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THE ESSENCE OF QUINTESSENCE AND THE COST OF COMPRESSION

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ABSTRACT

Standard two-parameter compressions of the infinite dimensional dark energy model space show crippling limitations even with current Type Ia supernova (SN Ia) data unless strong priors are imposed. First, they cannot cope with rapid evolution—our best fit to the latest SN Ia data shows late and very rapid evolution to $w_0 = -2.85$. However, all of the standard parameterizations (incorrectly) claim that this best fit is ruled out at more than 2 σ primarily because they track it well only at very low redshift, $z \lesssim 0.2$. Furthermore, they incorrectly rule out the observationally compatible region $w \ll -1$ for $z > 1$. Second, the parameterizations give wildly different estimates for the redshift of acceleration, which vary from $z_{\rm acc} = 0.14$ to $z_{\rm acc} = 0.59$. Although these failings are largely cured by including higher order terms (≥3 parameters), this results in new degeneracies and opens up large regions of previously ruled out parameter space. All of this casts serious doubt on the usefulness of the standard two-parameter compressions in the coming era of high-precision dark energy cosmology and emphasizes the need for decorrelated compressions with at least three parameters.

Subject headings: cosmological parameters — cosmology: theory

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1. INTRODUCTION

The issue of dark energy dynamics is perhaps the most pressing today in cosmology. There are claims both for and against dynamics (Bassett et al. 2002; Alam et al. 2004; Daly & Djorgovski 2004; Jassal et al. 2004). But it is a subject dogged by gauge problems (Maor et al. 2002; Wang & Tegmark 2004; Jonsson et al. 2004; Virey et al. 2004).

For instance, Riess et al. (2004) and Jassal et al. (2004) claim that current Type Ia supernova (SN Ia) data are inconsistent with rapid evolution of dark energy. Such conclusions must always implicitly refer to a finite dimensional subspace of the full dark energy model space, and broadening the class of models studied can (and in this case does) lead to a complete reversal of such conclusions. Figure 2 below provides an explicit counterexample.

The main result of this work is that compression of the dark energy space into low-dimensional subspaces, while convenient and easy to work with, can give seriously misleading conclusions. If one does not impose the weak energy condition (WEC), $w \geq -1$, then the results can border on the completely useless. The rest of this article delimits, as precisely as possible, the quicksands and danger areas in the use of two-parameter compressions.

As a first sobering example, consider constraints on $w(z)$ when we do not impose the WEC. One-parameter studies give constraints such as $-1.38 < w < -0.82$ at 2 σ (Melchiorri et al. 2003), suggesting that a model with $w = -5$ at $z = 2$ would be ruled out at more than 10 σ. Instead, a little thought makes it clear that if $w(z)$ can vary freely, then *there is no lower bound on* w for $z \geq 1$ since this merely changes how fast the already irrelevant and rapidly diminishing dark energy density decreases. If the rapid drop in w occurs at $z > 1$, this leaves essentially no observable trace (Bassett et al. 2002; Corasaniti et al. 2003). This is clearly reflected in the likelihoods in Figure 1 that allow for $w < -100$ at $z \sim 1$. How can we hope to cover such possibilities with simple one- or two-parameter compressions?

The dark energy literature overflows with one-, two-, and higher dimensional compressions of $w(z)$. Compressions also exist for $\rho(z)$ (Wang & Garnavich 2001; Wang & Freese 2004; Wetterich 2004), while decorrelated reconstructions of $w(z)$ have been proposed in Huterer & Starkman (2003) and Hu (2004).

The simplest parameterization, which describe the dark energy with a constant equation of state $w = \text{const}$, is well known to suffer from a severe bias (see, for instance, Maor et al. 2002 and Virey et al. 2004) in parameter estimation. Compressions invoking two parameters that somewhat alleviate this problem have been introduced in Efstathiou (1999) Linder (2003), and Jassal et al. (2004). However, as we will see, these models all struggle to describe rapid evolution. This is not surprising. With two parameters, one may fix w at $z = 0$ and w at high z, but one can do nothing about the time or the rapidity of the transition between the two extremes. Caldwell & Doran (2004) circumvented this by considering 13 different one- and two-parameter models, some exhibiting rapid transitions.

2. THE PARAMETERIZATIONS

For our study we consider two distinct classes of compressions. First are standard Taylor expansions of $w(z)$, and second

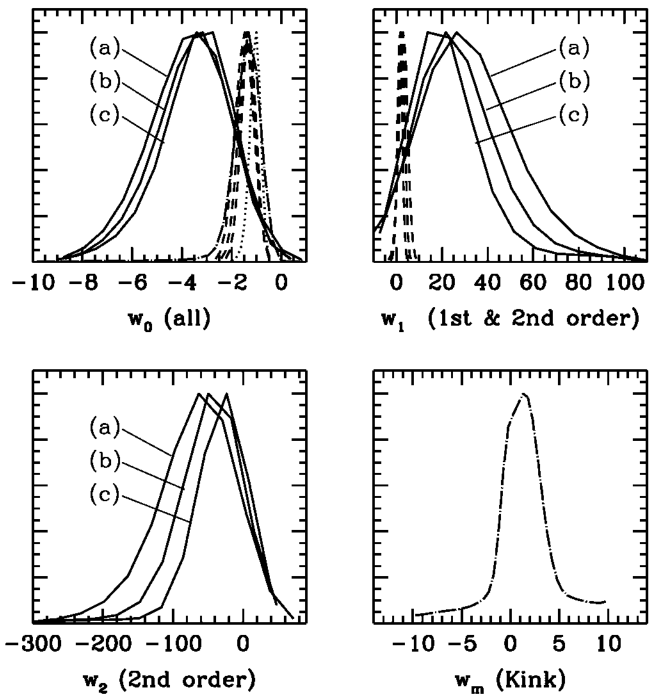

FIG. 1.—Expansion order is more important than parameterization. One-dimensional marginalized likelihoods for the different classes of parameterizations. The narrow curves are the likelihoods corresponding to a constant w, $n = 0$ (*dotted line*), and first-order expansions, $n \leq 1$ (*dashed lines*). The solid lines show the likelihoods at second order ($n \leq 2$); they are much wider because of degeneracies and because of the absence of a lower bound on w for $z \geq 1$. The curves correspond to expansions in (a) the scale factor, (b) log, and (c) redshift, respectively. Finally, the one-dimensional likelihood for w_m of the kink is shown (*dot-dashed line*) and exhibits strongly non-Gaussian wings that extend to very large values of $|w_m|$. For a definition of the order of the expansion, see eq. (1).

is the *kink*, a physically motivated compression. The Taylor expansions are all of the form

$$w(z) = \sum_{n=0} w_n x_n(z), \qquad (1)$$

where we consider four different choices for the "expansion" functions, $x_n(z)$. Namely,

$$x_0(z) \equiv 1; \;\; x_n \equiv 0, \; n \geq 1 \quad \text{(constant } w\text{)}, \qquad (2)$$

$$x_n(z) \equiv z^n \quad \text{(redshift)}, \qquad (3)$$

$$x_n(z) \equiv (1-a)^n = \left(\frac{1}{1+z}\right)^n \quad \text{(scale factor)}, \qquad (4)$$

$$x_n(z) \equiv [\log(1+z)]^n \quad \text{(logarithmic)}. \qquad (5)$$

To linear order ($n \leq 1$), these were first discussed by Huterer & Turner (2001) and Weller & Albrecht (2002), Chevallier & Polarski (2001) and Linder (2003), and Efstathiou (1999) for the redshift, scale-factor, and logarithmic expansion functions, respectively. Later we will consider their performance at higher order ($n \geq 2$).

The kink, on the other hand, is not an expansion. It is a four-parameter model that accurately captures the behavior of quintessence (Bassett et al. 2002; Corasaniti & Copeland 2003; Corasaniti et al. 2004). The extra parameters allow us to model very rapid transitions in $w(z)$, a freedom we will need:

$$w(a) = w_0 + (w_m - w_0)\frac{1 + e^{a_t/\Delta}}{1 + e^{(a_t - a)/\Delta}}\frac{1 - e^{(1-a)/\Delta}}{1 - e^{1/\Delta}}, \qquad (6)$$

where a is the scale factor, w_0 and w_m are the present and matter-dominated values of the dark energy equation of state, respectively, a_t is the value of the scale factor at the transition from w_m to w_0, and Δ controls the width of the transition. Other formulations of the kink, with relative merits, are discussed in Appendix A of Corasaniti et al. (2004). There are other parameterizations, but these are the most widely used today, and the lessons learned from these compressions will apply to many others in the literature.

3. CONSTRAINTS FROM SNe Ia

We use the current measurements of the luminosity distance from SNe Ia to compare the different parameterizations. In order to be conservative, we use only the gold sample of Riess et al. (2004), containing 157 data points. In our analysis, we assume a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe. The assumption of flatness is required to achieve reasonable error bars (see Bassett & Kunz 2004; Dicus & Repko 2004). Fortunately, this is now a data-driven assumption and particularly harmless for this study since we are primarily interested in testing compressions rather than deriving constraints. We will also assume the prior $\Omega_m = 0.27 \pm 0.04$. This can be justified from cosmic microwave background (CMB) data, and as shown in Kunz et al. (2004) and Corasaniti et al. (2004), the best-fit values for background FLRW parameters are not affected strongly by dark energy dynamics.

Our analysis methods are described in detail in Corasaniti et al. (2004). We use a Markov-Chain Monte Carlo code to find the constraints on the dark energy parameters for each parameterization. As usual, we marginalize analytically over the normalization of the luminosity distance, which takes care of the Hubble constant as well, leaving Ω_m as the only remaining parameter apart from those describing the equation of state.

Figure 1 shows the marginalized one-dimensional likelihoods for the parameters of the dark energy compressions. The main point of that figure is that the various parameterizations have similar likelihoods *at the same order* but that the likelihoods at different orders are completely different. Here, by order, we mean the maximum value of n in equation (1). Hence, "linear" or "first order" ($n \leq 1$) refers to the standard two-parameter expansions with only w_0, w_1 nonzero, while second order corresponds to $n \leq 2$ and has nonzero w_2.

We infer the "marginalized" limits on the redshift dependence of the equation of state by computing for a given parameterization the 95% confidence region over all models in the chains. We plot the result in Figure 2. We have also inferred the "maximized" limits by computing the highest and lowest $w(z)$ for the models in the chains with $\chi^2 < \chi^2_{\min} + 4$. As is expected for nearly Gaussian likelihoods, we found the marginalized limits to be consistent with the maximized ones. We have also checked that they coincide with limits from Gaussian error propagation. On the other hand, we found that the marginalized limits associated with the kink formula (*thick solid lines*) slightly differ from the maximized ones. This is because the likelihood is non-Gaussian and because the marginalized limits depend on the volume factor over which the likelihood

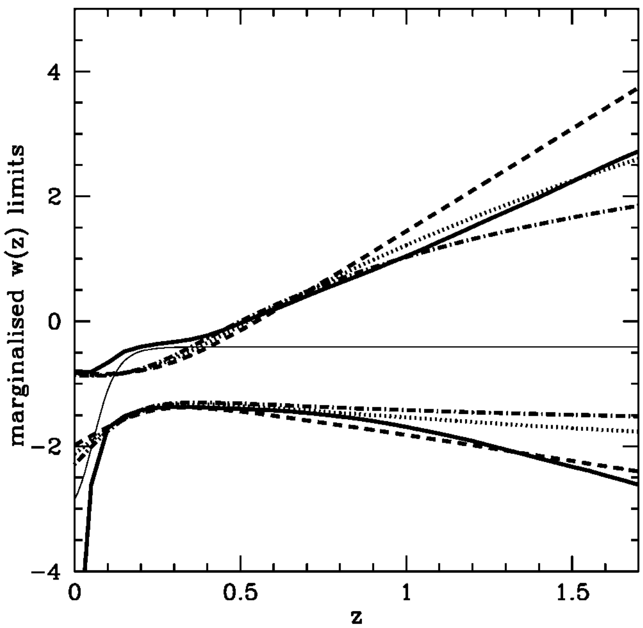

Fig. 2.—Parameterizations struggle with rapid evolution. Marginalized limits on $w(z)$ for the redshift (*dashed line*), scale-factor (*dash-dotted line*), logarithmic (*dotted line*), and kink (*thick solid line*) parameterizations. The best-fit kink solution is the thin solid line. It passes well outside the limits of all the first-order parameterizations both at $z \sim 0$ and at $z \sim 0.2$, showing their inability to capture rapid dynamics, which leads to their incorrectly ruling it out. The cosmological constant $w(z) = -1$ lies within the 2 σ limits and is perfectly consistent with the data. [*See the electronic edition of the Journal for a color version of this figure.*]

is integrated. As a consequence of this, models outside the marginalized limits cannot be ruled out, and only the maximized limits should be used as exclusion ones.

We now discuss the constraints derived from the kink formula, equation (6). The best fit to the data has a $\chi^2 = 172.6$ and is characterized by $w_0 = -2.85$, $w_m = -0.41$, $a_t = 0.94$, and $\log \Delta = -1.52$, corresponding to a rapidly varying equation of state with a transition from w_m to w_0 at $z_t = 0.1$. This best fit is shown in Figure 2 (*thin solid line*). As we have previously pointed out, the marginalized limits suffer from the integration over the marginalized parameters, and therefore no importance should be given to the fact that this best-fit model lies close to the 2 σ upper limit at $z \sim 0.2$.

Similar best-fit models were found in Alam et al. (2004), Huterer & Cooray (2004), and Hannestad & Mörtsell (2004). As can be seen from Figure 2, the best-fit model clearly exits the 2 σ limits from the two-parameter compressions, first from below, at $z \sim 0$, and then from above, at $z \sim 0.2$. This graphically illustrates the limitations of the standard parameterizations and shows how they artificially rule out models that should give the strongest signals for dark energy dynamics (Corasaniti et al. 2003).

It is not just one good fit to the data that violates the 2 σ limits of all the two-parameter compressions either. For instance, the model with $w_0 = -1.46$, $w_m = 0.16$, $a_t = 0.88$, and $\log \Delta = -0.7$ has $\chi^2 = 173.9$, while the model with $w_0 = -1.11$, $w_m = 6.13$, $a_t = 0.40$, and $\log \Delta = -.98$ has $\chi^2 = 175.9$. Both are excellent fits to the data but are supposedly ruled out by the $n \leq 1$, linear redshift, scale-factor, and logarithmic parameterizationz of equations (3)–(5).

The conclusion that rapid evolution of dark energy is ruled out by current data is therefore a "gauge" artifact. We have shown that rapid variations of the dark energy equation of state are perfectly consistent with, and in fact provide better fits to, the gold sample than do models without rapid transitions.

TABLE 1
Bayesian Evidence, BIC, AIC, and χ^2-Values of Best Fit for Different Parameterizations

Model	k[a]	χ^2	BIC	AIC	$-\ln E$
ΛCDM	1	177.6	182.7	179.6	93
w = const	2	177.6	187.7	181.6	96
Linear	3	174.5	189.7	180.5	99
Logarithmic	3	174.2	189.4	180.2	98
Scale-factor	3	174.0	189.2	180.0	98
Quadratic	4	172.1	192.3	180.1	100
Logarithmic II	4	172.2	192.4	180.2	100
Scale-factor II	4	172.3	192.5	180.3	99
Kink	5	172.6	197.9	182.6	96

[a] The total number of fitting parameters, which include the dark energy density Ω_{DE}.

The pathological behavior of ruling out models that are very good fits to the data can be rectified by the inclusion of higher order terms, $n \geq 2$. Indeed, since the data allow w_1 to be large in all cases, higher order terms in the redshift, scale-factor, and log expansions cannot be neglected. Therefore, we have extended our analysis in order to include second-order corrections ($n \leq 2$) to equations (3)–(5).

Comparing the likelihoods associated with the first-order parameterizations (*solid lines*) in Figure 1 with the second-order ones (*dashed lines*), we see that the allowed values of w_0 are significantly shifted toward more negative values, consistent with, but broader than, the kink confidence interval. Second, huge values of $w_1 \sim 50$ and $w_2 \sim -100$ are consistent with the data. As mentioned in § 1, this comes from the fact that w_1 and w_2 are strongly degenerate in all cases and that there is no lower bound on w at $z > 1$, illustrating the huge effect of imposing the weak energy condition $w \geq -1$. We have also considered a much higher order ($n \leq 6$) and found that severe internal degeneracies lead to finely balanced coefficients, with each order as important as the one before. This suggests that strong dark energy dynamics is not ruled out and that, consequently, higher order terms *must* be taken into account when using Taylor expansions. Table 1 summarizes the best-fit χ^2-values and compares the models based on Bayesian information criterion (BIC; Schwarz 1978), Akaike information criterion (AIC; Akaike 1974), and Bayesian evidence E (Sivia 1996). It is worth noting that fully degenerate parameters do not contribute to the evidence, so that, specifically, the kink model is less disfavored than the number of parameters naively suggests. For the same reason, we find that E grows very slowly when going to even higher order in the expansion-type parameterizations, although these cases are already disfavored by Bayesian statistics. The preferred parameterization is the ΛCDM model—it is indeed remarkable that a model with a single free parameter fits the data so well.

4. WHEN DID ACCELERATION BEGIN?

One of the key characteristics of dynamical dark energy is that the redshift at which the universe begins accelerating, z_{acc}, is characteristically different from that in the ΛCDM model with the same Ω_{DE} today. This is manifest in the CMB as a modified integrated Sachs-Wolfe effect (Bassett et al. 2002; Corasaniti et al. 2003) that is degenerate with reionization (Corasaniti et al. 2004). The SNe Ia offer the possibility to break this degeneracy, and therefore it is crucial to use a parameterization that can accurately estimate z_{acc} without bias. Using a simple linear expansion of the deceleration parameter q, Riess et al. (2004) estimated $z_{acc} = 0.46 \pm 0.13$. In Table 2, we compare this with the predictions of the various parameterizations for z_{acc}.

First, we notice that all of the parameterizations provide different best-fit values and 1 σ error bars for z_{acc}, ranging from $z_{acc} = 0.14$ for the redshift expansion to $z_{acc} = 0.59$ for the scale-factor expansion (see also Dicus & Repko 2004). The logarithmic, constant, and kink parameterizations all have similar best fits, but the first two have overly narrow error bars relative to the kink predictions. The largest error bars correspond to the scale-factor expansion, equation (4). This is a consequence of the different sensitivity of each parameterization to dark energy dynamics discussed in the previous section. Interestingly, the best fits for z_{acc} for all the parameterizations are lower than in the ΛCDM model. This suggests that a direct measurement of z_{acc} can provide strong constraints on dark energy dynamics.

TABLE 2

Best-Fit Values and 1 σ Confidence Intervals on z_{acc} at Linear Order ($n \le 1$)

Model	z_{acc}
ΛCDM	$0.66 \pm^{0.11}_{0.11}$
w = const	$0.36 \pm^{0.25}_{0.04}$
Linear in z	$0.14 \pm^{0.14}_{0.05}$
Logarithmic	$0.38 \pm^{0.42}_{0.08}$
Scale-factor	$0.59 \pm^{8.91}_{0.21}$
Kink	$0.45 \pm^{0.53}_{0.44}$

5. CONCLUSIONS

This Letter shows the limitations of standard one- and two-parameter compressions of the infinite-dimensional space of dark energy models. We have highlighted the dangers in using constraints derived with these parameterizations, particularly regarding the possibility of rapid evolution in the dark energy, which none of the standard compressions can follow, and in defining allowed regions of parameter space that depend sensitively on priors and, in particular, on whether the weak energy condition is imposed or not.

Rapid evolution provides a superlative fit to current SN Ia data (as measured by χ^2), despite claims to the contrary in the literature that were based on two-parameter compressions. Indeed, all of the two-parameter expansions we studied wrongly rule out such rapid evolution at 2 σ or more. In addition, the standard parameterizations also miss the fact that w has no lower bound at $z > 1$ if the weak energy condition is not imposed, artificially cutting out vast swathes of parameter space as a result of their innate limitations.

Further problems occur in estimating the redshift at which the universe began accelerating, z_{acc}. There is a nearly 300% variation in the best fit for z_{acc}, depending on parameterization. Interestingly, all the tested parameterizations gave best fits for z_{acc} below that of the ΛCDM model, providing unusual cross-parameterization evidence for dark energy dynamics. Nevertheless, when using Bayesian statistics for model selection, the cosmological constant is preferred over the other models.

The severe inadequacy of the standard two-parameter expansions leads us to consider higher order terms ($n \ge 2$) with one or more extra parameters, e.g., w_2. While this brings the rapid evolution models within the allowed region of parameter space, it leads to severe degeneracies (see Fig. 1) that may make the parameterizations impotent for constraining the space of theoretical dark energy models, particularly when $w < -1$.

We conclude that the confidence intervals inferred from standard two-parameter expansions often do not deserve that name and are typically untrustworthy, even with current data. The wealth and quality of dark energy data that we will acquire over the next decade will demand a significantly better performance.

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