Sussing merger trees: the Merger Trees Comparison Project


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Sussing Merger Trees: The Merger Trees Comparison Project

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ABSTRACT

Merger trees follow the growth and merger of dark-matter haloes over cosmic history. As well as giving important insights into the growth of cosmic structure in their own right, they provide an essential backbone to semi-analytic models of galaxy formation. This paper is the first in a series to arise from the Sussing Merger Trees Workshop in which 10 different tree-building algorithms were applied to the same set of halo catalogues and their results compared. Although many of these codes were similar in nature, all algorithms produced distinct results. Our main conclusions are that a useful merger-tree code should possess the following features: (i) the use of particle IDs to match haloes between snapshots; (ii) the ability to skip at least one, and preferably more, snapshots in order to recover subhaloes that are temporarily lost during merging; (iii) the ability to cope with (and ideally smooth out) large, temporary fluctuations in halo mass. Finally, to enable different groups to communicate effectively, we defined a common terminology that we used when discussing merger trees and we encourage others to adopt the same language. We also specified a minimal output format to record the results.

Key words: methods: numerical – galaxies: evolution – galaxies: haloes – dark matter.

1 INTRODUCTION

In the era of precision cosmology, numerous very large galaxy survey programmes are either currently underway or in development (just to name a few, BOSS, PAU, WiggleZ, eBOSS, BigBOSS, DESpec, PanSTARRS, DES, HSC, Euclid, WFIRST, etc.). The full power of these programmes to shed light on the nature of dark energy and dark matter can only be realized if the observational results are compared to theoretical expectations. Thus, the level of precision required can only be achieved if the theoretical framework is equally well controlled.

Numerical simulations underpin the theoretical predictions for structure formation and growth. They are required because the structures that host the galaxies we observe have densities well in excess of the mean and their growth is highly non-linear. Large simulations containing billions (soon to be trillions) of tracer particles have become common in recent years (e.g. Millennium, DEUS, Bolshoi, MillenniumXXL, Horizon4pi, Jubilee, see Kuhlen, Vogelsberger & Angulo 2012, for a recent review) and these models cover volumes that are well matched to aforementioned galaxy surveys covering increasingly large cosmological volumes. But accurate numerical
simulations are not the end of the story. In order to produce a mock galaxy catalogue, the structures present within these simulated volumes need to be identified and subsequently populated with galaxies.

By comparing the results obtained for a wide range of halo finding algorithms, Knebe et al. (2011) already quantified the errors introduced during halo identification. This project and its extensions to the related topics of subhalo detection (Onions et al. 2012) and stream finding (Elahi et al. 2013) are summarized in the review paper by Knebe et al. (2013b).

Once the set of haloes within a cosmological volume have been reliably identified, the second step is to populate them with galaxies. This can be done using the information from a single snapshot by relating the mass of a halo to the number of galaxies it contains. This is referred to as Halo Occupation Density or HOD modelling (e.g. Skibba & Sheth 2009). This, however, treats galaxies within each snapshot independently. To follow the self-consistent evolution of galaxies over cosmic time requires information about the growth and assembly of the haloes that host them. The ruleset that determines how the galaxies contained within these haloes form and evolve are known as semi-analytic models (SA models; for a review see Baugh 2006).

SA models rely on the accuracy of both the individual halo catalogues themselves as well as the framework that connects the halo catalogues from different snapshots together. For every object, this framework forms a tree structure, with many leaves and branches at early times eventually merging together to form a single trunk that represents the final galaxy (e.g. Lacey & Cole 1993; Roukema et al. 1997). The main aim of this paper is to compare and contrast the tree structures built from a common set of halo catalogues by 10 different tree building algorithms. We will examine the accuracy of the trees (how often they link unrelated haloes together) and the smoothness of the tree growth. Both can lead to unrealistic galaxy growth within an SA model.

The results presented in this paper arise out of the Sussing Merger Tree workshop, that took place on 2013 July 7. In advance of the workshop, participants were provided with a set of haloes (described in Section 3 below) and asked to return a merger tree that linked the haloes together over cosmic time in a way that best represents the growth of cosmic structure. We allowed participants to correct errors in their results that arose out of applying their code to this new data set (e.g. unusual data format; periodic boundary conditions) but gave them no feedback in advance of drafting the paper on how their results compared to those of other participants.

In this paper, we use a single set of halo catalogues from a cosmological box to test the basic properties of the merger trees and the mass-growth of haloes over time. During the course of the study presented here it became clear that tree building algorithms are often intimately tied to the algorithm used to generate the input halo catalogue, and so in that sense the comparison is not equally fair on all codes. While we adhered to this approach in general as it is the only way to enable an easy comparison between codes, we nevertheless allowed two codes to modify the halo catalogues (i.e. CONSISTENT TREES and HBT). We also allowed algorithms to convert between inclusive and exclusive particles lists (see Section 2, for a definition) where desired. Future papers will investigate the effect of changing the halo definition, snapshot spacing, mass resolution and eventually the effect on SA models.

In what follows, the terminology used throughout the paper will be specified in Section 2. Section 3 describes the halo data set that we use, and Section 4 gives an overview of the various codes that have participated in the comparison. We present results on the structure of the resultant trees in Section 5 and of their mass-growth in Section 6. Finally, we summarize our results in Section 7.

2 TERMINOLOGY

To avoid confusion, it is important that different researchers working on merger trees speak the same language. We define here the terminology used in this paper and would encourage others to adopt the same definitions.

(i) A halo is a dark-matter condensation as returned by a halo finder (in our case AHF). For the purposes of other definitions below, we assume that the IDs of the particles attributed to each halo by the halo finder are known.

(ii) Haloes may be spatially nested: in that case the outer halo is the main halo and the other haloes are subhaloes. Note that the assignment of main haloes and subhaloes is a function of the halo finder and one can envisage unusual geometries where this allocation is not obvious; nevertheless, the picture of subhaloes orbiting within larger ones ties in with our view of cosmic structure and is central to many SA models.

(iii) If particles are allowed to be members of only one halo, (i.e. particles in subhaloes are not included in the particle ID list of the main halo, and particles in overlapping haloes are assigned to just one of the two), then the haloes are said to be exclusive; otherwise they are inclusive (AHF falls into this latter category).

(iv) Haloes are defined at distinct snapshots. Snapshots correspond to particular values of cosmic time and contain the particle IDs, mass, location and velocity for each dark-matter particle in the simulation.

(v) For two snapshots at different times, we refer to the older one (i.e. higher redshift) as A and the younger one (i.e. lower redshift) as B.

(vi) A graph is a set of ordered halo pairs, \((H_A, H_B)\), where \(H_A\) is older than \(H_B\). It is the purpose of the merger-tree codes to produce a graph that best represents the growth of structure over cosmic time. \(H_A\) and \(H_B\) are usually taken from adjacent snapshots, but this is not a requirement as there are occasions where haloes lose their identity and then reappear at a later time.

(vii) Recursively, \(H_A\) itself and progenitors of \(H_A\) are progenitors of \(H_B\). Where it is necessary to distinguish \(H_A\) from earlier progenitors, we will use the term direct progenitor.

(viii) Recursively, \(H_B\) itself and descendants of \(H_B\) are descendants of \(H_A\). Where it is necessary to distinguish \(H_B\) from later descendants, we will use the term direct descendant.

(ix) In this paper, we are primarily concerned with merger trees for which there is precisely one direct descendant for every halo. Note that it is possible for haloes near the minimum mass limit to have zero descendants: we omit such haloes from our analysis.

(x) In the case that there are multiple direct progenitors, we require that precisely one of these be labelled the main progenitor – this will usually be the most massive, but other choices are permitted.

(xi) The main branch of a halo is a complete list of main progenitors tracing back along its cosmic history.\(^1\)

Over the course of writing this paper it became clear that there has been confusion in the past between what we call graphs and merger

\(^1\) We note that, for main haloes rooted at \(z = 0\), this main branch might more appropriately be called a trunk, but it seems unnecessary to introduce a new term for this specific purpose.
trees. Both are interesting in different contexts. We limit ourselves here to an investigation of merger trees which are the more relevant as an input to SA models.

3 INPUT HALO CATALOGUES

The halo catalogues used for this paper are extracted from 62 snapshots of a cosmological dark matter only simulation undertaken using the GADGET-3 N-body code (Springel 2005) with initial conditions drawn from the Wilkinson Microwave Anisotropy Probe 7 cosmology (Komatsu et al. 2011). We use 270$^3$ particles in a box of comoving width 62.5 $h^{-1}$ Mpc, with a dark-matter particle mass of $9.31 \times 10^8 h^{-1} M_{\odot}$. The snapshots are labelled 0, 1, 2, . . . , 61 from redshift 50 to redshift 0, as indicated in Fig. 1.

The main halo finder used in this paper is AHF (Gill, Knebe & Gibson 2004; Knollmann & Knebe 2009). It locates local overdensities in an adaptively smoothed density field as prospective halo centres. For each of these density peaks, the gravitationally bound particles are determined. Only peaks with at least 20 bound particles are considered as haloes and retained for further analysis. The halo mass $M_{200}$ is

$$M_{200} = 200 \rho_c(z) \frac{4\pi}{3} R_{200}^3,$$

where $\rho_c(z)$ is the critical density of the Universe as a function of redshift $z$ and $R_{200}$ is the radius enclosing a mean density that equals 200 times the critical density.

AHF generates inclusive data sets (i.e. particles in subhaloes are also included in the main halo). As an input to the tree-building codes, we provided the list of particle IDs associated with each halo, alongside information about the (kinetic plus potential) energy, position and velocity of each particle; we further made available the full halo catalogue containing, besides the usual mass, position and bulk velocity, an abundance of additional information (e.g. energies, centre offsets, shapes, etc.).

The participants were asked to run their merger-tree builders on the supplied data and return, for each halo, a list of progenitor haloes and (unless the halo was newly created) the ID of a single main progenitor. For the purpose of comparing merger-tree algorithms, we restricted participants to use only the information described above and did not give them access to the raw N-body data. However, they were allowed to alter the original halo catalogues by adding extra ‘fake’ haloes and removing some ‘unreliable’ haloes where they felt that was appropriate.

4 CODE DESCRIPTIONS

In this section, we briefly describe, in alphabetical order, the participating merger-tree codes. Furthermore, details of algorithms can be found in the accompanying references.

The participants were asked to build trees starting from our input halo catalogues described in Section 3. One of the features of a merger tree, as we define it, is that while an object can have multiple progenitors, only one descendant is allowed. But many of the algorithms tested did not, in the first instance, produce a tree. Instead they commonly built graphs that allowed multiple descendants of a single progenitor halo. To allow consistency and ensure a fair comparison, we required each author to modify their algorithm to return a tree. Nevertheless, the central process of linking haloes together between snapshots remains and exploring the various ways of achieving this is the main purpose of this paper.

We note that some of the participating codes required modification in order to allow them to take as input the AHF halo catalogues that we used for this comparison project. To facilitate analysis of the returned merger trees, we have defined a common, minimal data output format (described in the Appendix), and this has also required minor modifications to some of them.

4.1 Tree similarity

As a lot of methodology is similar across the various codes used here, we try to capture the main features and requirements in Fig. 2 and Table 1. Note that only a single code does not use particle IDs to link haloes between snapshots; that potentially makes it more widely applicable to legacy data but leads to problems with misidentification of haloes, as will be seen later in Section 5 below.

Many tree-codes make use of a merit function

$$M(H_A, H_B) = f(N_A, N_B, N_{A\cap B}),$$

where $N_A$ and $N_B$ are the numbers of haloes in haloes $A$ and $B$, respectively.

Figure 2. A summary of the main features and requirements of the different merger-tree algorithms. For details see the individual descriptions in the text.
Table 1. A summary of the features and requirements of merger-tree algorithms (for details see individual descriptions in the text). Columns: (i) code name; (ii) particle properties used to produce the merger trees; (iii) \textit{AHF} halo properties used to produce the merger trees (\(M_{200}\)-mass, \(r\)-position, \(v\)-velocity, \(V_{\text{max}}\)-maximum rotation speed of the halo); (iv) the merit function used to estimate descendants; (v) the merit function used to estimate the main progenitor; (vi) the number of consecutive snapshots used to determine descendants/progenitors at each snapshot.  

\[ M(H_A, H_B) = f(R_{A^C:B}) , \]  

where \(N_A\) and \(N_B\) are the number of particles in haloes \(H_A\) and \(H_B\), respectively, and \(N_{A^C:B}\) is the number of particles that are in both \(H_A\) and \(H_B\), or

\[ \mathcal{M}(H_A, H_B) = N_{A^C:B} / (N_A N_B) , \]  

for \(A\) and \(B\), being the 1/3 power of the \(R_{A^C:B}\). The \(f\) function aims at identifying the most likely progenitor/descendant of a given halo. A few of them use additional information such as, for instance, the binding energy of the particles, properties of the haloes or positions, velocities and masses, and the halo finder. In some cases, obvious inconsistencies arise between the expected positions, velocities and mass profiles as returned by the halo finder. From haloes in any given simulation snapshot, the expected positions and velocities of haloes at an earlier snapshot may be calculated. In some cases, obvious inconsistencies arise between the predicted and actual halo properties, such as missed satellite haloes (e.g. satellite haloes which pass too close to the centre of a larger halo) and spurious mass changes (e.g. satellite haloes which suddenly increase in mass due to temporary missassumption of particles from the central halo). These defects are repaired by substituting predicted halo properties instead of the properties returned by the halo finder. If a halo has no descendant, a merger is assumed to have occurred with the halo exerting the strongest tidal field across it, unless no such suitable halo exists in which case the halo is presumed to have been spurious and this branch is pruned from the merger tree. This process helps to ensure accurate mass accretion histories and merger rates for satellite and central haloes; full details of the algorithm as well as tests of the approach may be found in Behroozi et al. (2013).

4.2 CONSISTENT TREES (Mao & Behroozi)

The CONSISTENT TREES algorithm (Behroozi et al. 2013) first matches haloes between snapshots by identifying descendant haloes as those that have the maximum number of particles from a given progenitor halo. It then attempts to clean up this initial guess by simulating the gravitational bulk motion of the set of haloes given their known positions, velocities and mass profiles as returned by the halo finder. From haloes in any given simulation snapshot, the expected positions and velocities of haloes at an earlier snapshot may be calculated. In some cases, obvious inconsistencies arise between the predicted and actual halo properties, such as missed satellite haloes (e.g. satellite haloes which pass too close to the centre of a larger halo to be detected) and spurious mass changes (e.g. satellite haloes which suddenly increase in mass due to temporary missassumption of particles from the central halo). These defects are repaired by substituting predicted halo properties instead of the properties returned by the halo finder. If a halo has no descendant, a merger is assumed to have occurred with the halo exerting the strongest tidal field across it, unless no such suitable halo exists in which case the halo is presumed to have been spurious and this branch is pruned from the merger tree. This process helps to ensure accurate mass accretion histories and merger rates for satellite and central haloes; full details of the algorithm as well as tests of the approach may be found in Behroozi et al. (2013).

4.3 D-TREES (Helly)

The D-TREES algorithm (Jiang et al., in preparation) is designed to work with the \textit{SUBFIND} group finder, which (like \textit{AHF}) can occasion-
also have up to one main progenitor which may be up to \( N_{\text{step}} \) snapshots earlier.

4.4 HBT (Han, Jing)

The Hierarchical Bound Tracing (HBT) algorithm (Han et al. 2012) is a tracking halo finder in the sense that it uses information from earlier snapshots to help derive the latest halo catalogue. As such it naturally builds a merger tree. Starting from high redshift, main haloes are identified as they form. The particles contained within these haloes are then followed explicitly through subsequent snapshots, generating a merger tree down to main halo level at the first stage. To extend the merger tree down to subhalo level, HBT continues the tracing of merged branches, identifying the set of self-bound particles that remain for every progenitor halo. These self-bound remnants are defined as descendant haloes of their progenitors. With this kind of tracking, each halo has at most one progenitor, which defines its main branch. The main branch extends until the number of particles contained in the bound halo remnant drops below 20 particles. When this occurs a final tracking step is undertaken to determine which halo it has fallen into, adding minor branches to the tree.

The major challenge in this method is to robustly track haloes over long periods, and HBT has been specifically tuned to achieve this. In addition, the merging hierarchy among progenitor haloes is utilized to efficiently allow satellite–satellite mergers or satellite accretion inside satellite systems.

Note that HBT is not designed to be a general purpose tree builder for external halo catalogues. To generate the trees used in this paper, HBT was run using only the main haloes from the supplied catalogue as described in Section 3 as input. It then outputs its own list of haloes and calculates the relevant properties for them, as well as returning the merger tree built on top of these haloes.

HBT outputs exclusive haloes. In order to give a mass which matches that of \( \text{AHF} \) haloes as closely as possible, for each halo, we first calculate an ‘exclusive’ mass according to equation (1) using only particles from the halo itself. Then, we add to each halo the exclusive mass of all its subhaloes, to give an ‘inclusive’ mass, which we use throughout this paper.

4.5 JMERGE (Onions)

The JMERGE algorithm constructs a merger tree purely from aggregate properties (the position, centre-of-mass velocity and mass) of the haloes identified by a halo finder (i.e. it does not require the individual particle positions or particle IDs). It compares halo catalogues from two snapshots separated by a known time interval. For the two sets of haloes at snapshots \( A \) and \( B \), a new position is calculated for the centre of each halo by moving the \( A \) haloes forward in time by half the timestep, and the \( B \) haloes backwards by half the timestep assuming that they are moving at constant velocities. Then, starting from the most massive halo and working towards smaller masses, for each halo in \( A \), a best match on position is found to a halo in \( B \), together with constraints on the allowed change in mass and maximum circular velocity. Mass is allowed to shrink by a factor of up to 0.7, and to grow by a factor of up to 4. The search distance is limited to twice the radius at which the enclosed density is 200 times the background density plus four times the distance the halo has moved during the timestep. At this stage, each halo in \( B \) can only be claimed once. This process attempts to trace haloes growing over time.

For those haloes that do not find an unclaimed descendant in \( B \), two other processes are implemented. First, mergers are accounted for by finding so far unmatched haloes at time \( A \) that can accrete on to \( B \) targets already accounted for, whilst still limiting the total mass of the direct progenitors of each descendant to less than 1/0.7 times its mass. Secondly, haloes that cannot find a suitable match are deemed to be numerical artefacts and are pruned from the tree.

4.6 L-HALOTREE (Dolag)

L-HALOTREE was the first merger-tree algorithm to construct trees based on subhaloes instead of main haloes. The L-HALOTREE algorithm is described in the supplementary information of Springel et al. (2005) and the reader is referred there for further details. In short, to determine the appropriate descendant, the unique IDs that label each particle are tracked between outputs. For a given halo, the algorithm finds all haloes in the subsequent output that contain some of its particles. These are then counted in a weighted fashion, giving higher weight to particles that are more tightly bound in the halo under consideration, as listed in Table 1, and the one with the highest count is selected as the descendant. In this way, preference is given to tracking the fate of the inner parts of a structure, which may survive for a long time upon infall into a bigger halo, even though much of the mass in the outer parts can be quickly stripped.

To allow for the possibility that haloes may temporarily disappear for one snapshot, the process is repeated for Snapshot \( n \) to Snapshot \( n + 2 \). If either there is a descendant found in Snapshot \( n + 2 \) but none found in Snapshot \( n + 1 \), or, if the descendant in Snapshot \( n + 1 \) has several direct progenitors and the descendant in Snapshot \( n + 2 \) has only one, then a link is made that skips the intervening snapshot.

4.7 MERGERTREE (Knebe)

The MERGERTREE routine forms part of the publicly available Amiga halo finder (AHF) package. It is a simple particle correlator: it takes two particle ID lists (ideally coming from an AHF analysis) and identifies for each object in list \( B \) those objects in list \( A \) (at the previous snapshot) with which there \( N \) or more particles in common \((N = 10 \) for this comparison). Despite its name, therefore, it produces a graph mapping the connections between objects rather than a tree, as each halo can have multiple descendants.

MERGERTREE also identifies a unique main progenitor for each object in list \( B \) as found in list \( A \). It achieves this by maximizing a merit function (as shown in Table 1) This has proven extremely successful (Klimentowski et al. 2010; Libeskind et al. 2011; Knebe et al. 2013a). The code can hence not only be used to trace a particular object backwards in time (or forward, depending on the temporal ordering of files \( A \) and \( B \)), but also to cross-correlate different simulations (e.g. different cosmological models run with the same phases for the initial conditions).

To create an actual tree, we need to ensure that each halo has a unique descendant. This is guaranteed by running MERGERTREE in a novel mode that applies the same merit function in both directions when correlating two files. In practice this links haloes that share the largest fraction of particles between the two snapshots as well as forcing a choice between multiple possible descendants (of which now only the one maximizing the merit function in the direction \( A \rightarrow B \) is kept). The use of a merit function also eliminates any need for all the particles in the input halo catalogues to only belong to a single object: \( \mathcal{M}_{A,B} \) automatically takes care of particles that have been assigned to multiple objects.
4.8 SUBLINK (Rodríguez-Gomez)

SUBLINK (Rodríguez-Gomez et al., in preparation) constructs merger trees at the subhalo level. A unique descendant is assigned to each subhalo in three steps. First, descendant candidates are identified for each subhalo as those subhaloes in the following snapshot that have common particles with the subhalo in question. Secondly, each of the descendant candidates is given a merit function specified in Table 1. Thirdly, the unique descendant of the subhalo in question is the descendant candidate with the highest merit function.

Sometimes the halo finder does not detect a small subhalo that is passing through a larger structure, because the density contrast is not high enough. SUBLINK deals with this issue in the following way. For each subhalo from snapshot Sn, a ‘skipped descendant’ is identified at Sn+2, which is then compared to the ‘descendant of the descendant’ at the same snapshot. If the two possible descendants at Sn+2 are not the same object, we keep the one obtained by skipping a snapshot since, by definition, it has the largest score at Sn+2.

Once all descendant connections have been made, the main progenitor of each subhalo is defined as the one with the ‘most massive history’ behind it, following De Lucia & Blaizot (2007). This information is rearranged into fully independent merger trees.

4.9 TREEMAKER (Tweed)

The TREEMAKER algorithm was developed for the SA model Galaxies in Cosmological Simulations (GalICs) (Hatton et al. 2003). It was first used on Friends-of-Friends haloes (Davis et al. 1985), and later applied to main haloes and subhaloes extracted from a cosmological simulation with the AdaptaHOP group finder (Aubert, Pichon & Colombi 2004; Tweed et al. 2009). The code associates haloes from two consecutive time steps, listing all progenitors (including particles accreted from the background) and descendants (multiple descendants being allowed even if particles lost to the background are ignored). Here ‘background’ refers to particles not in any halo at the current time. This first step is completed by using the particle IDs as tracers to identify haloes. Under our scheme a particle can only belong to one single halo at a given step, meaning a particle in a subhalo belongs only to that subhalo and not to any enclosing halo.

In order to create a ‘usable’ merger tree, a simplification stage is required. Exactly one descendant per halo is selected and the list of progenitors updated to reflect this selection. Selecting this unique descendant requires the use of a merit function. The first versions of TREEMAKER used a shared merit function. For this study, we tested various modifications of this selection, but all gave similar results. We therefore include in this paper only the normalized merit function $M_1$ as shown in Table 1.

4.10 VELOCIRAPTOR (Elahi)

The halo merger-tree algorithm used in VELOCIRAPTOR is based on a particle correlator: that is the algorithm compares two (or more) exclusive particle ID lists and produces a catalogue of matches for each object in each list. Specifically, for each object $i$ in catalogue $A$, the algorithm finds all objects $j$ in catalogue $B$ that share particles, and calculates the strength of each connection using the merit function $M_1$ as shown in Table 1. The search for connections is done in both directions. Any connection with a merit function within Poisson fluctuations, $M_{A,B} \leq 1/(N_A N_B)$, is ignored. The connection that maximizes $M$ for $A \rightarrow B$ is deemed the unique descendant (note that the original code returned a graph that did not enforce this requirement of uniqueness). This approach is used as particle ID lists produced by VELOCIRAPTOR contain not only particles belonging to bound (sub)haloes but also those in physically diffuse tidal debris. Consequently, tracking object centres or weighting particles by a measure of how bound they are is meaningless. Note that tidal debris candidates, due to their physically diffuse nature, can be artificially fragmented into several VELOCIRAPTOR groups. For example, a single bound (sub)halo identified at time $A$ is found to be the progenitor of several tidal debris fragments at time $B$. Matching $B \rightarrow A$, the fragments identify the (sub)halo as the primary progenitor; however, the (sub)halo will identify the largest tidal fragment as its primary descendant. For the purposes of this paper, the other fragments are ignored. However, in the general merger graph produced by VELOCIRAPTOR, these fragments are flagged as secondary descendants if fragment shares $\geq 5$ per cent of particles with the primary progenitor.

4.11 YSAMTM (Jung, Lee & Yi)

The tree-making algorithm YSAMTM (Jung et al., in preparation) was developed to build dark-matter halo merger trees for the SA model ySAM (Lee & Yi 2013). It uses the particle information from two snapshot files or the particle IDs and locations from a pre-calculated halo catalogue. First the ‘shared mass’, the mass contribution of all progenitor haloes to each descendant halo, is calculated. At this stage, particles are matched between haloes in the two snapshots by using the particle IDs. Individual particles are only included in a single halo or subhalo and are not listed as members of the host halo of the subhalo. Secondly, in order to convert our graph into an actual tree that could be used by SA models, we define a unique descendant halo of each progenitor halo by determining which descendant halo has the most shared mass among all descendants of the progenitor halo, unless there exists a smaller halo which receives a larger fraction of its mass from the same progenitor. In this case, we determine that the smaller one is the most likely descendant halo of the progenitor even if its shared mass is not the largest amongst all the descendants. This avoids defining the smaller descendant halo as a newly formed halo when it contains many particles that were members of an existing halo in the previous snapshot. This process creates a true tree where one descendant halo can have multiple progenitor haloes, while each progenitor halo has a unique descendant halo. Among those progenitors, the main progenitor is determined by maximizing the merit function $M_2$ in Table 1.

5 TREE STRUCTURE

In this section, we look at the structure/geometry of trees. This includes a comparison of measurable quantities like the tree-length along the main branch, the tree-branching at every step and the general consistency of the tree (i.e. possible misidentification of descendants).

5.1 Length of main branches

The most basic requirement of a tree-building code is to trace haloes back in time. The length of the main branch gives a measure of how long single haloes can be followed through the complicated merger history of structure formation. Fig. 3 shows the number, $N$, of $z = 0$ haloes that have main branches extending for a given number of snapshots, $l$, for all haloes within three different mass-ranges: haloes with $M_{200c} \leq 10^{11} \, h^{-1} M_{\odot}$ (less than $\sim 100$ particles) are
increases. For $M > 10^{12} h^{-1} M_\odot$, the number of main branches per length is roughly constant from $l = 0$ until about $l = 30$ (corresponding to $z \approx 5$) and only drops to zero beyond $l \approx 50$ ($z \approx 10$). Thus, even in a hierarchical structure formation scenario, dwarf-sized haloes that survive to the current day have a wide variety of formation times.

One oddity in Fig. 3 is that most of the tree codes find a few large haloes with very short main branches which is in contradiction to the common picture of structure formation. Furthermore, investigation of these branches show that they are either truncated due to a non-identification by the halo finder, or are due to an error in the halo assignment of the tree building codes.

One such example is pictured in Fig. 4 which shows two similarly sized haloes merging almost head-on. The red and blue circles show the two haloes at $z = 0$ (right-hand column) and then traced back in time over several snapshots (successive columns to the left – note that we have chosen to omit Snapshot 58 as it added little to the plot). The AHF halo finder (and other halo finders) assigns most of the mass in overlapping objects to a single object, treating the other as substructure. Unfortunately, this assignment can change between snapshots so that haloes centred on the same clump of highly bound particles can fluctuate wildly in size. Different tree codes handle this in different ways, illustrated in the different rows of Fig. 4.

(i) Mergertree fails to find a match for the smaller of the two haloes at Snapshot 60 and does not seek a match at earlier times. This halo therefore has no links in its merger tree and appears to be created intact in the final snapshot. The other merit function codes that use just two snapshots (Treemaker, Velociraptor and V Samantha) behave in the same manner, as, in this case, does Jmerge.

(ii) Lhalotree does something similar, but due to its use of weighted function, it matches the smaller of the two haloes at $z = 0$ to the large one from the previous snapshot. While Lhalotree can cross-match haloes by skipping a snapshot, that is not applied here as a descendent halo exists.

(iii) D-Trees does the same as Lhalotree on Snapshot 60, but also manages to link together the larger of the two haloes between Snapshots 61 and 59. This results in a fluctuating mass for the both haloes, (low-high-low for red, high-low-high for blue).

(iv) Sublink also manages to cross-match the larger of the haloes between Snapshots 61 and 59 but chooses a different association for the halo in Snapshot 60, thus avoiding the large mass fluctuation. It links the smaller of the two haloes in Snapshot 61 directly to that in Snapshot 59, skipping over the intermediate snapshot.

(v) Consistent trees goes one step further and introduces a fake halo in Snapshot 60 to avoid a link in the merger tree that extends over more than one snapshot.

(vi) Finally, HBT redefines both haloes and outputs a smoother variation of mass over time.

From these descriptions, it may seem like the above is an ordered list of improving performance, from top to bottom. However, we stress that this is true only for this particular merging event and that different codes cope better in different situations. The purpose here was more to illustrate the variety of behaviours that are possible.

5.2 Branching ratio

Another interesting statistical quantity is the number of branches (i.e. the number of direct progenitors) at every node of the merger tree. This will depend upon the spacing between snapshots, and so the precise values are not important, but the differences between algorithms are still of interest.

In Fig. 5, we plot the number of tree nodes with $N_{\text{direct}}$ direct progenitors, including all haloes between redshift zero and two. In this range, the timestep $\Delta t$ between snapshots is roughly constant with $\Delta t \approx 0.4$ Gyr. The most common situation is to have a single progenitor (i.e. the halo existed in the previous snapshot but no merging took place), followed by zero progenitors (i.e. the halo appears for the first time). However, in some cases, and depending on the tree builder, the number of direct progenitors can exceed 20.

HBT has the lowest branching ratio, perhaps because it allows itself to modify the halo catalogue to extend the life of subhaloes. Jmerge also has a low branching number because its non-use of particle IDs gives it freedom to link together haloes that other algorithms classify as unrelated. Next come D-Trees and Consistent Trees which both use information extended over several timesteps to follow haloes that temporarily disappear (for instance when a subhalo comes close to the centre of its host halo).

Although multiple direct progenitors are rare, it can be seen that the choice of tree code can make a significant difference to the ability to follow substructures and hence to the length of time a subhalo exists before it is subsumed into the host halo.

5.3 Misidentifications

Most tree-building algorithms link together haloes on the basis of having particles in common. However, there are some that do not (in this paper, Jmerge), and there are occasions when this association is not clear-cut. So we wish to test how often an obvious misidentification occurs.

Figure 3. The length of the main branch for haloes identified at $z = 0$ (Snapshot 61). The ordinate is $l = 61 - S$, where $S$ is the snapshot number at the high-redshift end of the main branch. The upper, middle and lower panels show the halo mass ranges at $z = 0$, as indicated in the panel, which correspond to roughly <100, 200–500 and >1000 particles, respectively.
Figure 4. An example of the merger of two haloes where the fluctuation of centring and size causes difficulties for the merger-tree algorithms. The red and blue circles show two haloes selected at $z = 0$ (right-hand column) and then traced back in time over several snapshots (successive columns to the left – note that we have chosen to omit Snapshot 58 as it added little to the plot). The missing algorithms all return the same results as MERGERTREE, shown in the top row. See the main text for a commentary.
Figure 5. Histograms of the number of haloes with $N_{\text{direct progenitors}}$, using all haloes from $z = 0$ to $z = 2$.

Figure 6. Histograms of the displacement statistic, $\Delta_{\text{1r}}$, for main haloes and their main progenitor for which both of them have $M_{200} > 10^{12} h^{-1} M_\odot$. The vertical lines show the 90th and 99th percentiles for MERGETREE (but are approximately the same for all algorithms except HBT).

One way of doing this is to quantify how far haloes are displaced from their expected locations in moving from one snapshot to the next. This is hard to predict for sub-haloes that may be moving around inside a larger object and so we restrict our attention to main haloes only. To measure this deviation, we use the statistic

$$\Delta_{\text{1r}} = \frac{|r_B - r_A - 0.5(v_A + v_B)(t_B - t_A)|}{0.5(R_{200A} + R_{200B} + |v_A + v_B|(t_B - t_A))}$$

which stays small as long as there is approximately uniform acceleration and no error in the halo linking. Here $t$ is cosmic time, $r$ and $v$ are the haloes’ positions and velocities, and $R_{200}$ the radius that encloses an overdensity of 200 times the critical density. The subscripts $A$ and $B$ refer to two linked haloes along the main branch of any tree.

Fig. 6 shows a histogram of $\Delta_{\text{1r}}$ for each algorithm, for all main haloes and their corresponding main progenitors. Most algorithms agree on the bulk of the distribution, and this likely represents the true behaviour for the AHF haloes considered here, with deviations from $\Delta_{\text{1r}} = 0$ being caused by curved trajectories and/or merging of subhaloes. The difference in HBT’s result from the others is partly due to different tree-links but also because the HBT halo catalogue has an intrinsically lower $\Delta_{\text{1r}}$.

MERGE occasionally shows much larger deviations, suggesting that it does have a tendency to link together unassociated haloes. CONSISTENT TREES also shows large outliers in this test and Fig. 7 shows a typical example of how this comes about. Here, we see an interaction in which the assignment of main halo alternates between successive snapshots.

(i) Most algorithms (top row) link together the visually correct group of particles and have small $\Delta_{\text{1r}}$, but will have a large fluctuation in halo mass along the main branch.

(ii) MERGE requires smooth changes in mass and so it follows the main halo between Snapshots 58 and 59, leading to a large value of $\Delta_{\text{1r}}$.

(iii) CONSISTENT TREES follows the main branch across all three snapshots, giving large values of $\Delta_{\text{1r}}$ for both links. It (correctly) fails to associate the top-right halo in Snapshot 59 with the central one in Snapshot 58, so it removes the latter and creates a fake halo to take its place.

(iv) HBT resolves the situation by creating a halo catalogue in which the mass evolution is smoother. It also inserts an extra sub-halo on the bottom right that is not returned by any of the other algorithms.
5.4 The loss of particles during halo growth

During mergers (and, indeed, during quiescent evolution) particles can be lost from haloes. As a measure of this, we use the statistic

$$\Delta_N = \frac{N_{\cup A_i} - N_{\cup A_i \cup B_i}}{N_{\cup A_i}}, \tag{6}$$

where, for a given halo $B$, the union runs over all direct progenitors, $A_i$. Here $N$ is the number of particles in $\cup A_i$ and $B$ or common to them both, as indicated by the subscript.

The distribution function of the fraction of lost particles, $\Delta_N$, for haloes along the main branch with $M_{200} > 10^{12} \, h^{-1} M_\odot$ (corresponding to about 1000 particles) is shown in Fig. 8. Note the extensive wing on this plot that extends to $\Delta_N = 0.4$. For small values of $\Delta_N$, this is due to changes in the shape of the halo, and to natural particle orbits that result in material moving out across the radius (here $R_{200}$) used to define the edge of the halo. Large values of $\Delta_N$ can occur when haloes reduce their size significantly between snapshots. An example of this situation has already been shown in the third row of Fig. 4 which illustrates how the halo finder alternates between allocating most of the mass to one or other of two haloes as they fly by one another.

All halo finders roughly agree on the number of haloes for which $\Delta_N < 0.4$, but there are significant differences for larger values – these are most probably due to misidentifications. It is perhaps not surprising that JMERGE has occasional very poor matches, given that it does not use particle IDs, but rare examples of apparently erroneous links are found in many other algorithms too.

6 MASS GROWTH

In this section, we look at the mass evolution of haloes, primarily along their main branches, which is a key input for most SA models. While main haloes are expected to grow in mass through accretion and mergers, sub-haloes can lose mass through tidal stripping.

Consider first Fig. 9 which shows the mass evolution along the main branch for the red and blue haloes illustrated in Fig. 4. The large mass fluctuations seen on the right-hand side of this plot correspond to the right-most panels in Fig. 4 and illustrate how poorly constrained the mass evolution is during that merger. The strong variation between the results returned by different algorithms suggests that much of this mass variation is unphysical, and most SA models would struggle to cope with this kind of fluctuating mass behaviour.

6.1 Mass growth along the halo main branch

The logarithmic growth rate of main branch haloes, $d \log M / d \log t$ is approximated discretely by

$$\frac{d \log M}{d \log t} \approx \alpha_M(A, B) = \frac{(t_B + t_A)(M_B - M_A)}{(t_B - t_A)(M_B + M_A)}, \tag{7}$$

where $M_A$ and $M_B$ are the masses of a halo and its descendant at times $t_A$ and $t_B$, respectively. The distribution function of $\alpha_M$ is shown in Fig. 10 for every pair of main-branch haloes for which the mass of each exceeds $10^{12} \, h^{-1} M_\odot$ (corresponding to about 1000 particles).

As demonstrated in Fig. 10, most of the time haloes are growing but there is a significant proportion of the time (about 30 per cent) during which mass-loss occurs. Such a large fraction is unlikely to be due to stripping (as this result is restricted to high-mass main-branch haloes) but some apparent mass-loss can occur due to changes in the shape of haloes during their evolution, especially following a major merger.

Strong mass-loss, however, is unphysical and is due to failures in the halo-finding and linking process, as illustrated in Figs 4, 7 and 9. The halo evolution seen in the right-most columns of Fig. 4 correspond to the wings in Fig. 10.

6.2 Mass fluctuations of subhalo main branches

Abrupt fluctuations up and down in mass can be quantified with a statistic

$$\xi_M(k) = \arctan \alpha_M(k + 1) - \arctan \alpha_M(k - 1, k), \tag{8}$$
where \( \alpha_M \) is as defined in equation (7) and \( k - 1 \), \( k \) and \( k + 1 \) represent successive timesteps. This measures the change in the slope of the mass accretion rate between two consecutive steps and thus ranges from \(-\pi\) to \(\pi\). The main purpose of this statistic is to detect temporary mass fluctuations that occur either as a result of the natural growth process, or because of halo misidentification.

Large, negative values of \( \xi_M \) correspond to sharp temporary peaks in mass, and positive values to dips in mass. Somewhat surprisingly \( |\xi_M| \) exceeds \( \pi/3 \) 10 per cent of the time, and \( 2\pi/3 \) 1 per cent of the time. Thus, strong mass variations are relatively common. However, the presence of the strong mass variations seems to be a limitation of the halo finding algorithm rather than the merger-tree algorithms as evidenced by the great similarity between all merger-tree algorithms except HBT in Fig. 11. Note that the apparent discrepancy of HBT is because, for the purposes of this paper, they construct masses only from the supplied AHF halo catalogues. We have checked that, on applying HBT to the full simulation data, this discrepancy goes away.

7 DISCUSSION

This paper summarizes the results of a merger tree comparison project. The comparison was completed, and the paper drafted, in advance of the Sussing Merger Trees Workshop in Midhurst, Sussex in 2013 July. The aim of the workshop was not only to compare the existing status of merger-tree codes, but also to get people thinking about the desirable features of such codes, in particular for their use as backbones for SA modelling.

Ten different merger-tree builders contributed to this comparison project, as listed in Table 1. Although many of these adopted similar approaches, no two gave identical results.

In order to enable the comparison, we desired that each merger-tree code should use the same haloes as input. It soon became apparent that the halo finder can be intimately linked to the tree builder itself, and so some tree-building codes needed modification to enable them to take part. For two of the codes (CONSISTENT TREES and HBT), we had to allow modification of the halo catalogue. For this reason, and because the quality of a merger tree depends in some unspecified way upon the particular scientific use to which it will be put, we avoid making conclusive statements here about which algorithms perform better than others.

In Section 2, we defined some terminology that we used throughout the paper. This proved essential to get everyone talking a common language (for example, some algorithms did not initially return merger trees at all, in the sense that every halo did not have a unique descendent). We encourage other members of the community to use the same nomenclature.

7.1 Summary of results

Here, we present a brief summary of our findings.

(i) Imperfections in the halo finder can lead to great difficulties for tree-building algorithms. The particular halo finder that we used in this project was AHF, but we would expect similar behaviour with other halo finders and a study of this is presently under way.

(ii) The temporary loss of a halo during the merger of two haloes (see, e.g. Fig. 4) is disastrous for tree-building algorithms that examine only two adjacent snapshots. In such cases, it is possible for haloes containing over 1000 particles to apparently appear out of nothing between two adjacent snapshots.

(iii) Although they were working with the same input halo catalogue, different algorithms varied in their ability to link together subhaloes, leading to significantly different branching ratios for the trees.

(iv) Due to the limitations of the halo finder, codes that do not use particle IDs to link together haloes can occasionally produce clear misidentifications (see, e.g. Fig. 7).

(v) Even when haloes persist between snapshots, the halo finder will sometimes alter which of the two it treats as the main halo, and this can lead to large oscillations in mass. Different tree builders handle this in different ways.

(vi) The slope of the logarithmic mass growth curve, \( d\log M/d\log t \) has a very broad distribution with a peak around 0.5 to 1 but extending beyond the range \(-10–10\). Much of this is due to genuine fluctuations in mass, although the extremes are due to failures in the combined halo finder and tree builder.
We suggest that any optimal tree-building algorithm will require a high-quality input halo catalogue that minimizes ‘lost’ haloes and mass fluctuations, and in addition will possess the following:

(i) the use of particle IDs to match haloes between snapshots;
(ii) the ability to skip at least one, and preferably more, snapshots in order to recover subhaloes that are temporarily lost by the halo finder (for instance when they transit the centre of the host halo);
(iii) the ability to cope with (and ideally smooth out) large, temporary fluctuations in halo mass.

7.2 Future work

One of the main purposes of the workshop was to stimulate people into thinking harder about what makes a good merger tree. As a result of this, we have initiated projects on the following topics:

(i) Tree stability versus number of snapshots and their optimal spacing.
(ii) Which is the best halo finder to use for the purposes of tree building? The answer to this question may well vary from one tree-building code to another.
(iii) Related to the above, what is the best overdensity criterion to use when defining haloes?
(iv) How do the results change when applied to a large resimulation of a single halo with lots of nested substructure?
(v) What is the effect of the variation in merger trees on the resultant SA models?

It is our hope that a consensus will emerge, if not on a unique halo finding and merger-tree algorithm, at least upon the desirable features that such algorithms should possess in order to obtain stable results for the purposes of SA modelling.

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The authors contributed in the following ways to this paper: CS, AK, FRP, AS, PAT organized this project. They designed the comparison, planned and organized the data, performed the analysis presented and wrote the paper. CS is a PhD student supervised by PAT. The other authors (as listed in Section 5) provided results and descriptions of their respective algorithms; they also helped to proof-read the paper.

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APPENDIX A: THE TREE DATA FORMAT

In order to facilitate comparison and use of merger-tree data, it is our intention to define in a future paper a common merger-tree data format. This should make provision for: required minimal data to define a merger tree; desired fields to ease use; and the ability to include optional additional data that may prove useful. At the time
Table A1. The ASCII data format that participants were asked to use to return their merger-tree results.

<table>
<thead>
<tr>
<th>Information to be returned</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format version</td>
<td>=1 – an integer indicating the format version</td>
</tr>
<tr>
<td>Description</td>
<td>Name of code, version/date of generation; max 1024 characters</td>
</tr>
<tr>
<td>$N_{\text{halo}}$</td>
<td>Total number of haloes specified in this file</td>
</tr>
<tr>
<td>Halo ID$_1$, $N_1$</td>
<td>Halo’s ID and number of direct progenitors</td>
</tr>
<tr>
<td>Progenitor$_1$</td>
<td>Halo ID of main progenitor of halo Halo ID$_1$ (where $N_1 &gt; 0$)</td>
</tr>
<tr>
<td>Progenitor$_2$</td>
<td>Halo IDs of other progenitors of halo Halo ID$_1$</td>
</tr>
<tr>
<td>Progenitor$_{N_1}$</td>
<td>Halo ID of last progenitor of halo Halo ID$_1$</td>
</tr>
<tr>
<td>Halo ID$<em>{N</em>{\text{Halo}}}$, $N_{\text{Halo}}$</td>
<td>Halo’s ID and number of direct progenitors</td>
</tr>
<tr>
<td>Progenitor$<em>{N</em>{\text{Halo}}}$</td>
<td>Halo ID of main progenitor of halo Halo ID$<em>{N</em>{\text{Halo}}}$ (where $N_{\text{Halo}} &gt; 0$)</td>
</tr>
<tr>
<td>Progenitor$_2$</td>
<td>Halo IDs of other progenitors of halo Halo ID$<em>{N</em>{\text{Halo}}}$</td>
</tr>
<tr>
<td>Progenitor$<em>{N</em>{\text{Halo}}}$</td>
<td>Halo ID of last progenitor of halo Halo ID$<em>{N</em>{\text{Halo}}}$</td>
</tr>
<tr>
<td>END</td>
<td>String ‘END’ indicating the last line of the output file</td>
</tr>
</tbody>
</table>

of writing (prior to the Sussing Merger Trees Workshop) that format had not been defined and so we restrict ourselves to outlining here the minimal data format that was used for the work described in this paper.

We supplied each participant in the tree comparison project with a list of haloes, together with their properties (as described in Section 3) and an inclusive list of particle IDs. Each halo had a identifier (Halo ID) that was unique across snapshots.

We required participants to return their results in the ASCII format described in Table A1, where there is an entry for each halo. That contains enough information for us to be able to reconstruct the merger trees and, in conjunction with the original halo list, to follow the growth of haloes over time.

This paper has been typeset from a $\LaTeX$ file prepared by the author.