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Modelling Time Varying Volatility Spillovers and Conditional Correlations Across Commodity Metal Futures

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This draft: October 2017

Abstract

This paper examines how the most prevalent stochastic properties of key metal futures returns have been affected by the recent financial crisis using both mapped and unmapped data. Our results suggest that copper and gold futures returns exhibit time-varying persistence in their corresponding conditional volatilities over the crisis period; in particular, such persistence increases during periods of high volatility compared with low volatility. The estimation of a bivariate GARCH model further shows the existence of time-varying volatility spillovers between these returns during the different stages of such a crisis. Our results, which are broadly the same in relation to the use of mapped or unmapped data, suggest that the volatilities of copper and gold are inherently linked, although these metals have very different applications.

Keywords: Financial crisis, Metal futures, Structural breaks, Time-varying volatility spillovers

JEL Classification Codes: C32; Q02

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We would like to thank the editor and the anonymous referees for the valuable suggestions and comments on an earlier version of this paper. We are also grateful for their useful comments to Chris Brooks and participants in the 3rd Annual Conference of Studies in Economics and Finance, Brunel University London, the 10th Energy and Finance Conference on “Energy Finance-Challenges and Opportunities”, Cass Business School, City University of London, the 2nd Young Finance Scholars’ Conference, University of Sussex, the 2nd Annual Conference of the International Association for Applied Econometrics, University of Macedonia – Thessaloniki, and the 9th International Conference on Computational and Financial Econometrics, Senate House University of London.
1 Introduction

The financial crisis of 2007-08 and the European sovereign debt crisis that occurred afterwards sent a wave of panic throughout financial and commodity markets around the globe. Given the macroeconomic slowdown and the widespread fear of an international systemic financial collapse, an interesting issue is whether the main stochastic properties of the underlying financial time series of these markets and their cross-shock and volatility spillovers have been affected by the crisis. Karanasos et al. (2014) do indeed find a time-varying pattern in the persistence of the volatility of stock market returns, as well as their correlations, cross-shock and volatility spillovers during the period.

Surprisingly, the aforementioned impact in relation to the commodity futures markets has drawn less attention. To the best of our knowledge, the studies by Vivian and Wohar (2012) and Sensoy (2013) are the only ones to date to have examined the impact of the recent crisis on the volatility of commodity returns, even though they consider spot price data. Moreover, such studies have limitations in that they ignore the impact of the crisis on the cross-shock and volatility spillovers between the corresponding returns.

In this paper, we examine the impact of the recent financial crisis on two metals futures’ volatility dynamics and their associated cross-linkages: copper and gold. These metal futures are considered due to their sheer daily volumes. Gold is the main precious metal and has mixed demand characteristics. Its demand is determined by financial factors as it is a reserve currency for the world, as well as being a traded commodity whose price is longed and shorted continually in huge volumes. Gold is also affected by its pure consumer and market application in jewellery and electronics. Copper, on the other hand, is the main industrial metal, with huge applications in electronics, mainly in wiring. It is far more abundant in comparison to other metals, and hence it is a useful candidate metal to be considered for this analysis.

Consequently, the present paper makes several broad contributions to the existing literature. First, we make use of several modern econometric approaches for univariate and multivariate time series modelling, amongst which we consider the possibility of breaks taking place in the volatility dynamics of these metal futures returns to capture the different stages of the recent financial crisis. More specifically, we use a battery of tests to identify the number and estimate
the timing of breaks, both in the mean and volatility dynamics. Then, we use these breaks in the univariate context, by adopting an asymmetric generalised autoregressive conditional heteroscedasticity (AGARCH) model, to determine changes in the volatility persistence and in the multivariate one, by employing the recently developed unrestricted extended dynamic conditional correlation (UEDCC) AGARCH model of Karanasos et al. (2014), to analyse the volatility transmission and the correlation structure. It follows that the adopted univariate and multivariate frameworks are completely time-varying, and more strikingly, unlike the methods used in the existing literature the adopted bivariate model is sufficiently flexible and allows for volatility spillovers of either positive or negative sign.

Moreover, both chosen univariate AGARCH and bivariate (UEDCC) AGARCH models are further employed to examine respectively how the volatility persistence of the two considered returns is affected by their corresponding positive (e.g., increases in these metal futures) and negative (e.g., declines in these metal futures) returns and whether there are any regime-dependent shock and volatility spillovers between such returns. The former analysis will show the extent to which positive returns versus negative ones impact on volatility persistence for the considered metals, while the latter will help to discern shock and volatility spillovers associated with the exact movements of each metal future (e.g., upward or downward) to the other, and vice versa.

All in all, knowledge of the time-varying volatility persistence and the spillovers mechanism adopted in this paper could prove to be very valuable to investors since they could give rise to time-varying trading strategies, thereby minimising the risk exposure and maximising the returns.

Finally, unlike most relevant research studies on the linkages among commodity futures prices which do not take into account the abnormal volatility in the last weeks of life of the futures contracts, pointed out by Samuelson (1965) (see, e.g., Hamoudeh and Yuan, 2008; Bhar and Lee, 2011; Ewing and Malik, 2013; Beckmann and Czudaj, 2014; Sadorsky, 2014; among others), the present paper sheds light on the volatility dynamics of the considered metal futures and their interactions using two types of data: unmapped and mapped. The unmapped data is comprised of prices that have not been adjusted for differences in prices due to rollover or ‘basis’. Taking

1Rollover, or roll, occurs when the current contract of a commodity instrument expires and the next month contract then becomes the new front month contract. As this happens, the price of the instrument may ‘jump’ since the front month contract and next month contract do not have the same price at the time of rollover (for
into account the roll or basis alters the time series in such a way that econometric models’ best fit may change as a result. The use of the front month contract prices (at the time of trading in real time) indicates the time series as it would appear to a trader at the particular point in time. However, the use of mapped data will allow us to observe the true interactions between the commodities. The differences in the time series (mapped and unmapped) may be large or small and sometimes cancel each other out. Yet, they should be considered if a true ‘live’ trading time series is to be created.

Our results suggest that both copper and gold futures returns exhibit time-varying persistence in their corresponding conditional variances over the recent crisis, specifically such persistence is shown to increase during periods of high volatility compared with low volatility. The results of the bivariate UEDCC-AGARCH(1, 1) model, on the other hand, show the existence of a bidirectional mixed feedback between the volatilities of the two returns; that is, the conditional variance of copper returns affects that of gold returns negatively whereas the reverse effect is of the opposite sign. This mixed feedback between the volatilities of copper and gold is consistent with the fact that these two metals are so different in their values and uses. The results also suggest that the volatility transmission from gold returns to those of copper is time-varying; it shifts on the onset of the high uncertainty period induced by the European sovereign debt crisis along with the downgrade of the US government debt status and also over the low volatility period ensued afterwards based on optimism to resolving the debt crisis. Finally, the regime-dependent volatility spillovers analysis suggests that declines in copper prices induce positive volatility spillovers to gold returns. These time-varying volatility spillovers between the two metals further confirm the sensitivity of these metals and so are their associated cross-linkages to structural changes in volatility filtered through the financial system.

Overall, our results are broadly the same in terms of whether mapped or unmapped data are employed and, moreover, they are robust when different model specifications are considered, i.e., using constant conditional correlation instead of dynamic conditional correlation in the bivariate GARCH model, and by including an exogenous control variable, i.e., the VIX volatility index or squared returns of the US dollar exchange rate against the euro, of the US’ S&P 500 stock more details, see Samuelson, 1965). In this first analysis, therefore, the data have not been mapped to account for the rollover values. It has been discovered that taking into account the roll can significantly change the time series since these roll values can be significant in the commodities considered (Margaronis, 2015).
market index or of oil prices.

The remainder of this paper is as follows. Section 2 reviews the relevant literature. Section 3 describes our employed data and methodology. Sections 4 and 5 present our empirical results and a discussion, respectively. The final Section contains the summary and our concluding remarks.

2 A Review of the Relevant Literature

Modelling the stochastic properties of financial and commodity returns as well as their cross-shock and volatility spillovers has drawn much attention to the fields of financial and energy economics, given their important practical implications for investors. For example, understanding the stochastic properties of returns may help investors in terms of forecasting market movements, while strong linkages between financial and/or commodity returns would imply limited portfolio diversification opportunities for them.

Although there is a large body of literature that has examined the returns properties of international financial markets such as those of equity, foreign exchange, and bond, and their cross-shock and volatility spillovers (see, e.g., Aloui et al., 2011; Bubák et al., 2011; Coudert et al., 2011; Philippas and Siriopoulos, 2013; Caporale et al., 2014; among others), a very extensive literature has also been examining the returns characteristics of commodity markets as well as their dynamic interlinkages. Of this large and rapidly growing literature, various studies have explored the stochastic properties of commodity returns, including those of metals (see O’Connor et al., 2015 and Vigne et al., 2017 for recent surveys on precious metals). For example, Watkins and McAleer (2008) find that the conditional volatility of aluminium and copper returns have been time-varying when analysed over a long horizon, using a rolling AR(1)-GARCH(1,1) model. Choi and Hammoudeh (2010) instead employ a Markov-switching specification and also suggest that spot commodity returns (i.e., Brent oil, WTI oil, copper, gold and silver) exhibit different volatility persistence in response to financial and geopolitical crises. Vivian and Wohar (2012) conclude that the volatility persistence of spot commodity returns, including those of precious metals, remains very high even when structural breaks are accounted for. Sensoy (2013) further demonstrate that the volatility of palladium and platinum, unlike that of gold.
and silver, exhibited an upward shift during the turbulent year of 2008 using spot price data over the period January 1999 to April 2013. His results also provide evidence that gold has a uni-directional volatility shift contagion effect on all other precious metals while silver has a similar effect on platinum and palladium.

Arouri et al. (2012), on the other hand, use parametric and semiparametric methods and find strong evidence of long range dependence in the conditional returns and volatility processes for the daily precious metals (i.e., gold, silver, platinum and palladium). Whereas, Demiralay and Ulusoy (2014) have considered short and long trading positions and show that long memory volatility specifications under student-t distribution perform well in forecasting a one-day-ahead VaR for both positions.

Some studies have also considered the linkages across commodity prices and their returns and volatility. Ciner (2001) reports that gold and silver futures contracts traded in Japan are not cointegrated, using daily data over the period 1992 to 1998. Erb and Harvey (2006) further argue that commodity futures returns have been largely uncorrelated with one another, especially across the different sectors. However, using daily data of gold, platinum, and silver futures contracts traded in both the US and Japanese markets, Xu and Fung (2005) find evidence of strong volatility feedback between these precious metals across both markets. Choi and Hammoudeh (2010), using a dynamic conditional correlation model, also identify increasing correlations among all the considered spot commodity returns (i.e., Brent oil, WTI oil, copper, gold and silver) over recent years.

A large number of studies have further looked at the dynamic linkages across both financial and commodity markets. For example, Choi and Hammoudeh (2010) find evidence of decreasing correlations between spot commodity returns (i.e., Brent oil, WTI oil, copper, gold and silver) and the US’ S&P 500 stock market returns over recent years. However, Mensi et al. (2013), using a VAR GARCH model, show that there are significant spillovers in terms of shock and volatility between the S&P 500 stock returns and spot commodity market returns. In particular, their results reveal that past shock and volatility of such stock returns strongly influence oil and gold market returns. Cochran et al. (2012), on the other hand, suggest that the VIX index is an important factor in the determination of metal returns and their volatility, using spot price data on copper, gold, platinum, and silver over the period January 1999 to March 2009.
The impact of the macroeconomic performance on commodity prices and their returns and volatility has also drawn much attention. For instance, Tulley and Lucey (2007) confirm that the US dollar is the main macroeconomic variable which affects gold. Sari et al. (2010) also find that spot metal prices (i.e., gold, silver, platinum, and palladium) are strongly related to the dollar-euro exchange rate. Hammoudeh et al. (2010) further find that of major precious metals (i.e., gold, silver, platinum and palladium) silver volatility shows a strong reaction to that of the dollar-euro exchange rate. Hammoudeh and Yuan (2008), on the other hand, provide evidence that rising interest rates are found to dampen precious metals futures volatilities. In addition, Batten et al. (2010) have examined the macroeconomic determinants of four precious metals (i.e., gold, silver, platinum and palladium) and find that the gold price is greatly influenced by monetary variables but that of silver is not. Their results also provide supporting evidence of volatility feedback between the precious metals. More recently, Andreasson et al. (2016) provide strong evidence of nonlinear causal linkages of commodity futures returns with stock market returns and implied volatility.

As the existing literature suggests, unlike copper, empirical evidence in relation to gold has drawn much attention along with silver and some other metals and, more importantly, evidence related to exploring cross-linkages between copper and gold, specifically, is sparse compared to, for example, other metal pairs (e.g., gold and silver). Further, a few studies have analysed the impact of the recent crisis on the stochastic properties of metal returns; however, they consider spot price data and also disregard the time-varying cross-shock and volatility spillovers among such returns during the period. This paper aims to fill in the existing gaps by analysing the impact of the recent crisis on the volatility dynamics and the associated cross-linkages of two metal futures, namely copper and gold, and by using alternative econometric specifications and data compared to the wide existing literature, specifically the bivariate (UEDCC) AGARCH model (which is sufficiently flexible and allows for volatility spillovers of either positive or negative sign) and two types of data: mapped and unmapped.
3 Data and Methodology

This Section overviews the data we have used and outlines the methodology we have employed to study the different properties of the stochastic processes associated with gold and copper futures returns over the 2007-8 crisis. First, we provide a brief description of our data and the breaks identification method which we have adopted. Then, we describe the univariate and bivariate models which we have estimated.

3.1 Data Description and Breaks Detection Procedure

We use daily (mapped and unmapped) data on gold and copper futures prices which span the period January 3, 2007 to April 27, 2012. The unmapped data have been retrieved from Bloomberg.

Gold versus Copper

The precious metals are, and for many years have been, used as a reserve currency in times of financial turmoil where uncertainty lingers within economies (see, for example, O’Connor et al., 2015, for a recent survey on the financial economics of gold). When consumers are not confident in their currency they often buy gold or other precious metals. The reason for this is the precious metals’ value and demand. The increased volatility, liquidity and use as a reserve currency mean that gold prices will react to the market with little to no lag time. Precious metals are not really consumed (and if they are it is usually a small percentage, which is often recycled e.g. jewellery, watches, and used as wiring in expensive earphones or sound systems) and neither do they tarnish or rust. They also have value and demand worldwide, making them a very good substitute for a currency. Their price is therefore very difficult to be determined as they are traded very frequently by countless companies and individuals. The use of gold to hedge currencies has become increasingly popular lately, which adds yet another demand dynamic to its already complex demand characteristics. The induced demand that results from uncertainty in financial markets can cause behavioural changes in the price, hence impacting volatility.

In the case of copper and its heavy industrial use, the demand characteristics are very different. Rather than being exposed to many market participants who trade lower volumes each, the copper market tends to consist of fewer market participants who trade larger volumes each,
e.g. mining companies, electronics companies, of which there are limited numbers. Financial instability can be a major factor influencing the price of copper. Decreased demand for copper as world demand falls (especially for consumer goods in which copper is a major raw material) is therefore expected but as the non-industrial utilisation of copper rises, its demand characteristics are also subject to major changes. Over the years, the copper price has been subject to a huge amount of speculative trading (although far less significant than in the gold market) and this, combined with the uncertainty of financial markets, which typically causes the demand for copper to fall, can induce significant levels of volatility in the copper price. With a lower number of market participants, despite the very large volumes, the net positions placed in the copper market will differ significantly from those of gold due to the lower speculative nature and far less complex demand characteristics of the copper market. The recyclable nature of copper also makes it an interesting prospect to be analysed.

**Mapping Procedure**

Various procedures have been used to construct continuous futures series (see Ma et al., 1992). For example, Coakley et al. (2011) and Gutierrez (2013) roll contracts over to the next ones on the first business day of the contract month in analysing a wide range of futures. Martikainen and Puttonen (1996) roll the contract over to the next a week before the contract expires in analysing the Finnish stock index futures market. Hou and Li (2016) roll contracts over to the next ones ten working days before maturity in analysing both the S&P 500 and the CSI 300 stock index futures markets.

By contrast, the mapping procedure adopted in this paper is achieved by a specialist computer programme where the input for the programme is the entire set of monthly futures contract. The programme then takes the last (expiry) price of each contract and lines it up by date to the price of the second month contracts. As the programme uses a counter for both the price series and date series, mapping occurs when the counters match on the day before expiry. The front and second month prices on that date are then lined up and their difference gives the basis or rollover for that contract. Each roll value or basis value is stored and accumulated in order for a calculation of the cumulative roll or basis to be made (see, for details, Margaronis, 2015). Finally, we use continuously compounded returns \( (r_t) \) on these metal futures calculated as

\[
r_t = (\log p_t - \log p_{t-1}) \times 100,
\]

where \( p_t \) is the metal futures price at time \( t \).
Structural Breaks

Since the employed data span includes various economic and financial events causing behavioural changes due to confidence alterations in economies as a result of the financial crisis, the considered returns series are likely to contain breaks associated with such events. Examples may include the collapse of Lehman Brothers, the collapse and buy-out of Bearn Sterns and AIG, increased unemployment, quantitative easing and many more.

Given this, to account for the possibility of breaks in the mean and/or volatility dynamics of these returns we use a set of parametric and non-parametric data-driven methods to identify the number and timing of the potential structural breaks. In particular, we employ the procedures in Bai and Perron (2003) and Lavielle and Moulines (2000), and find that the stochastic behaviour of both returns yields four breaks during the sample period, roughly one every one and a half years on average (see Table 1). The predominant feature of the underlying segments is that it is mainly changes in variance that are found to be statistically significant. Moreover, all four breakdates for the two series are very close to one another, which apparently signifies economic events with a global impact. It follows that the detected breaks contrast to those of Vivian and Wohar (2012), who find limited evidence of common breaks for spot precious and industrial metals using the AIT (adjusted Inclan and Tiao, 1994) test statistics.

Figure 1 displays the four break points identified (Table 1) and the associated regimes for each metal futures ( unmapped) returns series. The graphs (available upon request) of the corresponding mapped returns exhibit a similar pattern. Overall, the identified breaks seem to well capture the changes in the volatility of both returns over the different stages of the recent crisis. For instance, the first break for gold returns observed on July 22, 2008 may be explained by the stock markets having suffered their steepest fall since January 2001, causing the Federal Reserve to make an emergency significant cut in rates soon after. By contrast, the first break for copper returns observed on September 29, 2008 can almost certainly be attributed to the rejection of the $700bn US banking sector rescue plan. Although this was revised soon afterwards, it caused the stock markets worldwide to collapse and instilled a great deal of fear

\footnote{Alternatively, we have adopted the two-stage Nominating-Awarding procedure of Karoglou (2010) (see also Karanasos et al., 2014 and Karanasos et al., 2016) to identify breaks that might be associated either to structural changes in the mean and/or volatility dynamics or to latent non-linearities that may manifest themselves as dramatic changes in the mean and/or volatility dynamics and might bias our analysis.}
and uncertainty into the world economies again.

[Insert Figure 1 about here]

Following the largest first-quarter loss ever announced in US history by AIG, the group received a significant amount in government rescue funds in 2009. This was followed by the Federal Reserve’s plans to buy $1.2tn of mortgage and government debt. These rescue plans by the Federal Reserve and the US government in addition to those implemented by the Bank of England, European Central Bank, and Bank of Japan in late 2008 and early 2009 to stimulate economic growth may explain the observed break on March 10, 2009 in gold returns as fear and uncertainty in financial markets were moderated. The same phenomenon is observed on June 25, 2009 (the second break for copper), where many large banks received the Troubled Asset Relief Programme (TARP) rescue funds, again showing how the intervention to aid the financial markets by propping up their major institutions instills confidence in the world economy, which therefore undeniably impacts on the commodity markets, especially the metals studied in this paper.

However, this relatively lower volatility period is interrupted by the identified third break for both returns. More specifically, the break on June 13, 2011 in gold returns can be explained by the European sovereign debt crisis, where the high volatility period spans from this date and along with the downgrade of the US government debt status in early August 2011. Likewise, the high volatility in copper returns in the period following the break on September 09, 2011 was related to financial markets’ sentiment linked to the European sovereign debt crisis and the slowdown in China’s economy in certain periods.

Finally, the breaks on August 10, 2011 and November 03, 2011 for gold and copper returns respectively do not exactly coincide with specific events. However, given the significance of the events prior to these dates, it is clear that at some point the economies of the world would begin recovering from the global financial crisis and also the uncertainty associated with the European sovereign debt crisis had eased based on optimism to resolving the debt crisis following these dates. Therefore, such dates may represent the beginning of some stability in markets, and hence the start of a relatively lower volatility regime.

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3For details on the rescue programmes implemented by the major central banks, the reader is directed to Fawley and Neely (2013).
3.2 Time Series Modelling

3.2.1 Univariate Models

The conditional mean of the considered metal futures returns ($r_t$) is specified as:

$$r_t = \mu + \varepsilon_t, \quad \varepsilon_t = \sqrt{h_t} \epsilon_t$$

(1)

where the innovation $\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0, h_t)$ is conditionally normal with zero mean and variance $h_t$ and $\{\epsilon_t\}$ is a sequence of identically and independently distributed standard normal variables, that is $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$. $\mathcal{F}_{t-1}$ is the filtration generated by the information available up through time $t-1$. Autoregressive terms (up to $k$ lags) are also considered in case there is persistence in the conditional mean of returns. Next, the dynamic structure of the conditional variance is specified as an AGARCH(1,1) process of Glosten et al. (1993) (one could also employ the asymmetric power GARCH (APGARCH) as in Karanasos and Kim, 2006). Moreover, Karanasos et al. (2014) find that the persistence of the conditional variances of financial returns such as those of equity indices are significantly affected by structural changes associated with financial crises and economic events over the last two decades. To this end, to examine the impact of the identified breaks on the persistence of the conditional variances of these metal futures returns, the conditional variance is specified as follows:

$$h_t = \omega + \sum_{l=1}^{n} \omega_l D_l + \alpha_{t-1} \varepsilon^2_{t-1} \varepsilon_{t-1} + \beta_d h_{t-1},$$

(2)

with

$$\alpha_{t-1} = \alpha + \gamma S_{t-1}^- + \sum_{l=1}^{n} (\alpha_l + \gamma_l S_{t-1}^-) D_l, \quad \beta_d = \beta + \sum_{l=1}^{n} \beta_l D_l,$$

where $S_{t-1}^- = 1$ if $\varepsilon_{t-1} < 0$, and 0 otherwise. The breaks for metal futures returns, $l = 1, ..., n$ (where $n = 4$), are given in Table 1, and $D_l$ are dummy variables defined as 0 in the period before each break, and 1 afterwards. Note that failure to reject $H_0 : \gamma = 0$ and $\gamma_l = 0, l = 1, ..., n$ (where $n = 4$), implies that the conditional variance follows a simple GARCH(1,1) process.
Furthermore, the stability conditions require $P_0, P_4 < 1$ where

$$P_n = \alpha + \beta + \frac{\gamma}{2} + \sum_{l=1}^{n} (\alpha l + \beta l + \gamma l/2), \ n = 0, \ldots, 4$$  
(3)

(we use the convention $\sum_{l=1}^{n} (\cdot) = 0$ for $l < n$). Clearly in the time invariant case only $P_0 < 1$ is required, which, when there are no asymmetries, is reduced to the well known condition: $\alpha + \beta < 1$.

Alternatively, to examine how the persistence of the conditional variances is affected by upward and downward shifts in these metal futures, we consider a simple GARCH(1,1) model which allows the dynamics of the conditional variances to switch across positive and negative returns. This is given by:

$$h_t = \omega + \omega^{-} D_{t-1}^{-} + \alpha \varepsilon_{t-1}^{-2} + \alpha^{-} D_{t-1}^{-} \varepsilon_{t-1}^{-2} + \beta h_{t-1}^{-} + \beta^{-} D_{t-1}^{-} h_{t-1},$$  
(4)

where $D_{t-1}^{-} = 1$ if $r_{t-1} < 0$, and 0 otherwise.

### 3.2.2 Bivariate Models

Having defined the univariate modelling, in this Section we use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of returns. Let $y_t = (r_{1,t}, r_{2,t})'$ represent the $2 \times 1$ vector of the two returns of metal futures. As before $\mathcal{F}_{t-1} = \sigma(y_{t-1}, y_{t-2}, \ldots)$ is the filtration generated by the information available up through time $t-1$. That is, we estimate the following bivariate AGARCH(1,1) model

$$y_t = \mu + \varepsilon_t,$$  
(5)

where $\mu = [\mu_i]_{i=1,2}$ is a $2 \times 1$ vector of drifts.

Let $h_t = (h_{1,t}, h_{2,t})'$ denote the $2 \times 1$ vector of $\mathcal{F}_{t-1}$ measurable conditional variances. The residual vector is defined as $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})' = e_t \odot h_t^{1/2}$, where the symbols $\odot$ and $^\wedge$ denote the Hadamard product and the elementwise exponentiation, respectively. The stochastic vector $e_t = (e_{1,t}, e_{2,t})'$ is assumed to be i.i.d with zero mean, finite second moments, and $2 \times 2$ correlation matrix $R_t = diag\{Q_t\}^{-1/2}Q_t diag\{Q_t\}^{-1/2}$ with diagonal elements equal to one and off-diagonal
elements being absolutely less than one. $Q_t$ is specified as follows (see Engle, 2002):

$$Q_t = [q_{ij;t}]_{i,j=1,2} = (1 - \alpha^{DCC} - \beta^{DCC})\bar{Q} + \alpha^{DCC} \varepsilon_t \varepsilon_t' + \beta^{DCC} Q_{t-1},$$

where $\bar{Q}$ is the unconditional covariance matrix of $\varepsilon_t$, and $\alpha^{DCC}$ and $\beta^{DCC}$ are non-negative scalars fulfilling $\alpha^{DCC} + \beta^{DCC} < 1$. A typical element of $R_t$ takes the form $\rho_{ij,t} = q_{ij,t} / \sqrt{q_{ii,t} q_{jj,t}}$ for $i, j = 1, 2$ and $i \neq j$.

Following Conrad and Karanasos (2010, 2015) and Karanasos et al. (2014), we impose the UEDCC-AGARCH(1,1) structure on the conditional variances (one could also use multivariate fractionally integrated APARCH models as in Karanasos et al., 2014):

$$h_t = \omega + (A + \sum_{l=1}^{n} A_l D_l + \Gamma S_{t-1}) \varepsilon_{t-1}^2 + (B + \sum_{l=1}^{n} B_l D_l) h_{t-1},$$

where $\omega = [\omega_i]_{i=1,2}$, $A = [\alpha_{ij}]_{i,j=1,2; i \neq j}$, $B = [\beta_{ij}]_{i,j=1,2; i \neq j}$; $A_l$ and $B_l$, $l = 1, \ldots, n$ (where $n = 4$), are cross diagonal matrices with nonzero elements $\alpha_{ij}^{(l)}$, $i, j = 1, 2$, and $\beta_{ij}^{(l)}$, $i, j = 1, 2$, $i \neq j$, respectively (the superscript in the parenthesis denotes an index); $\Gamma$ is a diagonal matrix with elements $\gamma_{ii}$, $i = 1, 2$, and $S_{t-1}$ is a diagonal matrix with elements $S_{i,i,t-1} = 1$ if $e_{i,t-1} < 0$, and 0 otherwise.

The model without the breaks for the shock and volatility spillovers and the asymmetries, that is $h_t = \omega + A \varepsilon_{t-1}^2 + B h_{t-1}$, is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of $|I - BL|$, denoted by $\varphi_1$ and $\varphi_2$, lie inside the unit circle. Similar conditions hold for the time-varying asymmetric version of the model. Following Conrad and Karanasos (2010) we also impose the four conditions which are necessary and sufficient for $h_t > 0$ for all $t$: (i) $(1 - b_{22})\omega_1 + b_{12}\omega_2 > 0$ and $(1 - b_{11})\omega_2 + b_{21}\omega_1 > 0$, (ii) $\varphi_1$ is real and $\varphi_1 > |\varphi_2|$, (iii) $A \succeq 0$ and (iv) $|B - \max(\varphi_2, 0)I|A > 0$, where the symbol $\succeq$ denotes the elementwise inequality operator. Due to the presence of asymmetry we also have to check cases iii) and iv), where now we replace $A$ by $A + \Gamma$. Similar conditions hold for the time-varying asymmetric version of the model, i.e., Eq. (8) below. Note that these constraints do not place any a priori restrictions on the signs of the coefficients in the $B$ matrix. In particular, these constraints imply that negative volatility spillovers are possible. When the conditional correlations are constant, the
model reduces to the UECCC-GARCH(1, 1) specification of Conrad and Karanasos (2010).

Moreover, we also amend the UEDCC-AGARCH(1, 1) model by allowing shock and volatility spillovers to vary across positive and negative returns:

$$h_t = \omega + A_{t-1} \varepsilon_{t-1}^2 + B_{t-1} h_{t-1},$$ (8)

where $A_{t-1} = A + 1 S_{t-1} + A^- D_{t-1}^-$ and $B_{t-1} = B + B^- D_{t-1}^-$. $A^- (B^-)$ is a cross diagonal matrix with nonzero elements $\alpha_{ij} (\beta_{ij})$, $i, j = 1, 2$, $i \neq j$; $D_t^-$ is a diagonal matrix with elements $d_{it}^-$, $i = 1, 2$, where $d_{it}^- = 1$ if $r_{it} < 0$, and 0 otherwise.

The quasi-maximum likelihood (QML) method of Bollerslev and Wooldridge (1992) is used in the estimation of the above univariate and bivariate specifications. Finally, we check the standardised residuals and their squares to determine, respectively, the adequacy of the conditional means and the conditional variances in these specifications to capture their associated dynamics.

4  Empirical Results

In this Section we outline our analysis, which is based on the breaks that we have identified, to discuss first the findings from the univariate modelling and then from the bivariate one.

4.1  Univariate Modelling Results

The QML estimates of the AGARCH(1, 1) model for copper and gold returns using mapped and unmapped data are displayed in Table 2 (the insignificant parameters are excluded). We allow the ‘numerator of the unconditional variance’ (the $\omega$’s) as well as the ARCH and GARCH parameters to change across the identified breaks, as in Eq. (2). The estimated models, at the 5% level, appear to be well-defined: there is no evidence of further linear or nonlinear dynamics to be captured. In a broad sense, the results seem not to be dissimilar with regard to the type of data used, mapped or unmapped. Margaronis (2015) find that small rolls or basis prove to yield similar time series for mapped and unmapped data sets. The differences in the results may be due to the explanations expressed earlier in this paper whereby small compensations required

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4The estimation of these models was implemented in RATS 8.1 with a convergence criterion of 0.00001.
over time to map data sets can accumulate to, and result in, large cumulative changes in the time series. The unmapped data are likely to include artificial ‘price jumps’ when contract roll over occurs, which are of course reflected in the returns.

Another remark is that copper returns are shown to exhibit asymmetric responses regardless of using mapped or unmapped data; however, this is not the case for gold returns. This finding is consistent with that of Hammoudeh and Yuan (2008) using the EGARCH model over the period January 1990 to May 2006.

As for the impact of the breaks, the results suggest that the $\omega$ for both types of metals returns is not significantly affected by the breaks. However, the dynamics of the conditional variances (i.e., the ARCH ($\alpha$) and GARCH ($\beta$) parameters) are shown to be time-varying, in line with the empirical findings in Vivian and Wohar (2012), who use spot price data. Specifically, the estimated ARCH parameter in copper returns becomes significant after the first break (September 29, 2008) (see $\alpha_1$), whilst this parameter in the case of gold returns decreases after the second break (March 10, 2009) ($\alpha_2$ is negative and significant at the 1% level regardless of whether mapped or unmapped are used). With regard to the GARCH parameter, it exhibits a time-varying pattern across the second (June 25, 2009), the third (September 09, 2011) and the fourth (November 03, 2011) break for copper returns and across the first (July 22, 2008), the third (June 13, 2011), and the fourth (August 10, 2011) break for gold returns (see the estimated $\beta_i$ parameters in Table 2). Moreover, as is shown from Table 3, the time-variation of the ARCH and GARCH parameters is also observed by allowing the dynamics of a GARCH (1, 1) process to switch across positive and negative metal futures returns (see the estimated $\alpha^-$ and $\beta^-$ parameters).

Table 4 reports the persistence of the conditional variances of the two types of metal futures returns (see Eq. (3) for its calculation). It is evident that both returns show time-varying persistence in their corresponding conditional variances irrespective of whether mapped or unmapped data are used. In particular, the persistence of the conditional variance of copper returns increases from 0.95 to 0.98 over the financial market uncertainty created as a result of the rejection of the $700bn US banking sector rescue plan in the US. Nonetheless, such persistence declines to 0.93 following the stimulus packages (i.e., the TARP rescue funds and other rescue plans) and then increases to 0.99 over the uncertainty period induced mainly by the European debt crisis.
and the downgrade of the US sovereign debt status before falling back to 0.93 over the lower volatility period following the break in late 2011. Regarding gold returns, the persistence of its corresponding conditional variance exhibits a similar pattern. It increases from 0.94 to 0.97 over the high uncertainty period following the first break (July 22, 2008), then it declines to about 0.91 over the capital purchase programme by the US Treasury Department and other rescue funds by the US government and major central banks. However, after the European sovereign debt crisis there is an increase in the persistence to unity before it declines to 0.94 following the relatively lower uncertainty period that ensued afterwards.

Table 5, by contrast, reports the time-varying pattern of the persistence of the conditional variances by allowing the GARCH (1, 1) process to switch across positive and negative futures returns. The results suggest that the persistence of the conditional variances originating from negative returns is higher than those of the positive counterparts, especially for copper returns, using mapped and unmapped data. In particular, negative returns are shown to increase the persistence of the conditional variances from 0.91 and 0.97 to around 0.98 and 0.99 for copper and gold returns, respectively.

To sum up, it is clear that the persistence of the conditional variances increases during periods of high volatility compared with low volatility. That is, such persistence responds to common factors such as events which induced high uncertainty in financial markets, even though the identified break points for each return series have slight differences in timing, which can be explained by how quick these metals react to such events. In a broad sense, our result of the time-varying persistence of the conditional volatility corroborates the findings of Watkins and McAleer (2008) and Choi and Hammoudeh (2010), who use rolling AR(1)-GARCH and Markov-switching specifications, respectively.5

4.2 Bivariate Modelling Results

We also apply the bivariate UEDCC-AGARCH(1, 1) time-varying model to estimate the shock and volatility spillovers structure between copper and gold returns using mapped and unmapped data. The results, reported in Table 6, provide evidence of strong conditional heteroskedasticity

5However, the finding is not consistent with that of Sensoy (2013), who conclude that gold volatility was not affected by the turbulent year of 2008 using spot price data.
in the two variables, irrespective of using unmapped (left panel) or mapped (right panel) data (the insignificant parameters are excluded). The estimated ARCH parameters ($\alpha_{11}$ and $\alpha_{22}$) are positive and significant. Copper returns exhibit asymmetric responses (the estimated $\gamma_{11}$ parameter is positive and highly significant). However, this is not the case for those of gold. These results are in line with those of the univariate ones. Furthermore, the results suggest the existence of bidirectional volatility spillovers between copper and gold returns. Specifically, it is shown that the volatility of gold returns affects that of copper returns positively (the estimated $\beta_{12}$ parameter is positive and significant at the 10% significance level), whilst the negative sign holds in the reverse direction (the estimated $\beta_{21}$ parameter is negative and significant at the 10% significance level); similar results [not reported] hold for the conventional [without breaks] model, as well. The negative volatility spillovers from copper returns to those of gold imply that volatility innovations in copper affect gold but they have a less persistent effect than the volatility innovations from gold itself (see Conrad and Weber, 2013; the estimation of volatility impulse responses is left for future research).

Regarding the impact of the breaks on the volatility transmission structure, the results indicate that there are shifts in the volatility spillovers from gold to copper after the third (June 13, 2011) and the fourth (August 10, 2011) break (see the estimated $\beta_{12}^{(3)}$ and $\beta_{12}^{(4)}$ parameters), regardless of using mapped or unmapped data. These two shifts correspond respectively to the high volatility period induced by the European sovereign debt crisis along with the downgrade of the US government debt status and the low volatility period followed based on optimism to resolving such a crisis. Strictly speaking, the results suggest that the volatility spillovers effect from gold to copper is sensitive to ‘structural changes’ in which such positive spillovers are shown to have diminished at the onset of the European sovereign debt crisis. That is, for the mapped returns this positive impact weakened in the period after the European sovereign debt crisis and before the low volatility period ensued afterwards. Interestingly, for this period the effect has turned to being negative for the unmapped returns. It is clear that the aforementioned ‘structural changes’ are filtered through the financial system and impact on the way commodities such as gold and copper behave. The mechanism by which this happens has been detailed elsewhere in this paper.

Evidently, metal futures volatility spillovers vary as structural breaks occur. The stabilisation
of the crisis over the years induced confidence in the world economies. The behavior of the world economy has a direct impact on metal markets and the structural breaks seen during this time of turmoil along with the findings of Mensi et al. (2013) support this. This is also complemented by the work of Cochran et al. (2012), where the analysis of the spot metal market and the VIX show similar mechanisms and impacts to those shown in this paper. The study by Batten et al. (2010), by contrast, show how influential macroeconomic factors can be on the price behaviour of gold. Batten et al. (2010) also look into the volatility feedback between precious metals and they find good supporting evidence of its existence, so that offering reassuring support for the findings of this paper.

Figure 2 shows the evolution of the dynamic conditional correlations between the two types of metal futures returns over the sample period. As is evident from Figure 2, the time-varying correlations between both returns are shown to be similar using mapped and unmapped data. Furthermore, Tse’s (2000) test statistics of the null hypothesis $H_0: \alpha^{DCC} = \beta^{DCC} = 0$ are 0.400 (with $p$-value of 0.527) and 0.315 (with $p$-value of 0.574) for unmapped and mapped data, respectively. These test statistics do not reject the constant conditional correlations between the two returns using the two types of data, even though the correlations between the two variables are shown to exhibit transitory shifts over the Lehman Brothers collapse and the phases of the European sovereign debt crisis. The results (available upon request) of the volatility spillovers were shown to be robust by using the UECCC-AGARCH(1, 1) specification.

The results of the regime-dependent volatility spillovers between the two metal futures returns, reported in Table 7, on the other hand, suggest that declines in copper prices generate positive volatility spillovers to gold, using mapped and unmapped data (the estimated $\beta^{21}$ parameter is positive and significant at the 5% level). This result indicates that negative shocks to copper result in an increase in the volatility of gold. Moreover, the corresponding dynamic conditional correlations (not displayed) were not much different from those shown in Figure 2.

Finally, it is noteworthy to indicate that we have further tested the robustness of our univariate and bivariate findings by including an exogenous control variable in the conditional variance equations of the considered metal returns such as the Chicago Board Options Exchange Volatil-
ity index (VIX), or squared returns of (i) the US dollar exchange rate against the euro, (ii) the US’ S&P 500 stock market index, or (iii) the West Texas Intermediate (WTI) crude oil spot prices.\textsuperscript{6} The empirical univariate and bivariate results (available upon request) were found to remain broadly unchanged. Furthermore, copper returns volatility showed a significant positive response to each of the considered exogenous control variables (where the impact was stronger in the mapped compared to the unmapped data), but this was not the case for gold returns volatility, which had no response to any of the considered control variables.

5 Discussion

From both the mapped and unmapped data results it is clear that there are bidirectional volatility spillovers between the two metals, where the conditional variance of copper returns affects that of gold returns negatively whereas the effect in the opposite direction is positive. This means that when the price of copper exhibits greater volatility the price of gold becomes more stable and its volatility falls. This is in line with the differences in the demand characteristics between the two metals, explained previously.

During times of financial turmoil, where uncertainty lingers and individuals and organisations tie their capital up in gold as a reserve currency, the price of gold is suddenly influenced more by all the new demand. Rather than trading gold to make profit on its price changes, people are suddenly inclined to buy gold and keep it until there is confidence and stability in the economies of the world. Also, the fact that gold is a precious metal and copper is a base means that the fluctuations in these metal prices will differ simply because of the differences in uses and therefore demand and demand characteristics.

This can also be understood by considering the products based on each of the metals. Products based on copper are generally less dear and are replaced with new ones at a much greater rate, which is not the case for products containing gold or made of gold. Since copper prices depend significantly on the state of the Australian mining sector, Chinese and South-East Asian demand and the demand of large world economies, the volatility exhibited can be due to uncertainties in these.

\textsuperscript{6}The data for the exogenous control variables were obtained from Datastream.
The positive spillovers from the conditional variance of gold returns to that of copper returns are consistent with the sheer volume and significance of gold in the world economy. Induced volatility in gold prices will almost certainly influence a wide range of world economic factors. With gold being a reserve currency, an increase in the volatility of gold implies an increased uncertainty in world economies. Copper, being the main industrial metal, is therefore hugely impacted by such uncertainty as industrial demand is based on economic and business confidence worldwide, hence the connection can be made. Uncertainty in such factors does not usually occur when economies are booming. In the case of the gold price, however, the opposite effect is seen due to its establishment as a reserve currency and its non-consumable nature. This could therefore explain the inverse relationship observed in the cross-volatility effects.

The links between the two metals in terms of their monetary value through foreign exchange rates could also be at play in their cross interactions. It is clear that while the two metals have, for the most part, very different applications, when a significant world event occurs impacting foreign exchange, volatility tends to be induced in most financial securities. However, given the relation of gold with foreign exchange as it is used as a reserve currency, it is clear that it may be affected with lesser lag than an industrial metal such as copper. The use of gold as a hedging tool in times of financial turmoil is common and is supported by Beckmann et al. (2015) and Wang and Lee (2011) among others, while the findings by Sensoy (2013) show gold having a uni-directional volatility shift contagion on all precious metals. Sensoy (2013) also supports the premise that precious metals are used in times of financial turmoil to hedge and diversify portfolios and as alternative investment vehicles.

6 Summary and Conclusions

In this paper, we have analysed how the recent financial crisis affected the principal time series properties of the underlying series of two metal futures, namely copper and gold. In particular, we have employed several univariate and multivariate models to examine how the volatility dynamics, including the volatility persistence and volatility spillovers structures of these two metal futures returns have changed due to the recent financial crisis, and based our analysis on non-parametrically identified breaks.
Our findings suggest that the volatility persistence of these metal futures returns exhibit a substantial time-variation over the recent financial crisis; in particular, such persistence is shown to increase during periods of high volatility compared with low volatility. This time-variation appears consistent across both metal futures returns and irrespective of whether we allow for positive or negative changes in the corresponding asset.

The estimation of the bivariate UEDCC-AGARCH model then shows the existence of a bidirectional mixed feedback between the volatilities of the two returns, i.e., copper returns volatility affects that of gold returns negatively while the reverse effect is positive, consistent with the fact that these two metals have very different uses or applications. The results also show that the volatility transmission from gold returns to those of copper shifts on the onset of the high uncertainty period created by the European sovereign debt crisis along with the downgrade of the US government debt status and also over the low volatility period ensued afterwards based on optimism to resolving such a crisis. The regime-dependent volatility spillovers analysis, on the other hand, suggests that declines in copper prices induce positive volatility spillovers to gold returns. Overall, these time-varying volatility spillovers between the two metals provide further evidence in terms of the sensitivity of such metals and their associated cross-linkages to structural changes in volatility filtered through the financial system.

From the results it may be concluded that there is indeed a systemic relationship between the two metals in spite of their very different applications and values. The volatilities of copper and gold are inherently linked, proved by the findings of the analyses carried out. The possible explanations for the findings have also been explored in depth, analysing the impacts of one market on the other, and of course other factors, including the implications of the financial turmoil for these markets.

Our findings have implications for other related research areas in the empirical finance and economics literature. First, we provide consistent empirical findings for the extensive literature on volatility persistence and cross-volatility spillovers among financial and/or commodity returns, which emphasises that these volatility structures exhibit a time-varying pattern (see, e.g., Watkins and McAleer, 2008; Choi and Hammoudeh, 2010; Karanasos et al., 2014; Adesina, 2017; Andriosopoulos et al., 2017; to name a few) driven by structural changes in volatility induced in the financial system. Our findings indicate that the considered metal futures are not
exceptions.

Second, our findings also have implications for the literature on rolling over futures contracts and/or the so-called expiration effect in futures markets, pointed out by Samuelson (1965). Since our findings on the time-varying volatility persistence and cross-volatility spillovers are broadly the same in relation to the use of mapped or unmapped data, they are consistent with previous related studies on the limited support for the expiration effect in commodity futures (e.g., Daal et al., 2006; Duong and Kalev 2008; Carchano and Pardo, 2009). Further, given that this paper provides thorough evidence on the impact of mapping in relation to the metal futures, future work could focus on analysing such an impact on the time series properties of other commodity futures (e.g., energy, grains, softs, etc.) traded in the US and outside the US, including emerging countries (e.g., China), thereby providing further evidence on this issue to the academic community as well as practitioners.
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Figure 1. Daily (unmapped) copper (left panel) and gold (right panel) metal futures returns over the sample period.

Figure 2. The dynamic conditional correlation between mapped and unmapped copper and gold returns.
### Table 1
The identified breakpoints in copper and gold returns

<table>
<thead>
<tr>
<th>Break</th>
<th>Copper</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29/9/2008</td>
<td>22/7/2008</td>
</tr>
<tr>
<td>3</td>
<td>09/9/2011</td>
<td>13/6/2011</td>
</tr>
<tr>
<td>4</td>
<td>03/11/2011</td>
<td>10/8/2011</td>
</tr>
</tbody>
</table>

### Table 2
The estimated univariate AGARCH (1, 1) models allowing for breaks in the corresponding conditional variances

<table>
<thead>
<tr>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.063</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.181</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.069</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.921</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.032</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td></td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.056</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Notes: Robust-standard errors are used in parentheses. $\alpha_l$ and $\beta_l$ indicate the estimated parameters of the break dummies where the break $l = 1, \ldots, 4$ (see Table 1). Insignificant parameters are excluded. $LB(5)$ and $LB^2(5)$ are Ljung and Box (1978) tests for serial correlations of five lags on the standardised and squared standardised residuals, respectively ($p$-values are reported in brackets). $^{a}$, $^{b}$ and $^{c}$ indicate statistical significance at the 1%, 5%, and 10% levels, respectively.
The estimated univariate GARCH (1, 1) models allowing the corresponding conditional variances to vary across positive and negative returns

Table 3

<table>
<thead>
<tr>
<th></th>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
<td>Gold</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.034</td>
<td>0.076$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.077$^a$</td>
<td>0.026$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.020$^b$</td>
<td>0.072$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$\alpha^-$</td>
<td>0.056$^a$</td>
<td>-0.056$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.891$^a$</td>
<td>0.900$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\beta^-$</td>
<td>0.090$^a$</td>
<td>0.095$^a$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$LogL$</td>
<td>-2929.7</td>
<td>-2277.0</td>
</tr>
<tr>
<td>$LB(5)$</td>
<td>8.688</td>
<td>3.608</td>
</tr>
<tr>
<td></td>
<td>[0.122]</td>
<td>[0.607]</td>
</tr>
<tr>
<td>$LB^2(5)$</td>
<td>1.404</td>
<td>0.558</td>
</tr>
<tr>
<td></td>
<td>[0.923]</td>
<td>[0.989]</td>
</tr>
</tbody>
</table>

Notes: Robust-standard errors are used in parentheses. The estimated model is specified as $h_t = \omega + \omega^- D_{t-1} + \alpha^2 \varepsilon_{t-1}^2 + \alpha^- D_{t-1} \varepsilon_{t-1}^2 + \beta \varepsilon_{t-1} + \beta^- D_{t-1} \varepsilon_{t-1}$, where $D_{t-1} = 1$ if $r_{t-1} < 0$, and 0 otherwise. $LB(5)$ and $LB^2(5)$ are Ljung and Box (1978) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively ($p$-values are reported in brackets). $^a$ and $^b$ indicate statistical significance at the 1% and 5% levels, respectively.

Table 4

The persistence of the AGARCH (1,1) models for copper and gold returns

Panel A. The persistence of the standard (without breaks) AGARCH (1,1) models

<table>
<thead>
<tr>
<th></th>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
<td>Gold</td>
</tr>
<tr>
<td></td>
<td>0.982</td>
<td>0.988</td>
</tr>
</tbody>
</table>

Panel B. The persistence of the AGARCH (1,1) models allowing for breaks in the conditional variances

<table>
<thead>
<tr>
<th>State</th>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
<td>Gold</td>
</tr>
<tr>
<td>0</td>
<td>0.956</td>
<td>0.943</td>
</tr>
<tr>
<td>1</td>
<td>0.981</td>
<td>0.975</td>
</tr>
<tr>
<td>2</td>
<td>0.935</td>
<td>0.909</td>
</tr>
<tr>
<td>3</td>
<td>0.991</td>
<td>1.018</td>
</tr>
<tr>
<td>4</td>
<td>0.932</td>
<td>0.941</td>
</tr>
</tbody>
</table>

Notes: State 0 covers the period preceding all breaks, while state 1 covers the period between breaks 1 and 2, state 2 covers the period between breaks 2 and 3, and so on (see Table 1 for the dates of the breaks). The persistence is given by: $P_n = \alpha + \beta + \gamma \sum_{l=1}^{n}(\alpha_l + \beta_l + \gamma_l/2)$, $n = 0, \ldots, 4$. 

32
Table 5
The persistence of the GARCH (1,1) models allowing the corresponding conditional variances to vary across positive and negative returns

<table>
<thead>
<tr>
<th>Returns</th>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Copper</td>
<td>Gold</td>
</tr>
<tr>
<td>$r^+$</td>
<td>0.911</td>
<td>0.972</td>
</tr>
<tr>
<td>$r^-$</td>
<td>0.984</td>
<td>0.991</td>
</tr>
</tbody>
</table>

Notes: $r^+(r^-)$ indicates the persistence of the conditional variance generated from positive (negative) returns. The persistence of the positive returns is calculated as $\alpha + \beta$, while that of the negative returns is calculated as $\alpha + \beta + \left(\frac{\alpha - \beta}{2}\right)$.

Table 6
Estimates of the bivariate UEDCC-AGARCH models allowing for shifts in shock and volatility spillovers between copper and gold returns

<table>
<thead>
<tr>
<th></th>
<th>Unmapped</th>
<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
</tr>
<tr>
<td>Conditional Mean Equation</td>
<td>0.060</td>
<td>0.075$^b$</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Conditional Variance Equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>0.017</td>
<td>$\beta_{12}$</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0.017$^b$</td>
<td>$\beta_{12}^{(3)}$</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.016$^c$</td>
<td>$\beta_{12}^{(4)}$</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>$\alpha_{22}$</td>
<td>0.038$^a$</td>
<td>$\beta_{21}$</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.929$^a$</td>
<td>$\alpha_{DCC}$</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>0.960$^a$</td>
<td>$\beta_{DCC}$</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>$\gamma_{11}$</td>
<td>0.067$^a$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td></td>
</tr>
</tbody>
</table>

LogL $-5208.3$      LogL $-5327.5$
$LB(5)_{Cop}$ 9.055  $LB(5)_{Cop}$ 3.910
$LB(5)_{Gol}$ 3.223  $LB(5)_{Gol}$ 3.702
$LB^2(5)_{Cop}$ 0.431  $LB^2(5)_{Cop}$ 5.972
$LB^2(5)_{Gol}$ 0.298  $LB^2(5)_{Gol}$ 3.823

Notes: Robust-standard errors are used in parentheses. Subscripts of the estimated parameters are defined as 1=copper and 2=gold. Therefore, $\alpha_{12}(\beta_{12})$ indicates shock (volatility) spillovers from gold to copper, whilst $\alpha_{21}(\beta_{21})$ indicates shock (volatility) spillovers in the reverse direction. $\alpha_{12}^{(l)}(\beta_{12}^{(l)})$ indicates the shift in shock (volatility) spillovers for the break $l$ (see Table 1) from gold to copper. Insignificant parameters are excluded.
LB(5) and LB^2(5) are Ljung and Box (1978) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values are reported in brackets). \(^a\), \(^b\) and \(^c\) indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7

Estimates of the bivariate UEDCC-AGARCH models allowing spillovers between copper and gold to vary across positive and negative returns

<table>
<thead>
<tr>
<th></th>
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<th>Mapped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\mu_1)</td>
<td>(\mu_2)</td>
</tr>
<tr>
<td><strong>Conditional Mean Equation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega_1)</td>
<td>0.020</td>
<td>0.073(^a)</td>
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<tr>
<td></td>
<td>(0.029)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>(\omega_2)</td>
<td>0.039(^b)</td>
<td>0.038(^b)</td>
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<tr>
<td></td>
<td>(0.016)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>(\alpha_{11})</td>
<td>0.016(^c)</td>
<td>-0.017(^a)</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>(\alpha_{22})</td>
<td>0.049(^a)</td>
<td>0.036(^a)</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>(\beta_{12})</td>
<td>(\beta_{12})</td>
<td>(\beta_{12})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conditional Variance Equation</strong></td>
<td>0.006</td>
<td>(\beta_{11})</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>(\beta_{22})</td>
<td>(\alpha_{DCC})</td>
<td>(\alpha_{DCC})</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.129)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(\log L)</th>
<th>(\log L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_1)</td>
<td>-5198.2</td>
<td>-5324.7</td>
</tr>
<tr>
<td>(\omega_2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{11})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha_{22})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{12})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{22})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Robust-standard errors are used in parentheses. Subscripts of the estimated parameters are defined as 1= copper and 2=gold. Therefore, \(\alpha_{12}(\beta_{12})\) indicates shock (volatility) spillovers from gold to copper, whilst \(\alpha_{21}(\beta_{21})\) indicates shock (volatility) spillovers in the reverse direction. \(\beta_{21}\) reports the shift in volatility spillovers from copper to gold (induced by negative copper returns). Insignificant parameters are excluded. LB(5) and LB^2(5) are Ljung and Box (1978) tests for serial correlation of five lags on the standardised and squared standardised residuals, respectively (p-values are reported in brackets). \(^a\), \(^b\) and \(^c\) indicate statistical significance at the 1%, 5%, and 10% levels, respectively.