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THE MINIMUM ENERGY EXPENDITURE SHORTEST PATH METHOD

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This article discusses the addition of an energy parameter to the shortest path execution process; namely, the energy expenditure by a character during execution of the path. Given a simple environment in which a character has the ability to perform actions related to locomotion, such as walking and stair stepping, current techniques execute the shortest path based on the length of the extracted root trajectory. However, actual humans acting in constrained environments do not plan only according to shortest path criterion, they conceptually measure the path that minimizes the amount of energy expenditure. On this basis, it seems that virtual characters should also execute their paths according to the minimization of actual energy expenditure as well. In this article, a simple method that uses a formula for computing vanadium dioxide (VO_2) levels, which is a proxy for the energy expenditure by humans during various activities, is presented. The presented solution could be beneficial in any situation requiring a sophisticated perspective of the path-execution process. Moreover, it can be implemented in almost every path-planning method that has the ability to measure stepping actions or other actions of a virtual character.

1. INTRODUCTION

Path planning plays an important role in computer animation. To perform locomotion sequences, characters are called on to start at a position P_S and move to a given goal position P_G. In general, path-planning techniques require the ability to execute a path with the shortest distance. This approach is characterized by the minimization of the distance while avoiding physical obstacles.

However, path-execution methods should be examined taking into account various other parameters. For example, when actual humans are called to perform locomotion sequences, they do not always choose the shortest path. Rather, longer paths may be chosen because of other factors, (either mental or physical), that may influence their final decisions about the optimal path. As a result, locomotion sequences do not always take the shortest path. Moreover, the shortest path does not always represent the path for which the output of physical energy is minimized.

The measurement of the energy expenditure of a given task is quite complex. The complexity of energy calculations is partly attributed to differences in the metabolism of different humans. For this reason, the mathematical models that describe the energy expenditure of human activities are generally based on empirical experiments and a set of assumptions. A variety of different techniques are available for computing the approximate energy expenditure associated with a given type of human exercise. However, to avoid complex computations, and to generate a method that can implement a variety of simple and efficient path-planning techniques, this article presents

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an approach for generating motion sequences based on a formula of approximating vanadium dioxide (VO_2) levels.

Specifically, energy requirements can be expressed in terms of the oxygen requirements of a given physical activity. Levels of VO_2 provide useful information for measuring the energy costs of physical activities, generally expressed in units of kilocalories (*kcal*). By measuring VO_2 levels, it is possible to predict the energy expenditure of a specified task. To examine the energy expenditure of a virtual character, we use a mathematical equation that returns an approximation of the VO_2 expenditure for different actions as a parameter to describe the expended energy of a character performing a given task. Hence, besides the measurement of the shortest-path distance, which is based on length units (e.g., centimeter, meter, kilometer, etc.), we propose that the VO_2 required for each step (in units of $ml * kg^{-1} * min^{-1}$) should also be considered.

By using the VO_2 parameter, this study introduces an energy minimization approach for the path-execution and planning process. Thus, we present a method in which the character's path is not the path that minimizes the distance, but the path that minimizes the amount of expended energy. Figure 1 illustrates the two approaches in which the path is based on geometrical measurements that minimize the amount of expended energy (a), and a second path that minimizes the path length (b). The presented method can be especially beneficial in cases when, during path planning, two or more paths have the same length. Hence, the quantitative value returned by the VO_2 parameter can be used as an additional measurement (or heuristic) for a planning algorithm [Mousas et al. 13]. Moreover, the presented approach employs the ability to execute a path based on different types of steps; each step represents either walking or stair-stepping activities. Thus, the final path will be the path for which the sum of the individual steps minimizes the amount of expended energy.

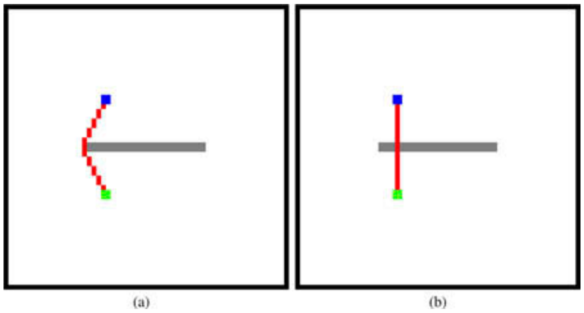

FIGURE 1. The presented approach computes the path that (a) minimizes the energy expenditure of the character rather than (b) the path with the shortest length. In both panels, the grey area denotes a single stair, the green dot is the starting position, the blue dot is the goal position, and the red line is the executed path.

The remainder of this article is organized as follows. In Section 2, we present work related to path-planning techniques. Section 3 presents the human factors that should be considered in path-planning processes. Section 4 examines the methodology for computing energy requirements based on VO_2 calculations for each individual step of the character. The path-execution process of the presented approach is presented in Section 5. Finally, conclusions and suggestions for future work are presented in Section 6.

2. RELATED WORK

Path-execution techniques have been examined thoroughly in recent years, and some of the most well-known approaches are discussed in this section. Path-execution techniques can be divided into four categories: the grid-based search, potential field, geometric, and sampling-based algorithms. Extended explanations of path-planning techniques can be found in Choset et al. [Choset et al. 05] and Latombe [Latombe 90].

One of the most well-known approaches for executing the shortest-path problem uses search-based algorithms such as A^*, introduced by Hart et al. [Hart et al. 68] in 1968; these algorithms are responsible for returning the shortest path, if one exists. The techniques have been thoroughly examined in cases of path planning, especially by the games-development community, which uses low-resolution grid solutions. On the other hand, flexible path-planning techniques, such as the potential field [Khatib 86] [Khosla and Volpe 88] method, were introduced approximately 20 years ago. In the potential-field method, the character is driven through an artificial potential field, which is defined by a free configuration space; one of the advantages of this method is its ability to execute smooth paths, however, the computation of such algorithms is relatively expensive, and the methods cannot be executed in real time. A dynamic perspective of potential fields was introduced by Treuille et al. [Treuille et al. 06] in a crowd-simulations system. In this case, a set of dynamic potential and velocity fields that guides all individual agents is generated over the domain.

Techniques based on roadmap methods, such as visibility graphs [Latombe 90], rapidly-exploring random trees [Kuffner and LaValle 00], and the probabilistic roadmap method [Kavraki et al. 96, Amato and Wu 96, Barraquand et al. 97], although not examining local minima, ensure the return of a path, if one exists. In addition, in recent years, techniques have been proposed that use Voronoi diagrams [Hoff et al. 99, Rohnert 91] to plan and execute paths. Recently, techniques such as the corridor map method [Geraerts and Overmars 07, Kamphuis and Overmars 04] have been determined to be beneficial for high-quality path planning, because they combine the ability to direct global motions using high-quality roadmaps and to control local motions by potential fields, thus providing local flexibility of the path.

The methods mentioned thus far are designed to generate collision-free paths, but they do not take into account the physical characteristics of the character. Although the mental and physical characteristics of the character are not absolutely necessary for executing a path, these factors may vary among different people. Hence, the execution process should be able to measure or predict how different humans decide to follow a path while walking within a constrained environment. Based on these considerations, the approach presented in this article follows a path on which the energy required from the character to follow the path is incorporated as a path-planning parameter. However, a preliminary study for path planning in which the physical characteristics of a virtual character are taken into account is proposed by Choi et al. [Choi et al. 11]. In this research, a deformable motion model is responsible for synthesizing a valid physical character's motion while the virtual character is planning within a cluttered environment. In addition to the deformable motion model, they implemented a probabilistic roadmap method for path planning in very challenging environments. Although, this approach of motion planning does not ensure the energy efficiency of the virtual character as presented in this paper.

From a review of the literature, we found only two approaches that mention the computation of expended energy in the completion of a given task by a virtual character. The first approach, proposed by Levine et al. [Levine et al. 11], refers to a space-time action locomotion controller; however, this approach is based on a cost function that assigns a lower cost per second to the waiting controller that is assigned to the character. Hence, to the best of our knowledge, energy minimization approaches such as that presented in this study have not yet been examined and implemented in path-planning algorithms. The second approach computes the energy required by a character during the path-planning process [Guy et al. 10]. This methodology tries to minimize the biomechanical energy of a character by assigning the corresponding path. Despite the similarity with the approach in this study, the main difference is that the presented approach tries to simulate the ability of a character that chooses to either avoid or mount a single stair (denotes as obstacle) in the three-dimensional space. This is achieved by measuring the energy that a character requires for each single step, which is something that is not provided in [Guy et al. 10], in which only the walking locomotion of the character is measured.

3. ANALYZING HUMAN LOCOMOTION

This section presents factors that should be considered in path-planning processes based on the energy expenditure of characters. Specifically, to implement a path-planning method for virtual characters, it is necessary to incorporate basic measurements that describe the energetic basis for the performance of actions related to locomotion.

The normal walking velocity of humans (v_{human}) has been estimated as 1.2 m/s based on the U.S. Manual of Uniform Traffic Control Devices (MUTCD) [Federal Highway Administration 03], and 1.1 m/s to 1.4 m/s based on Manual of Uniform Traffic Control Devices for Canada (MUTCDC) [Transportation Association of Canada 02]. The approach in this work requires the computation of the length of each single step of the character, because the human energy output per step depends on the step length. Previously, step length s_{length} has been empirically determined, based on the measurements of Grieve [Grieve 68], to obey a power law $s_{\text{length}} \approx v_{\text{human}}^{\beta}$, where the value of β in adults is approximately $\beta = 0.42$. In our calculations, the value of s_{length} is determined from estimations of character velocity; the number of steps can then be determined from the step length and the distance from the start to the goal position.

Our approach should also be able to compute a path, which minimizes the energy expenditure of the character, based on the goals and actions required, including stair-stepping actions as well. The factors that influence the latter actions must also be defined. First, based on Warren's empirical measurements of leg segments [Warren 84], a critical stair height $p_{\max}$, (i.e., the stair height that is impossible to climb bipedally), is defined as $p_{\max} = 0.88\,\text{m}$. Thus, the critical $p_{\max}$ value that describes "climbability" can be used as an upper limit of the height of a generated stair. However, although the stair height is required to solve the approximate energy use based on VO_2, the measurements of Warren Jr. [Warren Jr. 95] show that the optimal point p_0 that denotes the stair height, which minimizes the energy expenditure during stair stepping, should be $p_0 = 0.26\,\text{m}$. In addition, Templer [Templer 75] determines that the optimal stair stepping frequency s_f for stairs with a height $s_h = 10$ in ($\sim$ 0.25 m), is approximately 50steps/min.

The values mentioned above are important in our approach because they are related to energy expenditure. Although most of the values are empirically derived, they do provide a path-planning method that solves for the path with the least amount of expended energy; thus, it is possible to approximate a path expressed in terms of human energy consumption, on the basis of a sophisticated path-execution process.

4. COMPUTING HUMAN ENERGY

The computation of human energy expenditure and efficiency related to various locomotion tasks is a complex process. However, based on experimental measurements, researchers have designed formulas that approximate the amount of energy expended in different human activities. Various techniques that compute the metabolic requirements of an activity have been presented [Nishii 06, Atzler et al. 27, Garg et al. 78]. In [Nishii 06], the walking energy cost is determined using biomechanical models, whereas in [Atzler et al. 27, Garg et al. 78] metabolic cost calculations are based on empirical measurements. In addition, the ISO 8996 standard [International Organization for Standardization 04] has established methods for the determination of metabolic rates based on vanadium dioxide (VO_2) levels. The solution presented in our article uses a simple but effective method for computing human energy expenditure, based on a formula that uses the VO_2 consumption associated with various human activities.

The computation of the expended energy during a given task must be considered in terms of the individual components of the process. A path can be represented by a finite number of steps. This is quite important, especially in cases when the computation of the energy during walking, or walking over the stairs, is desired. Thus, our research considers the motions in terms of VO_2 consumption, which adequately describes possible actions in a quantitative manner. The energy expenditure based on VO_2, as introduced by the American College of Sports Medicine (ACSM) [Glass and Dwyer 07], is computed from Equation (1) as:

$$VO_2 = H + V + R, \tag{1}$$

where H and V denote the horizontal and vertical components of the activity, respectively, and R denotes the resting component (which is constant, $R = 3.5$).

4.1. Walking Step Energy Expenditure

When humans, or in this case virtual characters, are called on to perform walking motions, Equation (1) becomes:

$$VO_2 = 0.1 * v_{\text{walk}} + 1.8 * v_{\text{walk}} * h_{\text{fg}} + 3.5, \tag{2}$$

where v_{walk} denotes the velocity of the human during walking and h_{fg} is a fractional grade, which may be assigned a value of $h_{\text{fg}} = 0.13$, based on the protocol of Bruce [Bruce et al. 49]. In the presented approach, the computation of the energy expenditure is based on single steps; thus, the solution depends on optimization of Equation (1).

The optimization of Equation (1) requires the following assumptions. Based on MUTCDC [Transportation Association of Canada 02], v_{human} is assigned a value of 1.1 m/s. Substitution of v_{human} into the equation of Grieve [Grieve 68] ($s_{\text{length}} \approx v_{\text{human}}^{\beta}$) gives $s_{\text{length}} \approx 1.05$ m. These values allow the calculation of $v_{\text{walk}} = 1/0.95$ (step/sec). Additionally, because R is a constant value, it is not taken into account.

Based on these assumptions, the VO_2 for each simple walking step of the character is first computed. This calculation is important in the presented approach, because the shortest-path execution process is divided into single steps in which the amount of energy expended in the execution of the locomotion actions is minimized.

4.2. Stair Step Energy Expenditure

As mentioned, the advantage of computing VO_2 is that researchers can approximate the computation of various actions based on Equation (1), as well as on data in the ACSM [Glass and Dwyer 07]. Hence, the approximate energy expended by a human during stair stepping is described by

$$VO_2 = 0.2 * s_f + s_{\text{comp}} * 1.8 * s_h * s_f + 3.5, \tag{3}$$

where s_f is the step frequency (min^{-1}), s_{comp} is the stepping component, described by Equation (4), and s_h is the stair height in meters.

$$s_{\text{comp}} = \begin{cases} 1 & \text{if stepping up} \\ 0.33 & \text{if stepping down} \end{cases} \tag{4}$$

As previously described, for the computation of walking energy, the stair-stepping formula should be optimized to separately compute the expended energy at every step. In this case, based on Templer [Templer 75], the stair-step frequency for stairs with height $s_{h(0.25)} =$ 10 in (~ 0.25 m) is approximately $s_{f(0.25)} =$ 50steps/min. However, although the step frequency is related to the step height, the step frequency $s_{f(i)}$ for a given stair with height $s_{h(i)}$ is computed as $s_{f(i)} = s_{h(i)} * s_{f(0.25)}/s_{h(0.25)}$. Hence, Equation (3) becomes:

$$VO_2 = 0.2 + s_{\text{comp}} * 1.8 * s_h * s_{f(i)} + 3.5. \tag{5}$$

The result of computing VO_2 for both walking and stair stepping allows us to approximately calculate the expended energy in a quantitative way. Hence, even if empirical values are used for solving the energy expenditure for a single-task problem, Equation (5) closely approximates an actual human decision that takes into account different action tasks. In addition, it should be mentioned that approximate values are used in planning tasks that solve the shortest-path problem, rather than incorporating human motion data directly, which may extend each single parameter of the previously mentioned formulas.

Conversely, although implementations are desired to give to the character the ability to perform different actions, it is possible to assign to the search space all possible values that can be retrieved from the actual motion data. Hence, it should be mentioned that the values related to human velocity and stair height may be adjusted interactively, generating variations on an executed path. In both walking and walking-over-stair scenarios, our approach is considered a simpler execution process, because human velocity during walking is assigned with a value equal to one step per second, while during stair stepping it is based only on the step height and Templer's approximation for optimal step frequency [Templer 75]. Finally, it should be mentioned that another parameter that influences a human's path-execution process is stamina. This parameter can be computed as an incremental function assigned at each step of the character. This means that step climbing should be a tiring procedure for the character similarly to real humans. However, in our present research the stamina parameter is not taken into account.

5. PATH PLANNING

A character's path, or more specifically, a character's shortest path between two points, as based on the different methods described, can be considered as a discrete geometric measurement fulfilling parameters defined by the user. Even though the path can be executed by minimizing only the path length, attempting to generate a path based on more sophisticated computations that also employ physical measurements of human activity may change the final path. Hence, based on VO_2 computations, it is shown that the shortest path is not always the one with the minimum energy expenditure. It should be mentioned also that the presented approach is more applicable to cases in which path planning is required within a constrained environment (i.e., in environments with obstacles or stairs). Hence, having presented the computation process of the expended energy, based on walking or on stair stepping, the next section presents the implementation of our method.

5.1. Representation of the Environment

This study considers a two-dimensional scaled map of the environment, showing the locations of open (clear) areas, stairs, and obstacles. More specifically, as shown in Figure 2, the environment is represented by clear areas (A_{clear}) and occupied areas ($A_{occupied}$). The former consist either of areas free of obstacles (white areas), or of obstacles that can be negotiated, such as stairs (grey areas), and occupied areas ($A_{occupied}$), and the latter contains impassable obstacles that must be avoided (black

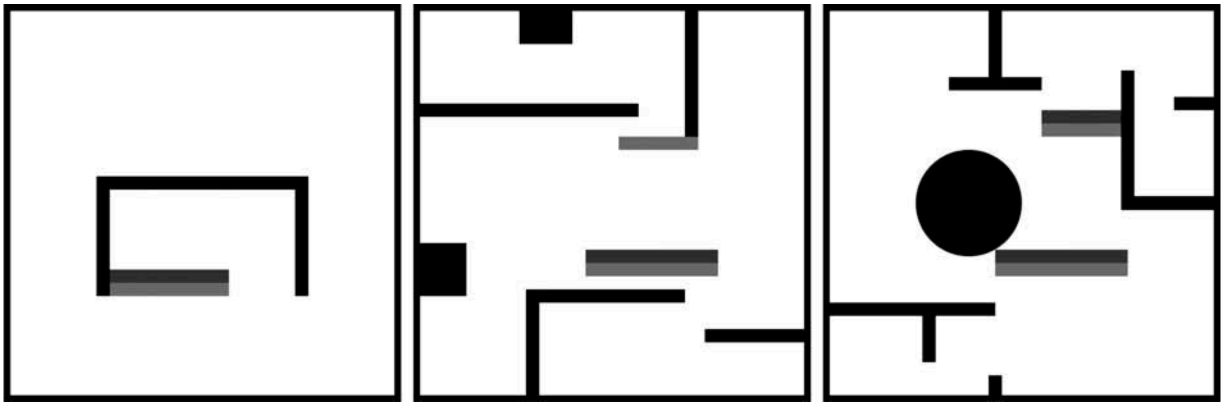

FIGURE 2. A sample environment generated for use in the study. The area in which the character is free to move is designated by white (floor areas) and grey (stairs) colors. Obstructions to movement are indicated in black (e.g., walls).

areas). Hence, the representation of the environment is:

$$A_{\mathrm{env}} = \{(x, z) \in \{A_{\mathrm{occupied}} | A_{\mathrm{clear}} \in \{A_{\mathrm{floor}} | A_{\mathrm{stair}}\}\}\}, \tag{6}$$

where (x, z) is the position at every pixel on the map, and the relation of each state in the environment is represented as $A_{\mathrm{env}} = A_{\mathrm{occupied}} \cup A_{\mathrm{clear}}$ and $A_{\mathrm{stair}} \subseteq A_{\mathrm{clear}}$.

5.2. Sampling States and Traversing

Based on a two-dimensional representation, the environment A_{env} is divided into possible next-step states. In the planning approach, the environment is subdivided into equal-sized grid cells, and each one represents a possible next step, disregarding the effect of step length s_{length}. The size of the environment is 600 × 600 pixels, and each grid cell has a size of 10 × 10 pixels. This results in a grid of 60 × 60 cells, in which each cell is tagged with information that describes the local A_{env} area. Hence, each of the generated grid cells c_i is used as the possible next state that the character can follow. Figure 3 illustrates the approach of sampling the environment into possible states. It should be noted that although cell c_i of the generated grid in the environment is located at the intersection of A_{occupied}, such that $c_i \cup A_{\mathrm{occupied}}$, cell c_i is not one of the possible states.

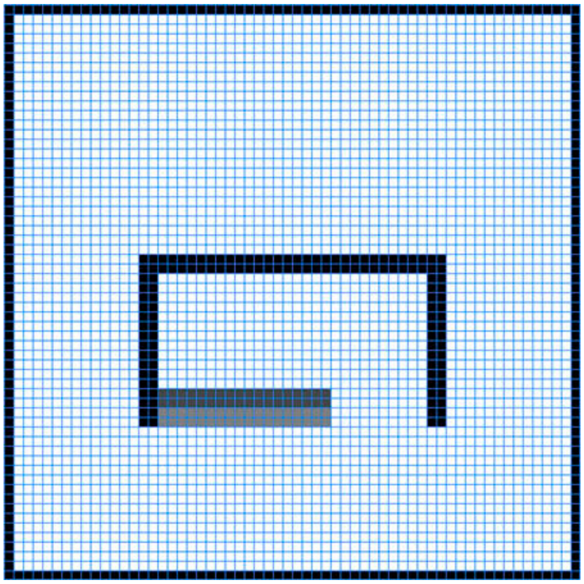

FIGURE 3. The generated grid that denotes the possible states of the character over the area A_{clear}.

After determining the sampling state that generates the possible steps of the character, the method then determines the degrees of freedom of movement based on the generated grid. Unlike an actual human, who can consider a stepping motion in any possible direction, the present method considers eight possible adjacent $(\pi/4)$ stepping directions within the terrain, as illustrated in Figure 4. In general, considerations of either four adjacent stepping directions or eight adjacent stepping directions traversing methods [Ferguson and Stentz 06, Ersson and Hu 01] are the most common approaches for grid-based path planning. However, this method uses the eight adjacent stepping directional traversing method, in order to execute a path in which the character has the potential to step into each neighboring cell. More complex traversing methods may be warranted when considering a complex approach in which the step size is adjusted to account for energy-expenditure computations.

The final step of the sampling process is to assign values for each potential next step

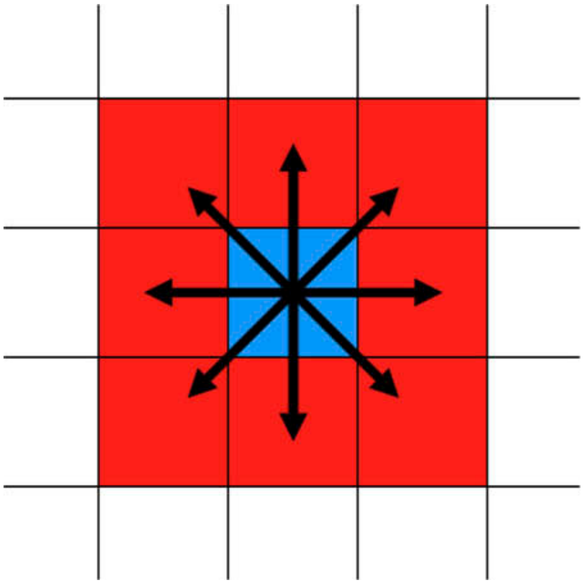

FIGURE 4. The eight adjacent stepping directions traversing method used in the presented path-planning process. The blue area denotes the position of the character and the red grid cells denote the possible next states.

of the character, based on the energy required for each next step as determined from VO_2 approximations. In this case, although our character is able to plan an action in the A_{clear} area, the neighbor steps are assigned values that denote the energy required by the character, respectively. Hence, for the process of assigning values, A_{clear} is represented as A_{floor} and A_{stair}, and the system computes the energy e_{ns} for each step based on the tag that is assigned to each step, as well as to each neighbor:

$$e_{\text{ns}} = \begin{cases} e_{\text{floor}} & \text{if current } A_{\text{floor}} \text{ and next } A_{\text{floor}} \\ e^{\text{up}}_{\text{step}} & \text{if current } A_{\text{floor}} \text{ and next } A_{\text{step}} \\ e^{\text{down}}_{\text{step}} & \text{if current } A_{\text{step}} \text{ and next } A_{\text{floor}} \\ e_{\text{cond}} & \text{if current } A_{\text{step}} \text{ and next } A_{\text{step}} \end{cases}, \tag{7}$$

where the conditions e_{cond} are taken into account for the condition in which both the current and next step are assigned as stair-stepping actions, and the energy is computed as follows:

$$e_{\text{cond}} = \begin{cases} e^{\text{up}}_{\text{step}} & \text{if current } A_{\text{step}} \text{ and next} \\ & \quad \text{higher } A_{\text{step}} \\ e^{\text{down}}_{\text{step}} & \text{if current } A_{\text{step}} \text{ and next} \\ & \quad \text{lower } A_{\text{step}} \\ e_{\text{walk}} & \text{if current } A_{\text{step}} \text{ and} \\ & \quad \text{next equal } A_{\text{step}} \end{cases} \tag{8}$$

Based on these conditions, the system assigns the necessary values to the corresponding traversing cases between neighboring cells automatically. Figure 5 illustrates all of the cases mentioned.

5.3. Minimum Energy Expenditure Path Execution

To determine the minimum-energy path, the approach first computes every possible path from the starting position P_S to the goal position P_G. The desired path Π can be characterized by executed states of the form $\Pi_{S \to G} = \{\pi_1, \ldots, \pi_n\}$, where n denotes the number of steps required to achieve the goal

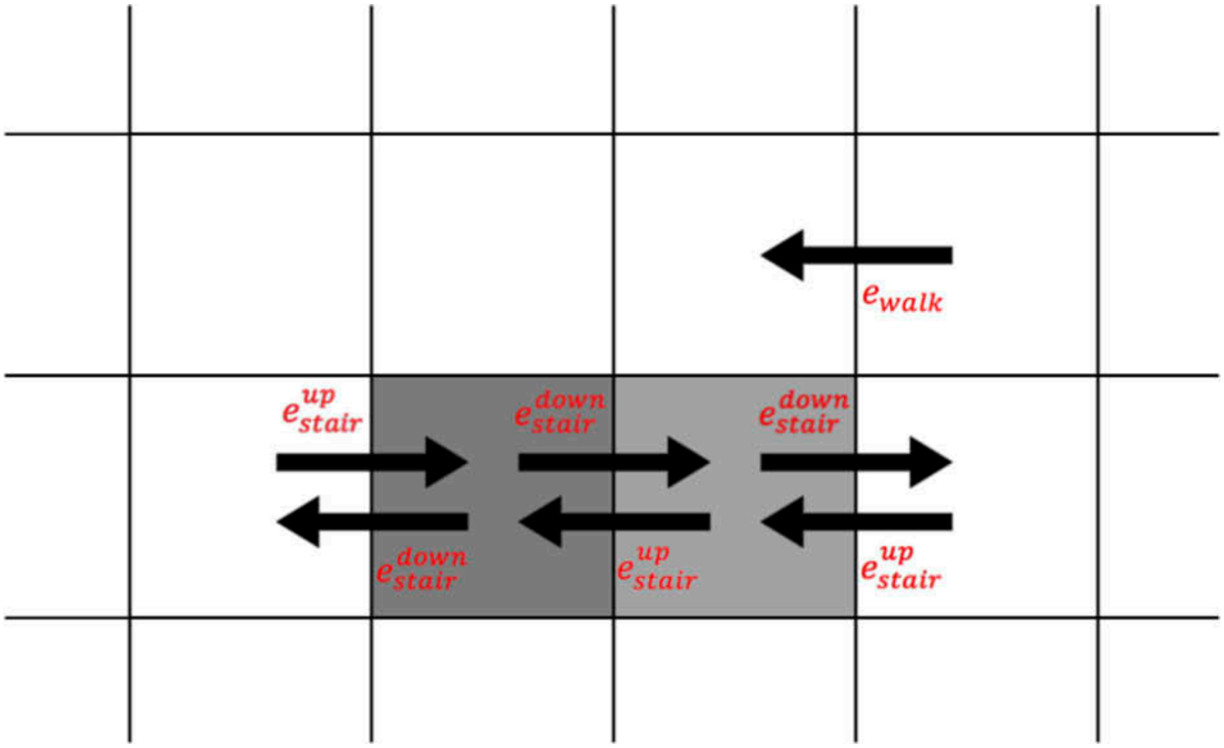

FIGURE 5. Assigning values to neighbor cells. The arrows show the direction of possible motion; light grey denotes a stair to a lower height; dark grey denotes a stair to a higher height.

position P_G. In this case, Dijkstra's algorithm to compute the shortest path, which is based on the possible states of the environment as well as the length of the path, results in the sum of the lengths of each simple step s_{length}; however, this approach returns a path for which the expended energy has not been computed.

Hence, implementation of Dijkstra's algorithm requires calculations of the energy expenditure of the character in moving from the current position to all possible candidate step positions. Thus, for each target state, an approximation of the VO_2 required to achieve the goal must be added to the algorithm. Then, rather than computing the minimum distance between π_i and π_{i+1}, which returns the shortest path length $L(\Pi_{S\rightarrow G})$, where $L(\Pi_{S\rightarrow G})$ is computed for all possible sampling states π, the approach computes the path $E(\Pi_{S\rightarrow G})$ which minimizes the energy expenditure E required to move from the π_i to π_{i+1} state. Considering that for any given state, the energy requirement of the character e_Π can be computed from VO_2, as previously presented for both stair stepping and simple stepping actions, the path extraction process should satisfy Equation (6)

$$E(\Pi_{S\rightarrow G}) = min \sum_{i=1}^{n} e_i. \tag{9}$$

With this simple approach, it is possible to compute an optimal path for the character, minimizing the total amount of energy. Some example paths computed using Dijkstra's algorithm, based on both the shortest length and the minimum energy expenditure, are illustrated in Figure 6.

5.4. Implementing Energy Constraints on Corridor Map Method

The presented solution can easily be implemented in various other path-planning methods. In this case, we consider the implementation of the well-known corridor map method (CMM) [Geraerts and Overmars 07]. In general, CMM consists of two phases, an off-line construction phase and an online query phase. More specifically, during the construction phase the system generates a roadmap graph, which consists of vertices and edges. Those vertices correspond to the collision-free position in a 2D or 3D environment. Conversely, the edges correspond to the local paths where the character may follow. Taking into account the clearance information, a corridor map is generated. During the online phase of the method, a backbone path is generated running the Dijkstra's shortest path algorithm. In this case, the character is modelled as a disc. Therefore, the clearance of the backbone path is at least the radius of the character, giving the ability to the character to traverse it.

The implementation process of the energy constraints over CMM is defined as follows. First, during the off-line phase of the CMM, the system defines the A_{clear} area where a path can be generated. During the online phase, rather than modeling the character as a disc whose radius must be at least equal to the length of the character s_{length}, the radius is constrained being equal to $s_{length}/2$. In addition, another constraint is defined during the pathfinding process, which is the corresponding energy assigned at each step of the character on the basis of the previously mentioned methodology (see Section 5.2). Then, by running the Dijkstra's shortest-path algorithm, it is possible to execute the minimum energy expenditure shortest-path algorithm over the CMM. Hence, it is possible not only to compute the necessary path that keeps the clearance provided by the CMM, but also the path that keeps the desired energy constraints as defined in the presented approach. Example scenarios that integrate the minimum energy expenditure shortest-path method with the CMM are illustrated in Figure 7.

5.5. Application

The proposed method was implemented in an interactive application, as illustrated in the accompanying video. After the definition of the position and the dimensionality of the stairs, the system generates the minimum energy path between start and goal positions, P_S and P_G, respectively. Examples of generated paths are

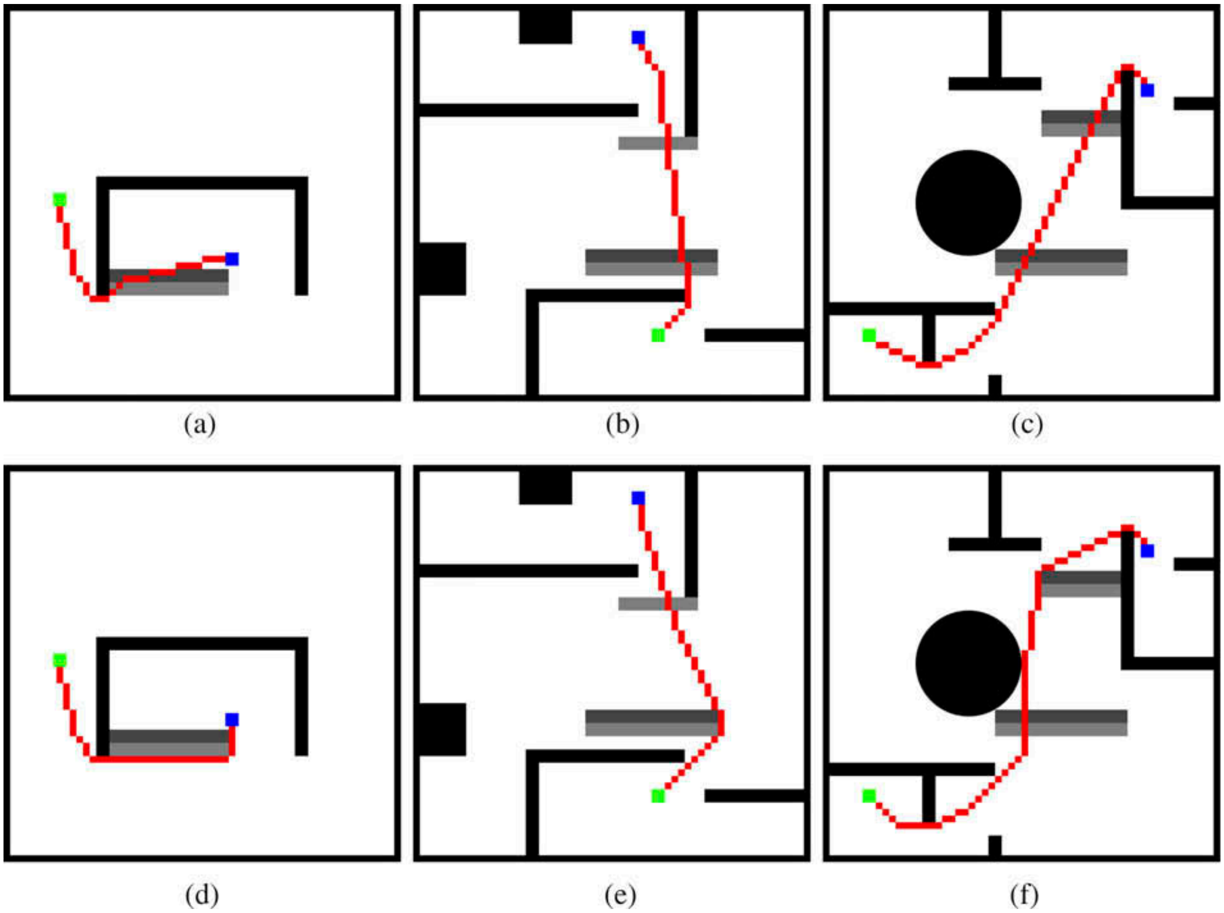

FIGURE 6. Example paths, between the start (green) and goal (blue) positions generated using Dijkstra's algorithm to measure (a, b, c) the path with the minimum length and (b) the path with the minimum expended energy. The stair height is $s_h = 0.25$ m for the light grey stairs and $s_h = 0.55$ m for the dark grey stairs. The difference between the heights of the two stair types is the height of the second step in a two-stair sequence.

illustrated in Figure 8. Finally, a simple locomotion controller similar to the one proposed by Treuille et al. [Treuille et al. 07] along with the path-following technique proposed in this article, gives the ability to produce the desired locomotion of the virtual character that follows the executed path. An example of the path-following method is illustrated in Figure 9.

6. CONCLUSION

This study introduces an energy parameter that can influence the calculation process of a character's shortest path. Our assumption in adding this parameter is that humans in their everyday life do not only consider solutions that minimize the distance to reach target positions, but also consider solutions that minimize their energy expenditure. Thus, by implementing a path-planning approach based on empirical formulas for VO_2 levels, which are related to the energy expended by actual humans in real life, it is shown that the shortest distance is not always the path that minimizes energy expenditure. The article also indicates that future path-planning methods could implement a variety of approaches in which not only the energy but also other physical characteristics of human characters could be modeled, such as the stamina of a character. Finally, it is shown that energy-based approaches can be implemented in nearly every path-planning method in which human motions are parameterized,

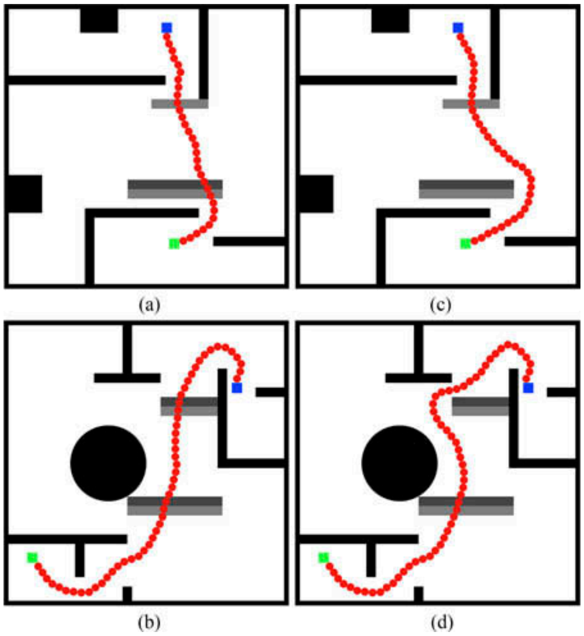

FIGURE 7. Integrating the minimum energy expenditure shortest-path method with the CMM. The shortest path according to (a, b) the CMM, and the (c, d) minimum energy expenditure approach over the CMM.

based on single steps rather than the incremental calculation of a root trajectory.

The approach introduced in this article is relatively simple, because only a few parameters influence the final result. To generate a more valid and robust implementation, our future plans incorporate motion-capture data and motion-trees generation such as those described in [Lau and Kuffner 06 and Mahmudi and Kallmann 12]. Another interesting extension would be the implementation of the proposed methodology for path planning in dynamic environments or for crowd simulations. For example, by considering a crowd as an energetic system, it should be possible to generate polite behaviors, which minimize the energy of the system, while the velocities of the characters increases or decreases to avoid local collisions with other characters. Finally, measuring the energy expenditure for virtual characters can also be applied in procedural virtual building prototyping methodologies. Specifically, the

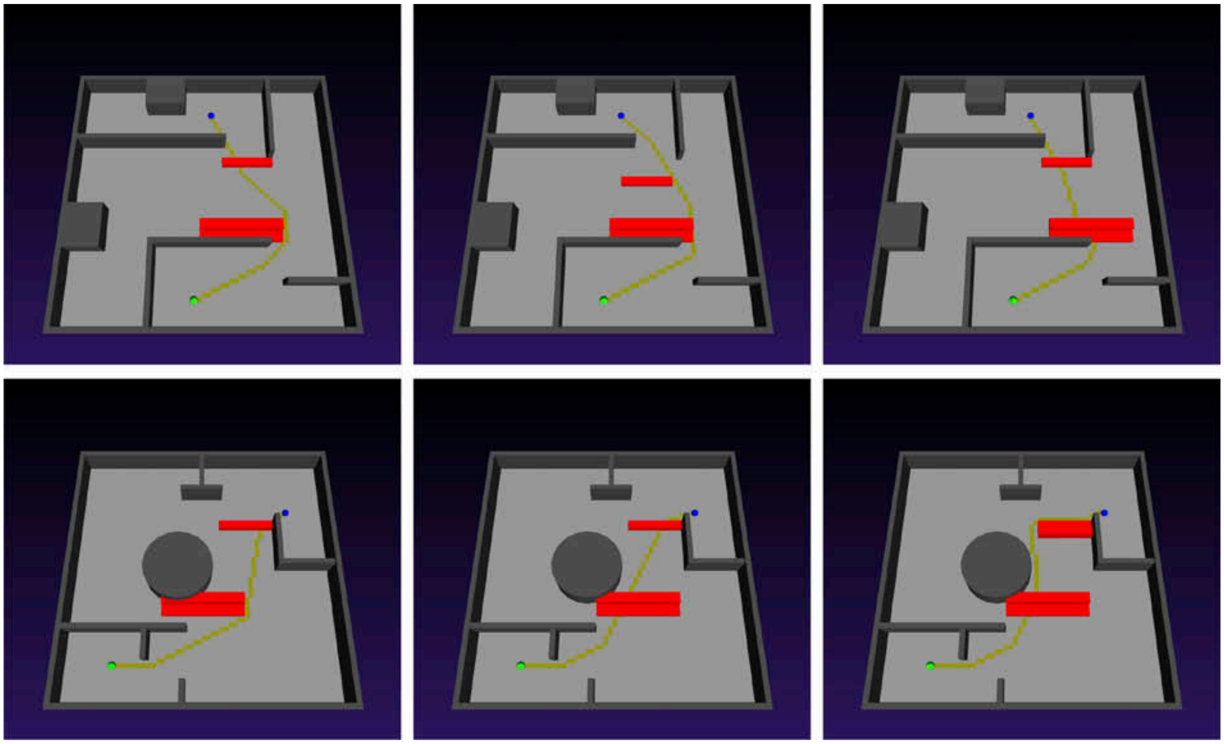

FIGURE 8. The three-dimensional implementation of the proposed solution. The above images represent the executed path based on minimum energy criteria.

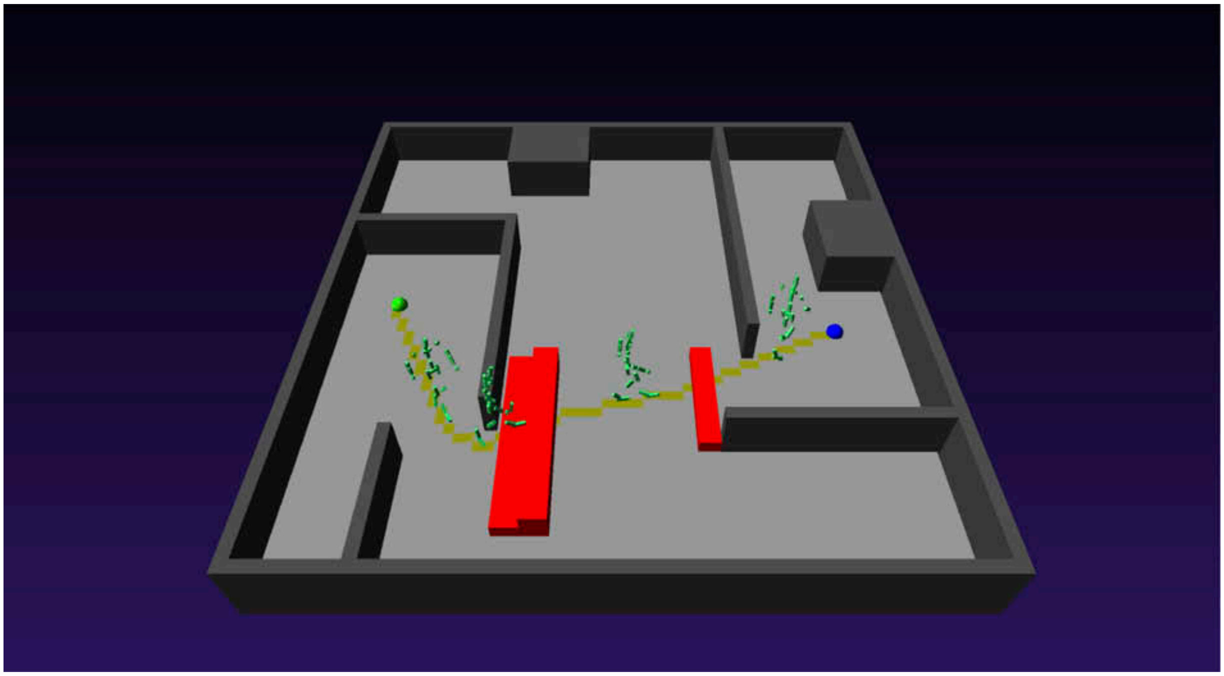

FIGURE 9. A character following the executed path, based on the implementation of a locomotion controller in conjunction with the proposed path-execution method.

measurements of the required energy for tasks such as walking and stair stepping can be considered for automatic generation of building interiors (i.e., ramps and stairs).

The main concern of the current solution is the ability to design a simulation procedure in which a virtual character decides whether it avoids a static obstacle located in the three-dimensional environment. The current solution is solved using the Dijkstra's algorithm with weighted regions. Because various other heuristics exist (e.g., A^*), and various types of terrains/environments can be generated, our methodology can be efficiently applied in almost every environment (e.g., similar to the one proposed by Jaklin et al.[Jaklin et al. 13]). Additionally, it can be executed with different heuristics by assigning the energy expenditure of simple stepping values to the path execution process, rather than computing the distance between the initial and the target position.

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