$ over the domain of the object language, so that all the elements of the domain are $=_D$-related to themselves. Moreover, some object language might be such that it contains some dyadic predicate, R, which has been interpreted as holding between a and a only if a is unambiguous and holds between nothing else. We may of course identify R of such an object language with $=_D$ of the metalanguage. This relation is transitive, symmetric, and satisfies (IND). Is this relation reflexive? That depends on what we take its field to be. It is reflexive over the elements of D. However, the proposed semantics is entirely consistent with it sometimes being the case that $\neg R(a, a)$. In fact, this is guaranteed if some names are ambiguous. So R is not universally reflexive. It is a corollary of this that R (interpreted as $=_D$ of the metalanguage) is not a relation of absolute identity. At least, not as that relation has been traditionally conceived. The hallmark of absolute identity is universal reflexivity just as

[3]The notion of interpretation in this semantics is still a coherent one, it is just that it cannot be understood as a function, except perhaps as a multi-valued function.

[4]Similarly, the J-values of dyadic predicates must be subsets of the set of all pairs of elements of the set $\wp(D)$, the J-values of triadic predicates must be subsets of the set of all triples of elements of the set $\wp(D)$, and so on.

[5]The following partial characterization of a model guarantees that we get the right result here:

(4) $M \vDash P(a)$ if and only if $J(a) \in J(P)$.

much as indiscernibility, a hallmark to which, amongst the critics of Relative Identity, Bueno at least is committed as his rejection of entities without identity conditions demonstrates (Bueno 2014: 329-331).

The above semantics also allows that a be interpreted as ambiguous between two different entities which are discernible within the object language itself. However, any interpretation which involves a name that has been so interpreted is contradictory and as a result, any sentence involving any name so interpreted is necessarily false on this semantics. So, while the semantics allows names to be mapped onto any set of elements, consistency demands that names be interpreted only as elements that are indiscernible within the object language. That is to say, the only models of languages for which NAI is true are those where the interpretations of the singular terms are $=_L$-related to themselves.

We may put the matter this way. Whereas classical semantics takes it that it is a condition on a genuine interpretation of names that their referents turn out to be absolutely identical to some element in the domain, the semantics of Strong Relative Identity takes it that the only condition on a consistent interpretation of names is that they be language-relatively-identical with some element in the domain and possibly with more than one element. In other words, in the semantics of Strong Relative Identity, $=_L$ replaces $=_D$ as the relation determining the appropriateness or inappropriateness of an interpretation in the semantics of Strong Relative Identity.

Finally, because the only limitation placed on the possible interpretations of a name, a, is that, to avoid contradiction, it must be indiscernible from some element of D, relative to the resources of the language being interpreted, this leaves open the possibility of an eventuality to which Geach attached great importance: that we come to be able to distinguish what we formerly could not when we add new predicates to our language. In other words, a Strong Relative Identity semantics will allow for the following state of affairs: in some language L_0, the referent of some interpreted term, a, bears the relation $=_{L_0}$ to itself, while in an extension, L_1, which differs from L_0 only in the presence of some monadic predicate not present in L_0 it will be the case that the referent of a does not bear the relation $=_{L_1}$ to itself, and this of course will show that a has not been interpreted as an absolutely self-identical entity in either language, because absolute identity is characterised by unrestricted indiscernibility and the referent of a has been shown to be non-self-indiscernible in L_1. This entails that no relation that a bears to a in the object language can be interpreted as $=_A$ of the metalanguage. Moreover, if no relation that a bears to itself can be interpreted as $=_A$, then no relation born to itself by any interpreted singular term in the object language can be interpreted as $=_A$, because that relation is characterized in part by (REF), so if there is one nameable entity in the language which fails to bear this relation to itself, then no entity nameable in the language bears it or else the language will be such that everything bears this relation to itself and that something does not bear this relation to itself.

Let us take stock: a Strong Relative Identity semantics allows interpretations of some languages such that the relation $=_D$ might be introduced into the object language, but will not be universally reflexive, and therefore not absolute identity, and the relation $=_A$ cannot be introduced into the language without contradiction. The reason for this, in brief, is that the semantics of Strong Relative Identity does not employ the relation $=_A$ to determine the appropriateness or inappropriateness of an interpretation. Rather, that semantics uses the weaker language-relative relation $=_L$ to do the job.

The opponent of Strong Relative Identity may of course maintain that the proposed semantics is illegitimate precisely because it allows interpretation to be a many-many mapping, while they hold that the relation between names and referents must be many-one. This is to say, they may hold that ambiguous reference is not real reference.

In response, it is worth noting what sort of ambiguity the Strong Relative Identity semantics involves. In all probability, the semantic picture will be joined to an unorthodox metaphysics according to which the notion of a singular object itself involves some language-relativity.[6] On such a view, the interpretation of a name would indeed be many-one, since the output would be a ($=_L$-self-related) language-relative single object. The ambiguity of the reference only comes out in a metalanguage sufficiently expressive to distinguish multiple elements which the object language could not distinguish. Is this sort of ambiguity really so objectionable that it renders the proposed semantics incoherent? It seems to me that to show that it is would require a powerful positive argument against the sort of ambiguity involved here. I suggest that none of the semantic objections have provided this.

Section V The Failure of The Arguments

We may now consider how the semantic picture sketched in the previous section might answer the objections to Strong Relative Identity we began with.

> **Objection 1**: An interpreted language requires a domain and a domain must be composed of discrete entities. This requires absolute identity.

Bueno (2014: 326) argues that for some class to count as a domain for a language, it must be composed of individuals such that each individual is distinguishable from other things and each individual can be re-identified. Meeting these conditions requires absolute identity, according to Bueno. Moreover, elsewhere, he argues that for some class to count as a domain for a language it must be "complete" and "adequate" with respect to that language (see *Internationales Zentrum für Philosophie NRW* 2017). A class is complete if and only if quantifiers range over each object of the domain so that all of the objects in the domain are quantified over. A class is adequate if and only if no object distinct from those in the domain is in the range of the quantifiers. Only classes with elements that are identical or distinct from one another absolutely can meet these conditions. Once again, this requires absolute identity, according to Bueno.

With respect to the first pair of conditions, I think it is clear how my proposed semantics satisfies them. I grant that there are a set of individuals, these are distinguished from another and can be re-identified in the metalanguage using the relation $=_D$. Names do not always map onto these individuals many-one or one-one, however. There are also language-relative objects, which are nothing over-and-above the aforementioned individuals, but just a less precise

[6] This is not an embarrassment to the proposed semantics, this is the implied metaphysical picture that Strong Relative Identity has been working with all along (see in particular Geach (1973) for the ontological relativity that his view commits him to).

way of talking about them. These entities are distinguished from one another and can be re-identified using the relation $=_L$.

The second pair of conditions is a bit more complicated. Consider the set of all the ultimately-precise entities, which I have labelled D, and assume a semantic treatment of variables parallel to that I proposed for singular terms in the previous section. Does the proposed semantics guarantee that this domain is complete and adequate? That depends on how we understand these conditions. If we take 'the objects of the domain' to mean the elements of the set D, then it is not the case that the quantifiers range over each of the objects in the sense that a variable is assigned to exactly one of X or Y or Z. It is, however, the case that each variable is mapped onto X or Y or Z (though it may be mapped onto more than one). Is that sufficient? I think it is. The significance of the completeness condition, or so it seems to me, is that it rules out domains with irrelevant parts. It may be complained that on the proposed semantics, some of the objects are indeed irrelevant. In a language L_n, involving only a name, a, a predicate, F, and a domain (X, Y) such that $F(X)$ and $F(Y)$, then a will be consistently interpreted in L_n as the language-relative object which, relative to L_n, is both X and Y. Surely, one might say, either X is irrelevant here or Y is. However, by including all the ultimately-precise objects about which a future language extension could come to distinguish in the domain, we are able to shed light on the process of precisification. This seems to me to be a strength rather than a weakness. The indistinguishable elements of D may be, in some sense, irrelevant for interpreting names in a given language, but they allow us to determine when one language has the same interpretation as an another language with respect to their common terms. Similarly, they allow us to determine if one interpreted term in one language corresponds to multiple interpreted terms in a more expressive fragment of the same interpreted language. The elements are not, then, wholly irrelevant.

With respect to the adequacy of the domain, again, taken a particular way, the proposed semantics does not meet the proposed condition. Taken another way, it does. It does not meet the condition in the sense that a name, a, can take as its value a language-relative object which is not an element in the set D. However, as I suggested above, it is a mistake to see this object as something over and above certain of the ultimately-precise entities which are in D. The language-relative entity which is the referent of a name is just another way of looking at the same reality as the elements of D to which it is $=_L$-related. If, as I imagine, the adequacy condition is supposed to rule out domains which fail to include everything that can be talked about, I think this condition has been met as well.

> **Objection 2**: Any interpretation of a singular term must be distinct absolutely from any other possible interpretation. This requires absolute identity.

The *prima facie* coherence of the semantic picture sketched in Section IV suggest to me that this objection is straightforwardly wrong. The business of interpretation requires that names be mapped onto indistinguishable elements of D, to ensure that our interpretation does not give us: $F(a)$ and $\neg F(a)$. The same goes for the assignment function and variables. However, language-relative identity is sufficient for these jobs. It guarantees that inconsistent interpretations and assignments are ruled out. As we have seen, it might be objected that without absolute identity, we get essentially ambiguous interpretations for our referring expressions. In a sense this is right, but this is exactly how the language works according to the Strong Theory of Relative Identity. On this semantics, names stand for entities that are individuated as far as is possible given the descriptive resources of the language and no further. Variables range over the same kinds of entities that names stand for. Once again, it seems to me that there is nothing incoherent about this picture. If there is, the arguments proposed by Dummett, Bueno, and others, have yet to show what it is.

Conclusion

In this paper, I have defended the Strong Theory of Relative Identity against what I take to be the most pressing objections to it. I think I have shown that a central component of Strong Relative Identity is at least coherent, because there is a coherent semantic treatment such that NAI is true of some languages. Moreover, I think this semantics sheds important light on the nature of ambiguous referring expressions and it is clear that most languages involve many ambiguous referring expressions. This includes even formal languages in so far as their referring expressions are intended to correspond to referring expressions of natural languages. For these reasons, I think, though it would require more argument than I have room for here, that this semantics gives the best account of how referring actually works in practice. That is to say, my own view is that NAI is true of most languages that are actually used. Most actual languages can express no universally reflexive relation that satisfies the Indiscernibility of Identity without contradiction. I said near the outset of the paper that, although my goal was only the limited one of demonstrating the coherence of NAI, I thought this made the whole package of Strong Relative Identity much more plausible. This is because, even for a language of which NAI is true, Quine?s dictum 'no entity without identity' rings true. At least so it seems for me. So, even for some ambiguous name, 'a', a must bear some relation to a. At the very least, it bears the language-relative, $=_L$, relations discussed in Section III. In short, if NAI is true of some possible languages, which I have argued, and if it is likely true of most languages we use, which I have suggested but not argued for, then I think some form of relative identity is likely to be true.

Bibliography

Alston, William and Jonathan Bennett, (1984). 'Identity and Cardinality: Geach and Frege' *The Philosophical Review* 93: 553-567.

Bueno, Otavio (2014) 'Why Identity is Fundamental' *American Philosophical Quarterly* 51 (4): 325-332.

Bueno, Otavio (2015) 'Can Identity Be Relativized?' in A. Koslow and A. Buchsbaum (eds.), *The Road to Universal Logic.* Springer.

Dummett, M., (1991). 'Does Quantification involve Identity?', in H.A. Lewis (ed.),*Peter Geach: Philosophical Encounters.* Dordrecht: Kluwer Academic Publishers.

Geach, P.T., (1972). *Logic Matters.* Oxford: Blackwell.

Geach, P.T., (1973). 'Ontological relativity and relative identity' in M Munitz (ed) *Logic and Ontology.* New York: New York University Press. 287-302.

Geach, P.T., (1991). 'Replies', in H. G. Lewis (ed.), Peter Geach: Philosophical Encounters. Dordrecht: Kluwer Academic Publishers.

Griffin, N., (1977), *Relative Identity.* Oxford: Oxford University Press.

Hawthorne, J., (2003). 'Identity' in M Loux and D Zimmerman (eds.) *The Oxford Handbook of Metaphysics.* Oxford: Oxford University Press. 99-130.

Internationales Zentrum für Philosophie NRW. (2017). Otávio Bueno - "Identity and Quantification". [Online Video]. 20 December 2017. Available from: www.youtube.com. [Accessed: 2 January 2018].

Le Poidevin, R., (2009), 'Identity and the composite Christ: an incarnational dilemma'. *Religious Studies* 45: 167-186.

Noonan, Harold W., (1997) 'Relative Identity' In B Hale and C Wright (eds.): *Philosophy of Language.* Oxford: Blackwell. 634-652.

Quine, W.V.O., (1964). 'Review of Reference and Generality'. *The Philosophical Review* 73, 100-104.

van Inwagen, P., (1988). 'And Yet There are Not Three Gods, But One God', in T Morris (ed.) *Philosophy and the Christian Faith* Notre Dame IN: University of Notre Dame Press. 241-278.

Zemach, E.M., (1974). 'In Defence of Relative Identity'. *Philosophical Studies* 26: 207-218.

Zemach, E.M., (1982). 'Schematic Objects and Relative Identity'. *Nous* 16: 295-305.