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## Stabilizing the dilaton in superstring cosmology

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We address the important issue of stabilizing the dilaton in the context of superstring cosmology. Scalar potentials which arise out of gaugino condensates in string models are generally exponential in nature. In a cosmological setting this allows for the existence of quasiscaling solutions, in which the energy density of the scalar field can, for a period, become a fixed fraction of the background density, due to the friction of the background expansion. Eventually the field can be trapped in the minimum of its potential as it leaves the scaling regime. We investigate this possibility in various gaugino condensation models and show that stable solutions for the dilaton are far more common than one would have naively thought. [S0556-2821(98)01320-4]

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### I. INTRODUCTION

Scalar fields in cosmology have been extensively studied over the past few years. One of the most intriguing areas in which they occur is superstring theory, where the presence of the dilaton field is vital. Its vacuum expectation value (VEV) determines both the gauge and gravitational coupling constants of the low energy theory and also fixes the scale of supersymmetry (SUSY) breaking through the gravitino mass,  $m_{3/2}$ . Therefore realistic models require a VEV of order one (in Planck units), and  $m_{3/2} \sim 1$  TeV.

Unfortunately, in string theory the dilaton potential is flat to all orders in perturbation theory, which of course means there is no way of obtaining a stable VEV for the field. This problem has to be overcome through some nonperturbative effect. The most promising possibility is through the formation of condensates of gaugino fields at an energy scale of around  $10^{14}$  GeV [1]. The resulting scalar potential for the dilaton is then a combination of exponentials and polynomials in the field. A detailed investigation of these condensate models has demonstrated the need for at least two condensates to form if the dilaton potential is to develop a minimum at a realistic value although with a negative cosmological constant (these are the so-called ‘‘racetrack’’ models) [2].

An alternative proposal has recently been suggested as a method of obtaining a minimum for the dilaton field, and it has the advantage of relying on only one gaugino condensate [3,4]. In this scenario the Kähler potential (which determines the kinetic terms of the dilaton in the action) requires string inspired nonperturbative corrections. A detailed analysis of these models [5] indicates that it is possible to have a minimum with zero or small positive cosmological constant. One additional positive feature that emerges is that the nonperturbative corrections can lead to a solution of the ‘‘moduli problem’’ for the dilaton [6] (fields with masses in the TeV range but which decay so slowly that they spoil nucleosynthesis), by giving it a huge mass.

Although attractive, both kind of models still have several

problems associated with them. One is the difficulty of achieving inflation which was carefully studied by Brustein and Steinhardt [7] a few years ago. Taking a model of multiple gaugino condensation (as those studied in [2]) they argued that the kinetic energy associated with the dilaton field would dominate over its potential energy until  $\phi$  (the canonically normalized field, related to the usual dilaton by  $\text{Re } S = e^\phi$ ) would settle near a minimum of its potential. This obviously excludes inflation from happening, at least with the dilaton as the inflaton field, leaving the possibility of  $\phi$  settling down to a minimum and then inflation being driven by other fields. However this second possibility also presented serious problems as the models they studied had a negative cosmological constant. Nonperturbative corrections to the Kähler potential can cure this latter problem, but in both cases the potentials are exponentially steep in the strong coupling regime. This would lead us to expect the dilaton to roll past the minimum rather than acquiring its VEV, which seems to be a major problem that superstring cosmology needs to address.

In this paper we turn our attention to the possibility that other matter fields rather than the dilaton drive the evolution of the Universe. Recent attention in cosmology has turned to the investigation of scaling solutions in models with exponential scalar field potentials [8–10]. These models are of particular interest because if the background dynamics are dominated by some matter source other than the field itself (i.e. radiation, dust, vacuum energy) then it is possible for the field to enter a scaling regime as it evolves down its potential. In this regime the friction term from the expansion of the Universe balances the kinetic energy of the field allowing it to enter this scaling era. Attractor solutions exist [8,10] where the energy density in the field becomes a fixed fraction of the total energy density.

This intriguing behavior can be applied to the case of the dilaton field arising from string theory. Under the assumption that it is evolving from somewhere in the strong coupling regime of its potential we show how, for a wide range of initial field values in the presence of a background dominated by a barotropic fluid, the dilaton enters a quasi scaling regime as it evolves down the potential, in spite of its steepness. This scaling behavior eventually ends as the field enters the minimum of its potential, by which time it has slowed

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down sufficiently for it to simply oscillate about it, losing energy and eventually becoming fixed with a realistic VEV.

In Sec. II we introduce the concept of scaling solutions with exponential potentials. In Sec. III we demonstrate how this can be successfully adapted to both the racetrack and modified Kähler potential models of gaugino condensates. Solutions are presented analytically and numerically showing how the dilaton field is stabilized in its minimum. We conclude in Sec. IV.

## II. SCALING SOLUTIONS WITH EXPONENTIAL POTENTIALS

In this section we will describe some general features concerning the cosmological evolution of scalar fields with exponential potentials. Let us consider a scalar field  $\phi$  with a potential energy density given by  $V = V_0 e^{-\lambda\kappa\phi}$ , with  $\kappa^2 \equiv 8\pi G$  and  $\lambda$  and  $V_0$  constants, which is evolving in a Friedmann-Robertson-Walker (FRW) Universe containing a fluid with barotropic equation of state  $p_\gamma = (\gamma - 1)\rho_\gamma$ , where  $\gamma$  is a constant ( $0 \leq \gamma \leq 2$ , for instance  $\gamma = 4/3$  for radiation domination or  $\gamma = 1$  for matter domination). The equations of motion for a spatially flat FRW model with Hubble parameter  $H$  are

$$\begin{aligned} \dot{H} &= -\frac{\kappa^2}{2}(\rho_\gamma + p_\gamma + \dot{\phi}^2) \\ \dot{\rho}_\gamma &= -3H(\rho_\gamma + p_\gamma) \end{aligned} \quad (1)$$

$$\dot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi},$$

subject to the constraint

$$H^2 = \frac{\kappa^2}{3} \left( \rho_\gamma + \frac{1}{2} \dot{\phi}^2 + V \right). \quad (2)$$

In the previous equations we have assumed that the only interaction between  $\phi$  and the other matter fields is gravitational. In what follows we set  $\kappa^2 = 1$  but it can easily be reinstated. For exponential potentials the asymptotic behavior of this system can be obtained analytically, and we shall focus on the solution for very steep potentials, namely  $\lambda^2 > 3\gamma$ , as these are of most interest in string theory. Let us first of all review the structure of the solutions which, as is well known [8], contain a late-time attractor solution. For that purpose it is useful to proceed as in [10], defining the variables  $x \equiv \dot{\phi}/\sqrt{6}H$  and  $y \equiv \sqrt{V}/\sqrt{3}H$  and using the logarithm of the scale factor  $N \equiv \ln(a)$  as the time variable. The previous system Eq. (1) becomes

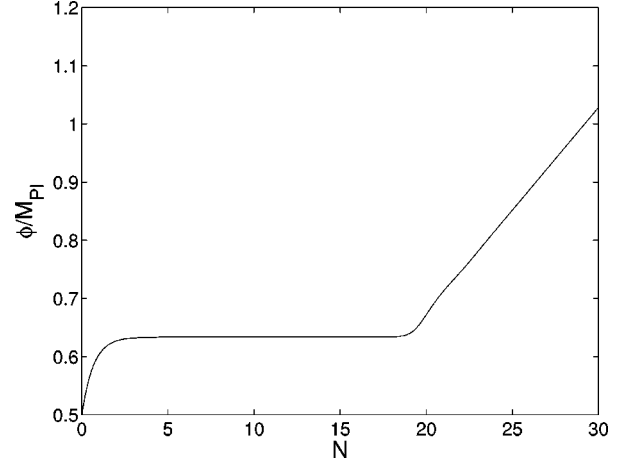


FIG. 1. Plot of the evolution of  $\phi$  vs  $N = \ln(a)$ . The scalar potential is given by  $V = e^{-\lambda\phi}$  with  $\lambda = 85$ ,  $\gamma = 1$  and the initial conditions are  $\phi_0 = 0.5$ ,  $\dot{\phi}_0 = 0.2$  and  $H_0 = 1$ .

$$\begin{aligned} x' &= -3x + \lambda \sqrt{\frac{3}{2}} y^2 + \frac{3}{2} x [2x^2 + \gamma(1 - x^2 - y^2)] \\ y' &= -\lambda \sqrt{\frac{3}{2}} xy + \frac{3}{2} y [2x^2 + \gamma(1 - x^2 - y^2)] \\ H' &= -\frac{3}{2} H [2x^2 + \gamma(1 - x^2 - y^2)], \end{aligned} \quad (3)$$

where a prime denotes a derivative with respect to  $N$ . In terms of these variables, the constraint Eq. (2) becomes  $x^2 + y^2 + \rho_\gamma/3H^2 = 1$  or, in other words, for  $\rho_\gamma \geq 0$  we have the bounds  $0 \leq x^2 + y^2 \leq 1$ .

The evolution of the  $\phi$  field in terms of  $N$  can be seen in Fig. 1, for  $\lambda = 85$ ,  $V_0 = 1$  and initial conditions  $\phi_0 = 0.5$ ,  $\dot{\phi}_0 = 0.2$ , and  $H_0 = 1$ . After an initial increase driven by the initial velocity of the field, we see how the friction term (or, in other words, the expansion of the Universe) dominates and freezes  $\phi$  at a constant value for a considerable amount of time, until the field reaches a scaling regime corresponding to the critical points  $x_c = \sqrt{3/2}\gamma/\lambda$  and  $y_c = \sqrt{3(2-\gamma)\gamma/2\lambda^2}$  [10]. This behavior can also be obtained by solving the system of equations Eq. (3) under the following assumptions:

Stage I (pre scaling regime): Since  $\lambda \gg 1$ , and  $H$  is decreasing,  $y$  is very small in this early stage of the evolution and can be neglected in Eq. (3). The equation for  $x$  now becomes

$$x' = -3x + \frac{3}{2} x [2x^2 + \gamma(1 - x^2)], \quad (4)$$

and its solution is

$$x = \left( 1 + \frac{1 - x_0^2}{x_0^2} e^{3(2-\gamma)N} \right)^{-1/2}, \quad (5)$$

where  $x_0$  is the initial condition for  $x$  (at  $N=0$ ). The solution for  $\phi$  can be obtained by integrating Eq. (5) (recall that  $x = \phi'/\sqrt{6}$ ), and is given by

$$\phi_1(N) = \phi_0 + \frac{2\sqrt{6}}{3(2-\gamma)} \left[ \sinh^{-1} \left( \frac{x_0}{\sqrt{1-x_0^2}} \right) - \sinh^{-1} \left( \frac{x_0}{\sqrt{1-x_0^2}} e^{-3(2-\gamma)N/2} \right) \right], \quad (6)$$

where  $\phi_0$  is the initial value of the field. As  $N$  increases this solution tends to a constant value  $\tilde{\phi}_0$  given by

$$\tilde{\phi}_0 = \phi_0 + \frac{\sqrt{6}}{3(2-\gamma)} \ln \left( \frac{1+x_0}{1-x_0} \right). \quad (7)$$

Obviously for zero initial velocity,  $\tilde{\phi}_0$  will reduce to  $\phi_0$ .

Stage II (scaling regime): as mentioned above, the scaling regime is defined by constant (i.e. critical) values for  $x$  and  $y$ . We then obtain  $H = H_0 e^{-3\gamma N/2}$  and

$$\phi_{\text{II}}(N) = \frac{1}{\lambda} \ln \left( \frac{2\lambda^2 V_0}{9H_0^2(2-\gamma)\gamma} \right) + \frac{3\gamma}{\lambda} N, \quad (8)$$

where now we have an explicit dependence on the characteristics of the potential (i.e.,  $\lambda$  and  $V_0$ ).

It is important to note that the background evolution is being determined by the additional matter fields present, given by  $\gamma$ . It is not the  $\phi$  field which is driving the evolution. The fact that the simultaneous evolution of the scalar field together with the background causes the former to reach a scaling regime, in spite of the steepness of its (exponential) potential, suggests that this could also be the case for the kind of potentials arising from SUSY breaking via gaugino condensation, where the superpotential depends exponentially on the dilaton field. In the next section we will apply these solutions to the gaugino condensation models studied in [2] and [5].

### III. SCALING SOLUTIONS WITH GAUGINO CONDENSATES

#### A. Two condensate potentials

Multiple gaugino condensation (or racetrack) models have been extensively studied in the literature [11,2]. Essentially the idea is to consider a strong type interaction in the hidden sector of our effective supergravity (SUGRA) theory, which is governed by a nonsemisimple gauge group. The superpotential of such models will then be expressed in terms of a sum of exponentials which conspire to generate a local minimum for the dilaton. To be more precise, the scalar potential in any  $N=1$  SUGRA model [12] is given by

$$V = e^K |W|^2 \left[ \left( K^i + \frac{W^i}{W} \right) (K^j)^{-1} \left( K_j + \frac{\bar{W}_j}{\bar{W}} \right) - 3 \right], \quad (9)$$

where  $K$  is the Kähler potential,  $W$  is the superpotential and the subindices  $i, j$  represent derivatives of these two functions with respect to the different fields. Given that we are interested in superstring derived models, and in particular in studying the hidden sector of the theory, both  $K$  and  $W$  will be dependent on the dilaton ( $S$ ) and the moduli ( $T_i$ ,  $i=1,2,3$ ) fields. In fact we know that in the case of orbifold compactifications the tree-level Kähler potential is given by

$$K = -\log(S + \bar{S}) - \sum_{i=1}^3 \log(T_i + \bar{T}_i), \quad (10)$$

and we will restrict our study to the case of a hidden sector interaction governed by two gauge groups,  $SU(N_1) \times SU(N_2)$ , under which we have  $M_1(N_1 + \bar{N}_1)$  and  $M_2(N_2 + \bar{N}_2)$  ‘‘quark’’ representations with Yukawa couplings to a set of singlet fields. For simplicity, we will assume an overall modulus  $T = T_1 = T_2 = T_3$  and a generic singlet field for each of the gauge groups,  $A_1$  and  $A_2$  respectively. In this case, the superpotential is given by

$$W = \sum_{i=1}^2 \left[ -\frac{d_i}{\eta(T)^{\beta_i}} A_i^{M_i/N_i} e^{-\alpha_i S} + h_i A_i^3 \right], \quad (11)$$

where  $\alpha_i = 8\pi^2/N_i$ ,  $d_i = N_i(32\pi^2 e)^{(M_i/N_i-1)}$ ,  $\beta_i = 2(3N_i - M_i)/N_i$ ,  $\eta(T)$  is the Dedekind function and the self coupling of the  $A_i$  field is set to  $h_i = 1$ . (Note  $N_i$  is not the same as  $N$ , the number of e-foldings defined earlier.)

These kind of models were thoroughly studied in [2], so let us summarize their main features: the presence of the  $\eta$  function, imposed by the requirement of target space modular symmetry and, in general, the  $T$ -dependence of the potential ensures the presence of a minimum for  $T \sim 1.2$  (in Planck units) [13], independently of the particular gauge groups and/or matter representations (provided the dilaton acquires a VEV); also there exist minima in the  $\text{Im } S$  direction if both condensates have opposite phases. Finally the existence was shown of many examples for which there is a minimum in the  $\text{Re } S$  direction at the phenomenologically acceptable value  $\text{Re } S \sim 2$  (remember that  $\text{Re } S = g_{string}^{-2}$ ), with a reasonable ( $\sim 1$  TeV) gravitino mass but always with a negative value of the potential energy. A typical example is shown in Fig. 2(a), which perfectly illustrates the problem that Brustein and Steinhardt pointed out in their paper: the steepness of the dilaton potential, which would prevent the field from settling down at its (negative) minimum, instead allowing it to run over the tiny maximum towards infinity.

We would like to study the evolution of the dilaton field (more precisely of its real part) before it settles to the minimum. We have already seen that a single exponential scalar potential can be solved analytically leading to scaling solutions. However there are major differences between this and the more realistic gaugino condensate models. First, the dilaton couples not only gravitationally but also directly to the matter fields. For simplicity, and to avoid making any assumptions about specific models, we will neglect this effect throughout this paper. Second, in the case of two condensates, the superpotential Eq. (11) contains two different ex-

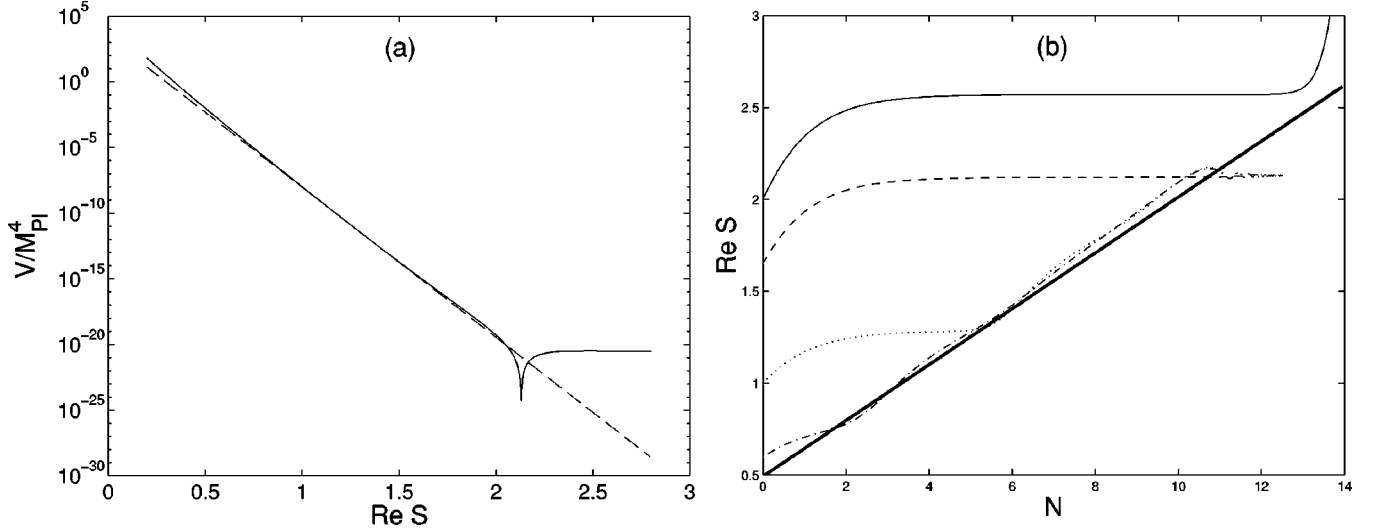


FIG. 2. (a) Solid line: plot of the scalar potential  $V$  (in logarithmic units) vs  $\text{Re } S$  for two condensates with gauge groups  $SU(6) \times SU(7)$  with  $M_1=2$  and  $M_2=8$  matter representations respectively; dashed line: our exponential approximation given by Eq. (12). (b) Evolution of  $\text{Re } S$  vs  $N$  for the two condensate potential plotted in (a) with  $H_0=1$ ,  $\gamma=4/3$ , the different initial positions being  $\text{Re } S_0=0.6$  (dot-dashes),  $\text{Re } S_0=1$  (dots),  $\text{Re } S_0=1.65$  (dashes),  $\text{Re } S_0=2$  (solid) and the initial velocities given by  $\text{Re } \dot{S}_0=\text{Re } S_0/4$ . The thick solid line represents the solution for the scaling regime Eq. (18), for the exponential potential of Eq. (12).

potentials of  $\text{Re } S$ , therefore the scalar potential Eq. (9) will have all the different terms coming from  $|W|^2$ . In particular there will be a mixture of exponential and polynomial terms in the dilaton. And finally, the differential equation for  $\phi$  written in Eq. (1) is meant to be obeyed by a canonically normalized field, and  $\text{Re } S$  is not. As it is well known, in the SUGRA Lagrangian the kinetic terms for scalar fields are given by  $K_i^j D_\mu \phi_i D^\mu \bar{\phi}^j$ , which in the case of  $\text{Re } S$  introduces an extra factor of  $1/(2 \text{Re } S)^2$ . Therefore, the correct procedure in order to study the evolution of the dilaton in an expanding Universe would be to solve the system in terms of the canonically normalized field  $\phi$ , with  $\text{Re } S \equiv e^\phi$  or, alternatively, to modify Eq. (1) accordingly in order to account for the non canonical kinetic terms.

Fortunately these problems can be overcome. First we note that, even though the two condensates in Eq. (11) have to be carefully fine-tuned to produce a minimum for  $\text{Re } S$  at the right value, for most of the evolution towards such a minimum only one of them (the one with a smaller  $\alpha_i$  value) will dominate. For simplicity, we also keep the matter fields constant at their minimum value  $A_{i \min} = (3N_i/d_i M_i)^{-2/\beta_i} \exp(-2\alpha_i S_{\min}/\beta_i)/\eta(T)^2$  during the evolution of  $\text{Re } S$ . Therefore in the region we are studying, the superpotential can be approximated by, for example, the first condensate; moreover it is also easy to show that, in Eq. (9), the term proportional to  $\partial W/\partial A_1$  dominates among those within the brackets. In conclusion, our scalar potential can be very well represented in the region before the minimum by the following expression:

$$V = \left| \frac{d_1 M_1 A_{1 \min}^{(M_1 - N_1)/N_1}}{\sqrt{6} N_1 (2 \text{Re } T) \eta(T)^{\beta_1}} \right|^2 e^{-2\alpha_1 \text{Re } S}, \quad (12)$$

which corresponds to considering only the dominant term mentioned above and setting  $\text{Re } S=1$  everywhere but in the

exponential. The result of such an approximation can be seen in Fig. 2(a), represented by the dashed line (with  $\text{Re } T=1.2$ , its value at the minimum), and it is good enough to justify our use of Eq. (12) when studying the evolution of  $\text{Re } S$  away from the minimum.

Concerning the second problem, that of  $\text{Re } S$  not being a canonically normalized field, we start by showing the exact numerical result of the evolution of the  $\phi$  field, plotted in Fig. 2(b) in terms of  $\text{Re } S = e^\phi$  versus  $N$ , for a radiation dominated Universe (i.e.  $\gamma=4/3$ ) and initial conditions  $H_0=1$  and  $\text{Re } \dot{S}_0 = \text{Re } S_0/4$ . The different lines correspond to different initial conditions for  $\text{Re } S$ . It is remarkable how the behavior in the first stages of the evolution is very similar to that shown in Fig. 1 for a pure exponential potential, the difference appearing after  $N \sim 11$  e-foldings and due to the presence of a minimum in this case. Depending on the initial position of the field it may or may not fall in the minimum, but what is clear is that if the field reaches the scaling regime the former will certainly happen, and that occurs, as we can see, for a very wide range of values of  $\text{Re } S_0$ , contradicting the general belief that only for a very narrow range of initial values, all around the minimum, would the dilaton settle at its minimum. In fact the top curve of Fig. 2(b), which is the only one that does not end up at the minimum, corresponds to an asymptotic value of the field  $\text{Re } \tilde{S}_0 \equiv e^{\tilde{\phi}_0}$  beyond the maximum of the potential.

We have checked that the scaling solution shown in Fig. 2(b) is very well represented by Eq. (8) in the case of the potential being given by Eq. (12), i.e.  $\lambda=2\alpha_1$  and  $V_0 = |d_1 M_1 A_{1 \min}^{(M_1 - N_1)/N_1} / \sqrt{6} N_1 (2 \text{Re } T) \eta(T)^{\beta_1}|^2$ , that is, as if  $\text{Re } S$  were a canonically normalized field. This approximate solution corresponds to the thick solid line shown in Fig. 2(b).

We turn our attention to explaining this scaling behavior of the non canonical dilaton field by trying to solve Eq. (1) analytically for this case. In terms of the normalized dilaton,  $\phi$ , the equations to solve are

$$\begin{aligned}
 x'_\phi &= -3x_\phi + \lambda e^\phi \sqrt{\frac{3}{2}} y_\phi^2 + \frac{3}{2} x_\phi [2x_\phi^2 + \gamma(1 - x_\phi^2 - y_\phi^2)] \\
 y'_\phi &= -\lambda e^\phi \sqrt{\frac{3}{2}} x_\phi y_\phi + \frac{3}{2} y_\phi [2x_\phi^2 + \gamma(1 - x_\phi^2 - y_\phi^2)] \\
 H' &= -\frac{3}{2} H [2x_\phi^2 + \gamma(1 - x_\phi^2 - y_\phi^2)],
 \end{aligned} \tag{13}$$

where  $x_\phi$  and  $y_\phi$  are the  $x$  and  $y$  of Eq. (3). Note that the presence of a more involved potential requires the replacement  $\lambda \rightarrow \lambda e^\phi$ . We can proceed as in the case of the pure exponential and solve for the two different stages defined before. Solving for Stage I is trivial, as the key point in this regime is to neglect any dependence on the potential and therefore the solution (for  $\phi$ ) is identical to that in Eq. (6). Then

$$\begin{aligned}
 \text{Re } S_I &= \text{Re } \tilde{S}_0 \left( \frac{x_0}{\sqrt{1-x_0^2}} e^{-(3/2)(2-\gamma)N} \right. \\
 &\quad \left. + \sqrt{1 + \frac{x_0^2}{1-x_0^2} e^{-3(2-\gamma)N}} \right)^{-\sqrt{8/3}(2-\gamma)}, \tag{14}
 \end{aligned}$$

where  $\text{Re } \tilde{S}_0 \equiv e^{\tilde{\phi}_0}$ , with  $\tilde{\phi}_0$  given by Eq. (7). However solving for the scaling regime of Stage II is much more complicated. To begin with, the form of the potential and the range of values of  $\phi$  we are interested in guarantees that  $x, y \ll 1$ , and therefore we can solve for  $H$ , obtaining the usual result  $H = H_0 e^{-3\gamma N/2}$ , once again indicating that the background fields are determining the evolution of the Universe. As for the other two equations, let us rewrite them in terms of  $\text{Re } S$  and a new variable  $x_S \equiv \text{Re } S' / \sqrt{6}$  (note that  $y_S = y_\phi$ ). Again in the same approximation of small  $x, y$  we have for the two first equations

$$\begin{aligned}
 x'_S &= -\frac{3}{2}(2-\gamma)x_S + \lambda \sqrt{\frac{3}{2}} (\text{Re } S)^2 y_S^2 \\
 y'_S &= -\lambda \sqrt{\frac{3}{2}} x_S y_S + \frac{3}{2} \gamma y_S.
 \end{aligned} \tag{15}$$

Obviously, for the scaling regime observed in Fig. 2(b) (i.e. Stage II) we must have  $x'_S = y'_S = 0$ . The second equation in Eq. (15) will give us then the expected solution for  $x_S$ , analogous to the pure exponential case

$$x_S^c = \sqrt{\frac{3}{2}} \frac{\gamma}{\lambda}, \tag{16}$$

whereas from the first equation we find that

$$y_S^2 = \frac{3(2-\gamma)\gamma}{2\lambda^2} \frac{1}{(\text{Re } S)^2}, \tag{17}$$

that is,  $y_S$  does not seem to reach a critical value but instead has a dependence on  $(\text{Re } S)^{-2}$ . However, given the size of  $\lambda$  ( $\geq 20$ ) and the range of values we are considering for  $\text{Re } S$  (between 0.3 and 2), the deviation of  $y_S$  from its expected critical value, given by  $y_S^c = \sqrt{3(2-\gamma)\gamma/(2\lambda^2)}$ , is not going to be significant, as it is obvious from Fig. 2(b). In any case let us compute this correction which modifies the pure scaling result by a factor of  $\epsilon(N)$ . Our ansatz is then

$$\text{Re } S_{II} = \frac{3\gamma}{\lambda} N + \frac{1}{\lambda} \ln \left( \frac{2V_0\lambda^2}{9H_0^2(2-\gamma)\gamma} \right) + \epsilon(N). \tag{18}$$

Substituting into Eq. (17) and using the definition of  $y_S$  [ $\equiv \sqrt{V_0} e^{-(\lambda/2)\text{Re } S} / (\sqrt{3}H)$ ] and the solution for  $H$  we obtain

$$\epsilon(N) = -\frac{2}{\lambda} \ln \left[ \frac{3\gamma}{\lambda} N + \frac{1}{\lambda} \ln \left( \frac{2V_0\lambda^2}{9H_0^2(2-\gamma)\gamma} \right) \right], \tag{19}$$

which is indeed a very small numerical correction to the standard result [in fact  $x'_S = \epsilon''(N) \sim 0$ ], and the only noticeable effect of having solved for a scalar field with an exponential potential but non minimal kinetic terms, a very encouraging result.

The pure exponential approximation will eventually break down as the dilaton approaches its minimum,  $\text{Re } S_{\min}$ . As can be seen from Fig. 2(b), if the field is in its scaling regime, it will not have enough energy to go over the maximum of the potential. Instead, it will oscillate around the minimum with an exponentially damped amplitude, settling down quickly to its final value. A simple estimation of the number of e-foldings needed to reach the minimum can then be obtained from the scaling solution Eq. (18) by equating  $\text{Re } S_{II}$  to  $\text{Re } S_{\min}$ . One obtains

$$N_{\min} = \frac{1}{3\gamma} \left[ \lambda \text{Re } S_{\min} - \ln \left( \frac{2V_0\lambda^2}{9H_0^2(2-\gamma)\gamma} \right) \right] \tag{20}$$

(where we have ignored the  $\epsilon$  correction as this result is accurate enough). Therefore once we have defined the example we are working with and the background, we will have  $N_{\min}$ . In fact, with this result we can also calculate  $H_{\min}$ ,

$$H_{\min} = H_0 e^{-(3/2)\gamma N_{\min}} = \sqrt{\frac{2V_0\lambda^2}{9(2-\gamma)\gamma}} e^{-(\lambda/2)\text{Re } S_{\min}}, \tag{21}$$

which, remarkably enough, is independent of  $H_0$ .

We have estimated  $N_{\min}$  and  $H_{\min}$  in a radiation dominated Universe ( $\gamma = 4/3$ ) for a number of hidden sector gauge groups with  $\text{Re } S \sim 2$  and  $m_{3/2} \sim 1$  TeV, obtaining the almost invariant result

$$N_{\min} \sim 11 \tag{22}$$

$$H_{\min} \sim 5.10^{-10} M_P.$$

Assuming a radiation dominated Universe, where  $H \propto T^2/M_P$ , this implies  $T_{\min} \sim 10^{13}$  GeV. The invariance of

this result can be more or less understood by rewriting Eq. (20) in terms of the gravitino mass and the gauge group factors of the two condensates. The requirement of the model to be phenomenologically viable fixes most of those parameters leading to a constant value for  $N_{\min}$  and, therefore,  $H_{\min}$  and  $T_{\min}$ .

From these results it is possible to estimate the range of initial conditions that will eventually lead to a stable dilaton. As long as the field enters the scaling solution before reaching its minimum, we can ensure that it will be stabilized. For this to happen, a sufficient condition is to take an initial value  $\text{Re } S_0$  between the scaling solution at  $N=0$  [i.e. the constant term in Eq. (18)] and  $\text{Re } S_{\min}$ , and an initial velocity such that the asymptotic solution  $\text{Re } \bar{S}_0$  in Eq. (14) is smaller than  $\text{Re } S_{\min}$ . Namely we obtain the following bounds for  $\text{Re } S_0$  and  $x_0$ :

$$\frac{1}{\lambda} \ln \left( \frac{2V_0 \lambda^2}{9H_0^2(2-\gamma)\gamma} \right) < \text{Re } S_0 < \text{Re } S_{\min} \quad (23)$$

$$0 < x_0 < \tanh \left[ \frac{\sqrt{6}}{4} (2-\gamma) \ln \left( \frac{\text{Re } S_{\min}}{\text{Re } S_0} \right) \right].$$

Some examples can be seen in Fig. 2(b). Note that this region gets smaller for decreasing  $H_0$ , as expected. There will also be a few initial conditions outside these limits that will still lead to a stable dilaton, such as initial values between the maximum and the minimum, values close to the lower bound on  $\text{Re } S_0$  or allowing for negative initial velocities. Still, even if we do not take these into account, it is clear that there is a sizable region in parameter space that allows the dilaton to evolve to its minimum and stay there.

A further possibility is to consider an inflationary scenario. This will correspond to the case of a very small (and changing)  $\gamma$ . Note that it is the barotropic fluid that is producing inflation, so that we are not considering any kind of dilaton driven inflation. We can see from Eq. (18) that for  $\gamma \approx 0$  the scaling solution is practically horizontal. It would take a large amount of e-foldings for this solution to reach the minimum of the potential. Furthermore, for some models of inflation,  $\gamma$  would be rapidly changing and one does not expect the field to follow the scaling solution exactly in these cases [10]. Nevertheless, a few e-foldings of inflation can open up a large region of parameter space. Unless the energy density is completely dominated by the dilaton, the almost constant  $H$  of an inflationary scenario will quickly freeze the field at a constant value that will then lead to a scaling solution during reheating ( $\gamma=1$ ). The bound for  $x_0$  in Eq. (23) still applies, but is maximized for  $\gamma=0$ . One then expects most of the region of parameter space to evolve to a stabilized dilaton if there is an initial period of inflation (even a small number of e-foldings is sufficient).

### B. One condensate with $K_{np}$

In this section we will perform the same analysis of the cosmological evolution of the dilaton field for a different class of models proposed more recently [3–5]. These consist of a single condensate which gets stabilized by the presence

of nonperturbative corrections to the Kähler potential, which form has been suggested in [14]. Therefore we are dealing now with a scalar potential given again by Eq. (9), where the superpotential is simply

$$W = \frac{C}{\eta(T)^6} e^{-\alpha \text{Re } S}, \quad (24)$$

with  $\alpha = 8\pi^2/N$  for a  $SU(N)$  group and  $C = -N/(32\pi^2 e)$ . The Kähler potential is now more involved,  $K = K_0 + K_{np}$ , where  $K_0$  is defined by Eq. (10) (with an overall modulus) and the nonperturbative correction is parameterized as

$$K_{np} = \frac{D}{B\sqrt{\text{Re } S}} \log(1 + e^{-B(\sqrt{\text{Re } S} - \sqrt{S_0})}), \quad (25)$$

that is in terms of three constants  $S_0$ ,  $D$ , and  $B$ , the first of which just determines the value of  $\text{Re } S$  at the minimum. Therefore this description is effectively made in terms of only  $D$  and  $B$ , which are positive numbers. It has been shown [5] that for a wide range of values of both parameters it is possible to generate a minimum at  $S_0$  with zero cosmological constant and  $T=1$ . The shape of these potentials is again very similar to that of the two condensate models.

Another feature of this ansatz for the nonperturbative corrections is that it is very well approximated in the region  $\text{Re } S < S_0$  by the following expression:

$$e^{K_{np}} = e^{-D(\sqrt{\text{Re } S} - \sqrt{S_0})/\sqrt{\text{Re } S}}, \quad (26)$$

and therefore the scalar potential is given by

$$V = \frac{e^{-D(1 - \sqrt{S_0}/\text{Re } S)}}{2 \text{Re } S} \times \left[ \frac{4(1 - D\sqrt{S_0}/\text{Re } S + 2\alpha \text{Re } S)^2}{4 + 3D\sqrt{S_0}/\text{Re } S} - 3 \right] |W|^2. \quad (27)$$

In this second example, even though we have a single condensate to start with, the dependence of the potential upon  $\text{Re } S$  is much more complicated, as can be expected from the form of the nonperturbative corrections to  $K$ , given by Eq. (26). To be able to write Eq. (27) in the usual exponential form  $V = V_0 e^{-\lambda \text{Re } S}$  we set  $\text{Re } S = 1$  everywhere except in the exponents for which we use a linear fit. We obtain

$$V \approx e^D \frac{2(D\sqrt{S_0} + 2\alpha)^2}{(4 + 3D\sqrt{S_0})} \left( \frac{C}{\eta(T)^6} \right)^2 e^{-(2\alpha + D/S_0)\text{Re } S}, \quad (28)$$

that is a pure exponential, where the exponent depends on the value of  $D$ . As it was mentioned before, for a given hidden gauge group, there exist a series of values of  $(D, B)$  for which  $V$  has a minimum at  $S_0$  with zero cosmological constant. That means that the cosmological evolution of a particular hidden sector interaction will be different depending on which values of the pair  $(D, B)$  we are considering.

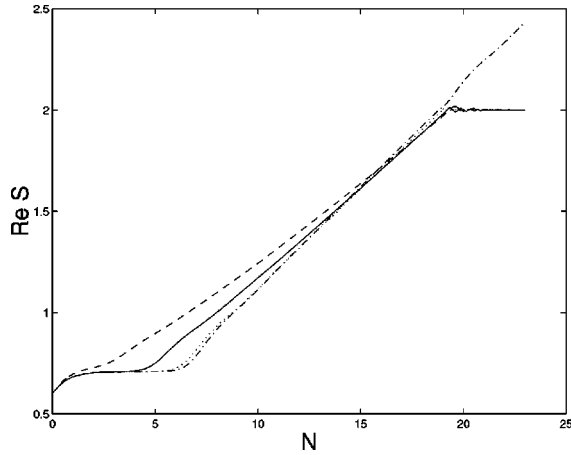


FIG. 3. Evolution of  $\text{Re } S$  vs  $N$  for the potential of Eq. (27) with gauge group  $SU(5)$  and initial conditions  $H_0=1$ ,  $\gamma=1$ ,  $\text{Re } \dot{S}_0=0.15$ ,  $\text{Re } S_0=0.6$ . The different lines correspond to different values for  $D$  in Eq. (25):  $D=1$  (dash-dotted),  $D=3$  (dotted),  $D=10$  (solid) and  $D=20$  (dashed).

This can be clearly appreciated in Fig. 3 where we plot the cosmological evolution of the dilaton field versus the number of e-foldings  $N$  for the same gauge group,  $SU(5)$ , and initial conditions  $H_0=1$ ,  $\gamma=1$ ,  $\text{Re } \dot{S}_0=0.15$ ,  $\text{Re } S_0=0.6$ .

Depending on the value of  $D$  (the corresponding  $B$  is fixed in order to have a zero cosmological constant at the minimum given by  $S_0$ ) we see that the field will fall into the minimum provided that this minimum is not too fine-tuned. This is determined by the magnitude of  $D$ , as was described in [5]; the smaller it is the bigger the value of  $B$  (and the amount of fine-tuning in the potential) is to set  $V=0$  at the minimum. Therefore in this particular case we see how for  $D \leq 1$  the minimum becomes too fine-tuned and is unable to stop the field from rolling past the maximum. Unfortunately these small  $D$  solutions are precisely the ones which correspond to a largest hierarchy between the dilaton and the gravitino masses [5], and therefore provide us with a solution to the ‘‘moduli problem’’ for this field. However, for the range of values of  $D$  which give a satisfactory stabilization of the dilaton we can generate ratios between those two masses of up to 300 which would, in most cases, be more than enough to solve the above-mentioned problem. Changing the value of  $\gamma$  would correspond to a change in the minimal value of  $D$  from which onwards the dilaton would fall in the minimum. That is, a bigger (smaller) value of  $\gamma$  corresponds to a bigger (smaller) value of the minimal  $D$ , and therefore to a smaller (bigger) hierarchy between the dilaton and gravitino masses.

A similar analysis to the one performed in the previous section for the two condensates model can be done here concerning the analytical solutions of the evolution equations [for which we use Eq. (28)]. Solving for Stage I is identical as before [see Eq. (14)], as the system of differential equations given by Eq. (3) does not depend on the characteristics of the potential in this regime. For Stage II we can obtain the analogue of Eq. (18) and in this case it is defined by  $V_0 = e^{D/2} (D\sqrt{S_0} + 2\alpha)^2 C^2 / [(4 + 3D\sqrt{S_0}) \eta^{12}]$  and  $\lambda$

$= 2\alpha + D/S_0$ . Expressions for the number of e-foldings needed to reach the minimum,  $N_{\min}$  as well as for  $H_{\min}$ , are also obtained by replacing in Eqs. (20), (21) the current expressions for  $\alpha$  and  $V_0$ . Once again, the fact that we are imposing a consistent phenomenology (i.e.,  $\text{Re } S \sim 2$  and  $m_{3/2} \sim 1$  TeV) at low energies implies that the numerical values of  $N_{\min}$  and  $H_{\min}$  are very similar to those obtained in the previous section. In particular we can express  $N_{\min}$  as

$$N_{\min} = \frac{-2}{3\gamma} \left[ \ln \left( \frac{m_{3/2}}{M_P} \right) + \ln \left( \frac{(D\sqrt{S_0} + 2\alpha)2\alpha}{\sqrt{(4 + 3D\sqrt{S_0})}} \right) - \ln \left( \frac{3H_0\sqrt{(2-\gamma)\gamma}}{2\sqrt{2}} \right) \right]. \quad (29)$$

For the example shown in Fig. 3, namely  $\gamma=1$ ,  $H_0=1$ , we obtain, to a very good approximation, the almost invariant value  $N_{\min} \sim 19$  which can be translated into  $H_{\min} \sim 10^{-13} M_P$ .

#### IV. CONCLUSIONS

We have examined the issue of the cosmological evolution of the dilaton field in gaugino condensation models of SUSY breaking. By studying the behavior of a scalar field  $\phi$  with an exponential potential, we have been able to obtain analytic expressions for its time evolution in the presence of the Hubble parameter and a dominating background consisting of other matter fields. It turns out that under such circumstances the field  $\phi$  tends to enter a scaling regime defined by constant values of both the potential and the velocity of the field relative to the expansion of the Universe, provided that this potential is steep enough.

Encouraged by these promising results we turned to apply them to the dilaton field in two particular models of SUSY breaking: multiple gaugino condensation (or racetrack) and one condensate with nonperturbative corrections to the Kähler potential. However two major differences arise with respect to the ideal case we had dealt with before: the shape of these potentials is not exactly exponential and the dilaton field is not canonically normalized. We have shown that these two problems can be easily overcome, and we have found very accurate analytic approximations to explain our numerical results. In both cases it is possible, contrary to the general belief, to stabilize the dilaton at its minimum for a large range of initial conditions (i.e., initial values for its position and velocity with  $\gamma \sim 1$ ) despite the steepness of the potentials in their strong coupling regime. A previous period of inflation (i.e.,  $\gamma \sim 0$ ) opens up even more the region of parameter space leading to a stable dilaton. Moreover, the number of e-foldings required to do so and the value of the Hubble parameter at the minimum seem to be fixed by the requirement of having a successful phenomenology at low energies.

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- [1] J. P. Derendinger, L. E. Ibáñez and H. P. Nilles, Phys. Lett. **155B**, 65 (1985); M. Dine, R. Rohm, N. Seiberg and E. Witten, *ibid.* **156B**, 55 (1985).
- [2] B. de Carlos, J. A. Casas and C. Muñoz, Nucl. Phys. **B399**, 623 (1993).
- [3] J. A. Casas, Phys. Lett. B **384**, 103 (1996).
- [4] P. Binétruy, M. K. Gaillard and Y.-Y. Wu, Nucl. Phys. **B493**, 27 (1997); Phys. Lett. B **412**, 288 (1997).
- [5] T. Barreiro, B. de Carlos and E. J. Copeland, Phys. Rev. D **57**, 7354 (1998).
- [6] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B **318**, 447 (1993); T. Banks, D. B. Kaplan and A. E. Nelson, Phys. Rev. D **49**, 779 (1994).
- [7] R. Brustein and P. J. Steinhardt, Phys. Lett. B **302**, 196 (1993).
- [8] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).
- [9] P. G. Ferreira and M. Joyce, Phys. Rev. Lett. **79**, 4740 (1997).
- [10] E. J. Copeland, A. Liddle and D. Wands, Phys. Rev. D **57**, 4686 (1998).
- [11] N. V. Krasnikov, Phys. Lett. B **193**, 37 (1987); L. Dixon, in *Proceedings of the A.P.S. Meeting*, Houston, 1990 (World Scientific, Singapore, 1990); J. A. Casas, Z. Lalak, C. Muñoz and G. G. Ross, Nucl. Phys. **B347**, 243 (1990); T. Taylor, Phys. Lett. B **252**, 59 (1990).
- [12] E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Nucl. Phys. **B212**, 413 (1983).
- [13] A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B **245**, 401 (1990); S. Ferrara, N. Magnoli, T. Taylor and G. Veneziano, *ibid.* **245**, 409 (1990).
- [14] S. H. Shenker, *Proceedings of the Cargèse School on Random Surfaces, Quantum Gravity and Strings*, Cargèse (France), 1990 (Plenum, New York, 1991); T. Banks and M. Dine, Phys. Rev. D **50**, 7454 (1994).