A novel kinematically redundant planar parallel robot manipulator with full rotatability

Article (Accepted Version)

Baron, Nicholas, Philippides, Andy and Rojas, Nicolas (2018) A novel kinematically redundant planar parallel robot manipulator with full rotatability. *Journal of Mechanisms and Robotics*, 11 (1). 011008 1-8. ISSN 1942-4302

This version is available from Sussex Research Online: <http://sro.sussex.ac.uk/id/eprint/81840/>

This document is made available in accordance with publisher policies and may differ from the published version or from the version of record. If you wish to cite this item you are advised to consult the publisher's version. Please see the URL above for details on accessing the published version.

Copyright and reuse:

Sussex Research Online is a digital repository of the research output of the University.

Copyright and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable, the material made available in SRO has been checked for eligibility before being made available.

Copies of full text items generally can be reproduced, displayed or performed and given to third parties in any format or medium for personal research or study, educational, or not-for-profit purposes without prior permission or charge, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

ASME Accepted Manuscript Repository

Institutional Repository Cover Sheet

Nicholas Baron
First Last

ASME Paper Title: A Novel Kinematically Redundant Planar Parallel Robot Manipulator With Full Rotatability

Authors: Nicholas Baron, Andrew Philippides and Nicolas Rojas

ASME Journal Title: Journal of Mechanisms and Robotics

Volume/Issue 11/1 Date of Publication (VOR* Online) 13/11/2018

ASME Digital Collection URL: <http://mechanismsrobotics.asmedigitalcollection.asme.org/article.aspx?articleid=2707516>

DOI: 10.1115/1.4041698

*VOR (version of record)

A Novel Kinematically Redundant Planar Parallel Robot Manipulator with Full Rotatability

Nicholas Baron¹, Andrew Philippides¹, and Nicolas Rojas²

¹School of Engineering and Informatics, University of Sussex, Brighton BN1 9RH, UK
{n.baron, andrewop}@sussex.ac.uk

²Dyson School of Design Engineering, Imperial College London, London SW7 2DB, UK
n.rojas@imperial.ac.uk

This paper presents a novel kinematically redundant planar parallel robot manipulator which has full rotatability. The proposed robot manipulator has an architecture that corresponds to a fundamental truss, meaning that it does not contain internal rigid structures when the actuators are locked. This also implies that its rigidity is not inherited from more general architectures or resulting from the combination of other fundamental structures. The introduced topology is a departure from the standard 3-RPR (or 3-RRR) mechanism on which most kinematically redundant planar parallel robot manipulators are based. The robot manipulator consists of a moving platform that is connected to the base via two RRR legs and connected to a ternary link, which is joined to the base by a passive revolute joint, via two other RRR legs. The resulting robot mechanism is kinematically redundant, being able to avoid the production of singularities and having unlimited rotational capability. The inverse and forward kinematics analysis of this novel robot manipulator are derived using distance-based techniques, and the singularity analysis is performed using a geometric method based on the properties of instantaneous centres of rotation. An example robot mechanism is analysed numerically and physically tested; and a test trajectory where the end-effector completes a full cycle rotation is reported. A link to an online video recording of such a capability, along with the avoidance of singularities and a potential application, is also provided.

1 Introduction

Planar parallel robot manipulators are used in a variety of different applications due to their strong load-carrying capabilities, precision, speed and accuracy; common applications include machine tooling and positioning devices [1]. However, one of the disadvantages of these robotic systems is that they have, in general, a limited range of rotational capability whereas their serial counterparts, on the other hand, do not. Full rotatability of an end effector is a desirable feature as this implies a larger workspace and better dexterity than those devices without this characteristic, it can be argued that this is indeed one of the reasons why serial manipulators

are more commonly used in industrial applications than their parallel counterparts. The cause of the limited rotatability of planar parallel robot manipulators is the encountering of mechanical interferences or singularities in full cycle trajectories.

The determination of singular configurations is a problem that has been studied greatly in the literature and many approaches to solving it have been proposed [2, 3]. The term *singularity* herein refers to the so-called forward kinematics singularities; these are particular configurations of parallel robot mechanisms where different solutions of the forward kinematics problem meet [4]. From the point of view of robot control, the problem of singularities is that parallel mechanisms that must be rigid when the actuators are locked, lose this characteristic in those particular configurations and the performance of the robot manipulator deteriorates as they are approached. A common solution to avoid singularities, and to increase the size of the robot workspace, is to redundantly actuate the parallel robot manipulator; this involves either replacing one or more of the passive joints in one of the legs with actuated joints or adding one or more additional actuated legs to the mechanism [5]. However, the problem with this solution is that the additional actuators or legs generate unnecessary internal forces or moments on the platform—since they are not required to make the robot manipulator rigid—which have to be compensated using advanced controllers and force sensors [6, 7, 8].

Another standard approach, usually called kinematic redundancy, is to introduce additional actuated joints to the existing legs of a non-redundant parallel robot manipulator [9, 10]. This type of redundancy enables the position and orientation of the platform to be set while, concurrently, the additional degrees of freedom provided by the added actuators and links can be used to avoid singular configurations and increase the workspace of the mechanism [11]. Nevertheless, this approach results in devices with serially connected actuators, what falls apart the parallel architecture, and the corresponding advantages, of the original manipulator; for example, the new architecture would be subject to the accumulation of actuator errors along the limbs with actuators connected in this manner. Moreover, despite the in-

crease of the size of the workspace, full rotatability is not achieved.

A different approach to redundancy of parallel robot manipulators for alleviating their limited rotational capability has been recently introduced [12, 13]. This strategy consists in using parallel architectures with more actuators than required by the task workspace, but which all contribute to make the robot manipulator rigid when locked; robot manipulators resulting from this approach are called kinematically redundant. Following this idea, a parallel architecture with unlimited rotational capability was proposed for the first time in [12]; this kinematically redundant robot manipulator is composed of two $\underline{R}\underline{P}\underline{R}$ legs connected to a common joint on the platform, along with two other similar legs connected to a revolute joint that is then connected to a second common joint on the platform. A rigid structure is obtained from this robot manipulator if and only if all four actuators are simultaneously locked. This resulting structure corresponds to three triads connected in cascade, which implies that the robot manipulator is not a fundamental truss [14]. A parallel robot manipulator architecture corresponds to a fundamental truss if it does not exhibit internal rigid structures, beyond local elements of single limbs, when the actuators are locked, such that its rigidity is not inherited from a more general architecture or resulting from the combination of other fundamental structures. Thus, for instance, the standard 3- $\underline{R}\underline{R}\underline{R}$ parallel robot manipulator architecture is a fundamental truss, but the 4- $\underline{R}\underline{P}\underline{R}$ redundant manipulator is not. The underline in this convention means that the corresponding joint is actuated.

It has been claimed that generic architectures of kinematically redundant planar parallel robot manipulators—as those corresponding to cases where the robot manipulator’s architecture is a fundamental truss—cannot achieve unlimited rotation capability [13]. However, in this paper, it is shown that this is not the case; a novel fundamental kinematically redundant architecture with such a characteristic is introduced. The fundamental topology of this robot manipulator implies that, just like the 3- $\underline{R}\underline{P}\underline{R}$ architecture, it can be used as a basis upon which future mechanisms can be developed and future research can be conducted. The proposed robot manipulator architecture consists of a moving platform connected to the base via four $\underline{R}\underline{R}\underline{R}$ legs and a ternary link, which is joined to the ground link by a revolute joint, via two other $\underline{R}\underline{R}\underline{R}$ legs. The robot manipulator is kinematically redundant as its degree of mobility (four) is the same as the number of actuated joints; and this value exceeds the number of degrees of freedom required to describe a pose of the end effector (three). The redundancy allows any pose to be attained within the workspace of the robot manipulator without producing a singularity and the novel architecture does not present mechanical interferences in full cycle trajectories; thus resulting in unlimited rotational capabilities of the end-effector. Although the architecture is similar to that in [13], it is novel as the two architec-

Figure 1: Kinematic diagram of the proposed robot mechanism. The architecture consists of a moving platform connected directly to the base via two $\underline{R}\underline{R}\underline{R}$ legs and connected to a ternary link, which is joined to the base by a passive revolute joint, via two other $\underline{R}\underline{R}\underline{R}$ legs.

tures represent different kinematic chains – if the corresponding graphs of the two mechanisms are compared it can be seen that they are not isomorphic. A method of identifying singular configurations and reconfiguring the robot manipulator such that they are avoided is presented in [15].

The rest of this paper is structured as follows. In section 2, the general architecture of the proposed kinematically redundant parallel robot manipulator is described and the mobility of the mechanism is calculated, along with a discussion of its fundamental characteristic. Section 3 presents the set of equations for solving the inverse kinematics of the robot manipulator using the bilateration method. This technique is then used in section 4 for solving the corresponding forward kinematics problem in closed form. The singularity analysis of the robot manipulator is performed in section 5 using a geometric technique based on the properties of instantaneous centres of rotation. In section 6 a test trajectory which demonstrates the robot manipulator’s rotational capability is tested using an example mechanism, the results of this experiment are then presented and discussed—a link to an online video recording of a full rotation, avoidance of singularities, and a potential application is also provided. Finally, we conclude in section 7.

2 Robot Architecture

The robot architecture, as exemplified in the instance shown in Fig. 1, consists of a moving platform ($P_{10}P_{11}$) that is connected to the base ($P_1P_2P_3$), and one ternary link ($P_3P_4P_5$), via four $\underline{R}\underline{R}\underline{R}$ legs; where $\underline{R}$ denotes a passive revolute joint and $\underline{R}$ denotes an actuated revolute joint. The moving platform is connected to the base, or ground link, directly via two of the legs, and to the ternary link via the other two legs. The ternary link

is connected to the base via a passive revolute joint and the legs are attached to the ternary link and the base via actuated revolute joints. Two of the legs are attached to a common passive revolute joint on the moving platform and the other two are connected to another common passive joint on it. The actuators are not serially connected and although two of them are attached to the ternary link, what increases the inertial properties of this part, the mechanism does not suffer from the accumulation of actuator errors along the limbs.

The proposed robot mechanism is, in general, rigid when the four actuators are locked; meaning that the links are unable to move with respect to the base or each other. This can be shown, for instance, by calculating its structural mobility, M , via the extended Chebychev-Kutzbach-Grübler formula [16]. According to this criterion, the structural mobility of a mechanism is

$$M = F - \sum_{i=1}^{\lambda} t_i \quad (1)$$

where $\lambda = J - L + 1$ is the number of independent closed-loops in the kinematic chain and t_i is the motion type of the i^{th} independent closed-loop ($t_i = 3$ in the planar case), with J , the total number of joints, L , the number of links, and F , the total number of degrees of freedom of the joints. Since the proposed architecture consists of 13 joints (counting twice the ternary joints of the platform), 11 links (including the base) and 13 degrees of freedom (as each revolute joint has one degree of freedom), its resulting structural mobility is 4. This result implies that in order for the mechanism to be rigid (*i.e.*, to have a mobility of zero), four of the joints need to be actuated. It is known that the structural mobility, which is a function only of structural parameters, is a lower bound of the total mobility of a mechanism; however, it has been proven that if M is computed using the extended Chebychev-Kutzbach-Grübler formula, it is unlikely that the structural mobility is different to the total mobility when a kinematic chain is selected at random [17].

The proposed architecture is fundamental, which implies that the robot manipulator, once the actuators are locked, does not exhibit rigid sub-structures beyond sub-components in a single leg. Thus, the rigidity of a fundamental parallel robot manipulator is not inherited from a more general architecture or resulting from the combination of other fundamental structures. For the case of the proposed robot mechanism, this can be proven by systematically analysing the kinematic chains formed by subsets of the set of joints, taking into account that rigid elements of an RRR leg do not contribute to general rigidity since these limbs are equivalent to an RPR leg when the actuators are activated, that is, they can be modelled as a line segment connecting the centres of the two end revolute joints. Since the introduced robot mechanism has three independent loops, $\lambda = 3$, there are only two fundamental structures that could be present, namely, a triad (*i.e.* a one-loop structure com-

posed of three links connected by revolute joints, $\lambda = 1$) or a pentad (*i.e.* a two-loop structure composed of two ternary links connected between them by binary links, all of them jointed by revolute kinematic pairs, $\lambda = 2$). Neither triads nor pentads that contribute to general rigidity are detected in the proposed robot mechanism.

In the instance of the introduced kinematically redundant planar parallel manipulator that is shown in Fig. 1, the robot manipulator is designed such that the end-effector is able to complete full rotations without encountering mechanical interferences. Moreover, the link which is the upper component of the left leg connected to the ternary link, the shortest link, is made to be able to complete a full rotation with respect to the platform; this characteristic is vital for the process of avoiding singularities as it is further discussed in section 5.

3 Inverse Kinematics

The inverse kinematics problem refers to the determination of the required values of the actuated joints in order to produce a given pose of the moving platform. Fig. 1 shows a schematic of the proposed architecture where the robot manipulator is depicted in terms of the centres of rotation of its kinematic pairs (joints) and the line segments connecting them (links); each centre has been labelled, from P_1 to P_{11} , and the sought values of the actuated joints are θ_1 to θ_4 . Since this mechanism is kinematically redundant with one extra degree of freedom, there are an infinite number of solutions to the inverse kinematics. However, if an additional condition is set, such as the orientation of the link defined by P_8 and P_{10} , to name one, then the number of solutions reduces to a finite number. The orientation of this link is given by α – this angle has been chosen to control the redundancy as opposed to, say, the orientation of the ternary link because it makes the method of singularity avoidance more straightforward. With this condition set, the positions of the joints can be found using, for instance, analytic geometry and trigonometric relations; here the bilateration method is used instead.

The bilateration method consists of finding the possible positions of an unknown point, P_k , if the distances between this point and two points whose positions are known, P_i and P_j , are known. $\mathbf{p}_{i,k}$, which is the vector going from P_i to P_k , is found by taking the matrix-vector product between the bilateration matrix, $\mathbf{Z}_{i,j,k}$, and $\mathbf{p}_{i,j}$, the vector going from P_i to P_j [14]. That is,

$$\mathbf{p}_{i,k} = \mathbf{Z}_{i,j,k} \mathbf{p}_{i,j} \quad (2)$$

where

$$\mathbf{Z}_{i,j,k} = \frac{1}{2s_{i,j}} \begin{bmatrix} s_{i,j} + s_{i,k} - s_{j,k} & -4A_{i,j,k} \\ 4A_{i,j,k} & s_{i,j} + s_{i,k} - s_{j,k} \end{bmatrix}$$

and

$$A_{i,j,k} = \pm \frac{1}{4} \sqrt{(s_{i,j} + s_{i,k} + s_{j,k})^2 - 2(s_{i,j}^2 + s_{i,k}^2 + s_{j,k}^2)},$$

with $s_{i,j} = d_{i,j}^2$ denoting the squared distance between the points P_i and P_j and $A_{i,j,k}$, the orientated area of the triangle defined by points P_i , P_j and P_k . The $\pm$ sign implies that $\mathbf{p}_{i,k}$ can point in one of two different directions; when positive, $\mathbf{p}_{i,k}$ points to the left of $\mathbf{p}_{i,j}$ and, when negative, it points to the right.

According to the notation of Fig. 1, the desired position and orientation of the platform can be represented by P_{10} and ϕ . Then, the vector $\mathbf{p}_{10,11}$ can be computed as

$$\mathbf{p}_{10,11} = d_{10,11} \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}, \quad (3)$$

where ϕ is the angle between the platform and the x-axis. By setting the value of α , the position of P_8 is found using basic trigonometry. The positions of the remaining points are found by applying the bilateration method, using equation (2). The position of P_4 is determined using the bilateration matrix $\mathbf{Z}_{3,8,4}$ and selecting an orientation of $\mathbf{p}_{3,4}$ by choosing the sign of $A_{3,8,4}$. P_5 is obtained from $\mathbf{Z}_{3,4,5}$ and since the orientation of $A_{3,4,5}$ is known, P_5 has a definite position. P_6 , P_7 and P_9 are determined with $\mathbf{Z}_{10,1,6}$, $\mathbf{Z}_{11,2,7}$ and $\mathbf{Z}_{5,11,9}$, and by selecting the orientations of their respective areas.

The above procedure computes the location of all joint centres. The values of the actuated joints, that is, the angles θ_1 , θ_2 , θ_3 and θ_4 , can then be computed using the arccosines of $\frac{\mathbf{p}_{1,6} \cdot [1,0]^T}{d_{1,6}}$, $\frac{\mathbf{p}_{2,7} \cdot [1,0]^T}{d_{2,7}}$, $\frac{\mathbf{p}_{4,8} \cdot \mathbf{p}_{4,5}}{d_{4,8}d_{4,5}}$, and $-\frac{\mathbf{p}_{5,9} \cdot \mathbf{p}_{5,4}}{d_{5,9}d_{4,5}}$, respectively.

4 Forward Kinematics

The forward kinematics problem consists of finding the feasible Cartesian poses of the moving platform once the actuators are fixed at particular values. A common method for solving this problem is to formulate the characteristic polynomial of the mechanism, which involves manipulating the kinematic equations of the system so that a single equation in terms of one variable is formed—this is usually called a closed-form solution [18]. Solving this polynomial gives, or leads to obtain, the feasible poses of the platform given the known geometric parameters, such as link lengths, and the actuator values. Additionally, the degree of the polynomial shows the maximum number of solutions to the forward kinematics. For example, when the actuators of the 3-RPR robot manipulator are locked a sextic polynomial is obtained, thus implying that up to 6 different configurations can be calculated; a proof of this feasible number of solutions is given in [19]. In this paper the bilateration method is used for formulating the characteristic polynomial of the proposed kinematically redundant planar parallel robot manipulator. To this end, the equivalent kinematic model shown in Fig. 2 is used. This model results from the fact that once the actuators are fixed at a given value, each of the RRR legs of the parallel robot manipulator can be represented by a line segment of known distance that connects the centres of the two end revolute joints. Looking back to the model pre-

Figure 2: Equivalent kinematic model used for solving the forward kinematics; it corresponds to the mechanism obtained when the robot actuators are fixed at particular values. This model also applies for a robot manipulator with type RPR legs.

sented in Fig. 1, once the values of θ_1 , θ_2 , θ_3 and θ_4 are fixed, the distances $d_{3,6}$, $d_{3,7}$ and $d_{6,7}$ can be calculated, as can $d_{3,8}$, $d_{3,9}$ and $d_{8,9}$; resultantly, P_3 , P_6 and P_7 , and P_3 , P_8 and P_9 form two triads. As these two triads are rigid structures, the robot manipulator can be modelled, when the actuator values are fixed, by the diagram shown in Fig. 2, where the triads represent the base and the ternary link, respectively, and the lower links of the legs to which they are attached. The moving platform is joined to these triads via four single links with passive joints at each end; which represent the upper links of the legs. Note that this equivalent model is also applicable to a mechanism with type RPR legs. Now, instead of directly calculating the Cartesian pose of the platform, the bilateration method is used firstly to determine the set of values of an unknown squared distance of the system, such as $s_{6,8}$, according to the notation of Fig. 2, that are compatible with the known geometric parameters of the mechanism. Following this approach, the characteristic polynomial is obtained as follows.

Firstly, using a sequence of bilaterations, an equation is formed which computes a single vector between two points whose distance is known, in this case $\mathbf{p}_{10,11}$, in terms of one of the two vectors that result from the unknown squared distance used as variable, in this case $\mathbf{p}_{6,8}$. This vector equation has to take into account all distance constraints in the mechanism. Thus, the fol-

lowing system of equations is obtained.

$$\mathbf{p}_{6,10} = \mathbf{Z}_{6,8,10}\mathbf{p}_{6,8}, \quad (4)$$

$$\mathbf{p}_{6,7} = \mathbf{Z}_{6,3,7}\mathbf{Z}_{6,8,3}\mathbf{p}_{6,8}, \quad (5)$$

$$\mathbf{p}_{6,9} = (-\mathbf{Z}_{8,3,9}\mathbf{Z}_{8,6,3} + \mathbf{I})\mathbf{p}_{6,8}, \quad (6)$$

$$\mathbf{p}_{7,9} = \mathbf{p}_{6,9} - \mathbf{p}_{6,7}, \quad (7)$$

$$\mathbf{p}_{7,11} = \mathbf{Z}_{7,9,11}\mathbf{p}_{7,9}, \quad (8)$$

$$\mathbf{p}_{6,11} = \mathbf{p}_{6,7} + \mathbf{p}_{7,11}, \quad (9)$$

$$\mathbf{p}_{10,11} = \mathbf{p}_{6,11} - \mathbf{p}_{6,10}, \quad (10)$$

with $s_{i,j} = \|\mathbf{p}_{i,j}\|^2$ and $\mathbf{I}$ denoting the 2-by-2 identity matrix, and taking into account that the orientation of the triangles defined by the revolute centres P_6 , P_7 , and P_3 ; and P_3 , P_8 , and P_9 is known. Rewriting the above system of equations in terms of $\mathbf{p}_{6,8}$ we obtain

$$\mathbf{p}_{10,11} = \mathbf{Q}\mathbf{p}_{6,8}, \quad (11)$$

where

$$\mathbf{Q} = (-\mathbf{Z}_{6,8,10} + \mathbf{Z}_{6,3,7}\mathbf{Z}_{6,8,3} + \mathbf{Z}_{7,9,11}(-\mathbf{Z}_{6,3,7}\mathbf{Z}_{6,8,3} - \mathbf{Z}_{8,3,9}\mathbf{Z}_{8,6,3} + \mathbf{I})).$$

Then, by the scaling property of bilateration matrices, we get that

$$\det(\mathbf{Q}) = \frac{s_{10,11}}{s_{6,8}}. \quad (12)$$

By eliminating the square roots involved in equation (12), a 14th-degree characteristic polynomial in terms of $s_{6,8}$ is finally obtained. The real roots of this polynomial correspond to the compatible values of $s_{6,8}$ for the geometric parameters and actuator values of the robot manipulator.

Finally, the feasible assembly modes of the parallel manipulator can be computed, for instance, by substituting the real values of $s_{6,8}$ into (12) along with each possible combination of orientations for the orientated areas $A_{6,8,10}$, $A_{6,8,3}$, and $A_{7,9,11}$; if the equation holds, then the corresponding assembly mode is feasible. Then for each of the detected assembly modes, a reference frame is introduced and the positions of the base joints, P_6 , P_7 and P_3 are designated. The resulting configuration from the positions of the remaining joints are then found by computing the following sequence of bilaterations

$$\mathbf{p}_{6,8} = \mathbf{Z}_{6,3,8}\mathbf{p}_{6,3}, \quad (13)$$

$$\mathbf{p}_{3,9} = \mathbf{Z}_{3,8,9}\mathbf{p}_{3,8}, \quad (14)$$

$$\mathbf{p}_{6,10} = \mathbf{Z}_{6,8,10}\mathbf{p}_{6,8}, \quad (15)$$

$$\mathbf{p}_{7,11} = \mathbf{Z}_{7,9,11}\mathbf{p}_{7,9}. \quad (16)$$

The sign of $A_{6,3,8}$ is the opposite of the sign of $A_{6,8,3}$.

As an example of the method described above; consider the mechanism with link lengths $d_{6,7} = 2$, $d_{3,6} = \sqrt{2}$, $d_{6,10} = \sqrt{17}$, $d_{3,7} = \sqrt{2}$, $d_{7,11} = \sqrt{17}$, $d_{7,8} = \sqrt{2}$, $d_{7,9} = \sqrt{5}$, $d_{8,9} = \sqrt{5}$, $d_{8,10} = \sqrt{5}$, $d_{9,11} = \sqrt{2}$ and $d_{10,11} = 4$, the following base joint positions: $P_6 = (2, 0)^T$, $P_7 = (4, 0)^T$ and $P_3 = (3, 1)^T$, and with the

Figure 3: Resulting configurations of the example used in the forward kinematic analysis.

oriented area $A_{3,8,9}$ being negative. The following characteristic polynomial is then obtained.

$$\sum_{i=0}^{14} k_i s_{6,8}^i, \quad (17)$$

where $k_0 = 6.11 \times 10^{17}$, $k_1 = -2.39 \times 10^{17}$, $k_2 = -3.41 \times 10^{16}$, $k_3 = 3.86 \times 10^{16}$, $k_4 = -1.00 \times 10^{16}$, $k_5 = 5.54 \times 10^{14}$, $k_6 = 4.42 \times 10^{14}$, $k_7 = -1.66 \times 10^{14}$, $k_8 = 3.08 \times 10^{13}$, $k_9 = -3.51 \times 10^{12}$, $k_{10} = 2.72 \times 10^{11}$, $k_{11} = -1.59 \times 10^{10}$, $k_{12} = 7.24 \times 10^8$, $k_{13} = -2.22 \times 10^7$ and $k_{14} = 3.20 \times 10^5$. The real roots of this polynomial are 4 and 5.04. The values of these roots, and the coefficients in the polynomial, are given to 2 decimal places. The resulting configurations of this example are depicted in Fig. 3.

5 Singularity Analysis

It is well known that the singularities of a standard 3-RPR mechanism can be determined geometrically by finding the configurations in which the lines that pass through the three legs of the robot manipulator intersect at a common point. In this section, a similar set of geometrical conditions are developed in order to determine if the proposed mechanism is in a singular configuration.

Singular configurations are those in which a mechanism of mobility zero ($M = 0$), which is generally rigid, loses its rigidity; this implies multiple problems for parallel robot manipulators such as loss of controllability and large actuation forces. The most commonly used method of identifying if a parallel robot manipulator is in a singular configuration is by formulating the relationship between the Cartesian velocities and the joint velocities of the robot manipulator in terms of Jacobian matrices; the robot manipulator is considered to be in a singular configuration when these matrices are not of full rank [12].

In this paper, the method used to determine if the robot manipulator is in a singular configuration is based on the properties of instantaneous centres of rotation (ICRs). The benefit of carrying out the singularity analysis by using this approach is that it gives a geometrical interpretation of the conditions which lead to the the production of a singularity, as opposed to a purely

Figure 4: Kinematic diagram of 3-RPR mechanism with the links numbered and with the ICRs of the $M = 1$ sub-mechanisms shown (links 1, 2, 3, and 5; links 1, 2, 4, and 5; and links 1, 3, 4, and 5).

mathematical description as that obtained from Jacobian matrices. The ICR between two rigid bodies that are moving relatively to one another is the point at which the absolute velocities of both bodies are equal [20]. Using ICRs, it can be seen that there are certain configurations where an $M = 0$ mechanism loses its rigidity when the $M = 1$ sub-mechanisms whose union composes the system are considered.

For instance, according to the notation of Fig. 4, in a 3-RPR parallel manipulator, which is rigid when the actuators are fixed at particular values, there exist three $M = 1$ sub-mechanisms whose collection generates the original kinematic chain, namely, the sub-mechanisms obtained when links 2, 3, and 4 are removed, respectively.

For each of these sub-mechanisms, the platform (link 5) is able to move relative to the base (link 1); implying that the ICR between the platform and the base can be found, that is, $\text{ICR}(1,5)$. Herein, the notation $\text{ICR}(i,j)$ will be used to denote the ICR between links i and j .

An effective way of determining the position of $\text{ICR}(1,5)$ for each of the sub-mechanisms is through the use of a bookkeeping system for $M = 1$ mechanisms, first presented in [21]. The system involves constructing what is called a *circle diagram* (also known as the auxiliary polygon derived from the Aronhold-Kennedy theorem on ICRs), shown in Fig. 5 for the 3-RPR robot manipulator depicted in Fig. 4, which details all the links in the mechanism by number and a known ICR between two links is denoted by a solid line drawn between them. An unknown ICR between two links, denoted by a dotted-line, can be found if this dotted-line is the common side of two triangles otherwise made up of solid lines. The geometrical location of this unknown ICR is found by drawing two lines, each of which pass through the two known ICRs of each triangle. The point at which these two lines intersect is the position of the ICR; note that if the lines are parallel, the ICR is positioned at infinity.

Following the above procedure, the positions of $\text{ICR}(1,5)$ for each sub-mechanism of the 3-RPR robot

Figure 5: Circle diagram for $M = 1$ sub-mechanism with link 4 removed (left) and link 3 removed (right) for the case of the 3-RPR robot manipulator. See text for details.

manipulator can be obtained as shown geometrically in Fig. 4. As long as these points are separate the robot mechanism is rigid; however, if they coincide, the platform is able to, instantaneously, rotate relative to the base about this point and hence the mechanism loses its rigidity. This corresponds to a singular configuration. It is important to highlight that the information provided by the $M = 1$ sub-mechanism created by removing link 2 (links 1, 3, 4, and 5) is redundant. The geometric conditions which cause the $\text{ICR}(1,5)$ of this sub-mechanism to coincide with that of the others are, generally, the same as the conditions which cause the positions of $\text{ICR}(1,5)$ for the other two sub-mechanisms to coincide with each other. The only exception to this, an instance where two of the $\text{ICR}(1,5)$ s are coincident but the third is not, is when one of the sub-mechanisms is itself in a singular configuration, however under these conditions it can still be verified that the entire system is in a singularity because the total degrees of freedom of one of the sub-mechanisms has increased, therefore the mobility of the robot manipulator also increases.

The same analysis is now carried out on the proposed kinematically redundant architecture using the equivalent mechanism when the actuators are locked; in this case, according to the notation of Fig. 6, four $M = 1$ sub-mechanisms can be detected, namely, the sub-mechanisms obtained when we remove (i) link 3, (ii) link 4, (iii) link 5, and (iv) link 6. Fig. 7 shows the circle diagrams used to determine the construction lines needed to find the ICR between the platform and the base for the $M = 1$ sub-mechanisms (ii) and (iii); similar to the case of the 3-RPR robot manipulator, the conditions resulting from the other sub-mechanisms are redundant.

The position of $\text{ICR}(1,7)$ of sub-mechanism (iii), shown in the left hand diagram of Fig. 7, is given by the point of intersection between the lines which pass through links 3 and 4. The case of sub-mechanism (ii), shown in the right hand diagram of Fig. 7, is slightly different. In this case, before the ICR between the platform and the base can be found, an additional unknown ICR must be determined since the dotted line which connects links 1 and 7 is not the common side of any two otherwise known triangles. Then, $\text{ICR}(2,7)$ needs to be found first, which is given by the point at which the lines that pass through links 5 and 6 intersect. $\text{ICR}(1,7)$

Figure 6: Kinematic diagram of proposed mechanism with links numbered. ICR(1,7) for the sub-mechanisms (ii) and (iii) is shown.

Figure 7: Circle diagram for $M = 1$ sub-mechanism with link 5 removed (left) and link 4 removed (right) for the case of the introduced kinematically redundant architecture.

is then obtained by finding where the line which passes through ICR(1,2) and ICR(2,7) intersects with the line which passes through link 3. A singular configuration in the kinematically redundant robot manipulator occurs either when the ICR(1,7) of these sub-mechanisms coincide, or when the position of one or more of these ICRs cannot be calculated. Following this, the distance, d , between the ICR(1,7) of two sub-mechanisms can be used to determine whether or not the robot manipulator is in a singularity; the robot manipulator is therefore in a singular configuration when the value of d is zero or cannot be calculated – corresponding to the instances where the ICR(1,7)s are coincident and where one (or more) of the sub-mechanisms is itself in a singularity, respectively. A similar approach is used in [22], where the proximity to a singularity is measured by comparing the incircle radius of the triangle created by the three construction lines of the mechanism with the maximum possible incircle radius.

A verification of this method is shown in Fig. 8, where the distance, d , between the ICR(1,7)s of sub-mechanisms (ii) and (iii) is plotted for a full rotation of the moving platform, along with the inverse of the (2-norm) condition number of the Jacobian matrix, $1/k(\mathbf{J})$. The Jacobian matrices $\mathbf{J}$ and $\mathbf{K}$ are used to relate the Cartesian velocities of the moving platform, denoted by the vector $\dot{\mathbf{c}}$, to the actuated joint velocities, denoted by

the vector $\dot{\mathbf{q}}$, such that

$$\mathbf{J}\dot{\mathbf{c}} = \mathbf{K}\dot{\mathbf{q}}. \quad (18)$$

For the proposed robot architecture it can be shown that

$$\mathbf{J} = \begin{bmatrix} (\mathbf{p}_{10} - \mathbf{p}_6)^T & (\mathbf{p}_{10} - \mathbf{p}_6)^T \mathbf{E} \mathbf{v}_{10} \\ (\mathbf{p}_{11} - \mathbf{p}_7)^T & (\mathbf{p}_{11} - \mathbf{p}_7)^T \mathbf{E} \mathbf{v}_{11} \\ (\mathbf{p}_9 - \mathbf{p}_3)^T \mathbf{N} \mathbf{G} \end{bmatrix},$$

where $\mathbf{p}_i$ denotes the vector from the origin to the point P_i , $\mathbf{v}_{10}$ and $\mathbf{v}_{11}$ denote the vectors from the centre of the moving platform to points P_{10} and P_{11} , respectively,

$$\mathbf{E} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} \frac{d_{3,8}}{d_{3,9}} (\mathbf{p}_{10} - \mathbf{p}_8)^T \mathbf{M} \\ (\mathbf{p}_{11} - \mathbf{p}_9)^T \end{bmatrix}^{-1},$$

$$\mathbf{G} = \begin{bmatrix} (\mathbf{p}_{10} - \mathbf{p}_8)^T & (\mathbf{p}_{10} - \mathbf{p}_8)^T \mathbf{E} \mathbf{v}_{10} \\ (\mathbf{p}_{11} - \mathbf{p}_9)^T & (\mathbf{p}_{11} - \mathbf{p}_9)^T \mathbf{E} \mathbf{v}_{11} \end{bmatrix}, \text{ and}$$

$$\mathbf{M} = \begin{bmatrix} \cos(\delta) & -\sin(\delta) \\ \sin(\delta) & \cos(\delta) \end{bmatrix},$$

with δ being the angle taken anticlockwise from the vector $(\mathbf{p}_9 - \mathbf{p}_3)$ to the vector $(\mathbf{p}_8 - \mathbf{p}_3)$. The above computation of the the 3×3 Jacobian Matrix, $\mathbf{J}$, can be obtained adapting, for instance, the method used in [13]. It is well known that the robot manipulator is considered to be in a singular configuration when $1/k(\mathbf{J})$ is equal to zero; Fig. 8 shows that d and $1/k(\mathbf{J})$ vanish for the same robot manipulator configurations. However, it should be noted that there are some inconsistencies with this Jacobian in some configurations where $\mathbf{N}$ is singular, in which the value of $1/k(\mathbf{J})$ equals zero but the robot is not physically in a singular configuration. In such circumstances d is calculated to be non-zero. Indeed, it can be verified that the robot is certainly not in a singularity by computing the rigidity matrix of the corresponding graph of the mechanism and calculating its rank [23].

The architecture of the robot manipulator for the results of Fig. 8 is that of the robot manipulator shown in Fig. 1, with: base joint positions $P_1 = (0, 0)^T$, $P_2 = (10, 0)^T$ and $P_3 = (3, 2)^T$, and link lengths $d_{1,6} = 5$, $d_{2,7} = 5$, $d_{3,4} = 6$, $d_{3,5} = 7$, $d_{4,5} = 11$, $d_{4,8} = 5$, $d_{5,9} = 5$, $d_{6,10} = 5$, $d_{7,11} = 5$, $d_{8,10} = 5$, $d_{9,11} = 5$ and $d_{10,11} = 1$; all values are given in mm. The test trajectory is a full rotation of the moving platform about point $(5, 6)^T$. The configurations at which d and $1/k(\mathbf{J})$ equal zero are the points at which the robot manipulator is in a singularity.

6 Experimental Results

In this section, an example trajectory is tested on an example mechanism both theoretically through the use of a simulation and experimentally by implementing a physical prototype. In both cases, the trajectory of the end-effector is predefined and the configuration of the robot manipulator at each stage along this trajectory

Figure 8: A comparison between d and the inverse of the condition number of the Jacobian, $1/k(\mathbf{J})$, of the proposed mechanism with RPR legs.

is calculated subsequently. A basic procedure to avoid singularities was implemented.

According to the notation of Fig. 1, the value of α is selected such that the robot manipulator always stays away from a singular configuration for the given pose of the end-effector. An appropriate value of α is identified by varying it between 0 and 2π and calculating the positions of all joints via inverse kinematics. The dimensions of the mechanism were selected carefully such that the link between P_8 and P_{10} is able to rotate fully around P_{10} at any point along the end-effector's trajectory. For each configuration, the positions of the ICR(1,7) of each sub-mechanism are determined and the distance between them, d , is calculated. If $d = 0$ the corresponding configuration is singular.

The test trajectory is a full rotation of the moving platform about the position $(100, 200)^T$; by following this trajectory, the robot manipulator demonstrates its rotational capabilities. For the reported numerical and experimental results, the following numerical values were used for the geometric parameters of the robot manipulator (all values are given in mm): the coordinates of the base joints are $P_1 = (-50, 50)^T$, $P_2 = (-100, 325)^T$ and $P_3 = (350, 175)^T$; the length of the links are $d_{1,6} = 175$, $d_{2,7} = 175$, $d_{3,4} = 150$, $d_{3,5} = 150$, $d_{4,5} = 150$, $d_{4,8} = 225$, $d_{5,9} = 200$, $d_{6,10} = 175$, $d_{7,11} = 175$, $d_{8,10} = 75$, $d_{9,11} = 200$, and $d_{10,11} = 100$.

For the physical prototype, shown in Fig. 9, four Herkulex drs-0601 servo motors were used for the actuated joints with centres P_1 , P_2 , P_4 and P_5 . Each of the links (including the ternary link) were 3D-printed from ABS (Acrylonitrile Butadiene Styrene) plastic. Ball bearings were used for all the passive revolute joints in the robot manipulator; each passive joint consists of a bolt passing through the ball bearing joint attached to each link. Wheels were attached to the bottom of the ternary link for support. The passive joint of the ternary link, at centre P_3 , consists of a bolt fixed to the base

Figure 9: Prototype of the novel kinematically redundant planar parallel robot manipulator. An online video of this prototype completing a 2π rotation about a single point, avoiding singularities, and performing a pick-and-place trajectory of full rotation can be seen at https://www.youtube.com/watch?v=J_F8eW-K8KI&feature=youtu.be.

which passes through a ball bearing fixed to the ternary link. An Arduino Mega 2560 was used to control the system.

Fig. 10 displays the d value as the robot manipulator completes the full rotation, both for the numerical simulation and the physical prototype, in which the final configuration of the mechanism is the same as the initial configuration. The experimental d values were obtained by measuring the positions of the joints using motion tracking cameras and then calculating the positions of the required ICRs using the method presented in section 5. The graph shows that, since d never goes to zero, the robot manipulator is able complete the full rotation without encountering a singularity. This is confirmed numerically, and the experimental validation is provided as well, showing that the full rotation is achievable without encountering mechanical interferences. An online video of the prototype of the robot manipulator completing the rotation can be seen at https://www.youtube.com/watch?v=J_F8eW-K8KI&feature=youtu.be. In the video, the rigidity of the robot manipulator is physically demonstrated during the rotation to show that it never moves into a singularity. The video also consists of an example of singularity avoidance and a pick-and-place trajectory of full rotation to demonstrate the robot manipulator's workspace.

7 Conclusion

This paper presents a novel kinematically redundant planar parallel robot manipulator whose architecture is a fundamental truss and is able to complete 2π rota-

Figure 10: Graph displaying d , the distance between the two ICR(1,7)s, against ϕ , the angular displacement of the end-effector along the rotation trajectory.

tions of the end-effector without producing singularities. Achieving such a combination of characteristics had remained elusive in the literature. Fundamental architectures of parallel manipulators are important because they constitute the general instance of a family of robot manipulators. For the proposed parallel manipulator, the bilateration method was used to solve the inverse kinematics and forward kinematics problems. The singularity analysis was carried out by describing the geometric conditions that lead to the loss of the rigidity of the robot manipulator by using a method based on the computations of instantaneous centres of rotations of the sub-mechanisms of mobility 1. An example trajectory with a full rotation of the moving platform was tested both numerically and experimentally, these results are reported and a link to an online video recording of the prototype performing the trajectory is also provided; the actuator values used to complete the predefined trajectory were calculated by solving the inverse kinematics. This video also consists of an example of singularity avoidance thanks to redundancy and a potential application of the full rotation capabilities. Directions of future work may include the development of algorithms for singularity avoidance in arbitrary trajectories, the optimisation of the workspace of the manipulator, the design of high-performance instances of the kinematically redundant robot manipulator – including the analysis of the out-of-plane stiffness, or the study of particular members of the family of robot manipulators defined by the proposed architecture.

Acknowledgements

This work was supported in part by the Engineering and Physical Sciences Research Council grant EP/R020833/1.

References

- [1] Merlet, J., 2006. *Parallel Robots*. Springer Science & Business Media, Dordrecht, The Netherlands, ch. 2.
- [2] Sefrioui, J., and Gosselin, C. M., 1992. “Singularity analysis and representation of planar parallel manipulators”. *Robotics and Autonomous Systems*, **10**(4), pp. 209–224.
- [3] Degani, A., and Wolf, A., 2006. “Graphical singularity analysis of planar parallel manipulators”. In *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*, IEEE, pp. 751–756.
- [4] Gosselin, C., and Angeles, J., 1990. “Singularity Analysis of Closed-Loop Kinematic Chains”. *IEEE Transactions on Robotics*, **6**, June, pp. 281–290.
- [5] Nokleby, S. B., Firmani, F., Zibil, A., and Podhorodeski, R. P., 2007. “An Analysis of the Force-Moment Capabilities of Branch-Redundant Planar-Parallel Manipulators”. In *Proceedings of the 2007 ASME Design Engineering Technical Conference*, ASME, Las Vegas, NV, USA, (Sept. 4-7, 2007), pp. 1013–1020.
- [6] Marquet, F., Krut, S., Company, O., and Pierrot, F., 2001. “ARCHI: A New Redundant Parallel Mechanism-Modeling, Control and First Results”. In *Intelligent Robots and Systems, 2001. Proceedings. 2001 IEEE/RSJ International Conference on*, pp. 183–188.
- [7] Marquet, F., Krut, S., Gascuel, O., Pierrot, F., et al., 2002. “Control of a 3-DOF Over-Actuated Parallel Mechanism”. In *DETC: Design Engineering Technical Conferences-CIE: Computers and Information in Engineering Conference*, pp. 1185–1191.
- [8] Harada, T., and Nagase, M., 2010. “Impedance Control of a Redundantly Actuated 3-DOF Planar Parallel Link Mechanism Using Direct Drive Linear Motors”. In *Robotics and Biomimetics (ROBIO), 2010 IEEE International Conference on*, pp. 501–506.
- [9] Ebrahimi, I., Carretero, J. A., and Boudreau, R., 2007. “3-PRRR redundant planar parallel manipulator: Inverse displacement, workspace and singularity analyses”. *Mechanism and Machine Theory*, **42**(8), Aug., pp. 1007–1016.
- [10] Cha, S.-H., Lasky, T. A., and Velinsky, S. A., 2007. “Kinematically-Redundant Variations of the 3-RRR Mechanism and Local Optimization-Based Singularity Avoidance”. *Mechanics based design of structures and machines*, **35**(1), Oct., pp. 15–38.
- [11] Cha, S. H., Lasky, T. A., and Velinsky, S. A., 2007. “Singularity Avoidance for the 3-RRR Mechanism Using Kinematic Redundancy”. In *Robotics and Automation, 2007 IEEE International Conference on*, pp. 1195–1200.
- [12] Gosselin, C., Laliberté, T., and Veillette, A., 2015. “Singularity-Free Kinematically Redundant Planar

- Parallel Mechanisms With Unlimited Rotational Capability”. *IEEE Transactions on Robotics*, **31**, Mar., pp. 457–467.
- [13] Schreiber, L.-T., and Gosselin, C., 2018. “Kinematically redundant planar parallel mechanisms: Kinematics, workspace and trajectory planning”. *Mechanism and Machine Theory*, **119**, Jan., pp. 91–105.
- [14] Rojas, N., 2012. “Distance-Based Formulations for the Position Analysis of Kinematic Chains”. PhD thesis, Universitat Politècnica de Catalunya, Barcelona, Apr.
- [15] Baron, N., Philippides, A., and Rojas, N., 2018. “A Geometric Method of Singularity Avoidance for Kinematically Redundant Planar Parallel Robots”. In *International Symposium on Advances in Robot Kinematics*, Springer, pp. 187–194.
- [16] Rojas, N., and Dollar, A., 2016. “Gross Motion Analysis of Fingertip-Based Within-Hand Manipulation”. *IEEE Transactions on Robotics*, **32**, Aug., pp. 1009–1016.
- [17] Müller, A., 2009. “Generic mobility of rigid body mechanisms”. *Mechanism and Machine Theory*, **44**(6), pp. 1240 – 1255.
- [18] Rojas, N., and Thomas, F., 2011. “The Forward Kinematics of 3-RPR Planar Robots: A Review And A Distance-Based Formulation”. *IEEE Transactions on Robotics*, **27**, Feb., pp. 143–150.
- [19] Hunt, K., 1983. “Structural Kinematics of In-Parallel-Actuated Robot-Arms”. *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, **105**, Dec., pp. 705–712.
- [20] Daniali, H., 2005. “Instantaneous center of rotation and singularities of planar parallel manipulators”. *International Journal of Mechanical Engineering Education*, **33**, July, pp. 251–259.
- [21] Hartenberg, R. S., and Denavit, J., 1964. *Kinematic Synthesis of Linkages*. McGraw-Hill, New York, NY, ch. 4.
- [22] Ebrahimi, I., Carretero, J. A., and Boudreau, R., 2008. “Kinematic analysis and path planning of a new kinematically redundant planar parallel manipulator”. *Robotica*, **26**(3), pp. 405–413.
- [23] Hendrickson, B., 1992. “Conditions for Unique Graph Realizations”. *SIAM Journal on Computing*, **21**(1), pp. 65–84.
- 2 Equivalent kinematic model used for solving the forward kinematics; it corresponds to the mechanism obtained when the robot actuators are fixed at particular values. This model also applies for a robot manipulator with type RPR legs. 4
- 3 Resulting configurations of the example used in the forward kinematic analysis. 5
- 4 Kinematic diagram of 3-RPR mechanism with the links numbered and with the ICRs of the $M = 1$ sub-mechanisms shown (links 1, 2, 3, and 5; links 1, 2, 4, and 5; and links 1, 3, 4, and 5). 6
- 5 Circle diagram for $M = 1$ sub-mechanism with link 4 removed (left) and link 3 removed (right) for the case of the 3-RPR robot manipulator. See text for details. 6
- 6 Kinematic diagram of proposed mechanism with links numbered. ICR(1,7) for the sub-mechanisms (ii) and (iii) is shown. 7
- 7 Circle diagram for $M = 1$ sub-mechanism with link 5 removed (left) and link 4 removed (right) for the case of the introduced kinematically redundant architecture. 7
- 8 A comparison between d and the inverse of the condition number of the Jacobian, $1/k(\mathbf{J})$, of the proposed mechanism with RPR legs. 8
- 9 Prototype of the novel kinematically redundant planar parallel robot manipulator. An online video of this prototype completing a 2π rotation about a single point, avoiding singularities, and performing a pick-and-place trajectory of full rotation can be seen at https://www.youtube.com/watch?v=J_F8eW-K8KI&feature=youtu.be. 8
- 10 Graph displaying d , the distance between the two ICR(1,7)s, against ϕ , the angular displacement of the end-effector along the rotation trajectory. 9

List of Figures

- 1 Kinematic diagram of the proposed robot mechanism. The architecture consists of a moving platform connected directly to the base via two RRR legs and connected to a ternary link, which is joined to the base by a passive revolute joint, via two other RRR legs. 2