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Quasistatic metamaterials: Magnetic coupling enhancement by effective space cancellation

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Metamaterials and transformation optics have greatly expanded the possibilities for controlling electromagnetic waves and static fields. Here we use magnetic metamaterials to solve a long-sought unsolved question in electromagnetic science and technology: increasing the magnetic coupling between distant circuits in the quasistatic regime. Magnetic coupling is the essential ingredient in many relevant applications. Read/writing heads and magnetic bits in hard disks, primary and secondary windings in electric transformers, or rotor and stator parts in motors and generators are all examples of magnetically coupled elements. Here, we present a novel and broadly applicable way to increase magnetic coupling between two distant elements, by using the properties of extreme anisotropic magnetic metamaterials. We analytically demonstrate how the magnetic coupling between emitting and receiving coils in a general system can be enhanced by surrounding them with magnetic metamaterials, exactly as if the distance between them has been reduced. The validity of the theoretical results will be confirmed by experimentally demonstrating that using magnetic metamaterials results in a boost on the efficiency in the wireless transmission of power between circuits, since this efficiency directly depends on the magnetic coupling between emitter and receiver.

Electromagnetic waves can be in general manipulated by materials, like glass lenses focusing light or optical fibers guiding lasers. Metamaterials have recently introduced a whole new toolbox for controlling electromagnetic waves, often designed by transformation optics [1, 2]. Metamaterials have also been applied to control static fields, both in the magnetostatic and electrostatic regimes [3–10]. In particular, magnetic cloaks have been proposed and experimentally realized using magnetic metamaterials [11–16], magnetic concentrators have allowed the spatial concentration of magnetostatic energy [17–20], and magnetic hoses the transfer of magnetostatic fields to long distances [21]. The solutions for static fields provided by magnetic metamaterials present unique advantages with respect to other regions of the electromagnetic spectrum. Firstly, magnetic and electric fields decouple in the static limit. Thus, an exhaustive control of magnetic fields can be achieved considering only the magnetic permeabilites. Secondly, the main ingredients for the magnetic metamaterials, namely passive non-resonant materials with tunable values of magnetic permeability, are readily available [3–5]. Even the most extreme values of zero and infinite permeabilities can be achieved in practice with superconductors and soft ferromagnets, respectively [3, 11]. Finally, no wavelength is involved in the static case, which poses no limitation to the size of either the metamaterial components or the full device. This set of properties makes magnetostatics an ideal case for actual metamaterials applications.

The metamaterials we use to increase magnetic coupling between circuits are extreme anisotropic magnetic metamaterials (with very large permeability in one direction and very small in the perpendicular one), forming magnetic concentrators. For the static field, using transformation optics, it was derived [17] that a hollow cylinder made of an ideal homogenous material with infinite radial permeability μ_ρ and zero angular permeability μ_φ would concentrate an externally applied magnetic field in the cylinder hole. Differently, concentrators for the full electromagnetic wave case require material in the concentration volume [22]. Remarkably, the concentrating shell has also the apparently opposite property: the field of a magnetic source placed in the hole is expelled to the cylinder exterior. The combination of both concentrating and expelling properties results in the possibility of concentrating static fields at a distance from the source [17–19]. Another important property of the metamaterial shells is that, when there is a field source in the hole, the field distribution in the hole volume is not perturbed by the shell [17]. The ideal extreme anisotropic homogenous material composing the concentrator does not exist in nature, but can be discretized into a practical set of alternating layers of available superconductor and ferromagnetic materials, with only a slight degradation of the theoretical performance [17, 19].

The response of magnetic concentrators for low-frequency electromagnetic waves has not been explored until now. This quasistatic regime assumes that all the electromagnetic sources change sufficiently slowly to disregard displacement currents. It involves systems with dimensions much shorter than the wavelength (hundreds of kilometers for frequencies up to kHz's), which gives validity to Ohm and Kirchhoff laws in circuits. Most of the electromagnetic technologies - turbines for energy generation, transformers, motors- work in the quasistatic regime (typically at 50 or 60Hz). The use of magnetic concentrators - derived for magnetostatic fields - in the quasistatic regime can be initially justified from Maxwell equations. It can be demonstrated that when a low frequency (neglecting displacement currents) oscillating magnetic field is applied in a region of space consisting only of non-conducting linear magnetic materials (including vacuum), then the solutions are simply those for the

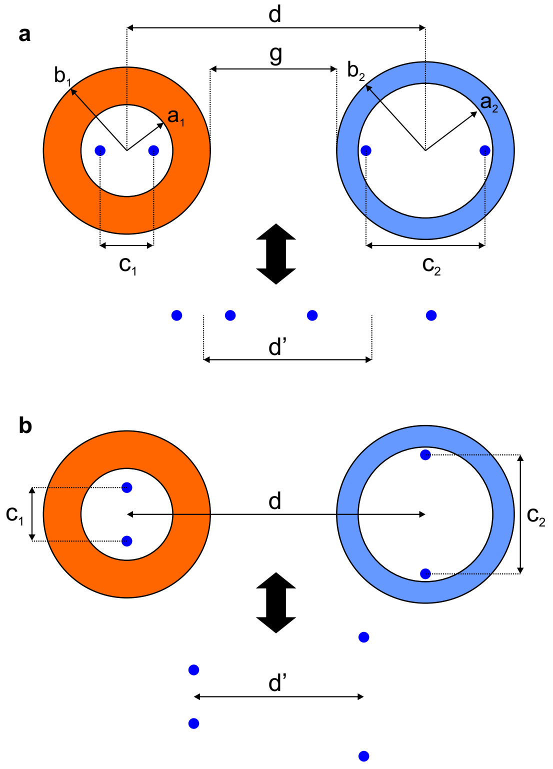

FIG. 1: Sketch of the (**a**) horizontal and (**b**) vertical configurations of the circuits (cross-sections), surrounded by the magnetic metamaterials (in orange and blue). Objects are very long in the perpendicular direction.

static case, oscillating at the frequency of the applied field. This should apply to the magnetic concentrators, since they can be simply made of passive linear magnetic materials [17, 19].

In this work we analytically demonstrate that the magnetic interaction between circuits in the quasistatic regime is enhanced when surrounding them with metamaterial concentrators, as if the distance between them has been effectively decreased. This is achieved without changing the circuits self-inductances. The reason for the latter property is that if current circulates in a circuit enclosed by a concentrating metamaterial shell, then the flux crossing the circuit is the same as if the circuit was in open space [17], i.e. the circuit self-inductance L is not modified by the shell (see Supporting Information for all theoretical details). This is relevant because, in general, magnetic materials strongly affect the field distributions and modify the L of the circuits. A common example is the high-permeability ferromagnetic core of transformers; the magnetic coupling between primary and secondary is increased at the prize of largely increasing the self-inductance of the coils as well.

Consider two circuits separated a distance d. Although our results are valid for any particular configuration of the circuits, we first assume that each circuit consists of a parallel pair of long wires separated by horizontal distances c_1 and c_2 for the primary and secondary circuits, respectively (see Fig. 1a). The mutual inductance per unit length between the two circuits is

$$M_{12} = \frac{\mu_0}{4\pi} \ln \left(\frac{4d^2 - (c_1 - c_2)^2}{4d^2 - (c_1 + c_2)^2} \right), \tag{1}$$

where μ_0 is the vacuum permeability.

We now consider that the primary circuit is surrounded by an ideal cylindrical metamaterial concentrating shell ($\mu_\rho \to \infty$ and $\mu_\varphi \to 0$) of inner and outer radii a_1 and b_1, respectively. We assume two equal circuits, $c_1 = c_2 = c$, for simplicity. From transformation optics [17] the field outside the shell is

$$\mathbf{H}(\rho, \varphi) = \left(\frac{a_1}{b_1} \right) \mathbf{H}_{\mathrm{s}} \left(\left(\frac{a_1}{b_1} \right) \rho, \varphi \right), \tag{2}$$

where $\mathbf{H}_{\mathrm{s}}(\rho, \varphi)$ is the field created by the source alone. Applying Eq. (2) the mutual inductance per unit length now becomes

$$M_{12}^{\mathrm{I}} = \frac{\mu_0}{4\pi} \ln \left(\frac{4d^2 - c^2(1 - \alpha)^2}{4d^2 - c^2(1 + \alpha)^2} \right), \tag{3}$$

where $\alpha \equiv b_1/a_1$. By comparing Eq. (3) with Eq. (1), we see that the effect in M_{12}^{I} of surrounding the circuit with the shell is to effectively increase the width of the primary circuit (distance between wires) from c to αc.

If a second concentrating shell with radii ratio $\beta \equiv b_2/a_2$ surrounds the secondary circuit, the mutual inductance is

$$M_{12}^{\mathrm{II}} = \frac{\mu_0}{4\pi} \ln \left(\frac{4d^2 - c^2(\beta - \alpha)^2}{4d^2 - c^2(\beta + \alpha)^2} \right). \tag{4}$$

This exactly corresponds to the mutual inductance between two circuits with larger spacings αc and βc separated a distance d. These results demonstrate that, regarding magnetic coupling, surrounding circuits with concentrators increases the mutual inductance as if the circuits had larger dimensions, but without changing their L's. For $d \gg c$, the improvement on the mutual inductance tends to $M_{12}^{\mathrm{II}}/M_{12} \to \alpha\beta$.

This enhancement of the mutual inductance, combined with the fact that the shells do not change the self-inductance of the circuits, exactly makes the system with concentrators magnetically equivalent to having the same bare circuits separated by a smaller distance d' (see Fig. 1a). This effective distance can be found through Eqs. (1) and (4) as

$$\frac{d'}{c} = \sqrt{\frac{4\left(\frac{d}{c}\right)^2 - (\beta - \alpha)^2}{4\alpha\beta}}, \tag{5}$$

where α and β are the radii ratios of the shells, c is the spacing between the wires in the circuits and d is the distance between their centers. Eq.(5) demonstrates that, in terms of the magnetic field, our metamaterials effectively cancel the space between the coils.

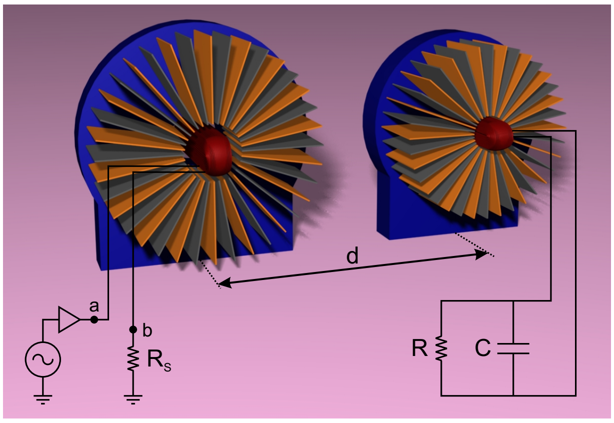

FIG. 2: Picture of the experimental setup to measure the wireless power transfer between the primary and secondary circuits surrounded by the magnetic metamaterials. Coils are shown in red, the ferromagnetic parts of the metamaterial shells in gray and the conducting ones in orange.

The improvement of the mutual inductance by concentrating shells can be rewritten as a function of the gap of free space g instead of the distance between the centers of the circuits d (see Fig.1a). Considering that the inner diameter of the shells exactly coincides with the size of the circuits (i.e. $2a_1 = 2a_2 = c$), the improvement on the mutual inductance for the same value of gap, $M_{12}^{\mathrm{II}}(d = g + b_1 + b_2)/M_{12}(d = g + c)$, is reduced but it is still larger than one. Therefore, although surrounding the circuits with concentrators in principle reduces the free gap, the overall effect is that using concentrators results in an increase of the coupling, for a given value of free gap.

The observed enhancement in M is also obtained for different orientations of the coils. When they are both vertically arranged (see Fig. 1b), M is enhanced to

$$M_{12}^{\mathrm{II}} = \frac{\mu_0}{2\pi} \ln\left(\frac{4d^2 + c^2(\alpha+\beta)^2}{4d^2 + c^2(\alpha-\beta)^2}\right), \tag{6}$$

which is the mutual inductance between two circuits having spacings between wires αc and βc . In the limit case $d \gg c$ the expression simplifies to $M_{12}^{\mathrm{II}} \simeq \frac{\mu_0}{2\pi}\alpha\beta\frac{c^2}{d^2}$.

The effective distance is in this case

$$\frac{d'}{c} = \sqrt{\frac{4\left(\frac{d}{c}\right)^2 + (\alpha-\beta)^2}{4\alpha\beta}}, \tag{7}$$

which shows that, for equal concentrators, the effective distance between the circuits is decreased by the radii ratio of the shells, $d'/d = 1/\alpha$. Thus, the effective distance between circuits can be significantly reduced by choosing large radii ratios for the shells. However, the minimum distance between shells at which they touch each other (assuming $2a_1 = 2a_2 = c$) is given by $d_{\min}/c = (\alpha+\beta)/2$ and, thus, the minimum effective distance is limited to $d'_{\min}/c = \sqrt{\alpha^2 + \beta^2/(2\alpha\beta)}$.

We next experimentally demonstrate the theoretically predicted enhancement of mutual inductance provided by the magnetic metamaterials. We choose the particularly relevant case of wireless power transmission. Motivated by the explosion of mobile technologies, there is a large interest in transferring power wirelessly [23, 24]. Recently, it was demonstrated that significant amounts of power could be efficiently transmitted to large distances (few times the coils radii) using self-resonant pair of coils in a strongly coupled regime [23]. The transmitted power and the efficiency of the energy transfer between the coils crucially depends on the mutual inductance between the circuits [25, 26].

We construct two concentrating shells, S1 and S2, the primary with radii ratio $\alpha = 4$ and the secondary $\beta = 5$. Even though the ideal material for the concentration shells required an extreme anisotropy, the properties can be effectively achieved by alternating homogenous isotropic ferromagnetic and conducting layers [17, 19]. The effect of the number of layers required to emulate the ideal behavior was studied in [18] for the static magnetic field; the number of shells we use here is within the adequate range. Also in the static case, it was show that, even though the metamaterials were designed in a cylindrical two-dimensional geometry, the required behavior in terms of guiding the magnetic fields was sufficiently preserved even when actual finite 3D samples were constructed [17, 19]. For the static experiments, superconducting layers placed between the ferromagnetic ones prevented angular permeability components μ_φ. Interestingly, here we show that in the quasistatic case superconductors can be simply replaced by conducting materials (we use copper), eliminating the complications of refrigeration.

Two elongated coils were placed inside the shells. Their self-inductances were measured by the resonance of an RLC circuit, both for the isolated circuits and when inside the corresponding shells. Experimental results (see Supporting Information for all experimental details) show that the shells do not significantly change the self-inductance of the coils, confirming our first theoretical prediction, even though they have finite length, have conducting parts, and are discretized into homogeneous isotropic materials.

In order to demonstrate the enhancement in the magnetic coupling we measure the wireless power transmission between the coils. The first coil C1 is connected to a primary circuit consisting of a signal generator and a power amplifier, in series with a shunt resistance. The second coil C2 is connected in parallel to a capacitor and a load resistance, forming a resonant secondary circuit (Fig. 2). The total power delivered to the system, W_{T}, and the power dissipated in the load resistance, W_R, can be calculated as [26, 27]

$$W_{\mathrm{T}} = \frac{V_a V_b}{2R_{\mathrm{s}}}\cos\phi_{a\to b}, \qquad W_R = \frac{1}{2}\frac{V_R^2}{R}, \tag{8}$$

being V_a (V_b) the amplitude of the voltage at point a

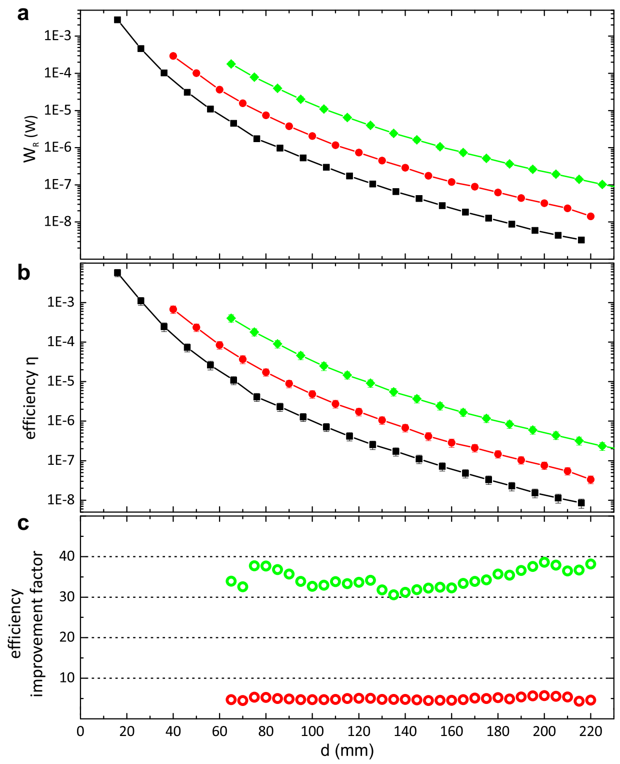

FIG. 3: (**a**) Measurements of the power dissipated in the load resistance, W_R, as a function of the distance between coils. (**b**) The efficiency of the power transfer is plotted together with the improvement achieved by using one or two concentrating shells (**c**). Black points are measurements of the two coils alone, red points correspond to use only the first shell S1 and green points to use the two shells.

(b), $\phi_{a\to b}$ the phase shift between the voltages at points a and b, and V_R the amplitude of the voltage drop in the load resistance (Fig.2). Finally, the efficiency of the transfer is calculated following the standard definition as $\eta = W_R/W_\mathrm{T}$ [26, 27]. Efficiency ideally increases as the square of magnetic inductance, $\eta \propto M^2$ [26]. M is thus the key parameter to boost efficiency.

Three kinds of measurements are performed. The wireless power transfer is measured as a function of the distance d between the emitting and a receiving coils, at the resonant frequency of the secondary circuit $f \simeq 30$kHz, in three situations: (i) bare coils, (ii) emitting coil C1 surrounded by the shell S1, (iii) both emitting and receiving coils C1 and C2 surrounded by shells S1 and S2, respectively.

The power dissipated in the load resistance and the efficiency of the transfer, η, are plotted for the three cases in Figs. 3a and b, showing how concentrating shells significantly increase both magnitudes. Actually, the improvement ratio of the efficiency reaches around 35 using the two shells and around 5 when using only one shell (Fig. 3c). The maximum theoretical improvement considering two ideal infinite concentrators would be $(\alpha\beta)^2 = 400$; the obtained value is less due to the finiteness and discretization reducing the performance of the concentrators [17], which indicates that there is still room for further experimental improvement.

Recently, a strategy has been proposed to enhance wireless power transmission using metamaterial "superlenses" [25–28]. They involve metamaterial slabs that exhibit effective negative permeabilities at the frequencies of interest. Apart from the difficulty of constructing such negative-μ media, there is the extra disadvantage that the metamaterial has to be placed around midway between both coils, which limits their applicability. Our results show that metamaterials made of available magnetic materials can boost wireless power efficiency while keeping the gap space between coils free.

An intriguing parallelism can be traced between our approach and 'statics optics', recently proposed to recreate in the full electromagnetic wave case some of the advantages of static fields [29–31]. There, using metamaterials with near zero permeability and permittivity values, similar properties as the quasistatic case are achieved, such as magnetic and electric fields being spatially distributed as if were static, while still temporarily dynamic. In these works, a whole region in space can be made to behave as a 'single point' electromagnetically [29], involving a kind of space cancellation. In our work, space is also (magnetically) canceled, improving the magnetic coupling at a distance. Our approach is thus complementary to static optics. It actually comes from the opposite direction, extending the results of magnetostatics to low-frequency waves.

In summary, we have presented a new paradigm for controlling low-frequency electromagnetic waves using magnetic metamaterials originally derived for the static case. We have analytically shown that mutual inductance between circuits can be largely increased when surrounding them with magnetic metamaterial concentrating shells, without increasing circuits self-inductances. Although we have focused here on a 2D cylindrical geometry, results can be straightforwardly extended to other geometries, like a 3D spherical symmetry. These properties have been experimentally demonstrated in the case of wireless power transfer, achieving very significant increases of both transmitted power and efficiency.

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