Sussex Research Online

Dynamics of assisted quintessence

Article (Published Version)

Dynamics of assisted quintessence

Soo A Kim,[1] Andrew R. Liddle,[1] and Shinji Tsujikawa[2]

[1]*Astronomy Centre, University of Sussex, Brighton BN1 9QH, United Kingdom*
[2]*Gunma National College of Technology, 580 Toriba, Maebashi, Gunma 371 8530, Japan*

We explore the dynamics of assisted quintessence, where more than one scalar field is present with the same potential. For potentials with tracking solutions, the fields naturally approach the same values—in the context of inflation this leads to the *assisted inflation* phenomenon where several fields can cooperate to drive a period of inflation though none is able to individually. For exponential potentials, we study the fixed points and their stability confirming results already in the literature, and then carry out a numerical analysis to show how assisted quintessence is realized. For inverse power-law potentials, we find by contrast that there is no assisted behavior—indeed those are the unique (monotonic) potentials where several fields together behave just as a single field in the same potential. More generally, we provide an algorithm for generating a single-field potential giving equivalent dynamics to multifield assisted quintessence.

I. INTRODUCTION

An attractive hypothesis for the fundamental nature of dark energy is quintessence—a scalar field evolving in a nonzero potential energy [1]. Such modeling has proven highly successful in implementing inflation models in the early Universe. It has however met with much less success in the present Universe, primarily due to the difficulty of obtaining equations of state close enough to the cosmological constant value $w = -1$ to satisfy observational bounds [2]. One strategy is to ensure that the quintessence energy density starts at such a low value that the field only begins evolving close to the present, but such tuning is hardly more satisfactory than a cosmological constant. The alternative is for the field to evolve significantly, ideally exploiting a ''tracker'' behavior rendering the late-time evolution almost independent of initial conditions. Unfortunately however this is viable only for very particular potentials: amongst monotonic potentials, exponentials do not give acceleration and power laws can give negative enough w only for exponents well below 2. Among the potentials regarded as giving satisfactory phenomenology, many in fact feature a minimum tuned to match the observed value of the cosmological constant.

In this paper we investigate whether this situation might be alleviated by allowing the quintessence to arise from several fields, which we assume to have the same potential energies. While our interest is primarily phenomenological, we note that such situations may arise from higher-dimensional theories; in fact there are many dynamical modulus fields in string theory corresponding to the size of compactified dimensions. In the context of early Universe inflation, such a set of fields has been shown to give the phenomenon of *assisted inflation* [3], whereby they may collectively drive inflation even if each individual field has too steep a potential to do so on its own, i.e.

yielding an effective equation of state closer to $w = -1$. It is therefore worth considering the possibility of assisted quintessence behavior, in order to see whether it may be better able to match observations. Assisted quintessence is particularly attractive in the context of tracking models, as each field will separately converge onto the tracking solution making it entirely natural that they all play a dynamical role.

In this paper we will investigate some aspects of assisted quintessence, mostly restricting ourselves to the simplest case where each field has the same potential and always assuming there are no interactions between fields. The most closely related paper is that of Blais and Polarski [4], who analyzed several multifield quintessence models both analytically and numerically, though with a different focus directed mainly at attempting to realize models where the acceleration is a transient phenomenon. We will primarily study the exponential and inverse power-law cases in detail, in the former case considering different exponents for the two potentials as already extensively analyzed by Coley and van den Hoogen [5]. In this paper we will also provide an algorithm for relating assisted quintessence to an equivalent single-field model under more general circumstances.

Our scenario is distinct from two types of scenario already extensively investigated in the literature. One is the double exponential potential models of Refs. [6,7], where there was only a single field (Ref. [6] briefly mentioned a multiple-field case but not with uncoupled fields). Another is the two-field models of dark energy which have received some attention recently as a way of crossing the cosmological constant boundary ($w = -1$) to give rise to a phantom behavior at late times [8–10]. Unlike our case, at least one of those fields must have a negative kinetic energy.

II. ASSISTED QUINTESSENCE

If one accepts the possibility of multiple fields, particularly sharing the same form of potential $V(\phi_i)$, then the idea of assisted quintessence emerges very naturally provided the potentials have tracker solutions for at least some values of the fields. Tracker solutions arise when the fields are initially subdominant as compared to a perfect fluid, the field contribution to the Friedmann equation then being neglected to give equations of the form

$$H^2 \simeq \frac{8\pi}{3m_{\rm Pl}^2}\rho_{\rm f}, \qquad (1)$$

$$\ddot{\phi}_i = -3H\dot{\phi}_i - \frac{dV}{d\phi_i}, \qquad (2)$$

where H is the Hubble rate, $m_{\rm Pl}$ is the Planck mass and a dot denotes the derivative with respect to a cosmic time t. Here the fluid density $\rho_{\rm f}$ might for instance be broken up into matter and radiation components $\rho_{\rm m}$ and $\rho_{\rm r}$. In this setup, the scalar fields are completely unaware of each other's existence (their normal channel of communication being via the Friedmann equation), and hence separately evolve onto the tracker solution in response to the fluid. At sufficiently late times, one would therefore have all the ϕ_i equal to each other, and hence of equal potential energy.

For an individual field ϕ with potential $V(\phi)$, whether or not there is tracking behavior can be determined from the value of the function

$$\Gamma \equiv \frac{VV''}{V'^2}, \qquad (3)$$

where a prime represents the derivative in terms of ϕ. Solutions converge to a tracker provided that it satisfies $\Gamma > 1 - (2 - \gamma)/(4 + 2\gamma)$ where the equation of state is $p_{\rm f} = (\gamma - 1)\rho_{\rm f}$, the interesting case however being $\Gamma > 1$ which is required for the field energy density to grow relative to the fluid allowing eventual domination [11].

The convergence of different fields to the same tracking solution does not in itself amount to assisted quintessence, as the fields are not generating any gravitational effect on the background evolution. The convergence does however set up the initial conditions for such a behavior. What is mainly of interest is what happens once the energy density of the fields, all evolving together, is no longer subdominant, and that is the situation addressed in the rest of this paper. We will therefore be considering the full Friedmann equation

$$H^2 = \frac{8\pi}{3m_{\rm Pl}^2}\left(\rho_{\rm f} + \sum_i \rho_{\phi_i}\right), \qquad (4)$$

where $\rho_{\phi_i} \equiv V_i(\phi_i) + \dot{\phi}_i^2/2$ and we assume spatial flatness throughout.

III. EXPONENTIAL POTENTIALS

We consider two fields ϕ_1 and ϕ_2 each with a separate exponential potential

$$V(\phi_1, \phi_2) = Ae^{-\lambda_1 \kappa \phi_1} + Be^{-\lambda_2 \kappa \phi_2} \equiv V_1(\phi_1) + V_2(\phi_2), \qquad (5)$$

where $\kappa^2 \equiv 8\pi/m_{\rm Pl}^2$. For generality, we will allow the potentials to have different slopes.

A. Assisted quintessence solutions

For the case where no matter is present, this system is exactly the original assisted inflation scenario of Liddle *et al.* [3], where the multiple fields evolve to give dynamics matching a single-field model with

$$\frac{1}{\lambda_{\rm eff}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}. \qquad (6)$$

For a single-field potential $V(\phi) = V_0 e^{-\lambda \kappa \phi}$ the scale factor evolves as $a \propto t^p$, where $p = 2/\lambda^2$. Then the multifield case Eq. (6) corresponds to an effective power-law index given by $p_{\rm eff} \equiv 2/\lambda_{\rm eff}^2 = \sum p_i$ [3]. The expansion rate is therefore more rapid the more fields there are. A particularly comprehensive analysis of multiple fields in exponential potentials has been given by Collinucci *et al.* [12].

As it happens, this assisted behavior continues to be valid in the presence of a perfect fluid with equation of state $p = (\gamma - 1)\rho$. This was first noted by Coley and van den Hoogen [5], who also allowed for the possibility of spatial curvature. This is because the method used to relate multifield dynamics to an equivalent single-field dynamics in Ref. [3] is a property of the scalar field sector alone, being valid in the presence of any other matter sources.

B. Critical points and stability

The critical points for this system were classified by Coley and van den Hoogen [5], and we will reiterate their results only briefly before embarking on some numerical analysis of the evolution towards them. In Ref. [5], spatial curvature was included as a degree of freedom and the fluid assumed to have $\gamma > 1$, whereas we assume spatial flatness and γ in the wider range 0 to 2.

To study the critical point structure and stability of the system, it is convenient to introduce the following dimensionless quantities [13]:

$$x_i \equiv \frac{\kappa \dot{\phi}_i}{\sqrt{6}\,H}, \qquad y_i \equiv \frac{\kappa \sqrt{V_i}}{\sqrt{3}\,H}, \qquad i = 1, 2. \qquad (7)$$

Then we obtain

$$\frac{dx_i}{dN} = -3x_i + \lambda_i \sqrt{\frac{3}{2}}y_i^2 + \frac{3}{2}x_i[2x_1^2 + 2x_2^2$$
$$+ \gamma(1 - x_1^2 - y_1^2 - x_2^2 - y_2^2)] \qquad i = 1, 2, \quad (8)$$

TABLE I.   The properties of the critical points in the presence of a barotropic fluid with $0 < \gamma < 2$. There are seven discrete points, while Case 2 corresponds to a circle of points parametrized by an angle θ. Case 2 is neutrally stable along the circle, but always unstable in at least one other direction. The last two columns show the effective energy density and equation of state of the two scalar fields combined.

Case	x_1	y_1	x_2	y_2	Existence	Stability	Ω_ϕ	γ_ϕ
1	0	0	0	0	All $\lambda_1, \lambda_2, \gamma$	Unstable	0	Undefined
2	$\cos\theta$	0	$\sin\theta$	0	All	Unstable	1	2
3	$\lambda_1/\sqrt{6}$	$(1 - \lambda_1^2/6)^{1/2}$	0	0	$\lambda_1^2 < 6$	Unstable	1	$\lambda_1^2/3$
4	$\sqrt{6}\,\gamma/2\lambda_1$	$[3(2 - \gamma)\gamma/2\lambda_1^2]^{1/2}$	0	0	$\lambda_1^2 > 3\gamma$	Unstable	$3\gamma/\lambda_1^2$	γ
5	0	0	$\lambda_2/\sqrt{6}$	$(1 - \lambda_2^2/6)^{1/2}$	$\lambda_2^2 < 6$	Unstable	1	$\lambda_2^2/3$
6	0	0	$\sqrt{6}\,\gamma/2\lambda_2$	$[3(2 - \gamma)\gamma/2\lambda_2^2]^{1/2}$	$\lambda_2^2 > 3\gamma$	Unstable	$3\gamma/\lambda_2^2$	γ
7	$\lambda_{\rm eff}^2/\sqrt{6}\,\lambda_1(\lambda_{\rm eff}/\lambda_1)(1 - \lambda_{\rm eff}^2/6)^{1/2}$		$\lambda_{\rm eff}^2/\sqrt{6}\,\lambda_2(\lambda_{\rm eff}/\lambda_2)(1 - \lambda_{\rm eff}^2/6)^{1/2}$		$\lambda_{\rm eff}^2 < 6$	Stable for $\lambda_{\rm eff}^2 < 3\gamma$ Unstable for $3\gamma < \lambda_{\rm eff}^2 < 6$	1	$\lambda_{\rm eff}^2/3$
8	$\sqrt{6}\,\gamma/2\lambda_1$	$[3(2 - \gamma)\gamma/2\lambda_1^2]^{1/2}$	$\sqrt{6}\,\gamma/2\lambda_2$	$[3(2 - \gamma)\gamma/2\lambda_2^2]^{1/2}$	$\lambda_{\rm eff}^2 > 3\gamma$	Stable	$3\gamma/\lambda_{\rm eff}^2$	γ

$$\frac{dy_i}{dN} = -\lambda_i \sqrt{\frac{3}{2}} x_i y_i + \frac{3}{2} y_i [2x_1^2 + 2x_2^2$$
$$+ \gamma(1 - x_1^2 - y_1^2 - x_2^2 - y_2^2)] \qquad i = 1, 2, \quad (9)$$

$$\frac{1}{H}\frac{dH}{dN} = -\frac{3}{2}[2x_1^2 + 2x_2^2 + \gamma(1 - x_1^2 - y_1^2 - x_2^2 - y_2^2)],$$
$$(10)$$

where $N \equiv \ln a$, together with the constraint

$$x_1^2 + y_1^2 + x_2^2 + y_2^2 + \frac{\kappa^2 \rho_{\rm f}}{3H^2} = 1. \qquad (11)$$

We define the density parameters Ω_{ϕ_i} and the equation of state w_{ϕ_i} for scalar fields, as

$$\Omega_{\phi_i} = x_i^2 + y_i^2, \qquad w_{\phi_i} = \frac{x_i^2 - y_i^2}{x_i^2 + y_i^2}, \qquad i = 1, 2. \quad (12)$$

We note that the density parameter for matter is given by $\Omega_m \equiv \kappa^2 \rho_{\rm f}/3H^2 = 1 - \Omega_{\phi_1} - \Omega_{\phi_2}$ from Eq. (11).

These equations are in fact valid for any uncoupled potentials, with λ_i defined by

$$\lambda_i \equiv -\frac{1}{\kappa V_i}\frac{dV_i}{d\phi_i}. \qquad (13)$$

Our case of exponential potentials corresponds to λ_i both constant. It is straightforward to extend our analysis to the case of a dynamically changing λ as studied in single-field models in Ref. [14].

The classification of critical points and their stability is given in Table I, and agrees with results in Ref. [5].[1] There are eight types of critical point, seven being discrete points and one a circular locus of critical points. They are readily

compared to the five types of critical point found for the single-field system in Ref. [13]. Our cases 1, 2 with $\theta = 0$ and π, 3 and 4 correspond to the five points in the single-field case for the field ϕ_1, with ϕ_2 playing no role. Our cases 1, 2 with $\theta = \pi/2$ and $3\pi/2$, 5 and 6 are the same solutions for the ϕ_2 field. Finally, cases 7 and 8 are critical points in which both fields play a role, and which have no direct analogue with the single-field case.

In the single-field case, the stable late-time attractor is either scalar field dominated (case 3 or 5) or a scaling solution (case 4 or 6) depending on the relative values of λ and γ [13]. Once a second field is added, the new degrees of freedom always render those solutions unstable. The late-time attractors instead become either the assisted scalar field dominated solution case 7 (for $\lambda_{\rm eff}^2 < 3\gamma$) or the assisted scaling solution case 8 (for $\lambda_{\rm eff}^2 > 3\gamma$).

To compare our results with Ref. [5], we note that under their assumptions the curvature always dominates the fluid at late times, and behaves like a $\gamma = 2/3$ fluid. Accordingly, they always find solutions with nonzero fluid density to be unstable to eventual curvature domination, whereas our assumption of spatial flatness renders them stable. Otherwise, the classification and stability shown in our table matches their Tables I and III. Additionally, they correctly describe the spatially flat case in their text.

C. Multifield phenomenology

For the exponential potential, the assisted quintessence phenomenon does indeed exist, with the extra fields being equivalent to a flatter single-field potential. Unfortunately this result does not seem particularly useful phenomenologically, as the scaling solutions do not give acceleration as required by observations, while the scalar field dominated solutions would long ago have made the matter density negligible.

There is however one scenario that might be of interest, which is to imagine there are a large number of exponential potentials with different initial conditions. As the Universe
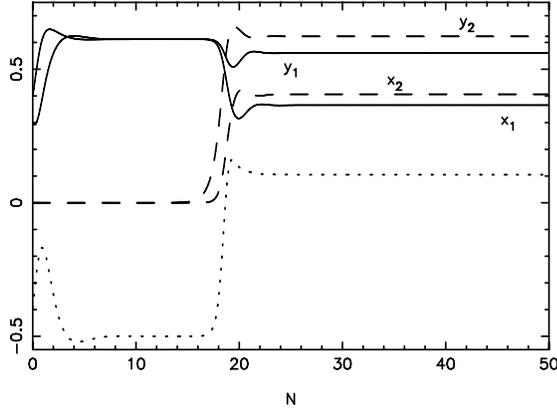
---

[1] We note that a full analysis has also been carried out for the case where one of the fields is of phantom type (opposite sign of the kinetic energy) [9], where the late-time behavior is domination by the phantom field.

FIG. 1.   A numerical integration for the case $\lambda_1 = 2$, $\lambda_2 = 1.8$ and $\gamma = 1$, with initial conditions chosen so that ϕ_1 dominates. Field 1 is in solid and field 2 dashed. The solution initially approaches the single-field scaling solution (case 4), but this becomes unstable once the second field becomes important, the late-time attractor being the multiscalar field dominated accelerating solution (case 7, $\lambda_{\rm eff} = 1.34$). The dotted line shows $-q = \ddot{a}/aH^2$, where q is the deceleration parameter.

evolves, more and more fields would join the assisted quintessence attractor, reducing $\lambda_{\rm eff}$. Eventually, this could switch the attractor from the scaling regime $\lambda_{\rm eff}^2 > 3\gamma$ into the regime of late-time scalar field dominance $\lambda_{\rm eff}^2 < 3\gamma$ [5].

Figure 1 shows a two-field example of such evolution, with the single-field scaling solution becoming unstable as the second field becomes important, switching the evolution into late-time scalar field domination. From Table I the first scaling regime corresponds to case 4 ($x_1 = y_1 = \sqrt{3/8}$, $x_2 = y_2 = 0$) with no acceleration, whereas the final stable attractor is case 7 ($x_1 = 0.365$, $y_1 = 0.560$, $x_2 = 0.406$, $y_2 = 0.623$) with acceleration. We checked that the values of the fixed points agree very well with numerical results.

However even if the initial conditions were fine-tuned to bring this transition into the recent past, it is hard to see how the equation of state w could reach a sufficiently negative value to be observationally viable, current observations indicating roughly $w \equiv \gamma - 1 < -0.8$ [2]. In fact, if we compare the two-field scenarios with a single-field scenario in which an accelerated expansion occurs at late times ($\lambda < \sqrt{2}$), we find that typically the two-field models give a larger present value of w (here w is the equation of state of scalar fields including the contribution of both ϕ_1 and ϕ_2). Figure 2 compares a two-field model with $\lambda_1 = 3$ and $\lambda_2 = 1$ against a single-field model with $\lambda = 1$, in both cases with the field with slope 1 coming to dominate only at the present epoch (the "thawing" regime as according to Ref. [15]). We see that the equation of state of the thawing field alone is indeed closer to -1 at the present (identified as when the matter density is 0.3 of critical) than
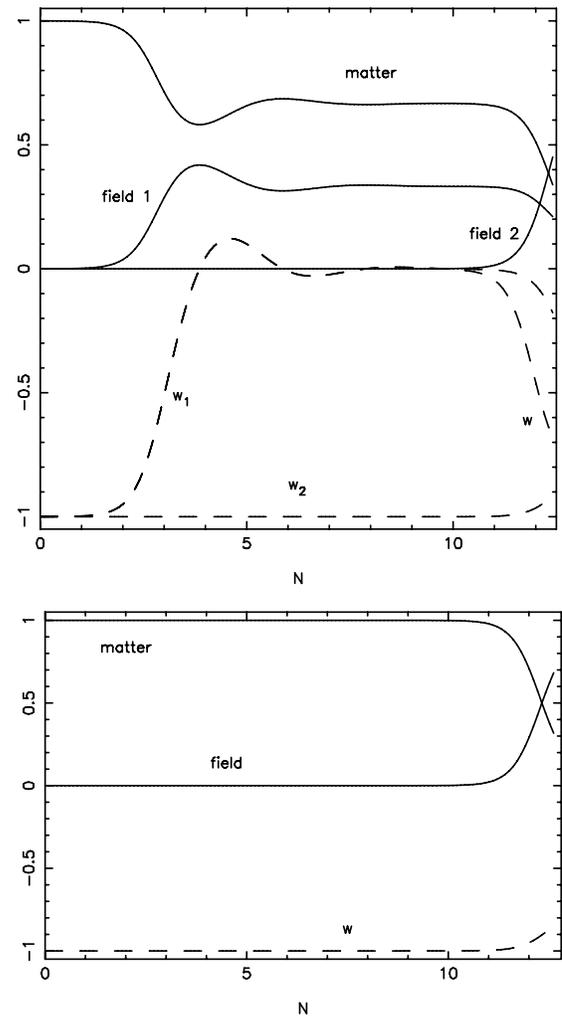

FIG. 2.   A comparison of two-field evolution for $\lambda_1 = 3$ and $\lambda_2 = 1$ (top) and one-field evolution for $\lambda = 1$ (bottom). The density parameters Ω_{ϕ_i} and Ω_m are shown in solid lines and the equation of state parameters w_{ϕ_i} and w in dashed lines. The region around $6 < N < 11$ corresponds to the scaling regime in which the energy density of the field ϕ_1 decreases proportionally to that of the fluid. The present epoch is defined in each simulation as the time when the matter density drops to 0.3 of critical.

in the single-field case, but the combined w of the two fields is larger.

IV. INVERSE POWER-LAW POTENTIALS

We now consider the inverse power-law case $V(\phi) = V_0 \phi^{-\beta}$, where we will take the same exponent and normalization for each field. Here one might hope that the assisted phenomenon would give rise to an effective $\beta_{\rm eff}$ which is smaller than the individual β, so that observationally viable models can be achieved in steeper potentials than the single-field case. Unfortunately that turns out not to be true, as we now see.

These potentials are favored because they exhibit tracker solutions, and it is therefore legitimate to suppose that after some early time we can take $\phi_1 = \phi_2 = \cdots \equiv \phi$. The Friedmann and scalar wave equations become

$$H^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\left(\rho_{\mathrm{f}} + nV_0\phi^{-\beta} + \frac{n}{2}\dot{\phi}^2\right); \qquad (14)$$

$$\ddot{\phi} = -3H\dot{\phi} + \beta V_0\phi^{-\beta-1}, \qquad (15)$$

where n is the number of fields. This is not yet equivalent to a single-field model, but can be made so (loosely following the method of Ref. [3]) by the redefinitions

$$\chi = \sqrt{n}\,\phi; \qquad W_0 = \sqrt{n^\beta}\,nV_0, \qquad (16)$$

which then gives the equations of a single-field model with potential $W(\chi) = W_0\chi^{-\beta}$.

This rescaling indicates that the assisted behavior renormalizes the amplitude of the potential in this case, but does *not* renormalize its exponent. In such models the amplitude has to be adjusted in order to give the present-day value of the matter density Ω_{m}, and having done that the multifield system then has identical dynamics to the single-field model, in particular, predicting the same present-day value of the equation of state w.

V. ASSISTED QUINTESSENCE DYNAMICS

We end with a more general construction for analyzing assisted quintessence, extending the analysis of the previous section to the case where the potential is arbitrary, but the same for all fields. Again we assume tracking behavior so that we can take $\phi_i = \phi$, giving

$$H^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\left[\rho_{\mathrm{f}} + nV(\phi) + \frac{n}{2}\dot{\phi}^2\right], \qquad (17)$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{\mathrm{d}V}{\mathrm{d}\phi}. \qquad (18)$$

To find an equivalent single-field system, we first note that the kinetic term in the Friedmann equation forces the correspondence

$$\chi = \sqrt{n}\,\phi. \qquad (19)$$

The equations can then be transformed into single-field form with potential

$$W(\chi) = nV(\chi/\sqrt{n}), \qquad (20)$$

which obviously has the desired effect in the Friedmann equation, but which also renders the fluid equation into single-field form. This formula therefore represents an algorithm for finding a single-field potential W which will generate the same evolution as multiple fields evolving together in the potential V.

For simplicity, we henceforth consider the two-field case, though the generalization is straightforward. The

correspondence then is

$$\chi = \sqrt{2}\,\phi; \qquad W(\chi) = 2V(\chi/\sqrt{2}). \qquad (21)$$

This allows us to ask what condition would have to be satisfied in order to have no assisted behavior, i.e. for W and V have the same functional form apart from an overall constant. This happens for potentials obeying the condition

$$V(\chi) = 2CV(\chi/\sqrt{2}), \qquad (22)$$

for all χ, where C is a constant. The general solution to this equation is

$$V(\chi) \propto \chi^{-\beta} \times f(\chi), \qquad (23)$$

where $f(\chi)$ is any function periodic in $\ln\chi$ with period $\ln\sqrt{2}$ (i.e. a Fourier series with this periodicity). While this can be any of an infinite class of potentials, having such a periodicity is clearly artificial. The only interesting case therefore is $f(\chi)$ equals a constant, giving the power-law potential. This proves that the inverse power-law potentials are the unique monotonic potentials which do *not* exhibit assisted quintessence behavior. In all other cases, the equivalent single-field potential has a different functional form. In the exponential case this algorithm correctly reproduces Eq. (6) for the case $\lambda_1 = \lambda_2$.

As a final point, we note that, under the correspondence Eq. (20), the tracking parameter $\Gamma_W \equiv WW''/W'^2$ is equal to $\Gamma_V = VV''/V'^2$ (the primes here being derivatives with respect to the arguments χ and ϕ, respectively). As one would expect, the tracking conditions on the multifield model and its single-field dynamical equivalent are the same.

VI. CONCLUSIONS

We have studied various aspects of assisted quintessence dynamics. Such dynamics arises naturally if there are several fields with the same potential, provided the potential exhibits tracking behavior for at least some stage of its early evolution.

Our most powerful result is Eq. (20), which provides a general algorithm for finding a single-field model which mimics the dynamics of a multifield assisted quintessence model. Applied to inverse power-law models, it shows that they are the unique (monotonic) potentials for which there is no assisted behavior, the collection of fields behaving as a single field in the same potential (up to overall normalization). All other potentials will exhibit assisted behavior, the exponential potential being an explicit example.

It should be possible to extend our analysis to the case of more general dark energy models in which the Lagrangian includes noncanonical kinematic terms, such as the tachyon, k-essence and ghost condensate. For theories whose Lagrangian p is a function of the field ϕ and $X \equiv (\nabla\phi)^2/2$, the existence of scaling solutions restricts the form of Lagrangian to be $p = Xg(Xe^{\lambda\phi})$, where g is an

arbitrary function and λ is a constant [16]. It would be certainly of interest to investigate whether the assisted behavior we found for the canonical scalar field with an exponential potential persists in such general dark energy models.

It is interesting that the multifield system can be analyzed so simply. Regrettably, however, we have not uncovered any scenarios where the assisted quintessence phenomenon appears to improve the situation with regard to the observations. In fact our results show the contrary; in potentially the most interesting scenario of the inverse power law it turns out that there is no assisted quintessence effect.

ACKNOWLEDGMENTS

[1] C. Wetterich, Nucl. Phys. **B302**, 668 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D**37**, 3406 (1988); E. J. Copeland, A. R. Liddle, and D. Wands, Ann. N.Y. Acad. Sci. **688**, 647 (1993); P. G. Ferreira and M. Joyce, Phys. Rev. Lett. **79**, 4740 (1997); Phys. Rev. D **58**, 023503 (1998); I. Zlatev, L. Wang, and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); A. R. Liddle and R. J. Scherrer, Phys. Rev. D**59**, 023509 (1999); L. Amendola, Phys. Rev. D**62**, 043511 (2000); V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); T. Padmanabhan, Phys. Rep. **380**, 235 (2003).

[2] M. Tegmark *et al.*, Phys. Rev. D**69**, 103501 (2004); U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, Mon. Not. R. Astron. Soc. **354**, 275 (2004); T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. **429**, 807 (2005); H. K. Jassal, J. S. Bagla, and T. Padmanabhan, Mon. Not. R. Astron. Soc. **356**, L11 (2005); P. S. Corasaniti, M. Kunz, D. Parkinson, E. J. Copeland, and B. A. Bassett, Phys. Rev. D**70**, 083006 (2004); U. Seljak *et al.*, Phys. Rev. D **71**, 103515 (2005).

[3] A. R. Liddle, A. Mazumdar, and F. E. Schunck, Phys. Rev. D**58**, 061301(R) (1998).

[4] D. Blais and D. Polarski, Phys. Rev. D**70**, 084008 (2004).

[5] A. A. Coley and R. J. van den Hoogen, Phys. Rev. D**62**, 023517 (2000).

[6] T. Barreiro, E. J. Copeland, and N. J. Nunes, Phys. Rev. D**61**, 127301 (2000).

[7] A. A. Sen and S. Sethi, Phys. Lett. B **532**, 159 (2002); I. P. Neupane, Classical Quantum Gravity **21**, 4383 (2004); L. Jarv, T. Mohaupt, and F. Saueressig, J. Cosmol. Astropart. Phys. 08 (2004) 016.

[8] B. Feng, X. Wang, and X. Zhang, Phys. Lett. B **607**, 35 (2005).

[9] Z.-K. Guo, Y.-S. Piao, X. Zhang, and Y.-Z. Zhang, Phys. Lett. B **608**, 177 (2005).

[10] W. Hu, Phys. Rev. D**71**, 047301 (2005); R. R. Caldwell and M. Doran, astro-ph/0501104; S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D**71**, 063004 (2005); X. F. Zhang, H. Li, Y. S. Piao, and X. M. Zhang, astro-ph/0501652.

[11] P. J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D**59**, 123504 (1999).

[12] A. Collinucci, M. Nielsen, and T. Van Riet, Classical Quantum Gravity **22**, 1269 (2005).

[13] E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D**57**, 4686 (1998).

[14] A. de la Macorra and G. Piccinelli, Phys. Rev. D**61**, 123503 (2000); S. C. C. Ng, N. J. Nunes, and F. Rosati, Phys. Rev. D**64**, 083510 (2001); E. J. Copeland, M. R. Garousi, M. Sami, and S. Tsujikawa, Phys. Rev. D **71**, 043003 (2005).

[15] R. R. Caldwell and E. V. Linder, astro-ph/0505494.

[16] F. Piazza and S. Tsujikawa, J. Cosmol. Astropart. Phys. 07 (2004) 004; S. Tsujikawa and M. Sami, Phys. Lett. B **603**, 113 (2004); B. Gumjudpai, T. Naskar, M. Sami, and S. Tsujikawa, J. Cosmol. Astropart. Phys. 06 (2005) 007.