Erratum: Quantum propagation of neutral atoms in a magnetic quadrupole guide

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The quantum number \( l \) in the wave function \( \psi(\rho, \phi) \) is in fact not an eigenvalue of the orbital angular momentum \( L_z \) of the motion of the atom around the center of the guide, as we had mistakenly assumed without checking, but an eigenvalue of the operator \( L_z - s_z \), where \( s_z \) is the \( z \) component of the total internal angular momentum of the atom that couples to the magnetic field. For a magnetic quadrupole field of the form of Eq. (1.1), \( \mathbf{B} = (4B_0x/R, -4B_0y/R, 0) \) the operators \( L_z \) and \( s_z \) do not individually commute with the Hamiltonian

\[ H = \mathbf{p}^2/(2m) + g \mu_B \mathbf{s} \cdot \mathbf{B}, \]

but the difference \( L_z - s_z \) commutes with \( H \) and is thus a conserved quantity, \( [L_z - s_z, H] = 0 \). Calculation shows that the quantum number \( l \) in the wave function \( \psi(\rho, \phi) \) is the eigenvalue of \( L_z - s_z \), e.g., for spin 1/2

\[
(1) \quad \frac{1}{\sqrt{2}} F_+(\rho) e^{i(\phi/2 + \pi/2)} = \sqrt{\frac{\hbar}{2}} \left( \frac{\phi}{2} + \pi/2 \right) F_+(\rho) e^{i(\phi/2 + \pi/2)} - \sqrt{\frac{\hbar}{2}} \left( -F_-(\rho) e^{i(\phi/2 + \pi/2)} \right)
\]

and similarly for the spin 1 wave function in Eq. (4.5). Although \( L_z \) and \( s_z \) do not commute with \( H \), they commute with \( L_z - s_z \) and with each other, which means that any eigenstate of \( L_z - s_z \) must be a linear combination of simultaneous eigenstates of \( L_z \) and \( s_z \). Since the eigenvalues of \( L_z \) are always integer, it follows that the eigenvalue \( l \) of \( L_z - s_z \) must be half integer for half integer spin \( s \) and integer for integer spin \( s \). Only then does the wave function have the correct transformation properties under rotations. For spin 1/2 a rotation by 2 \( \pi \) around the \( z \) axis, for example, transforms the wave function of the above eigenstate into

\[
\frac{1}{\sqrt{2}} F_+(\rho) e^{i(\phi/2 + \pi)} D_{\text{mm}}^{1/2}(0,0,2 \pi) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) + \frac{1}{\sqrt{2}} F_-(\rho) e^{i(\phi/2 - \pi)} D_{\text{mm}}^{1/2}(0,0,2 \pi) \left( \begin{array}{c} 0 \\ 1 \end{array} \right)
\]

Since the application of the spin rotation matrix \( D_{\text{mm}}^{1/2} \) causes the spinors to change sign, the \( \phi \) dependent prefactors must not change sign under \( \phi \to \phi + 2 \pi \). Thus \( l \) has to be half integer for spin 1/2, and not integer as was erroneously stated in Sec. III. It follows that \( l \) cannot be zero, which, as explained by Eq. (5.3), has the consequence that there are no exact bound states in the case of spin 1/2 and makes Sec. III A redundant. Solving Eqs. (3.11) for \( l = 1, 3/2, \ldots \) gives results that are qualitatively the same as those of Sec. III B. For \( l = 1/2 \) the energies and widths (\( e_i, B_i \)) of the first three resonances are (2.64, 0.34), (4.25, 0.34), and (5.62, 0.34), and for \( l = 3/2 \) they are (3.53, 0.11), (5.04, 0.15), and (6.34, 0.18), in units of \( \hbar^2 g / (2m) \). The similarity of the motion of spin 1/2 and of spin 1 atoms in the guide is underlined by the fact that Eqs. (3.7) for spin 1/2 and \( l = 1/2 \) are structurally the same as Eqs. (4.7a) and (4.7b) for spin 1 and \( l = 0 \) except for a replacement of \( G \) by \( 2G \).

The energies and widths for \( s = 1/2 \) and the unphysical case of integer \( l \) have been reproduced by a complex scaling calculation by Potvliege and Zehnle [1]. These authors agree that \( l \) should in fact be taken half integer [2]. The applicability of their method is independent of whether integer or half integer values of \( l \) are being considered.

Nothing changes for spin 1 in Sec. IV.

There are also two misprints. Contrary to what is stated in the text of Sec. II, we have used \( G = 2g \mu_B B_0 / R \) throughout the paper. The last paragraph of Sec. VII discusses the first excitation energy for an \( l = 0 \) atom of \( ^87 \text{Rb} \), and not \( l = 1 \) as originally stated.

We are indebted to Dr. John Stockton, Dr. Clifford Hicks, and Professor Hideo Mabuchi for pointing out to us that \( l \) is an eigenvalue of \( L_z - s_z \) and must be half integer for spin 1/2.
