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Fuel Consumption Minimisation, with Emissions Constraints, for Diesel Powered Cars

Gareth Pease, David Limebeer (IEEE Life Fellow), and Peter Fussey

Abstract—State of the art engine models are used to study the emissions production, and fuel consumption minimisation, of a typical diesel-powered road car operating on a variable-gradient road. The engine models, that have been fitted to measured test cell data, are used to represent both the performance and emissions generation characteristics of a typical diesel-fuelled car engine. Simple example are used to highlight the impact of elevation changes on the main structural features of fuel-optimal control problems (OCP). A typical semi-urban test route, with legislated speed limits and enforced stops, is used for performance evaluation purposes. The optimal functioning of a discrete-gear automatic transmission system, as opposed to a simple continuously variable transmission, is studied in detail. The main focus of this study is to evaluate the importance of three-dimensional (3D) road influences (gradient and camber), pre-imposed time-of-arrival constraints, enforced stops, and emissions constraints, on the fuel consumption and optimal driving of typical diesel-powered road vehicles. This paper proposes the use of multiple-phase optimal control to elicit a better understanding of ‘real’ driving situations, and motivates a move away from standardised drive cycles.

Index Terms—Engine calibration, nonlinear programming, trajectory optimisation, vehicle propulsion.

I. INTRODUCTION

Fixed drive cycles have remained the norm for engine testing and vehicle homologation for a number of years. More recently it has become apparent that there are shortcomings in these testing procedures. As a result, the road car industry is under scrutiny to ensure that emissions testing methods represent real-world driving and thus real-world emissions production. Engine calibration makes use of static lookup maps that determine engine operation under a prescribed set of torque-speed conditions [1]. These maps are typically generated with high-dimensional static engine models over fixed drive cycle scenarios. This testing procedure does not represent real world driving. The New European Drive Cycle (NEDC) is one of the most widely used in the European road vehicle industry. Under the new EU-6d emissions regulations [2], this will be replaced by the Worldwide harmonized Light vehicles Test Cycle (WLTC). A comparison between the NEDC and WLTC tests is given in Figure 1. In recent testing procedures portable emissions equipment is installed on the vehicle that measures the production of oxides of nitrogen (NO_x), carbon

Fig. 1: An example driving scenario in this paper (solid) compared to the NEDC (dashed) and the WLTC (dotted).

monoxide (CO), unburnt hydrocarbons (HC) and particulate matter (PM — HC and soot). This ensures that the real world emissions production falls within the legislative guidelines. As there is an element of real world driving incorporated into the vehicle testing and homologation, it is beneficial to be able to evaluate vehicle performance offline. However, it is not necessarily optimal to drive in the same manner when emissions constraints are introduced, and indeed faster route traversals may be used to ensure that the emissions constraints are met [3].

Optimal control studies, using dynamic engine and vehicle models, can be used for the offline evaluation of a number of different control strategies. The solution of these problems usually derives from Pontryagin’s Maximum Principle (PMP) [4]. Alternatively, solutions for simplified problems can be derived analytically; see for example [5], [6]. Optimal driving can be split into segments that comprise such things as maximum power acceleration, constant speed, coasting, and limit braking.

Dynamic programming (DP) can also be used to compute the optimal control solution, and is one of the most widely used solution techniques [4]. While this method is well suited to problems with few state and control variables, increasing the complexity of the model leads to the ‘curse of dimensionality’. The inclusion of terminal constraints in the cost function [7] facilitates the efficient use of DP to investigate the optimal trade-off between NO_x and fuelling. Optimal eco-driving cycles for hybrids was solved with DP in [8], where it is noted that the optimality of the solution depends heavily on the driver accepting the driving profile given. An online driver assistance system could be based on this concept. The optimal driving solution is computed online, and the driver is advised on the most efficient course of action (in terms of the engine torque and gearing) [9].

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Fig. 2: Exemplar driving route.

The optimisation of driving style and powertrain control for a vehicle equipped with a hybrid powertrain is studied in [10]. A short countryside route was considered in order to suppress the influence of traffic. This study highlighted the benefits of optimising both the power usage and the driving profile. One benefit was the removal of a prescribed velocity profile normally used for power split optimisation. Typical driving can be partitioned into a number of sub-problems each with their own solution structure. The results of the optimal control form the basis of eco-driving guidelines that depending on the scenario under consideration. In this context a number of different optimisation techniques may be used to advantage [11].

A pseudospectral method was employed to solve the optimal fuel economy optimal power split problem for a series hybrid electric bus in [12]. This was shown to be computationally more efficient than a standard DP approach. Improvements in solution accuracy are possible [13]. The authors of [14] presented a formulation of the eco-driving problem with particular emphasis on the high-fidelity modelling of both the vehicle and the road. The importance of including the road elevation into the problem was noted.

This paper outlines a novel method for generating fuel-optimal engine usage trajectories which can be utilised in the creation of optimal engine calibrations. These profiles will be the optimal driving solution based on the route in Figure 2, which includes both urban and fast motoring sections. The sensitivity of the resulting solution to emissions constraints, as well as the need to stop at junctions, will be examined. The comparative efficacy of a continuously variable transmission (CVT) and an automated manual transmission (AMT) will be analysed. This will form part of a set of non-linear constraints that can be recognised within a global pseudospectral solution framework. Additionally, in order to enforce legislative speed limits and enforced stops, the problem will be split into multiple phases.

The analysis of a simple energy OCP is presented in Section II. The optimal driving problem requires a high-fidelity

Fig. 3: The force system acting on the vehicle.

powertrain model and an accurate route model. These are described in Sections III and IV, respectively. The OCP is then formulated in Section V, along with the multiple-phase optimal control framework in Section V-A. The results are presented in Section VI.

II. ENERGY OPTIMAL CONTROL WITH VELOCITY CONSTRAINTS

The solution of fuel-optimal control problems have been studied extensively. As with many problems of this type, the optimal control solution is expected to be of *bang-bang*, or *bang-singular-bang* form [4]. One of the features of the eco-driving problem is the need to respect constraints on the vehicle speed. The aim of this paper is to study the problem of driving a vehicle over a fixed distance, within a specified time, while minimising the consumed energy.

To begin, consider the vehicle in Figure 3, which is initially driven on a road of constant inclination (constant θ). The reduced energy OCP can be formulated as

$$\min_{F(t)} E = \int_0^{t_f} F(t)v(t) dt \quad (1)$$

subject to

$$\dot{v}(t) = F(t)/M - K, \quad (2)$$

$$\dot{s}(t) = v(t), \quad (3)$$

$$v(t) \leq v_m, \quad (4)$$

$$0 \leq F(t) \leq F_m \quad (5)$$

where $K = g \sin \theta$; rolling resistance and aerodynamic losses are neglected ($F_r(t) = 0$). The total energy input E , is given by (1) and constitutes the cost function. The equations of motion are given by (2) and (3), equations (4) and (5) represent, respectively, velocity and force constraints. Upper limits on the velocity and tractive force are v_m and F_m , respectively. The boundary conditions on the states are

$$s_0 = 0, s(t_f) = s_f, \quad (6)$$

$$v(0) = 0, v(t_f) = v_f = \text{free}. \quad (7)$$

The control Hamiltonian [15] for the problem can be formed by appending the dynamic state equations and velocity inequality constraint to the cost function

$$\mathcal{H}(t) = F(t)v(t) + \lambda_v(t)(F(t)/M - K) + \lambda_s(t)v(t) + \mu(t)(v(t) - v_m(t)), \quad (8)$$

$\lambda_v(t)$ and $\lambda_s(t)$ are the co-state variables associated with each of the system state variables. The multiplier $\mu(t)$ appends the speed-related inequality constraint to the control Hamiltonian, which must satisfy the following conditions:

$$\mu(t) = 0 \quad \text{for } v(t) < v_m(t), \quad (9)$$

$$\mu(t) < 0 \quad \text{for } v(t) = v_m(t), \quad (10)$$

$$\mu(t) > 0 \quad \text{for } v(t) > v_m(t). \quad (11)$$

The sign of $\mu(t)$ ensures that violating the constraint (4) cannot decrease (8). Pontryagin's Maximum Principle (PMP) [15] states that an optimal control is one that minimises the Hamiltonian over the problem horizon, i.e.

$$F^*(t) = \arg \min_{F(t)} \mathcal{H}(v^*, s^*, F, \lambda_v^*, \lambda_s^*, \mu^*) \quad (12)$$

for $t \in [0, t_f]$. The superscript $(*)$ denotes the optimal trajectories for the control, state and co-state variables. The first-order necessary conditions for optimality are

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial v} &= -\dot{\lambda}_v, \\ \frac{\partial \mathcal{H}}{\partial s} &= -\dot{\lambda}_s, \end{aligned} \quad (13)$$

that is

$$\dot{\lambda}_v = -(F + \lambda_s + \mu) \quad (14)$$

$$\dot{\lambda}_s = 0 \quad (15)$$

which implies that λ_s is constant. The final velocity is free, and so there is a transversality constraint, $\lambda_v(t_f) = 0$. It follows from PMP that the optimal control is given by

$$F^*(t) = \begin{cases} 0, & \text{if } \Phi(t) > 0 \\ F_s(t), & \text{if } \Phi(t) \equiv 0 \\ F_m, & \text{if } \Phi(t) < 0 \end{cases} \quad (16)$$

where $\Phi = v + \lambda_v/M$ is the switching function. In the case of a singular arc, $\Phi(t) \equiv 0$ for some finite time interval, and so $\dot{\Phi} \equiv \ddot{\Phi} \equiv \dots \equiv 0$, must also hold on this time interval. In the present case the first derivative of the switching function is $\dot{\Phi} = -(K + (\lambda_s + \mu)/M)$. Since θ is fixed, λ_s is constant, see (15), and $\mu = 0$ in the case of an inactive speed constraint (as is assumed initially), $\dot{\Phi}$ is constant. We conclude therefore that the control never appears following differentiation of the switching function (with respect to time) and thus that singular arcs don't exist for this problem. The resulting control is *bang-bang* with a single switch.

At the start of the optimal control interval $F^* = F_m$, which implies that $\Phi(0) < 0$, because in the case that $F^* = 0$ the car would roll backwards down the incline. Since $\dot{\Phi}$ is constant, there is at most one control switch. In sum,

$$F(t) = \begin{cases} F_m, & t \in [0, \tau] \\ 0 & t \in (\tau, t_f] \end{cases} \quad (17)$$

with τ the thus far unknown switching time.

On the first sub-arc, the state equations can be integrated, observing (6) and (7), to give

$$v_1(t) = \left(\frac{F_m}{M} - K\right)t \quad (18)$$

$$s_1(t) = \left(\frac{F_m}{M} - K\right)\frac{t^2}{2}. \quad (19)$$

Similar expressions arise for the second sub-arc

$$v_2(t) = K(t_f - t) + v_f \quad (20)$$

$$s_2(t) = -\frac{K}{2}(t - t_f)^2 + v_f(t - t_f) + s_f. \quad (21)$$

Equating each of the states at the switching time $t = \tau$ gives

$$v_f = \frac{F_m}{M}\tau - Kt_f \quad (22)$$

$$0 = F_m\tau^2 - 2M(Kt_f + v_f)\tau + M(Kt_f^2 + 2v_ft_f - 2s_f). \quad (23)$$

Eliminating v_f gives

$$0 = F_m\tau^2 - 2F_mt_ft_f + M(2s_f + Kt_f^2). \quad (24)$$

As $\tau \leq t_f$, the smaller of the two roots of (24) must be taken

$$\tau = t_f - \sqrt{\frac{(F_m - MK)t_f^2 - 2Ms_f}{F_m}}. \quad (25)$$

The optimal cost is given by

$$E^* = \left(Ks_f + \frac{v_f^2}{2}\right)M, \quad (26)$$

which is the vehicle's total energy as it passes the finish line. Equation (25) contains information about the existence of solutions for the problem. Since $\tau \leq t_f$ must be real

$$(F_m - MK)t_f^2 - 2Ms_f \geq 0, \quad (27)$$

which implies that $F_m - MK > 0$. This is intuitively correct—the force available must be at least as much as required to overcome the vehicle's weight projection down the incline. Condition (27) also ensures there is enough time for the car to cover the course if driven at the maximum allowable force for the duration. In the case that (27) is an equality, $\tau = t_f$ and maximum force is required for the whole length of the course. The minimum arrival time is therefore

$$t_f^{min} = \sqrt{2s_f/(F_m/M - K)}. \quad (28)$$

Substituting into (22) gives the final velocity

$$v_f^{min} = \sqrt{2s_f(F_m/M - K)}, \quad (29)$$

which can be substituted into (26) to give $E^* = F_ms_f$.

The unconstrained arrival time case is also of interest. In this case the solution to the energy minimisation problem is $E^* = MKs_f$, which is energy required to raise the vehicle from the initial to the final elevation. Setting $v_f = 0$ results in the switching time

$$\tau = \sqrt{\frac{2MKs_f}{F_m(F_m/M - K)}}, \quad (30)$$

Fig. 4: Bang-singular-zero control (solid line), the switching function Φ (dashed line) and the velocity (dotted line).

and the minimum-energy arrival time

$$t_f^{en} = \sqrt{\frac{2F_m s_f}{K(F_m - MK)}}. \quad (31)$$

We will now consider the case when the velocity state constraint is active. When $v = v_m$ is constant, $F = MK$ follows from (2). Referring to (16) it can be seen that an active velocity constraint must occur on a singular sub-arc, which implies that for some finite period $\Phi(t) \equiv 0$. The second derivative of the switching function is given by

$$\ddot{\Phi} = -\mu^* = 0, \quad (32)$$

which implies that μ^* is also constant along the singular sub-arc. The constraint (4) is active, which implies that $\mu^*(t) < 0$. It follows, therefore, that the car accelerates up to v_m on the first sub-arc when $\Phi < 0$, the vehicle then progresses at constant speed v_m on a second sub-arc (when $\Phi = 0$), and coasts on a third sub-arc when $\Phi > 0$. The bang-singular-bang control strategy is shown in Figure 4. We can now use purely mechanics-based arguments to find the switching times and the state trajectory. The states on the first sub-arc are described by

$$v_1(t) = (F_m/M - K)t, \quad (33)$$

$$s_1(t) = \frac{(F_m/M - K)}{2} t^2. \quad (34)$$

For the singular arc, the velocity is constant and the driving force balances the weight projected along the incline. Thus

$$v_2(t) = v_m, \quad (35)$$

$$s_2(t) = \frac{(F_m/M - K)}{2} \tau_1^2 + v_m(t - \tau_1), \quad (36)$$

where τ_1 is the first switching time. For the third sub-arc the dynamics are given by

$$v_3(t) = K(t_f - t) + v_f \quad (37)$$

$$s_3(t) = s_f - \frac{K}{2}(t - t_f)^2 + v_f(t - t_f). \quad (38)$$

The first switch can be calculated using (33) with $v_1 = v_m$ to give

$$\tau_1 = \frac{v_m}{(F_m/M - K)}. \quad (39)$$

Equating (36) and (38) at τ_2 gives the second switching time as

$$\tau_2 = t_f - \sqrt{\frac{2}{K}((t_f - \tau_1)v_m + s_a - s_f)}, \quad (40)$$

in which $s_a = \frac{(F_m/M - K)}{2} \tau_1^2$ is the distance travelled on the first sub-arc. If this problem is to be soluble, t_f must satisfy

$$(t_f - \tau_1)v_m + s_a - s_f \geq 0, \quad (41)$$

which means that the terminal time must be long enough to allow the car to accelerate to v_m under the influence of F_m , and then move at v_m to the finish line at s_f .

It is easy to see that the energy usage is given by

$$E^* = F_m \frac{F_m/M - K}{2} \tau_1^2 + MK v_m (\tau_2 - \tau_1). \quad (42)$$

Once again consider the case where the arrival time is minimised and thus

$$t_f^{min} = \frac{v_m}{2(F_m/M - K)} + \frac{s_f}{v_m} \quad (43)$$

with corresponding energy expenditure

$$E^* = M(v_m^2/2 + K s_f). \quad (44)$$

In the minimum energy case (with a free arrival time), $v_f = 0$ and by (40)

$$t_f^{en} = \frac{F_m v_m}{2K(F_m - MK)} + \frac{s_f}{v_m} \quad (45)$$

$$\tau_2^{en} = t_f^{en} - v_m/K \quad (46)$$

The energy expenditure for the speed-limited case is $E^* = MK s_f$.

In the case that the road grade is variable, the lower bound on the propulsive force (5) changes and allows the vehicle to use conventional brakes. In this case the cost (1) becomes

$$E = \int_0^{t_f} \max(F(t), 0) v(t) dt. \quad (47)$$

The solution to this problem was computed numerically and is shown in Figure 5. Not surprisingly, the driving control strategy is more complex in the case of a variable gradient route with dissipative braking. As is evident from Figure 5, the optimal force law has a *bang-singular-zero-singular-zero* structure. The bang sub-arc sees the acceleration of the vehicle, as quickly as possible, up to the speed limit. The driving force on the first singular sub-arc keeps the vehicle at the speed limit by cancelling the decelerating gravitational influence on the car. On the zero sub-arc the driving force is set to zero allowing the car to coast towards the brow of the hill. The car then accelerates, under the influence of gravity towards the bottom of the valley. After the car has accelerated up to the speed limit, under the influence of gravity, a second singular sub-arc begins and the speed is held constant at its limit. The car then coasts towards the finish line on the second zero sub-arc. This example demonstrates how a control structure of

Fig. 5: The optimal force profile for the variable slope case. The road profile is plotted as a dashed line, the optimal tractive force in solid black and the variable component of weight in dotted black.

a mixture of singular arcs and bang-bang type switches is likely to feature in the full-vehicle version of this problem. The vehicle's speed profile is illustrated in Figure 6.

III. CAR MODEL

This section develops the vehicle and powertrain model that will be used in the remainder of the paper.

A. Vehicle Dynamics

Prior work that addresses the optimisation of a hybrid vehicle on a 3D track, in combination with a non-linear tyre model, is [14]. Here, a simple single-degree-of-freedom model of the vehicle longitudinal dynamics is used. The effects of pitch, roll and any rotational dynamics are neglected. A force balance between the tractive force $F(t)$ and resistive forces $F_r(t)$ (in Figure 3) can be expressed as

$$\dot{v}(t) = \frac{1}{M} (F(t) - F_r(t)), \quad (48)$$

Fig. 6: Optimal velocity profile for variable slope road.

where M is the vehicle mass. The resistive force is made up of three components: aerodynamic drag, rolling resistance and the component of the vehicle's weight projection along the slope:

$$F_r(t) = Mg(\mu \cos \theta(s(t)) + \sin \theta(s(t))) + \frac{1}{2} \rho C_d A v(t)^2. \quad (49)$$

The aerodynamic drag coefficient is C_d , A is the vehicle frontal area and ρ the density of air. The coefficient of rolling resistance is μ and g is the acceleration due to gravity. The elapsed distance is $s(t)$, while the road inclination angle is $\theta(s(t))$. Equation 48 represents the only vehicle system dynamics used here.

B. Powertrain Model

The engine dynamics will be considered 'fast', and will thus be neglected in the model used here; the engine data represents a prototypical mid-size diesel car engine.

1) *Fuel and Emissions Models:* In order to predict the engine's fuel consumption and emissions production, two-dimensional functional maps based on test bed data are used. The brake-specific fuel consumption (BSFC) is a useful measure of the engine operating efficiency and is given by the ratio of the fuel mass flow rate ($\dot{m}_f$) to the engine output power

$$\text{BSFC} = \frac{\dot{m}_f(\omega_e, \tau_e)}{\omega_e \tau_e}. \quad (50)$$

The engine brake power is defined as the product of engine speed (ω_e) and engine torque (τ_e). The map given in Figure 7 includes the thermal efficiency of combustion, frictional losses and pumping losses. In similar vein, the brake-specific (BS) emissions, which include the efficiencies relating to production of emissions, are shown in Figures 8 and 9.

For optimal control purposes, smooth maps for the prediction of fuel and emissions flows are required. The fuel mass flow rate can be represented by a biquadratic map [3]. The engine operating range is constrained by the idling speed ω_e^{idle} ,

Fig. 7: Brake specific FC (BSFC) map [g/kWh] including maximum torque line and lines of constant power.

Fig. 8: Brake specific emissions maps [g/kWh] (Left - unburnt hydrocarbons (HC), Right - NO_x); the maximum torque line is overlaid.

Fig. 9: Brake specific emissions maps [g/kWh] (Left - soot, Right - CO); the maximum torque line is overlaid.

the maximum engine speed ω_e^{max} , and zero torque and the maximum torque line $\tau_e^{max}(\omega_e)$. This limit is given by

$$\tau_e^{max}(\omega_e) = a_4\omega_e^4 + a_3\omega_e^3 + a_2\omega_e^2 + a_1\omega_e + a_0. \quad (51)$$

The coefficients a_i are computed using a curve fitting tool; the results of which were used to produce the τ_e^{max} curve in Figure 7.

The nonlinear response of emissions production to the engine operating condition is represented by Gaussian Process models of the type described in [16], [17]; further detail is available in [18]. This modelling method has the advantage that no black-box model is required, instead samples from a distribution of functions are used.

In general, Gaussian process models take the form

$$\mathbf{y} = f(\mathbf{x}) + \epsilon, \quad (52)$$

which relate the vector of inputs $\mathbf{x}$ to the vector of outputs $\mathbf{y}$ via the unknown, underlying stochastic process $f(\mathbf{x})$. The

output is measured, and some zero-mean Gaussian noise ϵ , is introduced. The variance of the noise is σ_n^2 . The function $f(\mathbf{x})$ is a static input-output relationship and is assumed to have zero-mean. The covariance for outputs $\mathbf{y}$ and $\mathbf{y}'$, corresponding to the inputs $\mathbf{x}$, $\mathbf{x}'$, is given by

$$\text{cov}(f(\mathbf{x}), f(\mathbf{x}')) = k(\mathbf{x}, \mathbf{x}'). \quad (53)$$

Consider a set of training data inputs $\mathbf{x}_i \in \mathbb{R}^{1 \times d}$ with corresponding output response, $\mathbf{y}_i \in \mathbb{R}^{1 \times p}$. This can be written succinctly in terms of matrices: $\mathbf{X} \in \mathbb{R}^{n \times d}$ and $\mathbf{Y} \in \mathbb{R}^{n \times p}$. Therefore, based on (52), the output data can be thought of as samples at each $\mathbf{x}_i$ taken from the distribution:

$$\mathbf{Y} \sim \mathcal{N}(\mathbf{0}, \mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I}). \quad (54)$$

The matrix $\mathbf{K}(\mathbf{X}, \mathbf{X})$ is an $n \times n$ matrix of the covariance between each of the training input points.

The joint distribution between the training data outputs $\mathbf{Y}$ and a new, noise-free test output $\mathbf{y}^* = f(\mathbf{x}^*)$ is

$$\begin{bmatrix} \mathbf{Y} \\ \mathbf{y}^* \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} \mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I} & \mathbf{k}(\mathbf{x}^*, \mathbf{X})^T \\ \mathbf{k}(\mathbf{x}^*, \mathbf{X}) & \mathbf{k}(\mathbf{x}^*, \mathbf{x}^*) \end{bmatrix}\right). \quad (55)$$

The entries of $\mathbf{k}(\mathbf{x}^*, \mathbf{X})$ are the covariance function between each of the training points and the test point $\mathbf{x}^*$. In order to predict from the distribution of functions, the joint Gaussian prior distribution is conditioned on the observations (Appendix A.2 in [18]):

$$\begin{aligned} \mathbf{y}^* | \mathbf{y}, \mathbf{X}, \mathbf{x} &\sim \mathcal{N}(\mathbf{k}(\mathbf{x}^*, \mathbf{X})^T (\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I})^{-1} \mathbf{y}, \\ &\mathbf{k}(\mathbf{x}^*, \mathbf{x}^*) - \mathbf{k}(\mathbf{x}^*, \mathbf{X}) (\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I})^{-1} \mathbf{k}(\mathbf{x}^*, \mathbf{X})^T). \end{aligned} \quad (56)$$

The expected value of the output, given the training data and the new test point is therefore the mean of this distribution and is given by

$$\mathbf{y}^* = \mathbf{k}(\mathbf{x}^*, \mathbf{X})^T (\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I})^{-1} \mathbf{y}. \quad (57)$$

Training data with fixed nonzero mean can be treated in the same way. The zero-mean Gaussian Process is applied to the difference between the observations and their mean (β). The predictive mean (57) becomes:

$$\mathbf{y}^* = \beta + \mathbf{k}(\mathbf{x}^*, \mathbf{X})^T (\mathbf{K}(\mathbf{X}, \mathbf{X}) + \sigma_n^2 \mathbf{I})^{-1} (\mathbf{y} - \beta). \quad (58)$$

The covariance function used for this work is the *squared exponential*:

$$k(\mathbf{x}, \mathbf{x}') = \sigma_F^2 \exp\left(-\frac{|\mathbf{x} - \mathbf{x}'|^2}{2\sigma_L^2}\right). \quad (59)$$

The characteristic length scale σ_L^2 and overall variance of the function σ_F^2 are estimated by maximising the log-likelihood of the predictions [18]. In the case for the emissions models used in this study, the input vector is made up of the engine speed and torque, $\mathbf{x} = [\omega_e, \tau_e]$. The now-scalar output y , is brake-specific (BS) NO_x , HC, soot or CO. The regression models are generated based on steady-state engine testbed data and are trained with the Statistics and Machine Learning Toolbox in MATLAB. The corresponding maps are those plotted in Figures 8–9.

2) *Power Transmission*: Power flows from the engine to the wheels via a transmission. This ensures that the engine and road wheels can be rotated at different speeds. A transmission is also required to enable the vehicle to move away from rest. Continuously variable transmissions (CVTs) are mechanical devices with no pre-determined gear ratios which, from an engine operation standpoint, allows any engine speed to be realised [11]. The major drawback to this type of power transmission are the high mechanical losses, for a belt driven CVT this is around 15%.

It is not uncommon to make a simplification in the evaluation of optimal control strategies by assuming a CVT rather than a discrete set of gears (see for example [6]). Automated manual transmissions (AMT) are implemented with velocity dependent shifting laws—a major improvement over CVTs are the lower power losses of approximately 5%.

Fig. 10: Engine speed as a function of forward velocity; the overall drive ratios are shown as dotted lines.

In order to evaluate the loss of information arising from the ‘CVT simplification’, the optimal driving strategy will be computed for both types of transmission. The 5-speed AMT with a forward velocity ($v(t)$) dependent shifting law takes the following form (see [19]):

$$\begin{aligned} N_g(v(t)) &\approx N_g^1 + \\ &0.5(N_g^2 - N_g^1)(1 + \sin(\arctan(\alpha_1(v(t) - u_1)))) + \\ &0.5(N_g^3 - N_g^2)(1 + \sin(\arctan(\alpha_2(v(t) - u_2)))) + \\ &0.5(N_g^4 - N_g^3)(1 + \sin(\arctan(\alpha_3(v(t) - u_3)))) + \\ &0.5(N_g^5 - N_g^4)(1 + \sin(\arctan(\alpha_4(v(t) - u_4)))). \end{aligned} \quad (60)$$

The N_g^i s for $i = [1, 2, 3, 4, 5]$ are the fixed gear ratios and the u_i s for $i = [1, 2, 3, 4]$ are the gear-shift speeds. The parameters α_i for $i = [1, 2, 3, 4]$ are chosen to make the gear change approximation sufficiently accurate and smooth for optimal control purposes. An example shifting law is plotted in Figure 10, the shift speeds u_i are chosen initially so that the shift occurs at approximately $\omega_e = 2000\text{rpm}$, but will later be optimised.

The rotational speed of the gearbox on the driveline side of the clutch is given by

$$\omega_g(t) = \frac{N_g(t)N_f v(t)}{r_w}, \quad (61)$$

where N_f is the final drive ratio and r_w is the radius of the rear wheel. To model the clutch disengaging the engine from the wheels, the following constraint is enforced

$$\omega_e(t) - \omega_e^{idle} - \max(\omega_g(t) - \omega_e^{idle}, 0) = 0. \quad (62)$$

From rest, $\omega_g(t) = 0$, and the engine speed is constrained to be equal to ω_e^{idle} until $\omega_g(t) = \omega_e^{idle}$ in first gear (Figure 10).

IV. ROUTE INFORMATION ESTIMATION

The route is taken from GPS data that is given as a latitude, longitude and elevation triple obtained from the mapping software available at www.mapmyride.com. The data follows the centreline of the road lane at random sampling intervals. The raw GPS data is transformed into a local Cartesian coordinate frame based on the UTM (WGS84 Geodetic) system.

The starting point of the route is deemed the origin of the coordinate frame. Using the method detailed in [20] for a two-dimensional track, the geodesic curvature, $\mathcal{C}(s)$, can be estimated. The geometry of the transformation between the Cartesian space to the spine coordinates can be written in the following differential form

$$dx = ds \cos \psi, \quad (63)$$

$$dy = ds \sin \psi. \quad (64)$$

where x and y are the local point coordinates and ψ is the orientation angle. The local curvature can be defined by the following relationship to the local angle

$$\mathcal{C} = \frac{d\psi}{ds}. \quad (65)$$

In the absence of noise, numerical differentiation of the data set should reveal the curvature. However, as the route is long, and the corners are tight, a smooth approximation of the curvature is required. Additionally, GPS data is not noise-free—a problem which is amplified by successive numerical differentiations. In order to alleviate this, an OCP can be posed that ensures that the boundary conditions are satisfied and that the noise is rejected. In order to assemble a complete set of equations, an additional dynamic state equation is required

$$\tilde{u} = \frac{d\mathcal{C}}{ds}. \quad (66)$$

For known coordinate pairs for the centreline (x_c, y_c) a cost function

$$J = \int_0^{s_f} (\tilde{u}(s)^2 + w_c((x_c - x(s))^2 + (y_c - y(s))^2)) ds. \quad (67)$$

is minimised.

As the route chosen is closed, the following boundary constraints must be satisfied

$$x(0) = x(s_f) \quad (68)$$

$$y(0) = y(s_f) \quad (69)$$

$$\mathcal{C}(0) = \mathcal{C}(s_f). \quad (70)$$

The weighting w_c provides flexibility between reconstructing the spine exactly, and ensuring the curvature does not change too rapidly. The results of the optimisation are plotted in Figure 12. Roundabouts feature as instances of very high curvature (inset 3 Figure 12).

The road inclination angle $\theta(s)$ ¹ can be estimated by considering an optimal filtering problem. This ensures that the data is sufficiently smooth for the purposes of optimal control. The problem can be posed as such:

$$\begin{aligned} \frac{dz}{ds}(s) &= \theta(s) \\ \frac{d\theta}{ds}(s) &= u(s). \end{aligned} \quad (71)$$

Continuity is also enforced at the beginning and end of the loop.

$$\begin{aligned} z(0) &= z(s_f) \\ \theta(0) &= \theta(s_f). \end{aligned} \quad (72)$$

Fig. 11: The optimal curvature (magnified 10 $\times$), $\mathcal{C}(s)$, and inclination, $\theta(s)$, as estimated around the route.

Once again a performance index that incorporates both the magnitude of the error between the GPS data and a regularisation term can be formed

$$J = \int_0^{s_f} (z(s) - z_{GPS}(s))^2 + w_e u(s)^2 ds. \quad (73)$$

The weighting factor w_e allows the exact reconstruction of the elevation to be traded off against a smoothing of the road gradient estimate. Figure 11 shows the curvature and elevation estimation estimates as a function of the route elapsed distance.

V. REAL DRIVING EMISSIONS OPTIMAL CONTROL PROBLEM

The minimum fuel control problem can now be assembled. It is convenient to transform the problem from the time domain to the distance travelled domain (s) so that route-specific information can be straightforwardly encoded in the problem setup. There is a one-to-one correspondence between time and the distance travelled provided $v > 0$, since $s = vt$. In differential form there holds:

$$\frac{d}{dt} = \frac{1}{v(s)} \frac{d}{ds}. \quad (74)$$

The s -domain formulation is equivalent in size, in terms of the number of states, as the t -domain. However, to properly implement distance based phenomenon such as discrete changes in state limits, or additional event constraints such as enforced stopping, the s -domain formulation is superior.

The OCP can be posed as a fuel minimisation problem by seeking to minimise the following integral

$$m_{fuel} = \int_0^{s_f} \dot{m}_f(\omega_e(s), \tau_e(s)) \frac{ds}{v(s)}, \quad (75)$$

in which $\dot{m}_f(\cdot, \cdot)$ is the fuel mass flow rate.

The state dynamics are given by Equation (48) following transformation into the s -domain:

$$\begin{aligned} \frac{dv(s)}{ds} &= \frac{1}{Mv(s)} \left(\frac{P_t(s)}{v(s)} - Mg(\mu \cos \theta(s) + \sin \theta(s)) \right. \\ &\quad \left. - \frac{\rho C_d A v(s)^2}{2} \right). \end{aligned} \quad (76)$$

¹The road gradient is $\theta(s) = \tan^{-1}(\frac{dz}{ds})$, but the small angle approximation $\theta(s) \approx \frac{dz}{ds}$ can be used in the OCP.

Fig. 12: Curvature estimate results. The solid black line denotes the data extracted from the GPS and the red line is route obtained from the curvature estimate for a weighing of $w = 10^{-2}$. The right-hand insets show the effectiveness of the estimate during the urban part of the course (1), and how it negotiates roundabouts (2 and 3). Stops are plotted as black crosses and the start-finish is a blue cross.

The tractive power $P_t(s)$, given by $F(s)v(s)$, along with the engine control inputs ω_e and τ_e , make up the control inputs for the problem. The following inequality constraints must hold:

$$\begin{bmatrix} \omega_e^{idle} \\ 0 \\ P_b \end{bmatrix} \leq \begin{bmatrix} \omega_e(s) \\ \tau_e(s) \\ P_t(s) \end{bmatrix} \leq \begin{bmatrix} \omega_e^{max} \\ \tau_e^{max}(\omega_e(s)) \\ P_e^{max} \end{bmatrix}. \quad (77)$$

The engine speed and torque are constrained to remain on the engine maps (see Figures 7, 8 and 9). The tractive power is bounded by the maximum engine power P_e^{max} and the maximum braking power P_b . The power balance between the powertrain and the car is modelled with the following inequality

$$\eta_X \omega_e(s) \tau_e(s) - P_t(s) \geq 0, \quad (78)$$

where η_X is the transmission efficiency for either the CVT or AMT.

Driver comfort and tyre usage constraints are captured by bounds on the maximum longitudinal and lateral accelerations; commonly referred to as a g - g diagram. A typical driver will not exploit fully the available acceleration limits [21]. An alternative description of the operating region is:

$$\left| \frac{a_x(s)}{a_x^{max}} \right|^n + \left| \frac{a_y(s)}{a_y^{max}} \right|^n \leq 1 \quad (79)$$

where n takes values between 1 and 2—corresponding to a rhombus and circle respectively. The lower the value of n , the

more conservative the driver is with the available accelerations—especially under cornering conditions. The longitudinal acceleration $a_x(s)$ is given by the right-hand side of (48). The lateral acceleration is given by

$$a_y(s) = \mathcal{C}(s)v(s)^2. \quad (80)$$

The legislative speed limits within each phase (p) of the route must also be respected:

$$v(s) \leq v_l^p(s). \quad (81)$$

The boundary conditions on the state are

$$v(0) = v(s_f) = \epsilon, \quad (82)$$

where ϵ is a small positive number to ensure the vehicle starts from rest, but avoids the problem breaking down when $v(s) = 0$. In the case of enforced stops at points $s = s_s$, additional boundary conditions arise:

$$v(s_s) = \epsilon. \quad (83)$$

Finally it is necessary to impose a number of integral constraints. The first of which is the upper bound on the arrival time

$$\int_0^{s_f} \frac{1}{v} ds \leq T. \quad (84)$$

The bound on T acts as a surrogate for the driving style, a lower arrival constraint implies more aggressive driving.

Particulate matter (PM) is made up of unburnt (HC) and soot according to the following mixing law [22]:

$$\dot{m}_{PM} = 1.024\dot{m}_{Soot} + 0.277\dot{m}_{HC} \quad (85)$$

The total quantity of PM created, along with total NO_x , HC and CO form the emissions constraints:

$$\int_0^{s_f} \omega_e(s) \tau_e(s) \text{BSNO}_x(\omega_e(s), \tau_e(s)) \frac{ds}{v(s)} \leq \overline{m}_{\text{NO}_x} \quad (86)$$

$$\int_0^{s_f} \omega_e(s) \tau_e(s) \text{BSHC}(\omega_e(s), \tau_e(s)) \frac{ds}{v(s)} \leq \overline{m}_{HC} \quad (87)$$

$$\int_0^{s_f} \omega_e(s) \tau_e(s) \text{BSCO}(\omega_e(s), \tau_e(s)) \frac{ds}{v(s)} \leq \overline{m}_{CO} \quad (88)$$

$$\int_0^{s_f} \omega_e(s) \tau_e(s) \left(1.024 \text{BSSoot}(\omega_e(s), \tau_e(s)) + 0.277 \text{BSHC}(\omega_e(s), \tau_e(s)) \right) \frac{ds}{v(s)} \leq \overline{m}_{PM}. \quad (89)$$

TABLE I: Table of the vehicle parameters used in the optimisation.

Symbol	Value
M	1750kg
$C_d A$	0.6m ²
ρ	1.2kg/m ³
a_x^{max}, a_y^{max}	0.1gm/s ²
μ	0.01
n	1
η_{CVT}	0.85
η_{AMT}	0.95
N_g	[4.48, 2.87, 1.84, 1.41, 1]

A. Multi-phase Numerical Optimal Control

The OCP of the previous section is solved numerically with a direct transcription method based on Legendre-Gauss-Radau (LGR) collocation and Radau's integration formula [23]. To facilitate the use of orthogonal polynomials, the independent variable (time) is transformed using the affine transformation $t = \frac{(t_f - t_0)}{2}\tau + \frac{(t_0 + t_f)}{2}$ for $\tau \in [-1, 1]$. The Lagrange objective function is thus:

$$J = \frac{t_f - t_0}{2} \int_{-1}^1 \mathcal{L}(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau, t_0, t_f, \mathbf{p}) d\tau. \quad (90)$$

The optimal control problem is minimise the cost functional J by determining the optimal state $\mathbf{x} \in \mathbb{R}^{n_x}$, the optimal control $\mathbf{u} \in \mathbb{R}^{n_u}$ and the optimal static parameters $\mathbf{p} \in \mathbb{R}^{n_p}$, subject to the system dynamics, path constraints and boundary conditions. Further details are available in [24] and [25].

In our study the OCP is discretized and solved using GPOPS-II [23], which uses *hp*-adaptive mesh refinement [26]. The large, sparse NLP is solved with the interior point algorithm such as IPOPT [27]. In this paper the problem is split into multiple phases, and additional constraints are used to ensure continuity of the state across each phase boundary, and the integral cost and integral constraints are then summed over all phases.

B. Regularisation

As demonstrated in Section II, the optimal control strategy is likely to contain singular arcs. In the case that the velocity constraints are active, the NLP will not be able to compute a solution. To overcome this difficulty a small regularisation cost is appended to the cost (90), and takes the following form

$$\mathcal{L}_r = \sum_{i=1}^{n_u} \epsilon_i \dot{u}_i^2. \quad (91)$$

Provided that ϵ'_i s are small, the control strategy and overall cost will not be significantly effected, and the control can be recovered [28]. Slew rate bounds may also be introduced, which serve to alleviate the numerical instability found in the computation of singular controls.

In short, the state, and control variables for the CVT optimisation are

$$\mathbf{x} = [P_t, v, \omega_e, \tau_e] \quad \text{and} \quad \mathbf{u} = [\dot{P}_t, \dot{\omega}_e, \dot{\tau}_e]. \quad (92)$$

When the AMT powertrain configuration is used, the engine speed and engine speed rate control and state are removed, and an additional parameter set is optimised (namely those shifting speeds in (60)):

$$\mathbf{p} = [u_1, u_2, u_3, u_4]. \quad (93)$$

The complete set of integral constraints is formed from (84) and (86)–(89).

C. Discontinuous Functions

Discontinuous functions pose a difficulty for the solution of the NLP due to the loss of gradient information. One way to overcome this problem is to break it down into multiple phases—distance-dependent speed limits are an example of relevance here. Another method is to ‘smooth’ the problem by replacing discontinuous functions with smooth approximations. For example, the $\max(x, 0)$ function in (62) has an undefined gradient at $x = 0$. A number of *sigmoid*-like approximations can be used to overcome this difficulty [20]. In this paper, the hyperbolic tangent function is chosen to approximate the $\max(x, 0)$ function as follows:

$$y = \max(x, 0) \approx \frac{x}{2} (1 + \tanh(kx)). \quad (94)$$

The derivative is well defined over the whole domain as

$$\frac{\partial y}{\partial x} \approx \frac{1}{2} (1 + \tanh(kx) + kx \text{sech}^2(kx)). \quad (95)$$

and at $x = 0$, $\frac{\partial y}{\partial x} = 1/2$. As $k \rightarrow \infty$, the RHS approaches the original $\max(x, 0)$ function. For the computations presented here, k takes values in the range $1 \leq k \leq 10$.

VI. RESULTS

We will now present fuel minimization results for a number of scenarios that represent ‘real’ driving situations. These will begin with a comparison of gearbox types and the optimization of the gearbox change speeds. The sensitivity of the solution to changes in emissions constraints will then be studied. Finally the influence of enforced on-route stops will be analysed. The

solutions use optimized driver and engine controls. The test route (see Figure 2) is broken down into twelve phases, each containing a different legislated speed limit. There are also fifteen potential enforced stopping points, which are shown on Figure 12.

A. Power Transmission Comparison

In the first study, fuel consumption is minimized for a maximum arrival time of $T = 1800$ s. The emissions constraints given in (86)–(89) are omitted. This problem is solved for both CVT- and AMT-equipped vehicles. In the CVT case the engine speed and torque are optimised. In the AMT case the engine torque and gearbox shifting speeds (u_i in Eq. (60)) are optimised.

The optimal speed and tractive power for each transmission is plotted in Figure 13. Engine power, and thus fuel, is required to ascend the hills, accelerate out of corners, and increase the vehicle's speed when the legislated speed limit increases. Braking is needed to slow the car to negotiate corners at 7km and 25km for example. Noticeable is the minimal use of the highest speed limits, which are avoided due to the low-speed turns at the end of each segment. In contrast to the example given in Section V, the maximum available power is not utilised on the route. Constraints on the maximum longitudinal acceleration and the relatively low speed limits obviate the need for maximum power.

Optimal engine operating points are compared in Figure 14. This clearly highlights the difference in the engine usage for the different transmission types. In the case of a CVT, the line associated with minimum fuelling are shown as circular scattered points.

For the AMT, the engine speed is constrained by the gear selected and the forward speed of the vehicle. This leaves the torque as the only free variable—this results in a more scattered set of engine operating points; see the crosses in Figure 14. The engine operating region is determined by the (optimized) gearbox switching speeds. The total FC for the CVT is 657 g compared to the AMT which used a total of 601 g. Despite the CVT operating along the locus of highest fuel efficiency, the increased mechanical efficiency of the AMT results in superior fuel economy.

Figure 15 compares the optimized gearing for each transmission. At the beginning and end of the route the CVT ratio is considerable higher than the AMT. This gives rise to increased acceleration for the CVT equipped vehicle. Also, the improved gearing offered by the CVT at higher speeds, ensures that lower engine speeds, in combination with higher torques, can be used to reduce the fuel consumption.

The regularisation cost (see (91)) accounts for approximately 0.05% of the total cost in both cases. The computation time for the CVT case is approximately 550 s, whereas for the AMT, this increases to approximately 2800 s. Additional processing time is needed due to the discontinuous nature of the engine idle speed constraint, and optimising the gear-speed shifting parameters given in (60). Both cases are initialised using the same mesh, consisting of 10 mesh intervals per phase, each with 2 collocation points. The CVT problem is solved with 21 mesh iterations, increasing the total number of mesh intervals to 186 (and 1054 collocation points). Coincidentally, the AMT problem also requires 21 mesh iterations, but ultimately exits with a total of mesh 213 intervals with 1205 collocation points. While the solution of the optimal control problem is simplified by neglecting the discrete nature of the

Fig. 13: Optimal velocity and power profiles for CVT (solid) and AMT (dotted) transmission systems. Legislative and curvature-related speed limits are plotted as a dashed black line on the velocity figure. The route elevation is plotted as a dot-dash curve on the power figure. Individual phases are represented by alternating light grey regions.

Fig. 14: Scatter of optimal engine operating points on the BSFC [g/kWh] map. The CVT results are plotted as circles with the AMT plotted as crosses.

gearbox, too much information regarding engine operation and emissions generation is lost. For the remainder of this study the vehicle is considered equipped with a discrete AMT, which makes use of the optimised gear-change speeds found here.

B. Emissions Sensitivity

This section considers the impact of enforcing the constraints on the emissions generated en route (see inequalities (86)–(89)). The gear-shift speed-change law is fixed to the minimum fuel solution found in the previous section. In this study each emissions type is constrained in turn to evaluate the sensitivity of the FC and optimal driving strategy for each case.

Figure 16 shows the effect of limiting the emissions components one-by-one to 90% of their unconstrained values. It is important to note that further restrictions on the emissions is likely to make the optimisation problem infeasible in some cases. With the ‘tight’ arrival time constraint being imposed, there is little scope for altering the power requirements. Consequentially, it is expected that only small changes in FC are achievable (also shown in Figure 16).

The first optimisation considers a limit of 90% of the unconstrained NO_x (black bars in Figure 16). A negligible increase in the FC is noted (as it is for the other cases in this section). The total CO and HC decreases by 2.5% and 2.9% respectively, while the PM increases by 5.6%. The increase in PM follows from the fact that the BS- NO_x and BS-soot maps have complementary minimum regions (see Figures 8 and 9),

Fig. 15: Gearbox ratios for CVT (solid) and the AMT (dotted).

and the generation of PM is dominated by the soot produced (85).

In the second optimisation case, a limit of 90% of the unconstrained CO is enforced (dark grey in Figure 16). Total NO_x is increased by 2.0%, while the PM decreases by 0.9%, and HC decreases by 10.8%. The coincidence of the regions of low BS-HC and BS-CO accounts for the large decrease in HC seen here.

Imposing a limit on the total HC (light grey in Figure 16) has the least effect on the FC (an increase of only 0.03%) implying that the region in which the engine is operated changes very little. This small difference results in changes of -0.6% in the total PM produced, and a change in the NO_x produced of +0.6%. This slight shift in engine operating point is enough to reduce the CO by 6.7%, which is consistent with the limited CO case.

The final case constrains the PM to 90% of that obtained in Section VI-A (the white bars in Figure 16). The large 10.0% increase in the NO_x is expected as discussed earlier. Abatements of 3.3% in CO, and 0.7% in HC, are also observed, which correlates with a move in engine operation to the higher power areas in the BS-maps.

In the results presented so far, behavioural changes in the route emissions have been explained by examination of BS-emissions maps. To complete the analysis it is necessary to consider not only the regions in which the engine is operated, but also how the power is utilised in these regions during driving. A comparison of the effects of the emissions constraints on driving, in terms of optimal tractive power and speed profiles, for each constrained optimisation are shown in Figure 17. In order to reduce the total NO_x output, the tractive power is kept below 20 kW. This reinforces the idea of avoiding the highest regions of BS- NO_x that occur at high power. To ensure that the speed is increased during highway driving, power is applied in small bursts in the 15 km to 16 km stretch.

When the PM constraint is introduced the application of power changes significantly. As with the restricted NO_x case, the optimal tractive power includes rapid changes between high and low engine powers. High power engine operation

Fig. 16: Total percentage changes in each of: fuel, NO_x , CO HC and PM for each of the constrained optimisations.

Fig. 17: Segments of the emissions-constrained optimal driving solution for the 5km—8km section, and the highway driving section 14km—18km. The baseline fuel-optimal solution is plotted in red. The 90% constraints cases on NO_x , CO, HC and PM are shown, respectively, as dot-dash, dotted, solid, and dashed.

is beneficial to soot reduction, but the speed limits have to be adhered to.

During the 6.25 km to 7 km period of driving, the opposing requirements of the PM and NO_x solutions can be observed. The application of high power is followed by a change to lower power in the PM constrained case. Whereas a longer and smoother application of power occurs in the NO_x limited case. Despite the limited HC and CO being similar in terms of their total outputs, the method of achieving this outcome requires different driving strategies. The limited HC case requires a driving strategy which is almost identical to the baseline. This corresponds to the negligible change in FC (see Figure 16). The CO-constrained case includes periods of zero engine power during which no emissions are produced. In this case it is preferable to coast for periods, rather than drive the engine hard as in the PM-limited case.

The results in this section illustrate the need to optimise simultaneously the driving strategy, and the engine operation to abate emissions production. Further reductions may be possible by relaxing the arrival time constraint, thereby allowing for more varied power and velocity profiles.

C. Enforced Stops

One can expect to encounter enforced stops along the route at intersections and give ways, which will impact on the FC and the production of emissions. These additional boundary conditions can be included in the problem by dividing the route into additional phases. In the stop-start study case adjacent phases may have the same speed limit, but are necessarily separated by the zero velocity boundary condition (83). The potential stopping places around the Oxford route are shown as a series of fifteen crosses in Figure 12 (the start-finish point is not counted). Traffic conditions, and accidents of timing, mean

TABLE II: The stops used in each of the comparison runs.

Run	Stop No.														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	*		*						*	*	*	*	*	*	
2						*	*		*			*		*	
3	*		*			*	*	*	*	*	*	*	*	*	*
4					*	*	*	*	*			*	*	*	*
5				*					*	*				*	*
6	*	*	*	*	*				*	*			*	*	*
7		*	*	*	*	*		*	*	*	*	*	*	*	*
8	*		*	*	*	*	*	*							
9	*	*	*	*	*	*				*		*	*	*	*
10	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
11	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*

that there is a degree of uncertainty associated with negotiating an urban road system. To investigate these influences on the FC and emissions, ten different sets of stops will now be studied as shown in Table II.

Enforced stops can influence the achievable route transit time and consequently the arrival time constraint must thus be relaxed to ensure that the route can be covered. The minimum arrival time for the route in the case that each enforced stop is respected is approximately 1850 seconds. We will therefore use an arrival time constraint of 2000 s (rather than the previous 1800 s).

Once again only the AMT will be considered as it reflects the more realistic driving scenario. As in the non-stopping unconstrained case with an AMT, the arrival time constraint is active, but increased to 2000 s. Clearly, even if the number of stops is increased, a minimum average velocity must be maintained. This results in increased tractive power effort and increased fuel consumption. The car must be accelerated more frequently to counteract the increased number of braking phases. In the case that there are no stops, the car does not

reach the speed limit between 6km and 11.5km, and between 14km and 23km. Power is applied more smoothly and rapid acceleration can be avoided. The results of three optimisations are presented in Figure 18.

The effect of including stopping events on the total route emissions and fuel consumption is shown in Figure 19. The inclusion of stopping events necessarily increases the number of braking events, as well as the need for higher peak speeds. This, in turn, leads to greater fuelling requirements due to increased numbers of prolonged applications of power. Increased power usage results in reduced HC—higher combustion temperatures remove some HC products. Total NO_x increases with the number of stops, this is once again linked to the higher engine power usage.

VII. CONCLUSIONS

This paper details an effective method for utilising route information in combination with state of the art engine modelling techniques to simultaneously optimise engine usage and driving style. 3D route information can be transformed into route geodesic curvature and elevation information, which is then utilised in a minimum fuel OCP. Gaussian process models for engine flow rate quantities form a set of integral constraints. Models of this form have the advantage of being accurate under interpolation and are good at capturing non-linear behaviours. A multi-phase optimal control problem is posed and solved, which represents real world driving situations for vehicles equipped with either continuously variable, or automated manual transmissions. Driving style changes are evident whenever new constraints are posed on the system. Fixed drive cycle testing lacks the flexibility to properly optimise FC and emissions predictions, which could result in poor engine calibration. While there is little flexibility for changing the total fuel consumption when the arrival time is tight, the production of emissions can change significantly with

driving style variations. If the arrival-time constraint is relaxed, greater flexibility in driving strategy can reduce emissions still further. It is not sufficient to ignore on-route stopping, because stopping causes a significant increase in FC, NO_x , CO, and PM.

The complex shape of the engine emissions maps, and the discrete nature of the gearbox, makes these studies computationally demanding. While this work is not directed specifically at real-time applications, it does have a role in providing ‘what’s possible’ benchmarking information in the development of real-time schemes such as receding horizon controllers. Powertrain configurations, which reflect new technologies, will be the subject of further work. In a move to reduce the number of diesel vehicles on the road, gasoline-electric hybrids offer one well known alternative. The flexible optimisation framework presented in this work allows easily for different powertrain configurations, for both gasoline and diesel internal combustion engines, as well as longer routes and greater route complexity.

VIII. ACKNOWLEDGMENT

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Fig. 18: Optimal velocity and power profiles for zero stops (black dotted), 7 random stops (blue solid) and all stops (red dot-dashed). Velocity limits are plotted as dashed black lines.

Fig. 19: Percentage changes in the total masses of fuel, NO_x, CO, HC and PM relative to driving without stopping.

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