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Abstract—Wireless sensor networks (WSNs) are autonomous networks of spatially distributed sensor nodes which are capable of wirelessly communicating with each other in a multi-hop fashion. Among different metrics, network lifetime and utility and energy consumption in terms of carbon footprint are key parameters that determine the performance of such a network and entail a sophisticated design at different abstraction levels. In this paper, wireless energy harvesting (WEH), wake-up radio (WUR) scheme and error control coding (ECC) are investigated as enabling solutions to enhance the performance of WSNs while reducing its carbon footprint. Specifically, a utility-lifetime maximization problem incorporating WEH, WUR and ECC, is formulated and solved using distributed dual subgradient algorithm based on Lagrange multiplier method. It is discussed and verified through simulation results to show how the proposed solutions improve network utility, prolong the lifetime and pave the way for a greener WSN by reducing its carbon footprint.

Index Terms—Green wireless sensor network (GWSN), wireless energy harvesting (WEH), wake-up radio (WUR), error control coding (ECC), subgradient algorithm.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) is a smart and intelligent infrastructure of uniquely identifiable devices capable of wirelessly communicating with each other through the Internet. Its technologies are used to monitor many aspects of a city in real time. For example, the networked heterogeneous devices connected in a smart structure are typically equipped with sensors, sink nodes, wireless transceivers, cloud servers and finite battery supply to monitor and send/receive data. With the development of Internet-of-Things (IoT), a wide range of intelligent and tiny wireless sensing devices have been massively deployed in a variety of application environments such as home automation, healthcare, surveillance, transportation, smart environments and many more.

Although the WSN systems possess tremendous potential but there are many dominant barriers in implementing such a grandiose scheme. For example, the limited battery capacity, on-chip memory and small transmit power, the lifetime of sensor devices, its processing capability and range of operation are curtailed [1]. Moreover, the sensor devices that are farther from sink nodes or that work as relay node are drained quickly of their battery and may negatively affect the overall system performance. The Information and Communication Technologies (ICT) is currently responsible for 2 to 4 % of the current total carbon emissions or footprint [2]. In the future, as the plethora of smart devices connected to each other utilizing WSNs will be deployed, the carbon footprint is going to increase manyfold and will be responsible for a larger percentage of carbon emissions [2]. The current systems are not equipped to deal with this issue. Hence, it is necessary to analyze the system lifetime by minimizing its total energy consumption and carbon footprint without degrading the desired application performance and reliability constraints. Motivated by the emerging concept of Green Wireless Sensor Network (GWSN) in which the lifetime and throughput performance of the system is maximized while minimizing the carbon footprints, our goal is to build an sustainable WSN system by supplying adequate energy to improve the system lifetime and providing reliable/robust transmission without compromising overall quality of service.

II. PRIOR RELATED WORKS, MOTIVATION AND CONTRIBUTION

Optimization methods have been extensively used in previous research works to solve for network lifetime of wireless sensor networks. Network lifetime maximization with flow rate constraint have been studied in many prior works. Kelly et al. was the first to propose two classes of distributed rate control algorithms for communication networks [3]. Madan et al. [4] solved the lifetime maximization problem with a distributed algorithm using the subgradient method. In [5], Ehsan et al. propose an energy and cross-layer aware routing schemes for multichannel access WSNs that account for radio, MAC contention, and network constraints, to maximize the network lifetime. But, the problems formulated and solved in all these approaches neither does take into account a proper energy model incorporating all the transceiver resources nor it involves the application performance trade-off due to increase in lifetime by decreasing rate flows.

System utility and network lifetime are problems that are related to each other in a reciprocal relationship meaning...
maximizing one will degrade the other. Chen et al. [6] analyzed the utility-lifetime trade-off in wireless sensor network for flow constraints. He et al. [7] followed a cross-layer design approach. Both of these papers take transmission rate as the sole indicator of the system throughput, which is not true as the reliability plays a vital role in determining the system performance. Reliability in the system can be improved by introducing error control schemes into the sensor nodes with multipath routing introduced by Lun et al. [8]. In [9], Yu et al. analyses the automatic repeat request (ARQ) as well as a hybrid ARQ scheme for WSNs. The ARQ scheme requires re-transmission if there is a failure of packet delivery which increases energy consumption of node. Xu et al. [10] describes a rate-reliability and lifetime trade-off for WSNs by taking theoretical end to end error probability of packets. Similarly, Zou et al. [11] has taken a joint lifetime-utility-rate-reliability approach for WSNs taking a generic error coding processing power model. Both [10] and [11] lack the inclusion and analysis of an error control scheme with their encoding/decoding powers as well as the delay performance of the overall system with error correction employed.

Energy harvesting is proposed as a possible method to improve the network lifetime and rechargeable batteries in WSNs by He et al. [12], Magno et al. [13], Deng et al. [14] and Kamalinejad et al. [15]. Practically, energy can be harvested from the environmental sources, namely, thermal, solar, vibration, and wireless radio-frequency (RF) energy sources [16]. While harvesting from the aforementioned environmental sources is dependent on the presence of the corresponding energy source, RF energy harvesting provides key benefits in terms of being wireless, readily available in the form of transmitted energy (TV/radio broadcasters, mobile base stations and hand-held radios), low cost, and small form factor implementation. Recently, dynamics of traffic and energy replenishment incorporated in the network power model has been an active research topic. Some of the challenges are addressed by [17], [18] and [19]. They assume battery energy to be zero at start, which may not be practical for many application scenarios that has sensors with rechargeable batteries. challenges caused by packet loss due to interference has also not been addressed.

Green networking of late in the past four to five years has attracted a lot of attention. Koutitas et al. [20] has analyzed a maximization problem based on carbon footprints generated in terrestrial broadcasting networks. In [21] Naem et al. have maximized the data rate while minimizing the CO\(_2\) emissions in cognitive sensor networks. But it is yet to be seen how much carbon emissions can be minimized while maximizing the utility and lifetime with reliability and energy harvesting constraints.

In this paper, we formulate and solve a joint maximization problem of system performance (measured by data utilization) and lifetime for wireless sensor network. The packet loss and data utilizations are incorporated to provide a more realistic data loss and utilization model for the WSN system. As energy is scarce resource for a WSN system, energy harvesting is adapted in the system model to increase its lifetime. We model the harvesting as a stochastically varying. Contrary to articles [17], [18] and [19], our model assumes that the battery starts with an initial energy and the network operations has to be sustained using harvesting and wake up radio (WUR), using harvesting from ambient RF energy rather than using a solar energy harvester which needs extra circuitry. The overall problem throws challenges in finding an optimal solution as the time-variation combined with retransmissions, packet loss and harvesting makes it complex. We, then provide a distributed solution to the problem by solving the data-utility and network lifetime separately. Our major contributions are summarized as follows:

1. We formulate the data-utility lifetime trade-off problem by taking an approximated lifetime function as well as the energy harvesting, wake up radio duty cycling and retransmissions into the utility function.

2. We propose a redundant residue number system based error correcting technique and compare it with ARQ and Bose-Chaudhuri-Hocquenghem (BCH). Innovatively, the packet error rate and delay are being included while computing lifetime and performance of the sensor network.

3. We show how the energy harvesting and error control coding can jointly reduce carbon footprints generated per year and make the network green.

As per the best knowledge of the author, this is the first paper that incorporates wireless energy harvesting and error control coding into the power model of the objective function. The rest of this paper is organized as follows. Section II presents the prior works. System model formulation is described in Section III. Section IV describes our error control coding based data transmission control. In Section V, we propose the WEH and WUR schemes for WSN system. In Section VI, we formulate the joint utility-lifetime trade-off problem and formulate a distributed solution based on subgradient method. Section VII shows our simulation plots followed by conclusion in Section VIII.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a network with static and identical sensor nodes denoted by \( N \). Sensor nodes collect data from the surrounding information field and deliver it to the sink node/collector node denoted by \( S \). As in [22], sensors communicate either in an uniformly distributed ring topology or randomly in a multi-hop ad-hoc topology. We assume that the sensor devices in an WSN system are transmitting over a set of links \( L \). We model the wireless network as a \{node, link\} connectivity graph \( G(Z, L) \), where the set, \( Z = N \cup S \), represents the source and sink nodes. The set of links, \( L \), represents the communication link between the nodes. Two nodes \( i \) and \( j \) are connected if they can transmit packets to each other with \( i \in N \) and \( j \in N_i \), where \( N_i \) is the number of outgoing sensor nodes from source to sink. Fig. 1 shows a sample connectivity graph with three sensor nodes \( \{i_1, i_2, i_3\} \), one sink node \( s_1 \) and six communication links \( \{1, 2, 3, 4, 5, 6\} \). The communication between node \( i_1 \) and \( s_1 \) is a single-hop transmission whereas between \( i_3 \) and \( s_1 \) denotes a multi-hop transmission with node \( i_2 \) acting...
as relay for data of node $i3$. The set of outgoing links and the set of incoming links corresponding to a node $i$ are denoted by $O(i)$ and $I(i)$ respectively. Thus, in Fig. 1, $O(i2) = (i3, i6)$ and $I(i2) = (i4, i5)$. Table I delineates the parameters used for the analysis of our scenarios.

### Table I
NOTATIONS USED IN THE PAPER

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$||_\infty$</td>
<td>$\infty$-norm</td>
</tr>
<tr>
<td>$||_p$</td>
<td>$p$-norm</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of Sensor Nodes</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of Sink Nodes</td>
</tr>
<tr>
<td>$i$</td>
<td>Outgoing Sensor Node</td>
</tr>
<tr>
<td>$j$</td>
<td>Incoming Sensor Node</td>
</tr>
<tr>
<td>$r_{ij}$</td>
<td>Rate of Information Flow between $i$ &amp; $j$</td>
</tr>
<tr>
<td>$R_{ij}$</td>
<td>Source rate</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Capacity of Link</td>
</tr>
<tr>
<td>$T_{network}$</td>
<td>Lifetime of Network</td>
</tr>
<tr>
<td>$E_{TX}$</td>
<td>Transmit energy [J/bit]</td>
</tr>
<tr>
<td>$E_{RX}$</td>
<td>Receive energy [J/bit]</td>
</tr>
<tr>
<td>$E_{PR}$</td>
<td>Processing energy [J/bit]</td>
</tr>
<tr>
<td>$E_{SN}$</td>
<td>Sensing energy [J/bit]</td>
</tr>
<tr>
<td>$P_{LS}$</td>
<td>Listening power [W]</td>
</tr>
<tr>
<td>$E_B$</td>
<td>Battery energy of Sensor</td>
</tr>
<tr>
<td>$P_h$</td>
<td>Harvested power</td>
</tr>
<tr>
<td>$W_{ij}'$</td>
<td>Wake-up-radio on-off signal</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Path loss exponent</td>
</tr>
<tr>
<td>$d$</td>
<td>Communication distance</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Packet Loss Rate</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Packet Success Rate</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Bit error rate</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Length of packet</td>
</tr>
<tr>
<td>$E(T)$</td>
<td>Expected number of retransmissions</td>
</tr>
<tr>
<td>$h$</td>
<td>Number of hops</td>
</tr>
<tr>
<td>$GF(2^b)$</td>
<td>Galois Field of $b$-bits</td>
</tr>
<tr>
<td>$U(.)$</td>
<td>Utility function</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>System design parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Lifetime approximation constant</td>
</tr>
<tr>
<td>$F_{CO2}$</td>
<td>Total Carbon footprint</td>
</tr>
</tbody>
</table>

#### A. Routing and Flow Conservation

We model the data transmission rates and routing of data in the network using flow conservation equation. Let $r_{ij}$ denote the rate of information flow from nodes $i$ to node $j$. Let $R_{ij}$ denote the total information rate generated at source node $i$ to be communicated to sink node $j \in N_i$. It is assumed that no compression is performed at the source node. Thus satisfying flow conservation constraint, we have the flow equations at the nodes for time slot $t$ as

$$\sum_{j \in N_i} (r_{ij}(t) - r_{ji}(t)) = R_{ij}(t), \forall i \in N, j \in N_i \quad (1)$$

The maximum transmission rate of a link is also known as its capacity $C_l$. For a given transmit power of node and bandwidth of the channel, this value is fixed and is a upper bound of $r_{ij}$ as $0 \leq r_{ij} \leq C_l$.

#### B. Energy Cost Model

The network lifetime is dependent on the power consumption of the sensor node $P_i$ per active duty cycle slot $T_i$ of a node. This involves the combined operations of sensing, processing and communication (receive/transmit). If a sensor node goes out of the service due to energy deficiency, then all the sensing services from that node are affected till the battery is replaced.

Radio transceiver is the one of the most power hungry block of a sensor device. The communication energy per bit per time slot $E_{comm}(t)$ consists of $E_{RX}(t)$ (receiver energy per bit per time slot) and $E_{TX}(t)$ (transmitter energy per bit per time slot). The computation energy includes $E_{PR}(t)$ (processing energy per bit per slot) and $E_{SN}(t)$ (sensing energy per bit per time slot). Let, $E_B(t) \geq 0$ is the total residual energy left in a sensor node operated by battery at time slot $t$. The power consumption in a time slot $t$ is modeled as

$$P_i(t) = \sum_{i \in N, j \in N_i} r_{ij}(t)E_{TX}(t) + \sum_{i \in N, j \in N_i} r_{ji}(t)E_{RX}(t) + \sum_{i \in N, j \in N_i} R_{ij}(t)E_{PR}(t) + \sum_{i \in N, j \in N_i} R_{ij}(t)E_{SN}(t) \quad (2)$$

From the communication energy model in [4], we modify our transmitter energy for transmitting one bit of data from $i \in N$ to $j \in N_i$ across distance $d$ as

$$E_{TX} = a_1 + a_2 \cdot d^\gamma \quad (3)$$

Where $\gamma$ is the path loss exponent varying from $\gamma \in [2, 6]$, $a_1$ and $a_2$ are constants depending on the characteristics of the transceiver circuit.

#### IV. Packet Loss and Data Re-transmission

A fundamental approach to reduce the packet loss is necessary to be integrated together with upper layer protocols to deliver reliable WSN management in an interfering environment. As often as the packets are failed to be delivered to the sink node, the re-transmission consumes extra energy from the battery source of the sensor node, thereby decreasing its lifetime substantially. We assume a TDMA based MAC.
protocol where retransmission occurs till time-out after which the packet is dropped. The packet loss is dependent on the data traffic. We propose to use the approach of Error Correction Coding (ECC) to improve transmission reliability. ECC adds redundancy to improve the transmission reliability thereby reducing the efficiency, it is still a more preferable solution, because it helps to improve both reliability and latency. In this paper, we describe an error coding scheme on the theoretical basis of Redundant residue number systems (RRNS). An analysis of the our proposed RRNS has been done in [23] that has been extended into our system model in this paper. We have briefly explained the coding scheme and its merits as compared to other lightweight coding schemes. Schemes such as Turbo-codes or viterbi codes need heavy resources for their implementation. So, they are not considered for this analysis in WSN. Reader is referred to our paper [23] for a detailed description.

Our main goal in this section is to introduce the current transmission scheme ARQ and give a brief description why ECC based schemes are advantageous. From ECC schemes of BCH and RRNS [23], our scheme provides improved performance and is thus incorporated for analysis in the optimization problem.

1) Analysis of packet error in ARQ scheme: In ARQ scheme, data is decoded by cyclic redundancy check (CRC) codes and the erroneous data is re-transmitted from the sender. Here we consider stop and wait ARQ method. Assuming the ACK bits are received without error, the packet error rate of the ARQ scheme is given by

\[ P_e^{ARQ} = 1 - (1 - P_b)^{L_P} \]  

where \( L_P \) is the packet length of the payload transmitted in a single transmission, \( P_b \) is the bit error rate. \( P_b \) for sensor nodes in IEEE 802.15.4 is given in [24].

2) Analysis of packet error in ECC schemes: Let us assume that we use a \((n, k, e)\) e-error control method with \( n-k \) redundant bits appended to the \( k \)-data bits. We further assume that the transmission of the packets between the sensor node and sink node is in bursts of \( n \)-bit data. Therefore, the packet loss rate at the sink node is given as

\[ P_e^{ECC} = 1 - \left( 1 - \sum_{i=0}^{n-k} \binom{n}{i} P_b^i (1 - P_b)^{n-i} \right)^{L_P} \]

where \( L_P \) is the packet length of the payload transmitted in a single transmission, \( P_b \) is the bit error rate. \( P_b \) for sensor nodes in IEEE 802.15.4 is given in [24].

ACK failed to reach the sensor from sink. Thus, for a single hop the expected number of re-transmissions is given by

\[ E(Tr) = \frac{1}{(1 - P_e)} \]

Where, \( P_e \) is the packet loss rate of ARQ or ECC schemes. Accordingly, packet loss rate for end-to-end in a h-hop scenario assuming each node transmission is independent of the other as per the TDMA based MAC protocol in Section VC

\[ E(Tr, h) = \frac{h}{(1 - P_e)} \]

Proof. See [4].

3) Redundant Residue Arithmetic based Error Correction scheme: A residue number system (RNS) is a non-weighted number system that uses relatively prime bases as moduli set over GF \((2^b)\) [25]. Owing to the inherent parallelism of its structure and its fault tolerance capabilities, shows fast computation capability and reliability. RNS is defined by a set of \( \beta \) moduli \( m_1, m_2, \ldots, m_\beta \), which are relatively prime to each other. Consider an integer data \( A \), which can be represented in its residues \( \Gamma_1, \Gamma_2, \ldots, \Gamma_\beta \)

\[ \Gamma_i = A \mod m_i, \quad i=1,2,\ldots,l \]

The maximum operating range of the RNS is \( \Theta \) given by (9). The corresponding integer \( A \) can be recovered at the decoder side from its \( \beta \) residues by using the Chinese Remainder Theorem [25] as

\[ A = \sum_{i=1}^{l} \Gamma_i \times M_i^{-1} \times M_i \]

where \( M_i = \Theta/m_i \) and the integers \( M_i^{-1} \) are the multiplicative inverses of \( M_i \) and computed apriori. One common modulus set \((2^{b-1} - 1, 2^{b-1}, 2^{b-1} + 1)\) with a power of two in the set makes it relatively easy to implement efficient arithmetic units. A redundant residue number system (RRNS) is defined as a RNS system with redundant moduli. In RRNS, the integer data \( X \) is converted in \( \beta \) non-redundant residues and \( \delta \beta \) redundant residues. The operating range \( \Theta \) remains the same and the moduli satisfy the condition \( m_1 < m_2 < \ldots < m_\beta < m_\beta + 1 < \ldots \). RRNS can correct up to \( \lfloor (\delta - \beta)/2 \rfloor \) errors. If we consider the popular modulus set, mentioned above, and add the redundant modulus \((2^{b}+1)\) to it, becomes the \((2^{b-1} - 1, 2^{b-1}, 2^{b-1} + 1, 2^{b} + 1)\) RRNS with capability to detect one error, it is explained extensively in [25]. Since the Chinese Remainder Theorem approach require processing large-valued integers, a suitable method for avoiding this is invoking the so-called base-extension (BEX) method using mixed radix conversion (MRC) [26] that reduces the computation overhead by minimum distance decoding.

Based on RRNS, we propose an online error detection and correction scheme for the GWSN systems. Fig. 2 shows the
encoding process of the data $A$ at the sensor node. A parallel to serial converter changes $A$ into its decimal representation. In a look-up-table (LUT), we store the modulus values of numbers $0 - 9$ and $10^n$ ($\chi \in 1, 2, \ldots, \kappa$) with respect to the $\delta$ moduli ($\beta$ non redundant moduli and $\delta - \beta$ redundant moduli). All operations are performed in parallel modulo channels without the need of transmission of information from one modulo channel to another. So, for $I$ moduli, we have $\delta$ modulo channels operating in parallel, all operations in each performs modulo of the particular modulus till $\delta$. Finally, we append the respective MAC IDs of the sensor devices at the front end of each set of packet data and transmit it to the gateway/sink node. Algorithm 1 shows the decoding process at the sink node/gateway. As can be seen, it first receives the packet and tries to recover the data. After the recovery of the data and the error moduli, it appends a 1-bit TRUE flag with the ACK signal and sends it to the sensor node to notify the reception of data, else it sends a 1-bit FALSE flag with ACK to the sensor node signifying to resend the packet data again. The sensor node in turn transmits the $\delta - \beta$ redundant residues again instead of sending the full $n$ bits of data again.

4) Packet loss statistics for different Error Correcting Schemes: We perform a theoretical analysis to find out the packet loss rate of the IEEE 802.15.4 based sensor. The systems signal to noise ratio is varied from 0dB to 20dB. The packet error rate is generated for BCH systems signal to noise ratio is varied from 0dB to 20dB. The sensor node in turn transmits the packet and tries to recover the data. After the recovery of the data, the error moduli, it appends a 1-bit TRUE flag with the ACK signal and sends it to the sensor node to notify the reception of data, else it sends a 1-bit FALSE flag with ACK to the sensor node signifying to resend the packet data again. The sensor node in turn transmits the $\delta - \beta$ redundant residues again instead of sending the full $n$ bits of data again.

![Algorithm 1 Algorithm For RRNS Decoding](image)

![Fig. 2. RRNS Encoding Process.](image)

![Fig. 3. Analytical results of different coding schemes for IEEE 802.15.4 based sensor.](image)

![Fig. 4. (a) Packet loss versus SNR for IEEE 802.15.4 based sensor at different coding schemes.](image)

![Fig. 5. (b) Plot of Expected No. of Packet Re-transmissions versus packet loss rate at different coding schemes for IEEE 802.15.4 based sensor.](image)
TABLE II
COMPARISON OF RRNS AND BCH SCHEMES

<table>
<thead>
<tr>
<th>Scheme Details</th>
<th>n</th>
<th>k</th>
<th>Error Correction</th>
<th>Code Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCH</td>
<td>63</td>
<td>16</td>
<td>11-bits</td>
<td>0.254</td>
</tr>
<tr>
<td>(n = 63; k = 16; e = 11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RRNS</td>
<td>64</td>
<td>28</td>
<td>16-bits</td>
<td>0.438</td>
</tr>
<tr>
<td>(2^{14} - 1, 2^{14} + 1, 2^{15} - 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCH</td>
<td>127</td>
<td>57</td>
<td>11-bits</td>
<td>0.449</td>
</tr>
<tr>
<td>(n = 127; k = 57; e = 11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RRNS</td>
<td>128</td>
<td>60</td>
<td>32-bits</td>
<td>0.469</td>
</tr>
<tr>
<td>(2^{30} - 1, 2^{30} + 1, 2^{31} - 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCH</td>
<td>127</td>
<td>8</td>
<td>31-bits</td>
<td>0.063</td>
</tr>
<tr>
<td>(n = 127; k = 8; e = 31)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1, 2^{31} - 1)(n = 128; k = 60; e = 32), the error correction capability is 32 bits in a burst of 128 bits. Whereas, in a similar BCH code of (n = 127; k = 57; e = 11), the error correction capability is way less at 11 bits in 127 bits of packet data. If we consider BCH codes with similar error correction capability (n = 127; k = 8; e = 31), the code efficiency is very poor, around \( \approx 47\% \), as compared to \( \approx 47\% \) in RRNS. Thus RRNS code simultaneously shows better efficiency and error correction capability as compared to the BCH and ARQ codes.

5) Network Lifetime Maximization through Energy Cost Model: By applying RRNS ECC scheme, the optimization problem has been modified here. The processing energy \( E_{PR} \) in (11) increases with redundancy \( P' = (n - k)/k. \) The re-transmissions consumes extra energy resources apart from the original transmission which is mandatory, hence incorporating the expected number of re-transmissions \( E(Tr, h_i) \) for \( h_i \)-hops into (11), we get power consumption as in time slot \( t \)

\[
P_i(t) = \sum_{i \in N, j \in N_i} r_{ij}(t)E_{TX}(t)(1 + E(Tr, h_i))
+ \sum_{i \in N, j \in N_i} r_{ji}(t)E_{RX}(t)(1 + E(Tr, h_i))
+ \sum_{i \in N, j \in N_i} R_{ij}(t)E_{PR}(t)(1 + E(Tr, h_i)P')
+ \sum_{i \in N, j \in N_i} R_{ij}(t)E_{SN}(t) + \sum_{i \in O(i)} P_{LS}(t)
\]

packet success rate \( P_s(t) \) affects the sample rate in the rate flow constraint as

\[
\sum_{j \in N, t=1}^{T_i} \sum_{j \in N, t=1}^{T_i} (r_{ij}(t) - r_{ji}(t) + P_s(t)R_{ij}(t)) \leq 0, \forall i \in N, j \in N_i
\]

The problem of maximizing the network lifetime can be stated as

\[
\begin{align*}
 & \max_{t \geq 0, E_B(t) > 0} T_i \\
\text{subject to} & \sum_{t=1}^{T_i} (P_i(P_r, h_i, t)) \leq \frac{1}{T_i} \cdot E_B(t), \\
& \sum_{j \in N_i, t=1}^{T_i} R_{ij}(t) - r_{ij}(t) - P_s(t)R_{ij}(t) \leq 0, \forall i \in N, j \in N_i \\
& E_{TX} = a_1 + a_2 \cdot d^\gamma, \gamma \in [2, 6] \\
& 0 \leq r_{ij} \leq C_i
\end{align*}
\]

In our model, we have considered a battery with a finite maximum capacity \( E_{B_{max}} \), where \( E_B(t) \leq E_{B_{max}}. \) Further, due to hardware limitations the total power consumption is upper bounded by maximum consumption \( P_{max} \) (i.e \( P_1(t) < P_{max}, \forall j \in N_i, \forall t \in T_i \)). Problem in (13) is not convex. By substituting \( s = 1/T_i \), we obtain a convex maximization problem in \( s \).

\[
\begin{align*}
 & \min_{s \geq 0} \quad s_i \\
\text{subject to} & \sum_{t=1}^{T_i} (P_i(P_r, h_i, t)) \leq s_i \cdot E_B(t), 1 \leq t \leq T_i \\
& 0 < E_B(t) \leq E_{B_{max}}, 1 \leq t \leq T_i \\
\text{Constraints in (13)}
\end{align*}
\]

V. WIRELESS ENERGY HARVESTING AND WAKE-UP RADIO SCHEME

A critical challenge in large scale implementation of WSNs technology and in a greater scope, IoT, is providing energy to the nodes. A more attractive energy harvesting approach is wireless (RF) energy harvesting which provides key advantages in virtue of being controllable, lower cost and smaller form factor implementation [15], [27]. In this section, enabling technologies for efficient wireless energy harvesting is presented. Also, an energy-efficient method to decrease the power consumption of nodes during the receive mode is discussed.

A. Wireless Energy Harvesting Networks

The wireless energy harvesting unit is in charge of receiving the transmitted waves and efficiently converting them into a stable waveform to recharge or to supply the node. In the context of our system, the wireless energy sources fall into two categories of dedicated sources and ambient sources [27]. A dedicated RF source is deliberately deployed to supply energy to the nodes at a designated rate and optimum frequency (e.g., sink node). An example of a dedicated source is the sink node in our system model. An ambient source, on the other hand, is a less predictable energy source happens to exist within the operation area of the network [28], but are not designed as a part of the network. Examples of ambient sources include TV and radio towers (static ambient source)
space equation which gives the available harvested power by power harvesting unit with reference to Friis free has significant implications on the overall performance of the ratio of the converted DC power to the RF input power, conversion efficiency (PCE) of the rectifier. PCE being the RF to DC comes with some energy loss in the internal RF power to a usable DC supply. The conversion from rectifier) constitute the core of the wireless energy harvesting and the processor. An RF-to-DC converter (also known as transceiver (RX, TX), sensors and sensor interface, storage unit (rechargeable battery), power management unit (PMU) and the processor. An RF-to-DC converter (also known as rectifier) constitute the core of the wireless energy harvesting unit. The rectifier is in charge of converting the received RF power to a usable DC supply. The conversion from RF to DC comes with some energy loss in the internal circuitry of the rectifier which quantified in terms of power conversion efficiency (PCE) of the rectifier. PCE being the ratio of the converted DC power to the RF input power, has significant implications on the overall performance of the power harvesting unit with reference to reference to Friis free space equation which gives the available harvested power by [29]

\[ P_H = P_{TX} \cdot P_L \cdot G_{TX} \cdot G_{RX} \cdot PCE \cdot \frac{\lambda^2}{(4\pi d)^2} \]  

where \( P_H \) is the available harvested power, \( P_{TX} \) is the transmitted power by the source, \( P_L \) is the path loss, \( G_{TX} \) is the transmitter antenna gain, \( G_{RX} \) is the receiver (node) antenna gain, \( PCE \) is power conversion efficiency of the rectifier, \( \lambda \) is the wavelength of the transmitted wave and \( d \) is the communication distance. In most applications, the RF transmitters are subject to regulatory requirements (in terms of frequency and maximum transmitted power), antenna gains are set by geometry obligations and the distance set by the network specification. All these limitations render the PCE as the only viable design parameter to enhance the performance of the WEH unit and consequently prolong the life time of the network nodes [30]. The PCE is optimized for a designated input power which corresponds to a specific communication distance. For longer (than optimal) distances \( (d_{ij}) \), the rectified power abruptly drops. When a receiver node \( i \) is in the energy harvesting mode, the power harvested \( (P_{H,i}) \) from base station server source in a time slot \( t \) can be calculated as follows

\[ P_{H,i}(t) = \frac{\eta \cdot P_{TX} \cdot |H_i(t)|^2}{d_{ij}^2}, 1 \leq t \leq T \]  

where, \( \eta \) is PCE and \( H_i \) denotes the channel gain between between source and receiver at time slot \( t \). As shown, the PCE is optimized for a designated input power (received form the antenna) which corresponds to an specific communication distance. Beyond this optimal point, the rectifier provides sufficient energy for storage or to drive the node circuitry. However, for longer distances from the sink node, the rectified power abruptly drops. In WEH-enabled nodes, PMU is in charge of managing the flow of energy to the storage unit, node circuitry and to the main receiver (RX). Aside from high efficiency, other key performance metrics of a WEH unit include high sensitivity (i.e., ability to harvest energy from small levels input power), wide dynamic range (i.e., maintaining high efficiency for a wide range of input powers), multi-band operation (i.e., ability to harvest wireless energy from wireless transmissions at different frequencies). Extensive studies exist in the literature investigating on techniques to improve the performance of WEH unit [27], [29]. The design presented in [31] studies techniques to enhance the efficiency of WEH unit and a multi-band approach to enable harvesting and different frequencies.

B. Wake-Up Radio Scheme

In a wireless sensor node, the receiver unit despite not being the most power hungry block, constitutes a significant portion of the overall energy consumption of the system. While similar to other building blocks, the receiver is practically called in to action only when its service is required. It has to keep listening to the communication channel for the commands from the sink node. An efficient solution to tackle the energy consumption during the idle listening mode is duty cycling (also known as rendez-vous scheme) in which the receiver maintains in deep sleep mode and only wakes up when there is a message to be received from the main transmitter (TX). There are three main classes of duty cycling, namely, synchronous, pseudo-asynchronous and asynchronous [32]. In the synchronous scheme, the transmitter and all the receivers pre-schedule designated time slots in which the receivers wake up for to receive the commands and fulfill the transmission. Such scheme imposes considerable overhead in terms of complexity and power consumption in order to establish time synchronization and leads to idle
energy consumption if there is no data to be received during the pre-scheduled time slots. In the pseudo-asynchronous scheme, the receivers wake up at designated time but a synchronization between the transmitter and receiver is not required. In the asynchronous scheme which the most energy efficient approach among the duty-cycling classes, the receivers spends most of their time in deep sleep mode and only wake up when interrupted by the transmitter. This interrupt message is generated by a wake-up radio (WUR). WUR is a simple and low-power receiver which keeps listening to the channel and only wakes up the main receiver when there is a request for transmission to the associated node [33]. This so called listening mode power \( P_{LS} \) consumption when integrated over the lifetime of the node is dependent on the amount of network utilized for given duty cycle. Let \( \alpha \in (0,1) \) be the system parameter that defines the amount of network utilization. The amount of energy consumption modeled in terms of \( \alpha \) in (11) is

\[
P_i(P_e, h_i, t) = \sum_{i \in N} r_{ij}(t)E_{TX}(t)(1 + E(Tr, h_i)) + \sum_{i \in N, j \in N} r_{ij}(t)E_{RX}(t)(1 + E(Tr, h_i)) + \sum_{i \in N, j \in N} R_{ij}(t)E_{PR}(t)(1 + E(Tr, h_i))P' + \sum_{i \in N, j \in N} R_{ij}(t)E_{SN}(t) + \sum_{i \in O(i)} \alpha(t)P_{LS}(t)
\]

(17)

Fig. 5 schematically compares the energy profile of a conventional transceiver versus that of a WUR-enabled transceiver. As shown in the figure, as compared to the conventional method, the main receiver (RX) in the WUR-enabled transceiver is activated only upon receipt of the wake-up command (WU) which is followed by the interrupt message generated by the WUR. The infrequent activation of RX facilitates a substantial energy conservation over the life-time of the wireless node. Obviously, WUR scheme is favourable only if the power consumption of the WUR is much smaller than that of RX (i.e., \( P_{WUR} << P_{RX} \) in Fig. 5(a)). WEH-enabled nodes provide a good opportunity for a very efficient implementation of WUR [34]. Fig. 5(b), shows the block diagram of one such implementation for on-off keying (OOK) WU message. As shown in the figure, the rectifier block of the WEH unit can be re-utilized to perform as a simple envelope detector while also providing energy supply for the rest of WUR circuitry [34].

C. Medium Access Control for WEH-WSN

In this section, the MAC protocols for WSN is presented which complements our wake-up radio design. MAC protocols for WSNs can be classified under contention-based and contention free schemes which are further divided into scheduled, random access, and duty-cycle based schemes. In scheduled MAC protocols (e.g., [35]), time slots are being assigned for each node to transmit so that idle listening mode can be eliminated, and collision can be avoided. However, this exchange requires additional overhead as well as a synchronization in time with a global clock, which is very tough to attain. Since the energy source is unpredictable in WEH-WSNs, it is difficult for nodes to exchange time schedules as they do not know future energy availability. Random access protocols also called contention based protocols, do not need to exchange schedules but incur additional idle time for the node to sense the channel before transmitting (like CSMA/CA schemes), and overhearing time to listen to packets not destined to itself (S-MAC, B-MAC) [35]. MAC protocols based on Time Division Multiple Access (TDMA) with wakeup and sleep periods have attracted considerable interest because of their low power consumption and collision free operation [36] [37] [38]. Due to its benefits of reduced collisions, scalability and bounded latency, TDMA is widely considered in wireless networks. TDMA partitions time into many fixed slots and nodes transmit data in their assigned slots, thereby avoiding collisions. The duty cycling concept is greatly efficient in terms of power saving. TDMA based protocols are more energy efficient, and the energy consumed is proportional to the length of the transmission cycle while the latency is proportional to the size of the network. Moreover, a single global clock is not needed for synchronization in wake-up duty cycled TDMA schemes.

ODMAC, an on-demand MAC protocol, was proposed to support individual duty cycles letting the nodes operate in the energy neutral operation state by exploiting the maximum harvested energy [36]. This state guarantees infinite lifetime as soon as there are not any hardware failures. However, it is hard to design the sensors to be always in this state since the dynamics of the environmental energy sources are hard
to predict. It exploits the fact that sensor nodes often have low traffic in order to remove the burden of idle listening by Carrier Sensing. A drawback of ODMAC is the lack of retransmissions, so the successful reception of packets is not acknowledged, which might result in discarding all the packets involved in collisions. Some other protocols proposed for EH-WSN are EH-MAC and ERI-MAC [37]. EH-MAC is an ID-polling-based MAC protocol proposed for multi-hop EH-WSNs and it achieves high channel performance in terms of network throughput and fairness. ERI-MAC is a receiver initiated protocol which dynamically adjusts the duty-cycle based on the energy harvesting state of the system.

To cater for the interference in the TDMA MAC model, our analysis is based on WUR scheme for low power duty cycling with transmission capacity provided with the incorporation of ECC codes.

D. Modeling Energy Harvesting and Wake-Up Radio

Let \( P^C_i(t) \) denotes the cumulative harvested energy in all the slots of node \( i \). For simplicity, we assume the harvested energy is available at the start of each interval \( t \). We also assume that the battery has finite capacity and harvested energy can only recharge till the maximum capacity of battery \( E_{B_{\text{max}}} \).

\[
P^C_{H_i}(t) = \sum_{x=1}^{t} P_{H_i}(x), (t \in 1, 2, ..., T_j)
\]

(18)

\( P^C_{H_i}(t) \) is a continuous increasing function that lies between points \((0, 0)\) and \((T_j, P^C_{H_i}(T_j))\) as shown in Fig. 6. The cumulative node energy \( P^C_{H_i}(t) \) for all \( t \in 1, 2, ..., T_j \) cannot be more than \( P^C_{H_i}(t) \). Using this constraint, the dynamic charging and discharging of battery can be modeled as

\[
E_B(t + 1) = E_B(t) - P_i(t) + P_{H_i}(t)
\]

\[
P^C_i(t) \leq P^C_{H_i}(t), \forall t \in 1, 2, ..., T_j
\]

(19)

To find an optimal energy consumption \((P^C_{i}(t))^*\), we need to find the upper and lower bound of consumed energy. (19) gives the upper bound on the consumed energy. Further, \((P^C_{i}(t))^*\) must satisfy that, the residual energy of nodes at all time slots i.e. \((P^C_{i}(t))^* - P^C_{H_i}(t)\) cannot exceed the battery maximum capacity \( E_{B_{\text{max}}} \), forms the lower bound of \((P^C_{i}(t))^*\). Thus the problem in (14), can be reformulated as

\[
\min_{s \geq 0} s_t
\]

\text{subject to} \sum_{t=1}^{T_i} (P_i(P_s, h_i, t) - s_t \cdot E_B(t) - P_{H_i}(t)) \leq 0,

1 \leq t \leq T_i

0 < E_B(t) \leq E_{B_{\text{max}}}, 1 \leq t \leq T_i

P^C_{H_i}(t) - E_{B_{\text{max}}} \leq P^C_i(t) \leq P^C_{H_i}(t),

\forall t \in 1, 2, ..., T_i

Constraints in (13), (16), (17) and (18)

(20)

VI. J. Utility & Network Lifetime Trade-off and Distributed Solution

Solving standalone maximization of network lifetime problem by varying the source rates will result in allocation of zero source rates to the node. Thus, it results in application performance of the system to be worst. Therefore, it is optimal to jointly maximize the network lifetime with the system’s application performance. We associate the network performance with the utility function \( U_i(.) \). In [3], it has shown that each node \( i \in N \) is related to a utility function and achieve different kind of fairness by maximizing the network utility. Thus the utility is a function of the node source rate \( R_{ij} \). Apart from source rates, packet success rate \( P_s \) also affects the overall system performance. Thus, the utility function has to be modified to accommodate the packet success rate and the payload data efficiency as \( U_i(R_{ij}, P_s) \).

Max-Min fairness maximizes the smallest rate in the network whereas the Proportional fairness favors the nodes nearer to the sink node. As given in [3], by aggregating the utility, the network lifetime can be solved in a distributed way with an approximated approach as \( P^*_{\text{src}}(.) = \left( \frac{1}{\epsilon + 1} \right), s_t^{\epsilon+1} \). Thus, the network lifetime problem in (20) becomes

\[
\min_{s \geq 0} \quad \left( \frac{1}{\epsilon + 1} \right), s_t^{\epsilon+1}
\]

\text{subject to} \quad \text{constraints in (20), (17) & (12)}

Using (21), we can now formulate a joint trade-off between maximizing utility and network lifetime simultaneously. Our method differs from other approaches in Section II as we consider a more practical scenario, incorporating path loss, fairness, packet loss statistics for error control schemes as well as energy harvesting and a event driven radio wake-up scheme. Thus the cross-layer joint maximization problem is given as

\[
\max_{(s, R_{ij}, r_{ij}) \geq 0} \quad \sum_{t=1}^{T_i} \alpha(t) \sum_{i \in N} \sum_{j \in N_i} U_i(R_{ij}(t), P_s(t))
\]

\[
- \sum_{t=1}^{T_i} (1 - \alpha(t)) \left( \frac{1}{\epsilon + 1} \right), s_t^{\epsilon+1}
\]

\text{subject to} \quad \text{constraints in (20), (17) & (12)}

We have introduced a system parameter \( \alpha \in [0, 1] \) in (17). It gives the trade-off between the utility and network lifetime. For \( \alpha = 0 \), the utility is zero and for \( \alpha = 1 \), network lifetime
is maximum with worst application performance. The maximization objective function is concave as \(U(.)\) is concave and network lifetime problem \(F_x^\nu(.)\) is convex. We try to solve the primal problem via solving the dual problem [22]. We keep the expected number of transmissions \(E(T_r, h_i)\) in hops \(h_i\) as constant and vary the rate \(r_{ij}\). The constraint set in (22) represents a convex set. According to slater’s condition for strong duality, if the non-linear constraints are strictly positive, duality gap between primal and dual problem is small. Thus the primal can be solved by solving the dual problem and the desired primal variables can be obtained. The dual-based approach leads to an efficient distributed algorithm.

### A. Dual Problem

To solve the problem in a distributed manner, we formulate the Lagrangian in terms of the Lagrange Multipliers \(\lambda\) and \(\mu\) by relaxing the inequality constraints in (22).

\[
L(\lambda, \mu, s, r_{ij}, R_{ij}, U(R_{ij}, P_s), t) = \sum_{i=1}^{T_i} \alpha(t) \sum_{j \in N} U_i(R_{ij}(t), P_s(t)) - \sum_{i=1}^{T_i} \left(1 - \alpha(t)\right) \lambda(t) r_{ij}(t) + \sum_{i=1}^{T_i} \mu_i(t) \right) + \sum_{i=1}^{T_i} \left(1 - \alpha(t)\right) \sum_{j \in N} \mu_i(t) \left(\frac{1}{\epsilon + 1}\right) \cdot \ell_i^{t+1}
\]

(23)

The corresponding Lagrange dual function \(D(\lambda, \mu)\) and the solution \(F^\nu\) is given by

\[
D(\lambda, \mu) = \sup_{s, r_{ij}, R_{ij}, U} L(\lambda, \mu, s, r_{ij}, R_{ij}, U(R_{ij}, P_s), t), (24)\]

subject to constraints in (20), (17) & (12)

\[
F^\nu = \min_{\lambda > 0, \mu > 0} D(\lambda, \mu) (25)
\]

The dual problem of (24) can be decomposed further into two different subproblems \(D_1(\lambda, \mu)\) and \(D_2(\lambda, \mu)\). Subproblem \(D_1(\lambda, \mu)\) is a rate control problem in the network and transport layer of the sensor networks. For all active links \(l \in L\), we substituted \(\sum_{i=1}^{T_i} \sum_{j \in N} \mu_i(t) \). Subproblem \(D_2(\lambda, \mu)\) gives the bound on the inverse lifetime. The objective function of the primal problem is not strictly convex in all its primal variables \(\{s, R_{ij}, r_{ij}\}\). The sub-dual problems \(D_1(\lambda, \mu)\) is only piecewise differentiable. Therefore, the gradient projection method cannot be used to solve the problem. We use the subgradient method [22] to solve the problem iteratively till a desirable convergence is reached.

\[
D_1(\lambda, \mu) = \max_{(R_{ij}, r_{ij}) \geq 0} \sum_{i=1}^{T_i} \sum_{j \in N} \alpha(t) \cdot U_i(R_{ij}(t), P_s(t)) + \sum_{i=1}^{T_i} \sum_{j \in N} \lambda_i(t) (r_{ij}(t) - r_{ji}(t) + P_s(t) R_{ij}(t)) + \sum_{i=1}^{T_i} \sum_{j \in N} \mu_i(t) \cdot (r_{ij}(t) E_{TX}(t) (1 + E(T_r, h_i))) + \sum_{i=1}^{T_i} \sum_{j \in N} \mu_i(t) \cdot (r_{ij}(t) E_{RX}(t) (1 + E(T_r, h_i))) + \sum_{i=1}^{T_i} \sum_{j \in N} \mu_i(t) \cdot (R_{ij}(t) E_{PR}(t) (1 + E(T_r, h_i) P')) + \sum_{i=1}^{T_i} \sum_{j \in N} \mu_i(t) \cdot (R_{ij}(t) E_{SN}(t) (1 + \alpha(t) P_{LS}(t))
\]

subject to

\[
E_{TX} = a_1 + a_2 \cdot d_{ij}, \gamma \in [2, 6]
\]

(26)

\[
P_{H_i}^C(t) - E_{Bmax} \leq P_C(t) \leq P_{H_i}^C(t), \forall t \in 1, 2, \ldots T_i
\]

\[
D_2(\lambda, \mu) = - \left\{ \max_{(s, E_B) \geq 0} \sum_{i=1}^{T_i} \sum_{j \in N} \sum_{t=1}^{T_i} \mu_i(s_i \cdot E_B(t) + P_{H_i}(t)) + \sum_{i=1}^{T_i} \left(1 - \alpha(t)\right) \sum_{j \in N} \mu_i(t) \left(\frac{1}{\epsilon + 1}\right) \cdot \ell_i^{t+1} \right\}
\]

subject to

\[
0 < E_B(t) \leq E_{Bmax}, 1 \leq t \leq T_i
\]

(27)

Let, \(s^*(\lambda, \mu), R_{ij}^*(\lambda, \mu), r_{ij}^*(\lambda, \mu), P_{H_i}^C(t)^*, P_s^*(t), P_{LS}^*(t)\) be the optimal solutions for problems (26) and (27). We define the following to obtain the distributed solution,

**Definition 1.** Let \(f: \mathbb{R}^n \rightarrow \mathbb{R}\) is a convex function. The subgradient of \(f\) at a point \(x' \in \mathbb{R}^n\) satisfy the following inequality with respect to a point \(y' \in \mathbb{R}^n\), \((\nabla f(x'))^T\) is the gradient of \(f\) at \(x'\)

\[
f(y') \geq f(x') + (y' - x') \nabla f(x')^T
\]

(28)

Using Definition 1, we write the update for dual variables at the \((\tau + 1)^{th}\) iteration as,

\[
\lambda_i(t, \tau + 1) = [\lambda_i(t, \tau) + \varphi_{\tau} \nabla \lambda D(\lambda, \mu)^T]^+,
\]

\[
\mu_i(t, \tau + 1) = [\mu_i(t, \tau) + \psi_{\tau} \nabla \mu D(\lambda, \mu)^T]^+
\]

(29)

\([.]^+\) is the projection on the non-negative orthant meaning \(z^+ = \max\{0, z\}\), \(\{\varphi_{\tau}, \psi_{\tau}\}\) are the positive step sizes and \(\{\nabla \lambda D(\lambda, \mu), \nabla \mu D(\lambda, \mu)\}\) are the gradients of dual problem in (25) w.r.t \(\lambda\) and \(\mu\).
B. Solution to GWSN Distributed Algorithm and its Convergence Analysis

The Lagrange multipliers \((\lambda_i, \mu_i)\) have cost interpretation to them. \(\lambda_i\) represents the link capacity cost and \(\mu_i\) denotes the battery utilization cost of sensor node \(i\). The gradients \(\nabla_{\lambda} D(\lambda, \mu)\) and \(\nabla_{\mu} D(\lambda, \mu)\) denote the excess link capacity and battery energy respectively. Problems \(D_1(\lambda, \mu)\) in (26) represent the maximization of the aggregate utility of the network in presence of flow constraints and energy spent in the network. The network lifetime problem \(D_2(\lambda, \mu)\) in (27) maximizes the revenue from battery capacities subtracting the lifetime-penalty function, resulting in reduction of lifetime. The procedure for solving the GWSN algorithm is outlined as follows:

**Algorithm 2 GWSN Distributed Algorithm**

- Initialize all the inputs \((E_{TX}, E_{RX}, E_{SN}, E_{PR}, P_{LS}, E_B)\) and step sizes \(\varphi_t \leftarrow 0.01, \psi_t \leftarrow 0.01 / \sqrt{T}, \forall t \leq 20\).
- Although the problem in \(D_2(\lambda, \mu)\) and \(D_2(\lambda, \mu)\) is convex, the solution is complex and difficult to implement due to the intricacies introduced by incorporation of optimal energy consumption \((P_i^C(t)^*)\), packet loss \((P_i^L(t)^*)\) and WUR \((P_i^D(t)^*)\). From (26) and (27), it is evident that \((P_i^C(t)^*)\) is dependent on optimal lifetime \((s_i^*)\) and sample rate \((R_i^*)\). Therefore we take \((P_i^C(t)^*)\) as some function \(g\) of lifetime and sample rate.
  \[ g(s_i^*, R_i^*) = f((P_i^C(t)^*)) \]

- We model \(P_i^D(t)^*)\ w.r.t the channel gain \(H_i(t)\) distributed as \(i.i.d\) with mean 0. Once the optimal \(s_i^*, R_i^*\) is found, \(P_i^D(t)^*\) is found using \(f^{-1}(g(s_i^*, R_i^*))\).
- The packet success rate \(P_s(t)\) is varied \(\in [0, 100]\) and system utility parameter \(\alpha(t)\) and overall node utilization \(U_i(R_i(t), P_s(t))\) determines the optimal listening power \(P_{\text{LS}}^i(t)\).
- Thus from all the previous assumptions mentioned above, the time coupling property of the node can be excluded and finding solution for \(\lim_{\tau \rightarrow \infty} \lambda(\tau), \mu(\tau)\) would be good \(\forall t \in \{1, 2, 3, ..., T_i\}\).
- The Lagrange multipliers can be updated by
  \[ \lambda_i(t, \tau + 1) = [\lambda_i(t, \tau) + \varphi_t \sum_{j \in N_i} (r_{ij}(t, \tau) - r_{ji}(t, \tau) + P_s(t, \tau) R_{ij}(t, \tau))]^+, \]
  \[ \mu_i(t, \tau + 1) = [\mu_i(t, \tau) + \psi_t \sum_{i \in N_j} \sum_{j \in N_i} (P_i(P_e, h_i, t, \tau) - s_i \cdot E_B(t, \tau) - P_{\text{H}}(t, \tau))]^+ \] (31)

- From (29), (31), it can be seen that as the flow \(r_{ij}\) exceeds the capacity of link \(C_{ij}\), the link cost and node energy cost increases. Thus higher link and node-battery prices result in greater penalty in the objective function in (26) forcing source rates \(R_{ij}\) & flows \(r_{ij}\) to reduce. Although higher node-battery cost (27) allow greater revenue for the same increase in battery capacities (by increasing ‘s’), there is a corresponding penalty incurred due to the consequent lower lifetimes.

Now we discuss the convergence of our distributed algorithm for GWSN. It is worth noting that the proposed algorithm takes into account the lifetime constraint, energy harvesting constraint, packet loss statistics and path loss into consideration. Thus it is necessary to analyze the convergence bounds.

**Lemma 2.** When \(\varepsilon \rightarrow \infty\), the network lifetime \(T_{\text{network}}\) determined by the optimal solution \(s^*\) of problem (22) approximates the maximum network lifetime of the wireless sensor network.

**Proof.** See Appendix A

Further, let us make the following two assumptions as below:

- **Assumption 1:** Let \(U_i(R_{ij}, P_s)\) be defined as \(\log_2(R_{ij}, P_s)\) which is an increasing and concave function, and its inverse and hessian exists.
- **Assumption 2:** Hessian of \(U_i(R_{ij}, P_s)\) is negative semidefinite and \(r_{ij}^\text{min} \leq r_{ij} \leq r_{ij}^\text{max}\).

Define \(\mathcal{T} = \max L\) as the maximum number of links that a sensor node uses. Let \(\overline{U} = \max U_i(R_{ij}, P_s)\) and \(\overline{R} = \max r_{ij}\), be the maximum rate flow of the node when transmitting information from \(i \rightarrow j\).

**Proposition 1.** If the assumptions 1 and 2 above hold and the step size satisfies \(0 < \varphi_t, \psi_t < \frac{2}{\sqrt{T}}\). Then starting from any initial rates \(r_{ij}^\text{min} \leq r_{ij} \leq r_{ij}^\text{max}\) & price \(\lambda_i, \mu_i \geq 0\), every limit point of the sequence \(\{s(\lambda, \mu), R_{ij}(\lambda, \mu), r_{ij}(\lambda, \mu)\}\) generated by GWSN Algorithm, is primal-dual optimal.

**Proof.** See Appendix B

**Lemma 3.** By the above distributed algorithm, dual variables \((\lambda_i, \mu_i)\) converge to the optimal dual solutions \((\lambda_i^*, \mu_i^*)\), if the stepizes are chosen such that

\[ \varphi_t(i) \rightarrow 0, \sum_{i=1}^{\infty} \varphi_t(i) = \infty, \psi_t(i) \rightarrow 0, \sum_{i=1}^{\infty} \psi_t(i) = \infty \] (32)

VII. SIMULATION RESULTS

To show the joint trade-off between maximizing utility and network lifetime in terms of system parameter \(\alpha\), path loss \(\gamma\), packet loss statistics \(\{P_i\}\), energy harvesting \(P_{\text{H}}\), we consider a WSN as shown in Fig. 7 with seven nodes distributed over a square region of 100m × 100m. The node at the middle of the network is taken as the sink node and the other six nodes are either source or source/relay nodes. Nodes \{1, 2, 3, 4, 5\} act as source nodes whereas nodes \{3, 6\} act as source node to deliver its own data and relay nodes for delivering nearest neighbor’s data to the sink node. The parameters taken for the simulation are depicted in Table III. The value of \(E_{TX}, \{a_1, a_2\}\) are chosen from [4] with \(\gamma=4\). \(E_{RX}\) and \(E_{SN}\) are taken from [39]. Processing energy \(E_{PR}\) is assumed to be same as the sensing energy \(E_{SN}\). Also, at
A. Convergence Plots

To show the convergence of our GWSN algorithm according to Lemma 2, and 3, we plotted in Fig. 8(a), the convergence of source node rates for different sensor nodes with respect to the number of iterations. We have chosen sensor node \(\{i, i', i''\}\), where \(\{i_1, i_5\}\) act as only sensor and relay node. The step size is taken as \(\varphi = 0.01\), where \(\varphi\) is the index of iteration. It can be observed that the step size plays a vital role as it controls the magnitude of oscillations near the optimal solution. The larger the step size, the faster the convergence but with more variations near the point of optimality whereas smaller step size reach a stable optimal solution with lesser fluctuations near the optimal. As predicted by our algorithm, sensor nodes that have lower lifetime \(\{i, i'\}\) are assigned higher rates, whereas nodes with higher lifetime \(\{i_3, i_6\}\) have lower rates being assigned to them. Fig. 8(b) shows the error in measuring the lifetime with respect to the coefficient \(\epsilon\).

\[
\text{Error in Approximating Lifetime} = \left| s - \frac{1}{\epsilon + 1}s^\epsilon + 1 \right|
\]

(33)

According to Lemma 2, if the coefficient \(\epsilon\) is large enough then the lifetime approximated by (22) is the maximum lifetime. Fig. 8(b) validates the point, as it can be seen that at \(\epsilon = 10\), we get less than 10% error in measurement of lifetime. For our Algorithm, we have initialized the value of \(\epsilon\) as 20 with less than 5% error in lifetime prediction.

B. Utility and Lifetime Trade-off with WEH and WUR constraints

The impact of the system design parameter \(\alpha(t)\) is shown in Fig. 9(a), 9(b) & 9(c). \(\alpha(t)\) is varied between 0.1 to 0.9. The network utility is computed as \((\sum_{i=1}^{6} log_2(R_{ij}P_s))\) which is the aggregate utility of all the nodes not including the sink node \(s_1\). The aggregate utility have been normalized with respect to the maximum utility of the network. Fig. 9(a) shows that the network lifetime decreases and the utility increases as the increment of \(\alpha\). On the contrary, we can observe that as the weighted system parameter \(\alpha\) decreases, the corresponding optimal network lifetime increases. It can be seen in Fig. 9(b) that the lifetime increases to 8.5s from 4.5s. Fig. 10(a) shows the harvested energy profile from (16) for the farthest node in the network. Replacing the optimal \(s_{ij}, P_{ij}\) in (30), \(P_{ij}^*(t)\) is found using \(f^{-1}(g(s_{ij}, P_{ij}))\) as shown in Fig. 10(b). Further, if wake-up radio scheme is applied with energy harvesting, the lifetime increases to ~10s as in Fig. 9(c). The network utility of the system also increases to 0.87 with energy harvesting and 0.97 with both energy harvesting and WUR. Hence, based on the desired performance of the system, designer can chose the value of \(\alpha\) and solve the set of equations for optimal lifetime and source node rates.

C. Impact of Error Control Coding on performance and lifetime

Fig. 9(d) shows the utility-lifetime trade-off with error coding applied. The system lifetime is further increased as compared to Fig. 9(a)-(c), to 14s and the network is more utilized at 91%. To visualize the impact of error coding on the performance of the system, we plot the network lifetime versus the packet loss rate \(P_e\) at \(\alpha = 0.1\). Fig. 11(a) shows the plot of network lifetime for different cases with packet loss rate varying from 0 to 20%. For a packet loss rate between 10% to 20% , the network lifetime increases more than 3 times with only energy harvesting and wake-up radio scheme. Whereas with the coding scheme applied,
Fig. 9. Simulation plots of Network Aggregate Utility - Lifetime trade-off for different $\alpha$.

(a) Network Aggregate Utility - Lifetime trade-off without WER, WUR and ECC.

(b) Network Aggregate Utility - Lifetime trade-off with WER and without WUR & ECC.

(c) Network Aggregate Utility - Lifetime trade-off with WER & WUR without ECC.

(d) Network Aggregate Utility - Lifetime trade-off with WER, WUR & ECC.

Fig. 10. Energy harvesting profile and allocated energy plots.

it doubles further giving a 6 times improvement. We evaluate the network lifetime of nodes $\{i_1, i_3, i_5, i_6\}$, where $\{i_1, i_5\}$ act as only sensor nodes and $\{i_3, i_6\}$ act as both sensor and relay node. The network lifetime is shown in Fig. 11(b) versus the system parameter $\alpha$ incorporating harvesting and coding at packet loss rate of 20%. As expected from (23), the lifetime of node $i_1$ is the least. Relaying of data from $i_5 \rightarrow i_6$ improves the lifetime of node $i_5$. Nodes $i_3$ and $i_6$ have a huge improvement in their lifetime owing to their proximity to the sink node from where they harvest energy according to (15). Even though the total energy consumption is increased, the harvested energy increase is sufficient enough to boost its lifetime.

<table>
<thead>
<tr>
<th>Sensors Type &amp; Model No.</th>
<th>$E_{PR}$ $E_{SN}$ $E_{comm}$ $E_{comm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration MMA72600Q</td>
<td>0.044 0.000027</td>
</tr>
<tr>
<td>Pressure 2200/2600 Series</td>
<td>0.044 0.00013</td>
</tr>
<tr>
<td>Light ISL 29002 18</td>
<td>0.047 0.00068</td>
</tr>
<tr>
<td>Proximity CP 18</td>
<td>0.047 0.267</td>
</tr>
<tr>
<td>Humidity SHT 1X</td>
<td>0.043 0.4</td>
</tr>
<tr>
<td>Temperature SHT 1X</td>
<td>0.94 1.5</td>
</tr>
</tbody>
</table>
D. Effect of Energy Harvesting and Error Correcting Codes on Practical sensor node TelosB

For analyzing the effect of our error correcting codes performance on node lifetime, we have taken real time sensor energy cost from [40] for different sensors as shown in Table IV. The Table shows different commonly used sensing devices, their $E_{PRI}$ and $E_{SN}$ energy cost normalized w.r.t communication energy $E_{comm}$ for common sensor mote TelosB (TelosB is a IEEE 802.15.4 compliant sensor mote that runs a TinyOS operating system with a CC2420 radio. http://www.willow.co.uk/TelosB_Datasheet.pdf).

Where, $E_{comm}$ is sum of $E_{TX}$ and $E_{RX}$. Using different energy cost of sensors from Table IV, we have plotted curve for TelosB mote. The battery power is taken as 9000 milli-Amphere-Hour (capacity of 2 standard 1.5 – volt batteries used in sensors). Fig. 12(a) is drawn for RRNS, BCH, and ARQ for a packet loss rate of 20% showing the estimated lifetime in days for the TelosB mote versus the total average power consumption $P_t$ from (11). For low power sensors i.e acceleration, pressure, light, proximity given in Table IV, TelosB motes lifetime increases by $\sim$1.7 times with BCH error scheme and more than doubles with RRNS error scheme. Whereas for power hungry sensor such as Temperature, the processing energy is higher, thus overpowering the effect of small number of retransmissions in error coding schemes.

One of the major overheads of error correcting codes in addition to transmission and reception of redundant bits is the delay associated with encoding and decoding of packets. Let us assume that $t^{ARQ}$ is the total time required for sending the packets to the sink node and receiving an ACK back. Further, if the decoding latency of a block code like $(n, k, e)$ BCH is $t_{dec}^{BCH}$. From [24], the decoding latency is given by

$$t_{dec}^{BCH} = (2ne + 2e^2)(t_{add} + t_{mult}) \left\lceil \frac{b}{b_m} \right\rceil \tag{34}$$

Here, $t_{add}$ and $t_{mult}$ are time required for additions and multiplications in GF ($2^k$), and $b_m$ is the number of bits of micro controller used in sensor nodes. In an 8-bit micro controller, $t_{add}$ take one cycle and $t_{mult}$ takes two cycles as computation time. The number of cycles depends on the frequency of the micro controller.

RRNS codes of form $((2^b-1, 2^b-1, 2^b+1, 2^b+1))$ needs $t_{dec}^{RRNS}$ as the total time required for sending the packets to the sink node and receiving ACK back. The decoding latency depends on the total additions and multiplications in the number of iterations $(\beta \beta)$. Depending on the value of $\beta$ for each step there are $2\beta$ multiplications and $\beta$ additions involved. Further, there are $(\beta \beta)$ number of moduli operations involved. Thus, the decoding latency for RRNS codes is

$$t_{dec}^{RRNS} = ((\beta) t_{add} + (\beta) t_{mult}) \left\lceil \frac{b}{b_m} \right\rceil + ((\beta) e) \left\lceil \frac{c}{b_m} \right\rceil \tag{35}$$

To analyze the effectiveness of the coding schemes, we have plotted the delay in sending one packet of data versus the packet loss rate of 10% and 20%. If we take $t^{ARQ} = \ldots$


50\text{ms}, from (34) and (35), delays of BCH(127, 57, 11) and RRNS(128, 60, 32) can be found as $t_{\text{delay}}^{\text{BCH}} = t_{\text{dec}}^{\text{BCH}} + t_{\text{dec}}^{\text{RRNS}} + t_{\text{dec}} ^{\text{ARQ}} \ast (n/k)$ and $t_{\text{delay}}^{\text{RRNS}} = t_{\text{dec}}^{\text{RRNS}} + t_{\text{dec}} ^{\text{ARQ}} \ast (n/k) + t_{\text{dec}} ^{\text{ARQ}}$. TelosB has a 16-bit microcontroller and its clock frequency is 8MHz. Fig. 12(b) shows the delay in milliseconds. It can be inferred that the coding schemes outperform the ARQ scheme in terms of total transmission delay. RRNS scheme has less delay compared to BCH coding due to its better coding rate and faster decoding. It can also be seen that as the packet loss rate increases, the delay gap between the three schemes increases. Thus RRNS has better performance in terms of lifetime improvement as well as lower delay as the packet loss rate increases in bad channel conditions.

E. Green Networking : Reduction in Carbon footprint

For network to be green, the carbon emissions has to be reduced. The index of measure of carbon emissions is $X_{gr}$ of CO$_2$ per year. For each packet loss in the network causes the data server station or the sink node to transmit back NACK to sensor node. The transmitting power ($P_{TX}^S$) of the data station depends on the fuel type from which the station derives its electrical power. Depending on the country, it can be coal or gas. Thus value of $X$ can be either 870 or 370 [20]. ($P_{TX}^S$) depends on the type of technology used. If we assume that the sink node data station runs on the Long Term Evolution (LTE) network and uses the static micro cell topology with radius 100m. Then from [41] and [20], the carbon footprint generated by sink node is

\[ F_{CO_2} = P_{TX}^S \cdot (E_{T,h}, + 1) \cdot 8.64 \cdot 10^{-3} \cdot X[KgCO_2/Year] \]

\[ P_{TX}^S = \left( \frac{P_T^B}{\mu_{PA}} \right) \cdot C_{T,X,static} + P_{SP,static} \]

\[ F_{CO_2}^B = B_u^{gear} \left( \frac{4.3}{30} \right) [KgCO_2/Year], \quad B_u^{gear} = \frac{365}{T_{\text{network}}} \]

Where, the notations are described in Table V. Apart from the sink node, the battery is also responsible for generation of carbon footprint. Typical AA batteries used in sensors have a end of life carbon emission of 4.3 K secular CO$_2$ per 30 batteries [42]. Thus, the carbon footprint [KgCO$_2$/Year] generated by number of batteries used is directly proportional to the total batteries used in a year ($B_u^{gear}$) and is given as

\[ F_{CO_2} = \frac{365}{T_{\text{network}}} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{TX}^B$</td>
<td>Power consumed by sink node base station server</td>
<td>2 W</td>
</tr>
<tr>
<td>$\mu_{PA}$</td>
<td>Power Amplifier efficiency</td>
<td>20%</td>
</tr>
<tr>
<td>$C_{T,X,static}$</td>
<td>Static transmitted power</td>
<td>0.8</td>
</tr>
<tr>
<td>$P_{SP,static}$</td>
<td>Static signal processing power</td>
<td>15 W</td>
</tr>
<tr>
<td>$C_{PS}$</td>
<td>Power supply loss</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The total carbon footprint ($F_{CO_2}$) is therefore the sum of carbon footprints in (36) and (37). To show the effectiveness of using ECC, WEH & WUR, we plot $F_{CO_2}$ for different packet loss rate of (0, 10, 20). We take $X$=370, the fuel for production of electricity as gas. The $T_{\text{network}}$ for different schemes ARQ, RRNS and BCH are taken from Fig. 12(a) at $P_i$=1mW. Fig. 13 shows the carbon footprint at different schemes. It can be seen that as the packet loss rate increases, the carbon footprint is tremendously reduced for RRNS and BCH. It is ~2.5 times lesser kgCO$_2$ per year at 10% packet loss and ~4 times lesser kgCO$_2$ per year at 20% packet loss. So, as the channel goes bad, the carbon emissions for normal scheme like ARQ increases tremendously, whereas incorporation ECC and harvesting the network becomes more greener.

VIII. Conclusion

Enabling technologies and schemes to facilitate green wireless sensor networks are presented. Wireless energy harvesting is investigated as a remedy to prolong the lifetime of sensor nodes and enable maintenance-free operation. Wake-up radio scheme is incorporated as an efficient solution to address the idle listening energy dissipation of sensor nodes. RRNS Error control coding is proposed to improve the reliability of the transmission and reduce re-transmission, hence, reducing energy consumption. A utility-lifetime maximization problem incorporating WEH, WUR and ECC schemes is formulated and solved using distributed dual subgradient algorithm based on Lagrange multiplier method. Simulation results verify the effectiveness of the proposed schemes in reducing the energy consumption and accordingly, carbon footprint of wireless sensor nodes, providing the means for a greener wireless sensor network.

APPENDIX A

PROOF OF THE LEMMA 2

We define $E^1$ and $E^2$ in $R^{N+[\ell]}$ as $E_{TX}(1 + E(T,h_1)) + E_{RX}(1 + E(T,h_1))$ and $E_{PR}(1 + E(T,h_1)P^T) + R_{ij}E_{SN}$ respectively. If we denote $\infty$-norm as $\| \cdot \|_\infty$ and $q$-norm as $\| \cdot \|_q$. the lifetime objective functions of (14) and (24) are represented by $\| E^1r + E^2R\|_\infty$ and $1/(\epsilon + 1)|| E^1r + E^2R\|_{\epsilon+1}$ respectively. Suppose $\{r^*, R^*\}$ be the optimal solutions for the two objective
functions. Then we have the following inequalities using
approximation of \( \| \cdot \|_\infty \) from [43]
\[
\| E^1 r^*_c + E^2 R^*_c \|_\infty \\
\leq \| E^1 r^*_c + E^2 R^*_c \|_{\ell+1} \\
\leq \| E^1 r^*_c + E^2 R^*_c \|_{\ell+1} \\
\leq \| N \|^{1/(\ell+1)} \| E^1 r^*_c + E^2 R^*_c \|_\infty
\]
(38)
The corresponding network lifetimes become
\( T_i = 1/\| E^1 r^*_c + E^2 R^*_c \|_\infty \) and
\( T_i = 1/\| E^1 r^*_c + E^2 R^*_c \|_\infty \). From (39) we have,
\[
\frac{1}{\| N \|^{1/(\ell+1)}} T_i \leq T^*_c \leq T_i
\]
(39)
At \( \lim_{\ell \to \infty} T^*_c = T_i \), and thus the lemma holds.

**APPENDIX B**

**PROOF OF THE PROPOSITION 2**

From (27), the gradient of the objective function \( D(\lambda, \mu) \) w.r.t \( \lambda_i \),
\[
\nabla \lambda D(\lambda, \mu) = \alpha \sum_{i \in N} \left( \sum_{j \in N_i} \nabla \lambda U_i(R_{ij}, P_j) - (1 - \alpha) s_i \cdot \nabla \lambda s_i \right)
\]
(40)
By Definition 1 and Assumption 1, we can find the error in the cost estimation of the link price \( \lambda_i \) when iteration \( c \to c+1 \)
\[
\| D(\lambda(c+1)) - D(\lambda(c)) \| \leq \| \nabla \lambda D(\lambda) \| \| (\lambda(c+1) - \lambda(c)) \| \\
\leq \| \nabla \lambda D(\lambda) \| \| (\lambda(c+1) - \lambda(c)) \|
\]
(41)
From the above inequalities, we see that function is Lipschitz. Thus the solution generated with step size \( \phi_c \) is optimal [43]. Let the update at each iteration \( c \) is given by \( \Delta(\lambda(i)) \). Then,
\[
[\Delta(\lambda(0))] = \left[ \frac{r_{ij}(c)}{\alpha \nabla \lambda U_i(R_{ij}, P_j)} \nabla \lambda D(\lambda) \right] \leq \frac{R}{\alpha} \| \nabla \lambda D(\lambda) \|
\]
(42)
\[
\frac{\| \nabla \lambda D(\lambda) \|}{\| \lambda(c) \|} \Delta(\lambda(0)) \leq \frac{R}{\alpha} \| \nabla \lambda D(\lambda) \| \leq \frac{R}{\alpha} \| \nabla \lambda D(\lambda) \| \leq \frac{1}{\alpha} \| \nabla \lambda D(\lambda) \| ^2
\]
(43)
According to [43], the step size satisfies
\[
0 < \phi_c < \frac{2}{R^2 U R}.
\]
Similarly, the step size bound can be proven for \( \psi_c \).

**REFERENCES**


