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Bound on four-dimensional Planck mass

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In this paper we derive a bound using data from cosmic rays physics on a model recently proposed to solve the hierarchy problem by lowering the Planck scale to the TeV region without the introduction of extra dimensions. We show that the nonobservation of small black holes by AGASA implies a model independent limit for the four-dimensional reduced Planck mass of roughly 488 GeV.

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Different four-dimensional models designed to address the hierarchy problem which predict strong scattering cross-sections in the tera-scale have recently been proposed [1–3]. The most remarkable feature of these models is the possible formation of quantum black holes in high energetic collisions of particles. The LHC, which will start to operate in the coming months, will be able to probe this energy domain, but cosmic rays experiments are already sensitive to strong interactions of cosmic rays neutrinos with nuclei in the Earth atmosphere. Anchordoqui et al. [4,5] have derived bounds on the scale of quantum gravity for extra-dimensional models using data from AGASA. The aim of this work is to derive a similar limit on the scale of quantum gravity in four dimensions following the work of Anchordoqui et al. very closely. We shall first summarize the model proposed in [1].

The strength of the gravitational interaction is renormalized by matter field fluctuations [1,6,7]. One finds that the effective Planck mass depends on the energy scale $\mu$ as

\[ M(\mu)^2 = M(0)^2 - \frac{\mu^2}{12\pi} (N_0 + N_{1/2} - 4N_1). \]  

(1)

where $N_0$, $N_{1/2}$, and $N_1$ are the numbers of real spin zero scalars, Weyl spinors, and spin one gauge bosons coupled to gravity and where $M(0) = 1.2209 \times 10^{19}$ GeV. Related calculations have been performed in string theory and lead to the same behavior for the running of the Planck mass [8].

If the strength of gravitational interactions is scale dependent, the true scale $\mu_*$ at which quantum gravity effects are large is the one at which

\[ M(\mu_*) \sim \mu_* . \]  

(2)

This condition means that fluctuations in spacetime geometry at length scales $\mu_*^{-1}$ will be unsuppressed. It has been shown in [1] that the presence of a large number of fields can dramatically impact the value $\mu_*$.

For example, it takes $10^{32}$ scalar fields and or Weyl spinors to render $\mu_* \sim$ TeV, thereby removing the hierarchy between weak and gravitational scales. The most striking feature of this model is that small black holes will form in particles collisions with center of mass energies of the order of 1 TeV. We shall now derive a bound on the scale of quantum gravity that applies to this model but also more generically to quantum gravity in four dimensions and we thus use $M_P$ instead of $\mu_*$ in the sequel.

Following the work of Anchordoqui et al. [4,5], we use the observation of quasihorizontal showers by AGASA [9,10] which translates into an upper bound on the number of small black holes of 3.5 [4] produced during the run time $T = 1710.5$ days of the experiment. These small black holes would be produced in collisions of high energetic Earth-skimming neutrinos with nuclei in the Earth atmosphere. The cross-section $\nu N \rightarrow BH$ is given by

\[ \sigma(E_\nu, x_{\text{min}}, M_R) = \int_0^1 2dz \int_{(x_{\text{min}}M_R)^2/(y(z)^2x_{\text{min}})}^1 dx F(4) \times \pi r_s^2(\sqrt{3}, M_R) \sum_i f_i(x, Q), \]  

(3)

where $M_R = M_P/\sqrt{8\pi}$ is the reduced Planck mass, $x_{\text{min}} = M_{\text{BH}}^\text{min}/M_R$ is the ratio of the minimal black hole mass which can be created to the reduced Planck mass, $F(4)$ is the Eardley Giddings correction which describes the fact that not all of the energy of the partons is available for black hole formation [11], $y(z)$ is the inelasticity function calculated in [12] following the work of Eardley and Giddings [11], $s = 2x_m N E_\nu$ where $m_N$ is the nuclei mass and $E_\nu$ is the neutrino energy. The functions $f_i(x, Q)$ are the parton distribution functions (we use CTEQ5 for which an unofficial MATHEMATICA version is available on the web page of the CTEQ collaboration). We take $Q \sim 1000$ GeV, as noted in [4], the choice of $Q$ does not impact much the outcome of the calculation. Finally, $r_s$ is the Schwarzschild...
The black holes produced in the reaction $\nu N \rightarrow BH$ can be charged under $U(1)$, $SU(3)$ but they could in principle also be neutral under these two gauge symmetries. We shall consider both semiclassical black holes (for which the construction of Eardley and Giddings applies i.e. $x_{\text{min}} \geq 3$) and what we call quantum black holes [1] ($x_{\text{min}} \geq 1$) which only decay to a couple of particles. The three particles final state is strongly suppressed with respect to the two particles final state because of phase space. This is important for collider experiments. However, in the case of AGASA which is sensitive to a suppression of the neutrino flux due to new strong interactions between neutrinos and nuclei, it is not important if black holes decay visibly or invisibly and we can thus sum over all the possible intermediate black holes.

Following [4] we consider the flux of guaranteed cosmogenic Greisen-Zatsepin-Kuzmin (GZK) neutrinos which originate from the collision of high energy protons on the cosmic microwave background photons producing a delta resonance which then decays to a charged pion among other particles. This pion decays to a lepton and a neutrino. This is the famous GZK mechanism for the suppression of the spectrum of high energetic cosmic rays above $10^{19}$ eV. The number of black holes expected is given by

$$N(M_R) = \frac{\sqrt{\delta}}{4\pi M_R}.$$  

(4)

It is worth mentioning that the mechanism proposed by Lykken et al. [14] who have studied the possibility that neutrinos would annihilate through gravitational interactions with e.g. supernovae neutrinos on their way to Earth does not yield a sizable suppression of the GZK neutrino flux in our case. The gravitational interaction is too weak to suppress the flux in a sizable manner. Similarly the GZK production mechanism for the neutrinos is not affected by the new gravitational interaction.

Requesting that $N(M_R) < 3.5$ black holes, we find a bound on the scale of four-dimensional quantum gravity $M_R > 488$ GeV (i.e. for the reduced Planck mass) and $M_P > 2.4$ TeV for the Planck mass. We stress that the neutrino flux is poorly known. For example, the estimate for the neutrino flux given in [15] is an order of magnitude smaller than the one we are using. Since the cross-section depends on $M_R^{-1/4}$, the bound on $M_P$ would be smaller by a factor two. Furthermore, as pointed out in [16,17], the composition of the cosmic rays is not yet determined, so heavier nuclei could be the dominant component, in which case the neutrino flux will be significantly smaller than those assumed in the literature we relied on. One should also keep in mind that the bound we obtained relies on a number of assumptions, e.g. we are assuming that one can extrapolate the cross-section for semiclassical black hole to the case of quantum black hole (see [18] for a criticism of the assumptions made in [4,5]).

It is remarkable that this bound is independent on the details of the model proposed in [1]. It is also independent on assumptions about quantum gravity such as possible violation of symmetries, e.g. violation of Lorentz invariance, which leads to much tighter bounds (see e.g. [19]). We note that our bound also applies to the model proposed in [20] where gravity remains weak, but where a new scalar similar to a dilaton which stabilizes the Planck mass can lead to strong rescattering effects. It is worth mentioning that if the inelasticity function and the Eardley Giddings construction of the new gravitational interaction.

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