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Supply Chain Finance: Optimal Introduction and Adoption Decisions

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Supply chain finance (SCF) can improve supply chain performance by facilitating longer payment terms for buyers and better access to financing for suppliers. In spite of these clear benefits, there is empirical evidence for some hesitation and resistance to SCF adoption, manifesting in an often substantial time lag between a buyer’s introduction of SCF and its adoption by all targeted suppliers. Observed adoption processes often resemble the s-shaped Bass-curve, suggesting that successful early adoptions support adoption decisions by other suppliers. Based on these observations, we consider supplier SCF adoption decisions within a diffusion model, to obtain insights regarding a buyer’s optimal SCF introduction decisions in terms of timing and payment terms. We find that initial payment terms and procurement volume strongly affect the optimal timing of SCF introduction and optimal payment term extensions. The degree to which the buyer can influence suppliers in their adoption decisions affects the optimal introduction timing, but not optimal payment terms. Interestingly, our results suggest that, in spite of the clear benefits, many buyers might be well-advised to postpone their SCF implementations.

Key words: Supply chain finance, reverse factoring, operations management and finance interface, diffusion model

1 Introduction

Trade credit granted by suppliers is an important source of financing. In the UK, for instance, 80% of all business-to-business transactions are made on trade credit (Summers & Wilson 2002). Even buying firms with strong credit ratings prefer trade credits to bank loans as this improves their net working capital (Petersen & Rajan 1997). However, from a supply chain perspective this approach is suboptimal if suppliers have weaker credit ratings and thus pay higher interest rates than their

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customers. An interesting solution to this problem is a practice called supply chain finance (SCF), sometimes also referred to as reverse factoring (Tanrisever et al. 2012). With SCF a supplier delivers to a buyer and provides trade credit by allowing payment due dates. Once the buyer has checked the delivery she confirms the invoice release to her financial institution. Based on this confirmation, the supplier receives the due amount directly from this financial institution, minus some interest based on the buyer’s credit rating. The buyer eventually pays the loan principle after expiration of the payment terms. Both parties can significantly profit from using SCF. Suppliers with weak credit ratings benefit from low interest rates. Compared with traditional factoring, SCF is less expensive and does not involve recourse. Buying firms, in turn, use SCF to extend their payment terms even further and thus obtain more trade credit and improve their working capital (Tanrisever et al. 2012). This is possible without worsening their upstream supply chain’s liquidity because SCF provides the supplier with the necessary funds.

However, even though firms could substantially benefit from SCF, its implementation is often delayed. As an illustrative example, consider the adoption process of a large German firm in the industrial automation industry, which introduced SCF in July 2010 and immediately started to invite its relevant suppliers to participate. Figure 1 depicts the number of suppliers onboarded over time. It clearly resembles the so called Bass curve (Bass 1969): initially, the number of suppliers using SCF grows slowly, then the growth accelerates before it eventually declines. Wuttke, Blome, Foerstl & Henke (2013) explored the adoption process of SCF through a series of six rigorous case studies in European production firms, and they empirically derived two reasons for the observed patterns. First, SCF requires internal clarifying. Procurement officers, who are supposed to use SCF in their daily routines but have not been involved in the SCF implementation process before, need to be persuaded of using SCF. Both the purpose of and processes related to SCF thus needs to be clarified. The more suppliers actually use SCF and the more successful cases there are, the faster clarifying takes place.

The second reason discovered by Wuttke, Blome, Foerstl & Henke (2013) is called upstream dissemination and requires a closer look at the suppliers’ typical internal decision and incentive structure. Procurement managers of buying firms communicate with sales managers of suppliers, whose incentives are based on two outcomes: increased prices and reduced payment terms. However, adopting SCF leads to neither of these goals. Yet, it requires efforts related to understanding the

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process and identifying knowledgeable colleagues. As a consequence, in the absence of explicit incentives for sales people that encourage exploring SCF, suppliers may not consider SCF adoption in spite of obvious benefits. At the beginning, SCF is new to most of the suppliers’ CFOs and CEOs, too, and it might take exposure to a certain number of successful SCF business cases to convince them of the benefits of SCF, and to start the internal process of evaluating and then promoting SCF adoption. Given the financial benefit of SCF, any such decision for SCF adoption promotion in turn increases other suppliers’ executives exposure and willingness to explore SCF, so that all suppliers will consider SCF and adopt it, if their evaluation demonstrates financial benefits. In essence, a similar word-of-mouth effect as described by Bass (1969) applies.

![Figure 1: Number of suppliers using SCF (data from a German firm in the industrial automation industry)](image)

While the aforementioned firm introduced SCF in July 2010, it could have also waited until more of its suppliers had been exposed to successful SCF business cases through other buying firms’ initiatives, which would have accelerated the diffusion process. Besides a faster introduction process, waiting brings the added benefit of lower introduction costs as the platform technology matures. Yet, waiting also implies foregone profit. This trade-off motivates our main research question: When should buying firms introduce SCF?

A second decision of buying firms within this context regards the extension of payment terms. While payment term extensions do not affect whether the supplier is in consideration of SCF – essentially a supplier will evaluate SCF adoption if the CFO has seen sufficient successful cases
they affect whether a supplier’s evaluation will lead to a positive outcome and SCF adoption. Longer payment terms increase the buying firm’s benefits, but they also reduce the attractiveness of SCF for suppliers, suggesting a trade-off between per-supplier benefit for the buyer and the number of suppliers eventually using SCF. This leads to our second research question: How much should a buying firm extend payment terms? The objective of our study is thus to make normative predictions on optimal SCF introduction decisions of buying firms.

We consider a sequential game with a buyer (female) and a set of suppliers (male). Along with introducing SCF, the buyer proposes payment term extensions. As long as the supplier is not in consideration of SCF, he will not adopt it. If he is in consideration because his CFO has seen enough successful cases, he will evaluate the offer and accept it if it is in his economic interest. We utilize a social contagion model (cf. Bass 1969) to capture that suppliers’ consideration of SCF depends on their exposure to successful SCF cases. We then analyze the economic impact of SCF by studying the impact of payment term extensions. We characterize both, the optimal introduction time and the optimal extension of payment terms. Finally, we extend the buyer’s decision problem to a game where we consider several buying firms sharing several suppliers.

Our paper contributes to the literature on the finance-operations interface as it comprises several novel perspectives. First we shed light on the importance of timing decisions by buying firms. We show that it is often not optimal to introduce SCF immediately but rather to wait. This provides a formal explanation for the often observed hesitation by buyers who argue that their suppliers would not be ready yet. In fact, we find that each buyer should adopt SCF once a specific fraction of her suppliers are persuaded of the SCF concept. Emphasizing the importance of timing complements former research that primarily assessed the SCF performance based on the assumption that all suppliers are fully persuaded right from the start. Second, our research provides structural results that help to characterize optimal strategies and different types of buying firms: those that should introduce SCF immediately, those that should wait, and those that should never introduce SCF. Third, our SCF introduction framework allows us to explore the role of the influence that buying firms can exert over suppliers. While their influence will not lead to greater payment term extensions, buyers can affect the adoption pace by suppliers. We show that positive influence is a necessary condition for buying firms to introduce SCF immediately and that more influence leads to earlier introduction in general. Finally, we characterize optimal payment term extensions, which
are central to the allocation of benefits between buyers and suppliers.

2 Literature review

Two streams of research are related to our work: the operations-finance interface and the innovation diffusion. Next, we review each stream in turn. The intertwining of financial and operational decisions has recently received an increasing attention in the literature (Wuttke, Blome, Foerstl & Henke 2013, Wuttke, Blome & Henke 2013, Protopappa-Sieke & Seifert 2010, Gupta & Dutta 2011, Pfohl & Gomm 2009, Hofmann 2005, Jamal et al. 2000) where the main focus lies on how financial restrictions and decisions influence the operational performance of a supply chain. Wuttke, Blome & Henke (2013) and Wuttke, Blome, Foerstl & Henke (2013) are both based on multiple case studies providing empirical insights into the supply chain and finance interface. We incorporate several observations of Wuttke, Blome & Henke (2013) into our study to understand the decision processes of most suppliers, who only take SCF into consideration if they have been exposed to enough successful SCF cases. The main difference to both publications is that we use a normative model to derive insights on optimal adoption strategies whereas they explore the actual adoption process qualitatively. The studies of Protopappa-Sieke & Seifert (2010) and Gupta & Dutta (2011) focus on the cash management problem where Protopappa-Sieke & Seifert (2010) consider jointly cash and inventory decisions and Gupta & Dutta (2011) derive payment schedules. While both papers relate financial flows and supply chains, their questions differ from ours, as they do not consider SCF. Jamal et al. (2000) consider the question of optimal payment term delays but likewise do not consider SCF. The work of Pfohl & Gomm (2009) provides a general framework to evaluate joint supply chain efforts to improve financing, and Hofmann (2005) provides conceptual insights into the operations and finance interface; we, on the other hand, derive specific normative insights on optimal adoption decisions.

A rather promising area within this literature stream is the exploration of innovative supply chain financing solutions such as SCF. There exists a significant number of papers that state the importance of SCF and examine the benefits generated for the involved parties. Klapper (2006) studies the role of reverse factoring in developing countries using empirical methods and concludes that it is an important source of financing. Shang et al. (2009) note the relevance of SCF for
implementing coordination mechanisms in decentralized serial inventory systems. Tanrisever et al. (2012) study the quantitative implications of SCF. They analyze in particular the effect of SCF on operational decisions under demand uncertainty, showing that SCF is most beneficial in supply chains where the credit spread between a buying firm and its suppliers is high. Despite potentially further effects of SCF on operational measures, the key performance drivers of SCF are thus credit rating difference and payment term extensions.

Closely related to our work is Dello Iacono et al. (2015) who use a systems dynamics simulation framework to exploratively study the SCF adoption process. To simulate the latest financial crisis, they compare a regular with an exceptional market scenario; they find that SCF is not necessarily a win-win situation if implementation costs are high. Our work differs from that of Dello Iacono et al. (2015) in various dimensions. First, we use a social contagion model to capture the suppliers' consideration of SCF and we provide analytical solutions to a set of differential equations to specifically study the impact of diffusion parameters and firm properties such as the buyer’s influence over his suppliers and procurement volume respectively. Second, we provide optimal introduction strategies in terms of timing and extension of payment terms. Third, we assume that the buyer has to account for the suppliers’ benefits when considering extending payment terms and cannot demand arbitrary extensions. Wandfluh et al. (2015) study the impact of both internal collaboration and buyer-supplier collaboration on buyer-supplier finance and find a positive relationship. While they focus on a broad set of management activities, we specifically consider SCF as a particular approach. Moreover, our study provides insights on the adoption process. Raghavan & Mishra (2011) study a supply chain with financially constrained buyer and supplier. They show that a third party lender can be better off by not only financing the buyer but also the supplier. While they provide motivation for SCF providers to offer SCF, we rather focus on other constituents as we show optimal introduction and adoption decisions of buyers and suppliers.

Methodologically, our work draws from the literature on the diffusion of innovation, which departed from Bass (1969). We introduce heterogeneity among the population of potential SCF-adopting suppliers as we distinguish between a global contagion process among all firms and a local process among a certain buying firm’s suppliers. In a certain sense this relates to the study of innovation diffusion among heterogeneous customers as observed in marketing (e.g., Midgley 1976). Similar to Kalish (1985), who distinguishes between the process of innovation awareness diffusion
and adoption diffusion, we consider two sub-processes on the suppliers’ level. We distinguish between the underlying process of the diffusion of SCF consideration and the suppliers’ actual adoption decisions once they are in consideration of SCF. Finally, while, so far, scholars have focused on the role of typical marketing instruments such as pricing of and advertising for durable goods innovations (e.g., Russell 1980, Kalish 1985, Krishnan et al. 1999, Tapiero 1983), we incorporate the decisions of timing and splitting benefits through the mechanism of payment term extensions. Through these three extensions we are able to study the role of both the global and local diffusion processes on the buying firm’s optimal introduction time. In particular, splitting both processes allows us to understand the impact of a buyer’s influence over her suppliers.

3 Model

We take the perspective of a single buyer and study when she should introduce SCF and by how many days she should extend upstream payment delays to her suppliers. Let $\tau \in [0, \infty)$ denote the time at which the buyer introduces SCF, where $\tau = \infty$ refers to the case of never introducing. At the same time, the buyer determines the new upstream payment delay, $d_s$, that exceeds the current payment delay $d_0$, which is independent of SCF and thus exogenous in our model. A supplier begins to use SCF at the smallest $t \geq \tau$ when two conditions are met: (i) there have been sufficient successful cases so that the supplier considers SCF and (ii) given the proposed extended payment delays the SCF introduction is economically beneficial to the supplier. Consistent with our observations before, the primary reason for a supplier to consider SCF is his exposure to successful SCF cases. As long as the buyer has not yet introduced SCF, a supplier can only be exposed to SCF through other suppliers’ adoptions. In addition there are some external sources of exposure such as banks or publications. These sources form the basis of what we refer to as global exposure. After the buyer has introduced SCF, his procurement officers will seek to inform the supplier regarding SCF, adding pressure to adopt SCF. We refer to this process as local exposure because it takes place between the buyer and her suppliers.
3.1 Global Exposure

The number of suppliers for each buyer and the number of firms in general is large and we approximate exposure to SCF through a continuous function. Let $F(t)$ denote the fraction of all firms successfully exposed to SCF at time $t$ with derivative $f(t) \equiv F'(t)$. Based on Bass’ (1969) description of the s-shaped diffusion process which resembles a logistic function, we assume:

\[ \frac{f(t)}{1 - F(t)} = p + qF(t), \]

(1)

where $p$ and $q$ are constant external parameters. Parameter $q$ can be interpreted as social contagiousness, since it reflects how fast information spreads when managers of two firms meet. It captures that suppliers do not automatically consider SCF just because they see a successful case, but rather that there is a probability (related to $q$) that they will. Parameter $p$ can be interpreted as a fixed probability that contributes to creating willingness to consider SCF. This reflects, for instance, information that suppliers obtain from banks, publications, or other third parties that do not depend on the actual fraction of suppliers having adopted SCF. We assume that the number of suppliers for each single buyer is relatively small compared to the number of overall firms, so no single buyer can significantly influence the global diffusion process. In Section 5, we show that global diffusion follows an s-shaped process, even if individual buyers influence the global diffusion process, and strategically consider the consequences of their introduction decisions.

3.2 Local Exposure

The local exposure diffusion process takes place among the suppliers of the focal firm. Let $G(t, \tau)$ describe the fraction of the buyer’s suppliers that would consider SCF and let $g(t, \tau) \equiv \frac{\partial G}{\partial t}(t, \tau)$. Since the buyer can accelerate the diffusion process, the external parameter in the local diffusion process might be higher than the external parameter in the global diffusion process, $p$. We assume the buyer does not influence her suppliers before using SCF, but that, after the introduction of SCF, she influences her suppliers with a constant, exogenous level of $a_0 > 0$. We capture this through the influence function.
\[
\begin{aligned}
a(t, \tau) := \\
\begin{cases}
0, & \text{if } t \leq \tau \\
0, & \text{if } t > \tau .
\end{cases}
\end{aligned}
\]  \tag{2}

The local diffusion of consideration among the buyer’s suppliers can then be expressed as

\[
g(t, \tau) = \frac{p + a(t, \tau) + qF(t)}{1 - G(t, \tau)}. \tag{3}
\]

This equation is an extension of the original Bass equation in two respects. On the one hand, the diffusion process is adjusted by mixing social contagion among all firms and the buyer’s suppliers. On the other hand, the buyer’s influence increases the external parameter. In line with Bass (1969), we consider an initial value problem with \( F(t = 0) = G(t = 0, \tau) = 0 \), meaning that initially no firm is considering SCF. As implied by the term \( qF(t) \) in the local diffusion equation, it is necessary to solve the system consisting of the interdependent ordinary differential equations (1) and (3) with variable coefficients. In general, such a system does not necessarily have a closed-form, simple solution. However, in this specific case both ordinary differential equations can be solved as the following lemma states.

**Lemma 1** The unique solution to the ordinary differential equations of consideration diffusion as an initial value problem with \( F(t = 0) = G(t = 0, \tau) = 0 \) is \( F(t) = \frac{1 - e^{-(p+q)t} + \frac{1}{q/e^{-(p+q)t} + 1}}{q/e^{-(p+q)t} + 1} \) and \( G(t, \tau) = 1 - (1 - F(t))e^{-a_{0}\max(0, t-\tau)} \).

These equations reflect \( G(t, \tau) = F(t) \) if \( t \leq \tau \) and \( G(t, \tau) > F(t) \) otherwise, implying that local diffusion follows global diffusion until the buyer introduces SCF, but is more advanced from that moment on. This lemma is central to the following analysis, as it identifies a closed-form description of the local consideration diffusion process, relating it to both the buyer’s influence and the global process.

### 3.3 Suppliers’ Adoption Decisions

To examine the SCF adoption decision any of the buyer’s suppliers, supplier \( j \), we need to consider the related interest rates. Let \( i_s \) denote the interest rate offered to suppliers under SCF\(^1\) and \( i_0^j \) the

\(^1\)Note that typically \( i_s \) is the same for all suppliers because it depends on the buyer’s credit rating and not the suppliers’.
supplier’s standard interest rate according to his credit rating. Once supplier \( j \) is in consideration of SCF, he adopts SCF if his net financing cost difference, \( i^j_{sd}d_0 - i_s d_s \), is positive. We assume there is a known distribution \( \Phi \) of the interest rates of the focal firm’s suppliers with density \( \varphi \). We assume \( \Phi \) to be an increasing failure rate (IFR) distribution. Many important distributions have the IFR property (e.g., normal, gamma and uniform) and the assumption of an IFR function is widely used in many operations and supply chain management contexts (Cachon 2003, Lariviere 2006, Porteus 2002). Exposure to firms using SCF is quite random, and we assume that interest rates are independent of whether or not a supplier considers SCF. The fraction of suppliers considering SCF adoption then is

\[
M(d_s, i_s) \equiv 1 - \Phi \left( \frac{d_s i_s}{d_0} \right). \tag{4}
\]

### 3.4 Buyer’s Introduction Decisions

The buyer understands the dynamics of SCF adoption and thus can anticipate the number of suppliers adopting SCF when she sets \( d_s \) and \( \tau \). The marginal costs of onboarding a single supplier are negligible for buyers in most SCF programs, so we only consider the buyer’s one-time implementation cost \( c(t) \). As these costs depend on the bank’s experience with SCF, we approximate them with a non-increasing, convex and positive function. The buyer’s benefits are driven by the extension of payment terms, \( d_s - d_0 \). Let \( V \) denote the buyer’s total procurement volume and \( i_b \) the buyer’s interest rate. Usually banks charge a small premium, \( i_t \), on each transaction, so the interest rate for all suppliers under SCF is \( i_s = i_b + i_t \). In this case, the buyer’s optimization problem is

\[
\max_{\tau, d_s} \Pi(\tau, d_s) = \int_0^\infty (i_b (d_s - d_0) V \cdot M(d_s, i_s) G(t, \tau)) e^{-\lambda t} dt - c(\tau)e^{-\lambda \tau} \tag{5}
\]

\[s.t. \quad \text{Equations (1) - (4)}\]

where \( \lambda = ln(1 + i_b) \) is a continuous discount factor.

### 4 Analysis and discussion

We next analyze the optimal payment terms before turning to the optimal time of SCF introduction for buyers.
4.1 Optimal Extension of Payment Terms

The profit in (5) is affine in $i_b(d_s-d_0)M(d_s,i_s)V$, so the buyer seeks the value of $d_s$ which maximizes this term, independent of the introduction time. Hence, we first characterize the properties of the optimal extended payment terms $d_s^*$ before turning to the optimal introduction time.

**Proposition 1** There exists an optimal payment term period, $d_s^*$, which is a unique and strictly positive interior maximum of $\Pi(\tau,d_s)$.

All proofs can be found in the appendix. Even though it is impossible to express $d_s^*$ in closed form, we can provide several insights into its behavior with respect to the initial payment terms $d_0$ and the buyer’s interest rate $i_b$.

**Corollary 1** Ceteris paribus, the optimal absolute extension of payment terms $d_s^*-d_0$ (i) increases in the initial payment delay $d_0$ and (ii) decreases in the buying firm’s interest rate $i_b$. Also, (iii) the accepting fraction of suppliers conditional on their consideration $M(d_s^*,i_s)$ increases as the buyer’s interest rate $i_b$ decreases.

This corollary states that firms with higher initial payment terms and lower interest rates should demand more extended payment terms. The economic rationale behind this corollary is that suppliers with longer payment terms have to finance a higher amount of trade credit. Since higher initial payment terms increase the supplier’s cost without SCF, we find that suppliers with high initial payment terms have a higher incentive to adopt SCF. Hence, these suppliers will benefit more from reducing the interest rate and thus are willing to adopt SCF even if payment terms will be longer. This finding suggests that SCF is particularly suited to supply chains that are inefficient in the sense that the members with the highest cost of capital finance a large amount of trade credit. Moreover, as Corollary 1 shows, when the optimal extended payment terms increase in response to a decreased interest rate $i_b$, the fraction of suppliers that actually use SCF increases, too. From the suppliers’ perspective the positive effects of lower financing costs outweigh the negative effects of the corresponding longer extensions of payment terms. Please note that the aforementioned findings do not only hold for the absolute difference $d_s^*-d_0$, but also for $d_s^*$, as can be seen in the corresponding proof.
Note that there are several factors in our model which we assume to be exogenous and constant, for instance interest rates and purchasing volumes. Keeping everything else fixed, it is straightforward to see that the buyer’s profit increases in $V$. The effect of interest rates is two-fold. The supplier’s benefit from SCF increases in his external financing costs but decreases in the SCF interest rate. Accordingly, the market potential is positively or negatively, respectively, affected by changes in interest rates.

4.2 Optimal Introduction Timing

In this section we address the question of when a buyer should introduce SCF.

Proposition 2

Buyers can be grouped into three disjoint sets: (i) buyers that never introduce SCF ($\tau^* = \infty$), (ii) buyers that eventually introduce SCF ($0 < \tau^* < \infty$), and (iii) buyers that immediately introduce SCF ($\tau^* = 0$).

The proof to Proposition 2 highlights the main factors governing the categorization of a buyer into one of the three groups. Set (i) refers to non-adoption which turns out to be the optimal strategy for firms that face relatively high adoption costs but would only gain very little by introducing SCF. A driver of this result is the assumption that the one-time implementation costs, $c(\tau)$, decrease over time but remain strictly positive. Set (ii) refers to postponed SCF implementation. While these buyers introduce SCF eventually, they wait at least some time and are not first movers. When a buyer belongs to this category, still the optimal time must be identified: if the buyer introduces SCF early, there is a long horizon over which she can benefit from extended payment terms, at least with those suppliers who consider SCF early on. Moreover, a buyer who introduces SCF at an early stage can use her influence to accelerate the local diffusion process, which leads to earlier consideration of SCF by suppliers, thus causing a greater number of suppliers to use SCF at a certain point in time. In contrast, the buyer might be better off waiting due to decreasing costs, delaying expenses, and waiting for more suppliers to be familiar with SCF.

Set (iii) refers to immediate adoption and bears an interesting aspect. The initial value problem stated in (3) implies that at $t = 0$ no supplier considers SCF; therefore, introducing SCF right at $t = 0$ triggers costs of $c(0)$ without leading to any immediate benefits, because yet not supplier is in consideration of SCF. Nevertheless, as shown in Proposition 2 there are buyers who should
introduce SCF immediately. The economic rationale behind this strategy is the anticipation of the acceleration of organizational readiness among the supplier base. Even though these buyers do not immediately benefit from extended payment terms, they benefit from shifting future benefits to earlier points in time. To understand the timing decision and characterize the buyers more precisely, we use the following corollary.

**Corollary 2** A buyer should introduce SCF later (in the extreme case never), ceteris paribus, if she has (i) a low procurement volume or (ii) low initial payment terms. (iii) While a buyer should introduce SCF later, ceteris paribus, if she has low influence, low influence per se is never sufficient for not introducing and even very high influence does not guarantee that introduction is optimal.

The economic rationale behind Corollary 2 relates to the buyer’s benefits. Condition (i) implies that there is less volume to benefit from. As we have shown in Corollary 1, the optimal payment term extension of the buying firm increases in the initial terms. Since the buying firm’s benefits are due to the extension of payment terms, its benefits are greater when initial payment terms are higher. As a consequence, these buyers with lower benefits should wait longer before introducing SCF, to delay expenses. Ultimately, if discounted future benefits are below introduction costs for certain buyers at any time, they should not introduce SCF at all.

A quite different rationale underlies the role of influence. As we have seen in Corollary 1, optimal extensions of payment terms do not depend on influence. The direct benefits of SCF, that is the benefits related to working capital costs, are thus not affected by influence. Put differently, just because a buying firm is more influential does not allow it to extend payment terms further. Nevertheless, influence accelerates the diffusion process. So influence affects benefits indirectly as it affects their timing. As a consequence of Corollary 2, each buyer who would eventually introduce SCF would do the same even with extremely low influence, but at a later date.

Figures 2a - 2c numerically illustrate the profit functions of the three buyer types. Keeping everything else fixed, we varied the buyers’ interest rate \( i_b \in \{0.03, 0.045, 0.06\} \). In each subfigure there are three buyers who differ with respect to their level of influence \( a_0 \in \{0.03, 0.06, 0.12\} \). The essence of the aforementioned results is that for each buyer there is a critical fraction of suppliers that need to consider SCF before she should introduce it. This fraction, which grows over time, is depicted on the horizontal axis. Hence, the figures show the profit a buying firm would gain.
through SCF if it introduced SCF at the time when it observes a certain fraction of its suppliers are ready for SCF.

In Figure 2a all buyers have negative profits and will thus never introduce SCF. Their influence does not have any impact on profit, as they would not introduce SCF in any case. Figure 2b depicts buyers who will eventually introduce SCF. For this group higher influence implies that it is optimal to introduce SCF at a lower fraction of SCF-considering suppliers. Finally, immediate introducers are shown in Figure 2c. For all of these it is optimal to introduce immediately, even though no supplier is yet considering SCF. Among them, the buyer with the highest influence ($a_0 = 0.12$) has the highest profit.

Figure 2: Illustration of Proposition 2 and Corollary 2.

$d_0 = 45, V = $20,000,000, C = $500,000, q = 0.09, p = 0.001.$
5 An SCF introduction game with $n$ buyers

We assumed that the behavior of each individual buyer does not affect the global diffusion process so that a buyer does not consider her own impact on the global diffusion process. In this section we derive some insights into the strategic interactions that arise when there are $n$ buying firms that consider an SCF introduction, and where all firms are aware of the impact of their decision on other buyers’ decisions. Let $\tau = (\tau_1, \ldots, \tau_n)$, $d_s = (d_{s,1}, \ldots, d_{s,n})$, and $a = (a_1, \ldots, a_n)$ denote the introduction time, payment terms, and the influence vectors respectively. We endogenize the external parameter $p$ by defining a function $\tilde{p}(t; \tau, a)$, which assigns to each time, $t$, and any given set of introduction decisions and influences the current external diffusion parameter. Let the indicator function be denoted by $1_{\tau_i \leq t}$ which is 1 if $\tau_i \leq t$ and 0 otherwise. Then we define

$$\tilde{p}(t) := p_0 + \frac{1}{n} \sum_{i=1}^{n} 1_{\tau_i \leq t} a_i$$

(6)

with exogenous $p_0 \geq 0$. This exogenous component captures the fact that some suppliers might consider SCF for other reasons. This leads to the following $n+1$ diffusion equations,

$$\frac{f(t)}{1 - F(t)} = \tilde{p}(t) + qF(t) \quad \text{and}$$

$$\frac{g_i(t, \tau)}{1 - G_i(t, \tau)} = \tilde{p}(t) + a_i(t, \tau) + qF(t) \quad (i = 1, \ldots, n).$$

(7)

The strategy vector $(d^*_s, \tau^*)$ is a Nash Equilibrium if it satisfies the following condition for all buyers $i = 1, \ldots, n$:

$$(d^*_{s,i}, \tau^*_i) \in \arg \max_{(d_{s,i}, \tau_i)} \Pi_i(\tau^*_i, d^*_s | \tau^*_1, \ldots, \tau^*_{i-1}, \tau^*_{i+1}, \ldots, \tau^*_n)$$

$$= \arg \max_{(d_{s,i}, \tau_i)} \int_{\tau_i}^{\infty} (i_b(d_s - d_0) V_i \cdot M_i (d_{s,i}) G_i(t, \tau)) e^{-\lambda t} dt - c_i(\tau_i) e^{-\lambda \tau_i}$$

(8)

subject to (4), (6), and (7).

**Proposition 3** If $\tau^*_i$ increases in $\tau_j$ for all $i, j \in N_n$, $i \neq j$, and $\Pi_i$ are quasi-concave in $\tau_i$, then there exists a Nash equilibrium in pure strategies.
This proposition supposes that $\tau^*_i$ increases in $\tau_j$ ($i, j \in \mathbb{N}_n, i \neq j$). In economic terms this refers to the situation where each buyer would introduce SCF earlier if fellow buyers were to do the same. This seems plausible because whenever a buyer uses the waiting strategy until a certain fraction of her suppliers has achieved organizational readiness, then earlier introduction by fellow buyers should have a positive effect on her own introduction time. However, this technical condition is not automatically met. The game theoretical argument can be translated into practical reasoning as follows. Consider a buying firm that would introduce SCF immediately if other firms sufficiently postponed introduction because it has enough influence to accelerate the diffusion process on its own. However, it also knows that there are few suppliers willing to use SCF at the beginning. Now, if instead the other firms introduced SCF early, this firm would anticipate this and postpone its own introduction to share the burden of accelerating the diffusion with them. Therefore, because other firms introduce earlier, this firm introduces later. In this case, the condition is not met and the proposition cannot be applied. Numerically, such special cases can be constructed, but in most of the cases that we analyzed the condition was met. In practical terms, such cases require a very influential buyer which appears rather exceptional so that typically a buyer would introduce SCF early if other firms do the same.

Since there is no closed form solution to the above system of $n + 1$ intertwined differential equations, we construct a tâtonnement process simulation to identify a pure-strategy equilibrium following the idea of Fudenberg & Tirole (1991). We begin with an initial distribution of adoption times, $\tau^0$. Then we consider buyer $i = 1$ given this vector and calculate $\tau^1_i$. Next we update $\tau^1 \leftarrow (\tau^1_1, \tau^0_2, \ldots, \tau^0_n)$. Subsequently we look at buyer $i = 2$ given $\tau^1$ and update $\tau^1 \leftarrow (\tau^1_1, \tau^1_2, \ldots, \tau^0_n)$. We proceed until $i = n$. We then continue with the next tâtonnement step, again starting with buyer $i = 1$ to update $\tau^2$. We repeat the entire procedure until changes in the introduction times are of the order of the numerical imprecision of our simulation; in this example the precision was reached after 23 steps. The two charts in Figure 3 show the results for this process. Here 8 buyers are identical except for their influence.

This extension and our numerical study highlight two main results. First, as depicted in Figure 3a, the global diffusion process remains s-shaped. Hence, our mathematical model is consistent with observed practice. Second, consistent with the results derived in the main part of the paper, we see that influential buyers introduce earlier (cf. Figure 3b). As problems (5) and (8) are structurally
equivalent with respect to $d_s^*$, the insights of Proposition 1 and Corollary 1 remain valid. We can generalize our results from the analysis section to the case where buyers could actually influence the global diffusion process and where they consider this influence strategically.

Figure 3: Tâtonnement simulation.

6 Conclusion

The primary contribution of this study is to advance the knowledge on the introduction of SCF, which is increasingly gaining attention in theory and practice. Motivated by observations of real introduction processes, we take a diffusion perspective. Our paper revolves around the SCF consideration by suppliers and extends the previous literature that focuses on the supply chain and finance interface. Our paper has several implications.

First, we demonstrate the importance of timing decisions in the implementation of SCF. Clearly, SCF is not beneficial for all buying firms and in cases where it is beneficial, some buyers should wait before introducing it. In this sense we reject the view of several management reports that claim SCF is always beneficial and buyers should introduce SCF as soon as possible. More precisely, we study the impact of procurement volume, the firm’s interest rate, and initial payment terms on SCF profitability and optimal introduction time. Thus, particularly buyers with high procurement
volumes should introduce SCF early. Especially in industries characterized by long initial payment terms we would expect to see early SCF introduction by buyers. Buyers who are early introducers will also benefit most from SCF. A particular focus of our model is on a buyer’s influence over her suppliers. In contrast to the traditional paradigm that processes will be used in supply chains if the net benefits are positive for all actors, we explicitly capture the concept of consideration diffusion to understand the ramifications arising when certain actors first have to be convinced.

Our study also generates some insights regarding the allocation trade-off that buyers face when they introduce SCF. On the one hand, buyers prefer to extend payment terms substantially. On the other hand, excessive extensions reduce the number of suppliers for whom SCF would then be attractive. Our model characterizes buyers who could most substantially extend payment terms. These buyers are characterized by low interest rates (i.e., good credit rating), high procurement volume, and long initial payment terms. We find that for these buyers extending payment terms still leads to high fractions of suppliers who use SCF once they are organizationally ready. Our paper has some limitations. In order to allow for analytically tractable results we assume certain variables, such as interest rates or procurement volumes, to be time-invariant. It is also not clear how variables should change over time. In terms of interest rates, for instance, cyclical behavior seems realistic. Therefore, we chose to keep exogenous variables time-invariant and focus on structural insights. One possible way to solve models under further assumptions would be using simulation approaches such as those used by Dello Iacono et al. (2015); however, such analyses are beyond the scope of our paper.

Appendix A: Proofs

**Lemma 1.** (1) and (3) together with the values \( F(t = 0) = G(t = 0, \tau) = 0 \) form an initial value problem. Therefore, in this proof we need to identify functions that satisfy (1) and (3) as well as the initial values.

(i) (1) is of the Ricatti type and known to have a unique solution as initial value problem. This solution is \( F(t) = \frac{1-e^{-(p+q)t}}{q+pe^{(p+q)t}+1} \), as formally derived in Bass (1969). Please note that the same term can be expressed by

\[
F(t) = 1 - \frac{p+q}{q+pe^{(p+q)t}} \tag{9}
\]

which we will use from now on frequently throughout the proofs.

(ii) (3) is equivalent to

\[
\frac{dG}{dt}(t, \tau) + (p + a(t, \tau) + qF(t)) G(t, \tau) = p + a(t, \tau) + qF(t)
\]
This is a linear, first-order and inhomogeneous ordinary differential equation with disturbance function \( p + a(t, \tau) + qF(t) =: \Psi(t) \). We are interested in finding a homogeneous \((G_h)\) and a particular \((G_p)\) solution, such that all solutions have the form \( G = G_h + G_p \). To find \( G_p \), we use the method of variation of the constant. For our specific problem, these functions are then given by

\[
G_h(t, \tau) = c_1 e^{-\int \Psi(x) dx}, \quad \text{and} \\
G_p(t, \tau) = e^{-\int \Psi(x) dx} \int e^{\int \Psi(y) dy} \Psi(x) dx = 1,
\]

where \( c_1 \) is a constant of integration. Hence, \( G(t, \tau) = G_h(t, \tau) + G_p(t, \tau) = c_1 e^{-\int \Psi(x) dx} + 1 \). Using the indicator function \( 1_{\tau \leq t} \) which is 1 if \( \tau \leq t \) and 0, otherwise, we have

\[
\int \Psi(t) dt = \int (p + a(t, \tau) + qF(t)) dt \\
= pt + \int a_0 1_{\tau \leq t} dt + \ln(p e^{(p+q)t} + q) - pt + c_2 \\
= a_0 \max(0, t - \tau) + \ln(p e^{(p+q)t} + q) + c_2,
\]

where \( c_2 \) is another constant of integration. Hence we have,

\[
G(t, \tau) = Ce^{-a_0 \max(0, t-\tau)-\ln(p e^{(p+q)t} + q)} + 1 \\
= Ce^{-a_0 \max(0, t-\tau)} p e^{(p+q)t} + q + 1
\]

where \( C \) is a constant of integration that captures \( c_1 \) and \( c_2 \) and that will be determined through the initial value. The initial condition is \( G(t = 0, \tau) = 0 \), so the unique solution is determined by \( C = -(p + q) \). Using (9) we have

\[
G(t, \tau) = 1 - (1 - F(t)) e^{-a_0 \max(0, t-\tau)}
\]

\( \square \)

**Proposition 1.** First we show that the optimal decision \( d_s^* \) is not a boundary point. This follows from \( \Pi(d_0, \tau) = 0 \), \( \Pi(\infty, \tau) = 0 \), and \( \Pi'(d_0, \tau) = M(d_0) > 0 \). Hence \( d_s^* \) must be an interior solution and satisfy the necessary condition \( \frac{\partial}{\partial d_s} \Pi(d_s, \tau) = 0 \) and a sufficient condition.

\[
\frac{\partial}{\partial d_s} \Pi(d_s, \tau) = \frac{d}{dd_s} ((d_s - d_0) M(d_s)) \int_\tau^T i_b V \cdot G(t, \tau) e^{-\lambda t} dt \bigg|_{t > 0} = 0 \\
\Leftrightarrow \frac{d}{dd_s} ((d_s - d_0) M(d_s)) = 0 \Leftrightarrow -\varphi \left( \frac{d_s}{d_0} i_s \right) + \left( \frac{d_0}{d_s - d_0} i_s \right) = 0. \tag{10}
\]

The proposition makes three claims: existence of \( d_s^* \), uniqueness, and that it is a maximum. (i) Existence follows immediately from the discussion above. (ii) Uniqueness. Let \( d_s^* \) be a solution to (10), then it is an extreme point. Assume \( \exists d_{s**} \neq d_s^* \) such that \( d_{s**} \) is also a solution of (10).
Then it is either $d_s^{**} > d_s^*$ or $d_s^{**} < d_s^*$. Let us begin with the first case. The term $-\frac{\varphi\left(\frac{d_s}{d_0}\right)}{1 - \Phi\left(\frac{d_s}{d_0}\right)}$ decreases in $d_s$, because $\Phi(\cdot)$ is IFR by assumption. Furthermore, $\left(\frac{d_0}{(d_s-d_0)i_s}\right)$ decreases in $d_s$ because $\frac{d}{dd_s}\left(-\frac{d_0}{(d_s-d_0)i_s}\right) = -\frac{d_0}{(d_s-d_0)^2i_s} < 0$. It follows that

$$-\frac{\varphi\left(d_s^{**}/d_0\right)}{1 - \Phi\left(d_s^{**}/d_0\right)} + \left(\frac{d_0}{(d_s^{**}-d_0)i_s}\right) < -\frac{\varphi\left(d_s^{**}/d_0\right)}{1 - \Phi\left(d_s^{**}/d_0\right)} + \left(\frac{d_0}{(d_s^{**}-d_0)i_s}\right) = 0.$$

This is a contradiction to the assumption that $d_s^{**}$ is a solution of (10), hence the assumption $d_s^{**} > d_s^*$ is wrong. Similarly, it follows that $d_s^{**} < d_s^*$ leads to a contradiction. It follows that $d_s^*$ is unique. (iii) Maximum. From (10) if follows $\frac{\partial}{\partial d_s} \Pi\left(\tau, d_s\right) < 0$ if $d_s > d_s^*$ and $\frac{\partial}{\partial d_s} \Pi\left(\tau, d_s\right) > 0$ if $d_s < d_s^*$. Hence, there is a switch of signs in the first derivative from positive to negative, indicating a unique maximum.

**Proof of Corollary 1.** Assertion (i) follows if $\frac{dd_s}{dd_0} > 1$ holds. We apply the implicit function theorem on the first derivative of $\Pi$ which implicitly defines $d_s^*$:

$$\frac{dd_s}{dd_0} = -\frac{\frac{\partial^2 \Pi}{\partial d_0 \partial d_s}}{\frac{\partial^2 \Pi}{\partial d_s^2}} = \frac{2d_s^* \varphi(\cdot) + i_s d_s^*-d_0 d_s^* \varphi'(\cdot)}{2\varphi(\cdot) + \frac{i_s}{d_0} (d_s^*-d_0) \varphi'(\cdot)} =: \frac{N}{D},$$

with numerator $N$ and denominator $D$. We will prove that both are positive, and therefore the fraction is also positive.

(a) $N > 0$. $\Phi(\cdot)$ is IFR, i.e., $\frac{d}{d(\cdot)} \frac{\varphi(\cdot)}{1-\Phi(\cdot)} \geq 0 \Rightarrow \frac{\varphi(\cdot)(1-\Phi(\cdot)) + \varphi^2(\cdot)}{(1-\Phi(\cdot))^2} \geq 0$.

$\Rightarrow \varphi(\cdot) (1-\Phi(\cdot)) + \varphi^2(\cdot) \geq 0$. By the first order condition in (10) we know that $1 - \Phi(\cdot) = (d_s^*-d_0) \frac{i_s}{d_0} \varphi(\cdot)$. Combining both, the IFR assumption and the first order condition, it follows that $\varphi'(\cdot) \frac{d_s^*-d_0}{d_0} i_s + \varphi(\cdot) \geq 0$. We can thus simplify the numerator as follows:

$$N = \frac{2d_s^*}{d_0} \varphi(\cdot) + i_s d_s^*-d_0 d_s^* \varphi'(\cdot) > d_s^* \left(\frac{\varphi(\cdot)}{1-\Phi(\cdot)} + \frac{i_s d_s^*-d_0}{d_0} \varphi'(\cdot)\right) \geq \left(\varphi(\cdot) + \frac{i_s d_s^*-d_0}{d_0} \varphi'(\cdot)\right) \geq 0. \quad (12)$$

(b) $D > 0$. $D = 2\varphi(\cdot) + \frac{i_s}{d_0} (d_s^*-d_0) \varphi'(\cdot) > \varphi(\cdot) + \frac{i_s}{d_0} (d_s^*-d_0) \varphi'(\cdot) \geq 0$. This proves that $N > 0$ and $D > 0$.

Finally, the proof of $\frac{dd_s}{dd_0} > 1$ follows by contradiction: Assume $\frac{dd_s}{dd_0} \leq 1$, which would imply $N \leq D$ and by (11) also $\frac{2d_s^*}{d_0} \varphi(\cdot) + i_s d_s^*-d_0 d_s^* \varphi'(\cdot) \leq 2\varphi(\cdot) + \frac{i_s}{d_0} (d_s^*-d_0) \varphi'(\cdot) \leq 0$, as can be seen by subtracting the right term from the left term, since $d_s^* > d_0$. This contradicts (12), hence $\frac{dd_s}{dd_0} > 1$.

Assertion (ii) is equivalent to $\frac{dd_s}{dd_0^*} \leq 0$. Similar to Assertion (i) we have,

$$\frac{dd_s}{dd_0^*} = -\frac{\frac{2d_s^*-d_0}{i_s} \varphi(\cdot) + \frac{d_s^*-d_0}{d_0} d_s^* \varphi'(\cdot)}{2\varphi(\cdot) + \frac{i_s}{d_0} (d_s^*-d_0) \varphi'(\cdot)} =: -\frac{N}{D} \text{ and } D > 0.

N = \frac{2d_s^*-d_0}{i_s} \varphi(\cdot) + \frac{d_s^*-d_0}{d_0} d_s^* \varphi'(\cdot) > d_s^* \left(\frac{\varphi'(\cdot) d_s^* - d_0}{d_0} i_s + \varphi(\cdot)\right) \geq 0.
Hence \( D > 0 \) and \( N > 0 \). Therefore, \(-\frac{N}{D} < 0\).

Assertion (iii) is equivalent to \( \frac{d}{ds} M(d_s^*(i_s), i_s) < 0 \):

\[
\frac{d}{ds} M(d_s^*(i_s), i_s) = \frac{\partial}{\partial s} d_s^*(i_s) \frac{d}{ds} M(d_s^*(i_s), i_s) + \frac{\partial}{\partial i_s} M(d_s^*(i_s), i_s)
\]

\[
= - \left( \frac{\partial}{\partial i_s} d_s^*(i_s) i_s + d_s^* \right) \varphi(\cdot) = - \frac{\varphi^2(\cdot)}{2\varphi(\cdot) + \frac{1}{d_0} (d_s^* - d_0) \varphi'(\cdot)} < 0,
\]
similar to the reasoning in Assertion (i).

\[\square\]

**Proposition 2.** We will use from now on the following convention: \( \frac{d}{d\tau} \Pi(\tau, d_s^*) \equiv \Pi'(\tau) \) and \( \frac{d^2}{d\tau^2} \Pi(\tau, d_s^*) \equiv \Pi''(\tau) \). This proof is structured as follows. We begin by calculating \( \Pi'(\tau) \). Then we prove the three assertions noting that (i) holds if \( \Pi'(\tau) > 0 \), \( \forall \tau \geq 0 \) is true for some buyers, (ii) holds if \( \Pi'(0) > 0 \) and \( \Pi'(\tau) < 0 \) for \( \tau \to \infty \) for some buyers, and (iii) holds if \( \Pi'(\tau) < 0 \), \( \forall \tau \geq 0 \) for some buyers. We use this specific approach, as it allows us to derive several equations which are relevant for proving Corollary 2. Note that

\[
\frac{\partial}{\partial \tau} G(t, \tau) = \begin{cases} 0, & \text{if } t \leq \tau \\-a_0(1 - F(t))e^{-a_0(t-\tau)}, & \text{if } t > \tau \end{cases}, \quad (13)
\]

and define \( \tau := \frac{i_s(d_s^*-d_0)V\cdot M(d_s^*)}{\lambda} \). Applying the Leibniz rule yields:

\[
\Pi'(\tau) = \frac{d}{d\tau} \left( \int_\tau^\infty (i_b(d_s-d_0) V \cdot M(d_s^*)) G(d_s, t, \tau) e^{-\lambda t} dt - c(\tau)e^{-\lambda \tau} \right)
\]

\[
= \kappa \left( \frac{d}{d\tau} \left( \int_\tau^\infty (1 - (1 - F(t))e^{-a_0(t-\tau)}) e^{-\lambda t} dt + \left( -\frac{c(\tau)}{\lambda \varepsilon} + \frac{c(\tau)}{\varepsilon} \right) e^{-\lambda \tau} \right) \right)
\]

\[
= \kappa e^{-\lambda \tau} \left( -F(\tau) - a_0 \int_\tau^\infty (1 - F(t)) e^{-(a_0+\lambda)(t-\tau)} dt - \frac{c'(\tau)}{\lambda \varepsilon} + \frac{c(\tau)}{\varepsilon} \right).
\]

This integral does not have a closed form anti-derivative.

Assertion (i). We will prove that there are buyers with \( \Pi'(\tau) > 0 \), \( \forall \tau \geq 0 \). Since \( c > 0 \), there exists such that \( 0 < \tau < c(t), \forall t \). So we can assume that \( -\frac{c(\tau)}{\lambda \varepsilon} + \frac{c(\tau)}{\varepsilon} > 1 \) may hold for some buyers, which means that costs are relatively high or declining fast enough. Moreover, for the last integral in (14) we note that if \( t \geq \tau \) then \( \frac{1}{q + pe^{(p+q)t}} \leq \frac{1}{q + pe^{(p+q)\tau}} \) and find the following upper bound:

\[
\int_\tau^\infty (1 - F(t)) e^{-(a_0+\lambda)(t-\tau)} dt \leq \int_\tau^\infty \left( \frac{p + q}{q + pe^{(p+q)t}} \right) e^{-(a_0+\lambda)(t-\tau)} dt = \frac{1 - F(\tau)}{a_0 + \lambda} \quad (15)
\]

With that we can estimate the lower bound for \( \Pi'(\tau) \) in (14),

\[
\Pi'(\tau) = \kappa e^{-\lambda \tau} \left( -F(\tau) - a_0 \int_\tau^\infty (1 - F(t)) e^{-(a_0+\lambda)(t-\tau)} dt - \frac{c'(\tau)}{\lambda \varepsilon} + \frac{c(\tau)}{\varepsilon} \right)
\]

\[
> \kappa e^{-\lambda \tau} \left( -F(\tau) - a_0 \frac{1 - F(\tau)}{a_0 + \lambda} + 1 \right) = \kappa e^{-\lambda \tau} \frac{\lambda(1 - F(\tau))}{a_0 + \lambda} > 0.
\]
We will prove that there are buyers with \( \Pi'(0) > 0 \) and \( \Pi'(<\infty) < 0 \) for \( \tau \to \infty \). Let \( L := a_0 \int_0^\infty (1 - F(t)) e^{-(a_0 + \lambda)t} dt \). Then \( \frac{a_0}{a_0 + \lambda} < 1 \). Hence, with appropriate parameters we can construct parameters satisfying
\[ L < -\frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} < 1. \]
Moreover, \( \Pi'(0) = \kappa \left( -a_0 \int_0^\infty (1 - F(t)) e^{-(a_0 + \lambda)t} dt - \frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} \right) \). But \( -\frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} > L \) implies \( \Pi'(0) > 0 \). Due to \( -\frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} < 1 \), \( \Pi'(\tau) < 0 \) as \( \tau \to \infty \).

 Assertion (ii). We will prove that there are buyers with \( \Pi'(\tau) < 0, \forall \tau \geq 0 \). Again we start with a bound on the integral in (14), this time the lower bound. If \( t \geq \tau \) then \( e^p(t-\tau) \geq 1 \) and \( \frac{q + pe^{(p+q)\tau}}{a_0 + \lambda + p + q} \) as the denominator becomes larger and \( p, q \geq 0 \) in the diffusion model. As both sides of the inequality are strictly positive, integrating does not affect the direction of the inequality. Hence,
\[
\int_\tau^\infty (1 - F(t)) e^{-(a_0 + \lambda)(t-\tau)} dt = \int_\tau^\infty \frac{p + q}{q + pe^{(p+q)t}} e^{-(a_0 + \lambda)(t-\tau)} dt \\
\geq \frac{p + q}{q + pe^{(p+q)\tau}} \int_\tau^\infty e^{-(p+q)(t-\tau)} e^{-(a_0 + \lambda)(t-\tau)} dt = \frac{1 - F(t)}{a_0 + \lambda + p + q}
\]

Let \( U := \frac{a_0}{a_0 + \lambda + p + q} > 0 \). Then we can construct parameters satisfying \( 0 < -\frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} < U \). We need to show that under this condition \( \Pi'(\tau) \leq 0, \forall \tau \). This is sufficient for the proof because it implies that at each point in time the buyer should not wait because of decreasing profit. It is \( \Pi'(\tau) = \kappa e^{-\lambda t} \left( -F(\tau) - a_0 \int_\tau^\infty (1 - F(t)) e^{-(a_0 + \lambda)(t-\tau)} dt - \frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} \right) \); (16) implies \( \Pi'(\tau) \leq \kappa e^{-\lambda t} \left( -F(\tau) - a_0 \frac{1 - F(\tau)}{a_0 + \lambda + p + q} \right) \).

Since \( -\frac{c'(t)}{\lambda c} + \frac{c(t)}{\tau} < \frac{a_0}{a_0 + \lambda + p + q} \), it is \( \Pi'(\tau) < \kappa e^{-\lambda t} \left( -F(\tau)(\lambda + p + q) \right) \). This proves the claim that buyers exist, who should immediately introduce SCF.

\[ \square \]

**Corollary 2.** Assertions (i) and (ii). Consider (14). If there is an inner optimum at \( \tau^* \) then \( \Pi''(\tau^*) < 0 \). Hence the implicit function theorem implies \( \frac{d_\tau}{d_\tau} < 0 \iff \frac{d_\Pi}{d_\Pi} < 0 \). We can directly infer that \( \frac{\partial^2 \Pi}{\partial \tau \partial a_0} < 0 \) from (14). With \( \tau \) as defined in Proposition 2 we observe \( \frac{\partial \tau}{\partial \tau} > 0 \) and \( \frac{\partial \tau}{\partial a_0} > 0 \). Using the chain rule shows that \( \frac{d_\tau}{d_\tau} < 0 \) and \( \frac{d_\tau}{d_\Pi} < 0 \), which are Assertions (i) and (ii), respectively.

For the extreme case of \( \tau^* \) being a boundary solution, note that as \( \tau \) goes to zero, the sufficient condition \( \tau^* < c(t)/\lambda t \) for rejecters as stated in the proof of Proposition 2 holds for any cost function satisfying our assumptions. Hence, the extreme case statements of assertions (i) and (ii) are also shown.

**Assertions (iii).** This assertion consists of two claims: (a) if firms are introducers then more influential firms introduce earlier and (b) changes in influence cannot make rejecters to introducers and vice versa. We prove (a) by showing that \( \frac{d_\tau}{d_0} > 0 \). As before, we apply the implicit function theorem to the first derivative of \( \Pi \) in some neighborhood of \( \tau^* \), where we know that \( \frac{\partial^2 \Pi}{\partial \tau^2} < 0 \).

Hence, it suffices to analyze \( \frac{\partial^2 \Pi}{\partial \tau \partial a_0} \), for which we have
\[
\frac{\partial^2 \Pi}{\partial \tau \partial a_0} = \kappa e^{-\lambda t} \int_0^\infty - (1 - F(t + \tau)) (1 - a_0 t) e^{-(a_0 + \lambda)t} dt.
\]
One can split this integral into two intervals, \([0, 1/a_0]\) and \([1/a_0, \infty)\). The integral over the first integrand is positive, and the integral over the second integrand is negative. Overall, with the estimations of upper and lower bounds derived above, it follows that the sum is negative, so \(\frac{\partial^2 \Pi}{\partial \tau \partial a^*} < 0\) and \(\frac{\partial \tau^*}{\partial a_0^*} < 0\).

Claim (b) follows directly from the sufficient condition for rejecters that follows from Proposition 2 stating that if \(\frac{i_b(d^* - d_0) V \cdot M(d^*)}{\lambda} < \frac{c'(t)}{\lambda} + c(t), \forall t\) a firm is a rejecter. Obviously this condition is independent of \(a_0\).

**Proposition 3.** We apply the Theorem of Debreu, Glicksberg, and Fan (Fudenberg & Tirole 1991, page 34) to our game. The buyers have strategy spaces \(S_i = [d_0, \infty[ \times [0, \infty[\). As in the single-buyer game, we know that \(d_{s,i}^*\) does not depend on \(\tau\), hence it can be computed upfront and independently for each buyer. Hence there is a value \(D_s > 0\) defined by \(D_s := \max_{i \in N_n} d_{s,i}^* \in \mathbb{R}\) such that \(d_{s,i}^* \leq D_s (i \in N_n)\). Therefore, all strategies involving \(d_{s,i}^* > D_s (i \in N_n)\) are dominated and can be deleted. We can split the buyers into two groups: introducers and non-introducers. The latter group consists of buyers as described in Proposition 2. Independent of what other buyers do, for them non-adopting is optimal. Therefore, we can limit our discussion to introducing buyers and assume all other players choose \(\tau = \infty\) and do not interact with other introducers. Consider for each buyer a value \(\tilde{\tau}_i^*\) which corresponds to the solution where \(\tau_j = \infty \forall j \neq i\). Since \(\tau_i^*\) increases in \(\tau_j\) and \(\tau_j < \infty\), \(\tau_i^* \leq \tilde{\tau}_i^* < \infty\). Hence there is a value \(T \geq 0\) such that \(T := \max_{i \in N_n} \tilde{\tau}_i^* \in \mathbb{R}\). Similarly, all strategies involving \(\tau_i^* > T (i \in N_n)\) can be omitted. Deleting dominated strategies we obtain an equivalent game with strategy spaces \(S_i = [d_0, D_s] \times [0, T]\). These spaces are non-empty, compact, convex subsets of the Euclidean space \(\mathbb{R}^2\). Moreover, the payoffs \(u_i\) are continuous in \(s\) and quasi-concave in \(s_i\). The existence of an equilibrium in pure strategies then follows from the Theorem of Debreu, Glicksberg, and Fan. □

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