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A quasistatic magnetic cloak

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Abstract. Cloaking a three-dimensional object in free space from electromagnetic waves has recently become a theoretical possibility, although practical implementations can only be made in reduced schemes. If static fields are involved, requirements are less restrictive and some practical realizations have been possible. Here we present a third regime between the full wave and the static cases. We experimentally demonstrate that a cloak constructed under the dc conditions can keep cloaking properties for applied magnetic fields oscillating at low frequencies (up to hundreds of Hz). Because electromagnetic technology works at these frequencies, applications of our ideas to present technology are discussed.

The combination of transformation optics with the development of metamaterials has resulted in new ways of controlling electromagnetic waves, allowing the design and fabrication of objects that seemed impossible to make, such as electromagnetic cloaks or perfect lenses [1–3]. Cloaks, in particular, have been theoretically devised [2, 4, 5] and experimentally realized in several regions of the electromagnetic spectrum [6–12]. However, some problems have prevented the actual realization of fully working cloaks. First, because most cloak designs are based on space transformations, they require fine-tuned values of permittivity ε and permeability μ, often anisotropic, highly inhomogeneous and even singular [13]. Besides, cloaks for electromagnetic

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waves achieve their useful functionalities by structuring functional matter on the subwavelength scale of the involved wave [6], which is difficult to realize in practice for most electromagnetic waves—e.g. the relevant scale is hundreds of nanometers for visible light or infrared. Finally, bandwidth limitations and three-dimensional implementation are two main problems that need to be solved for real applications of cloaking [13].

A very convenient direction to avoid these difficulties [13] is cloaking of static (dc) fields [14], in both magnetic [14–17] and electric cases [18–20]. In the static limit one only needs to control a single parameter (ε or μ but not both), because electric and magnetic responses decouple. For this reason, the four Maxwell equations governing electromagnetic fields reduce to only two (the two magnetostatic or the two electrostatic ones). Thus, for the static case, alternative solutions to those resulting from the full electromagnetic wave case can be found [21]. Also, a dc design should work at an arbitrary scale since there is no limiting intrinsic wavelength [16]. Good examples in this line are the experimental realizations of an exact magnetic cloak [16] and a dc electric cloak and illusion [18, 19].

However, a strict dc case has less relevance than a full electromagnetic wave case. For many applications and from a fundamental point of view, it is important to study whether the simple but powerful solutions for the dc case can be extended to other frequency regions. Obviously, a direct translation of the dc results to optical, infrared or even microwave regions is impossible, because the electric and magnetic effects are intertwined. But is the cloaking behavior obtained at dc valid for low-frequency waves? Here we construct a dc magnetic cloak in the form of a superconductor–ferromagnet bilayer and measure its response to ac fields at low frequencies (such as those mostly used in the generation and consumption of energy in our society, 50–60 Hz). We will see that the magnetic cloaking properties achieved by the cancellation of the effects of the superconductor and the ferromagnet are maintained in the ac case, in spite of the non-ideal behavior of the constituting materials as evidenced by the presence of ac losses. This reveals the existence of a third regime between the full electromagnetic case (ruled by the four Maxwell equations) and the strict static case, in which the simpler solutions found for the two magnetostatic equations apply even for slow oscillating fields.

In [16], we designed and built a dc magnetic cloak that consisted of an interior ideal superconducting (SC) layer surrounded by an ideal ferromagnetic (FM) layer. We demonstrated a relation between the dimensions of the layers and the permeability of the FM one so that the external field distortion caused by the SC and the FM layers was *exactly* canceled for a uniformly applied magnetic field. Based on this, we built a cloak made of a few turns of SC strip above which some turns of FM alloy were wound. Despite the short length of the cloak (limited by the width of the strip), field measurements near the cloak showed a small field distortion of the applied field. Because the inner layer was SC, the magnetic signature of an object inside the cloak would not leak outside [15, 16].

For this work, we build a new SC–FM bilayer magnetic cloak as in [16]. The cloak is made longer in order to reduce the end effects. Four SC strips are used to build the interior layer and an FM foil (made of steel with 18% Cr and 9% Ni) is the exterior one as shown in figure 1, with dimensions $R_1 = 10.7\,\mathrm{mm}$, $R_2 = 12.1\,\mathrm{mm}$ and $L = 70\,\mathrm{mm}$.

To characterize the cloak in the dc regime, we measure the field near the exterior surface of the cloak when a perpendicular dc uniform field of 21 mT is applied (see figure 1(b)), and also the response at other distances (figure 1(c)). A remarkable reduction of the field distortion is shown for the bilayer as compared with the cases when only the SC or the FM layers are present. Nevertheless, the cancellation is not perfect, probably due to the imperfections of the

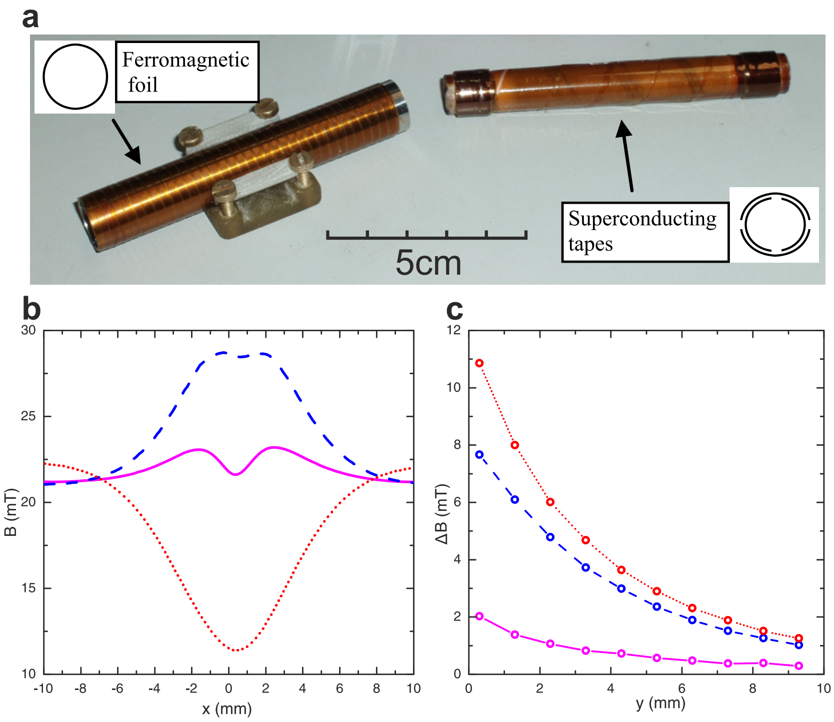

Figure 1. (a) Picture of the two parts forming the magnetic cloak. (b) Measured vertical field components near the cloak when there was only the SC (dotted) and the FM (dashed) layers. When the bilayer was complete (solid), the measured field approximated to the applied one. At other heights (c) the difference between the maximum and the minimum values of the field along a horizontal line were significantly smaller when the SC–FM bilayer was complete.

constructed bilayer; the four strips forming the interior SC layer have a different response from a solid SC cylinder and the effective permeability of the exterior layer made of turns of FM foil does not exactly fit the required theoretical value [16].

Our goal in this work is to study the response of the magnetic cloak to an applied ac magnetic field. For this purpose, we apply the calibration free method for ac magnetization loss measurement described in [26]. An oscillatory input current i_{ac} with frequency f is supplied to a copper coil that creates an applied magnetic field proportional to i_{ac}. A set of auxiliary coils is used to extract from the voltage measured on this coil the signal exclusively caused by the sample, u_{s}. Electromagnetic losses in the sample are $P_{\mathrm{s}} = i_{\mathrm{ac}} \cdot u_{\mathrm{s,loss}}$, $u_{\mathrm{s,loss}}$ being the resistive part of the voltage with frequency f (first harmonic) in phase with i_{ac}. The inductive part of the signal, $u_{\mathrm{s,ind}}$, is shifted by $\pi/2$ rad with respect to the input current and is proportional to the time derivative of the magnetic flux in the pick-up coil due to the presence of the sample inside it. In the experiment, both $u_{\mathrm{s,loss}}$ and $u_{\mathrm{s,ind}}$ as well as i_{ac} were measured by a lock-in technique at different ac field amplitudes H_{m} from zero to 12 mT and at different frequencies from 18 to 144 Hz.

Measurements were performed for three cases: (i) the SC layer only; (ii) the FM layer only; and (iii) the SC–FM bilayer. Experimental results are shown in figure 2. In figure 2(a) the values of $u_{\mathrm{s,ind}}$ for the SC and the FM parts alone have opposite signs and non-negligible values even at low fields. However, for the SC–FM bilayer the values are close to zero for fields up to 7 mT.

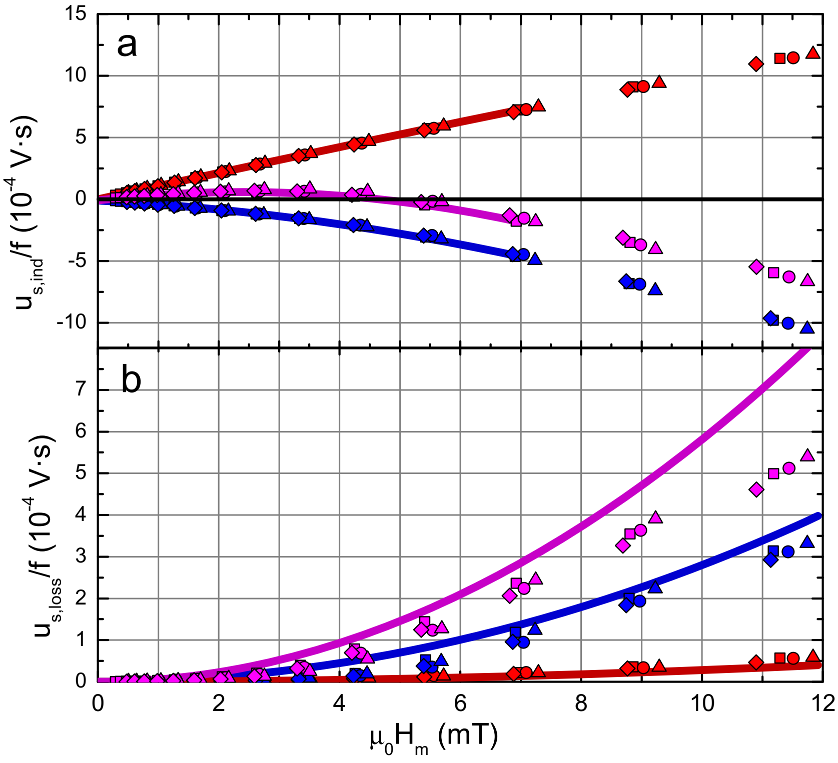

Figure 2. (a) Measured voltage component $u_{\mathrm{s,ind}}/f$ for SC, FM and SC–FM (red, blue and purple symbols, respectively) together with the corresponding fitting curves (solid lines). (b) Measured component $u_{\mathrm{s,loss}}/f$ (symbols) with plots of equation (5) using the fitting parameters. In both figures, square, circle, triangle and diamond symbols correspond to measurements at different frequencies of 144, 72, 36 and 18 Hz, respectively.

This indicates that the contributions to $u_{\mathrm{s,ind}}$ coming from the two constituting parts cancel each other, in analogy with the cancellation in the dc case. However, for the resistive part, $u_{\mathrm{s,loss}}$ (figure 2(b)), the signal from the bilayer increases continuously and is larger than that of its two constituting parts separately. Since this part of the signal is related to the dissipated energy, this means that the bilayer has a hysteretic behavior characterized by a certain magnetization loop with a non-zero width, which would make the bilayer detectable. Is it therefore physically impossible to achieve complete cloaking behavior (both resistive and inductive signals zero) for our bilayer, or instead this depends on the particular materials used?

To answer this question and, in general, to interpret these measurements more deeply, we need to deduce the basic features of the magnetization curves for the SC, FM and SC–FM cases, preferably by a procedure that works for both SC and FM materials. Interestingly, we can model the behavior of the constituting materials through a quasistatic magnetic approach. Although having different physical origins [22, 23], the magnetization of SC and FM at low applied fields can be both described by the Rayleigh model [24], for which the ascending and descending branches of the magnetization loop are

$$M = (\chi_a + \eta H_{\mathrm{m}})H \pm \frac{1}{2}\eta(H^2 - H_{\mathrm{m}}^2), \qquad (1)$$

where M is the magnetization of the sample, H is the applied magnetic field, H_m is the maximum applied field and χ_a and η are two independent parameters called initial susceptibility and Rayleigh constant, respectively. The maximum magnetization is achieved for $H = H_\mathrm{m}$, when $M(H_\mathrm{m}) = \chi_a H_\mathrm{m} + \eta H_\mathrm{m}^2$. The value of the remanent magnetization is $M(0) = \frac{1}{2}\eta H_\mathrm{m}^2$. In addition, the hysteresis loss can be calculated as the area of the loop, which is found to be $W = \frac{4}{3}\eta H_\mathrm{m}^3$.

The measured signals u_s arise from the fundamental component (first harmonic) of the sample magnetization, whose loop is an ellipse [25] expressed by

$$H = H_\mathrm{m} \cos\theta, \tag{2}$$

$$M = M' \cos\theta + M'' \sin\theta, \tag{3}$$

with θ varying from 0 to 2π, where M' and M'' are the magnetization at the maximum applied field [$M' = M(H_\mathrm{m})$] and the remanent magnetization [$M'' = M(0)$], respectively.

The relation between the two parameters of the Rayleigh loop and those of the fundamental loop can be obtained by equating the magnetization at the maximum applied field $M(H_\mathrm{m})$ and also the remanent magnetization $M(0)$ of both cases. Also taking into account that the measured voltages are proportional to the magnetization of the sample [27], one has

$$-\alpha \frac{u_\mathrm{s,ind}}{f} = M' = \chi_a H_\mathrm{m} + \eta H_\mathrm{m}^2, \tag{4}$$

$$\alpha \frac{u_\mathrm{s,loss}}{f} = M'' = \frac{1}{2}\eta H_\mathrm{m}^2, \tag{5}$$

where α is a calibration constant that depends on the sample volume and on the susceptometer details. These relations allow us to determine the free parameters of the Rayleigh loops corresponding to the SC, FM and SC–FM cases at low applied fields. By fitting the measured signal $u_\mathrm{s,ind}/f$ to a second-grade polynomial, we can determine $-\chi_a/\alpha$ and $-\eta/\alpha$ through equation (4) (solid lines in figure 2(a) for the fitting curves). These parameters are used to calculate $u_\mathrm{s,loss}/f$ through equation (5), which are plotted in figure 2(b) as solid lines together with the measured values showing consistency at low fields.

Using these adjusted parameters, we plot the corresponding Rayleigh loops for the SC, FM and SC–FM cases given by equation (1). In figure 3, we present these loops for different values of maximum field, $\mu_0 H_\mathrm{m} = 3$, 5 and 7 mT.

The calculated loops allow us to interpret the experimental data. First, the cancellation of $u_\mathrm{s,ind}$ at low field for the SC–FM case arises from the fact that the cloak is designed so that the magnetic moments of the SC and the FM parts compensate each other leading to a negligible magnetic response of the whole device at dc; interestingly, this cancellation persists at low ac field amplitudes. This can be seen in the loops in figure 3; the magnetization values at the maximum (and minimum) applied field $H = H_\mathrm{m}$ ($H = -H_\mathrm{m}$) of the SC and the FM parts separately are approximately equal with opposite signs so that the SC–FM bilayer has a negligible magnetic signature, which makes $u_\mathrm{s,ind}$ tend to zero. For fields larger than 7 mT the contribution of the FM to $u_\mathrm{s,ind}$ dominates so that the compensation is lost making the bilayer fully magnetically detectable.

The non-negligible signal $u_\mathrm{s,loss}$ measured even at low fields for the SC–FM case implies losses during the ac cycle. We experimentally observe that most losses come from the

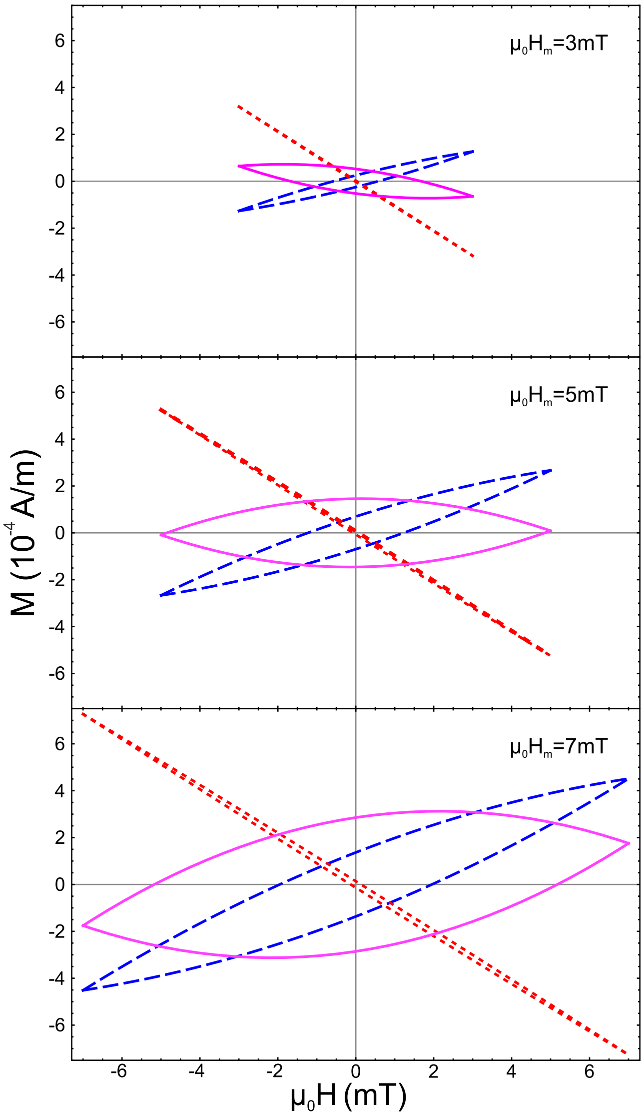

Figure 3. Calculated Rayleigh loops using equation (1) and the fitting parameters for different values of maximum field H_m (SC, FM and SC–FM in dotted, dashed and solid, respectively).

ferromagnet, which is not surprising when we notice that the differences between ascending and descending branches of the FM curves in figure 3 are about one order of magnitude larger than for the SC ones. In the combined SC–FM case, the magnetic flux expelled by the SC shell is accommodated by the FM material causing the observed blowing of the SC–FM loops. At the studied frequencies, these losses should mainly arise from magnetic hysteresis rather than eddy current loss associated with the electrical conductivity, because $u_{s,loss}$ is basically proportional to the frequency (figure 2). At the maximum (and the minimum) applied field, the compensation of the FM and the SC parts is almost perfect but in all other points of the ac cycle the behavior of magnetic flux penetration differs for the two materials so that their contributions no longer cancel and a net magnetic moment appears (resulting in the SC–FM loops having a significant width, directly related to the dissipation in the sample and proportional to the measured $u_{\mathrm{s,loss}}$). If materials with small hysteresis are selected[4], we expect that the cancellation of the FM and SC parts of the cloak would be significantly better for the whole ac cycle, so that the width of the SC–FM loops would be very small and the measured $u_{\mathrm{s,loss}}$ very much reduced. In this case there would be a cancellation of the two components of the measured voltage so that the cloak (and its content) would be basically magnetically undetectable through all the ac field cycle.

Our results may have practical applications in electromagnetic technology, which typically operates at the frequencies studied in this work, along several lines. One line could be protecting sensitive electromagnetic equipment from magnetic fields created by a nearby device; because one could achieve shielding without field distortion both devices could be working close to each other without affecting their respective field distribution, contrary to conventional ways of shielding equipment (e.g. by high-permeability ferromagnets or by superconductors) which necessarily imply a modification of the surrounding magnetic fields. Also, experimentally confirming that the required magnetic behavior of SC and FM materials persists at non-zero low frequencies can be used in other applications beyond cloaking. Recently, magnetic flux concentrators for enhancing the sensitivity of magnetic sensors have been presented [21]. These designs also allow the transfer of magnetic energy from a source to a given distant point separated by empty space, with possible applications in wireless transmission of energy. Because they are based on the same constituting materials, the results of this work may be relevant to achieve a full experimental realization of these magnetic energy harvesting and concentration ideas for non-zero frequency magnetic fields.

To sum up, we have experimentally demonstrated that, in addition to the full electromagnetic wave and the simpler dc cases, there exists a third regime of cloaking systems where fields are actually oscillating but still the advantages of the dc case can be used. An SC–FM bilayer has been constructed and tested at frequencies up to 144 Hz. We have observed that the permeability of the FM does not change appreciably at low frequencies and the SC keeps shielding the field well. Good cloaking behavior is observed, but only at the applied field amplitude (both minimum and maximum) and not for the rest of the cycle because of the magnetic hysteresis of the employed materials. With specially selected more linear materials a cloaking behavior extended along the whole ac cycle can be expected, meaning that using these ideas a bilayer (and its enclosed objects) would not be detected by a magnetic detector. The important advantages of applying transformation optics to the dc case can thus be extended to the low-frequency range of the electromagnetic spectrum, opening the way for novel applications in electromagnetic technology.

Acknowledgments

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[4] Commercial materials exist with very low values of remanence and coercivity like some steels. Another possibility would be using superparamagnetic nanoparticles embedded in a non-magnetic matrix.

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