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Cosmological perturbations and the reionization epoch

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ABSTRACT

We investigate the dependence of the epoch of reionization on the properties of cosmological perturbations, in the context of cosmologies permitted by *WMAP*. We compute the redshift of reionization using a simple model based on the Press–Schechter approximation. For a power-law initial spectrum we estimate that reionization is likely to occur at a redshift $z_{\rm reion} = 17^{+10}_{-7}$, consistent with the *WMAP* determination based on the temperature-polarization cross power spectrum. We estimate the delay in reionization if there is a negative running of the spectral index, as weakly indicated by *WMAP*. We then investigate the dependence of the reionization redshift on the nature of the initial perturbations. We consider chi-squared probability distribution functions with various degrees of freedom, motivated both by non-standard inflationary scenarios and by defect models. We find that in these models reionization is likely to occur much earlier, and to be a slower process, than in the case of initial Gaussian fluctuations. We also consider a hybrid model in which cosmic strings make an important contribution to the seed fluctuations on scales relevant for reionization. We find that in order for that model to agree with the latest *WMAP* results, the string contribution to the matter power spectrum on the standard 8 h^{-1} Mpc scale is likely to be at most at the level of 1 per cent, which imposes tight constraints on the value of the string mass per unit length.

Key words: cosmic microwave background – cosmology: theory.

1 INTRODUCTION

The reionization history of the Universe gives important insight into the epoch of structure formation in the Universe, with the initial reionization believed to have been caused by light from the first generation of massive stars (Couchman & Rees 1986; Cen & Ostriker 1992; Fukugita & Kawasaki 1994; Tegmark, Silk & Blanchard 1994). Over recent years, increasingly sophisticated techniques have been employed to estimate the reionization epoch in cosmological models (Haiman & Loeb 1997; Miralda-Escudé, Haehnelt & Rees 2000; Cen 2002; Fukugita & Kawasaki 2003; Somerville, Bullock & Livio 2003; Wyithe & Loeb 2003; see Barkana & Loeb 2001 for a review).

Two important observations have begun to shed light on the reionization epoch, while painting a somewhat contradictory picture. The detection of Gunn–Peterson troughs (Gunn & Peterson 1965) in the absorption spectra of distant quasars suggest a very late reionization for the Universe, at a redshift $z \sim 6$ (Becker et al. 2001; Fan et al. 2003; White et al. 2003). By contrast, the measurement by the *WMAP* satellite of temperature–polarization correlations in the microwave background strongly suggests a higher optical depth from reionized electrons, implying that reionization took place at a higher redshift in the range from $z = 11$ to $z = 30$ (Kogut et al. 2003). It has been argued (Hui & Haiman 2003) that such a high redshift of reionization would lead to an intergalactic medium (IGM) temperature at $z \simeq 3$ which is lower than observations permit. In combination, these results suggest that the full history of ionization might be quite complicated (Cen 2003; Ciardi, Ferrara & White 2003; Hui & Haiman 2003) with more than one source of ionizing radiation.

Until recently, one of the key uncertainties in determining the reionization epoch was the choice of cosmological model, which determines the time of initial structure formation leading to the production of ionizing radiation. For example, Liddle & Lyth (1995) carried out the first survey of cosmological parameter space and found a wide range of predictions for different models compatible with then-existing observations. These uncertainties have largely been removed by the establishment of a standard cosmological model based on a spatially flat low-density Universe with inflationary perturbations. However, there remain significant uncertainties in the reionization epoch from the nature of the seed fluctuations. We will study two effects, one being a change in the short-scale power spectrum (due either to a running spectral index from inflation or from a topological defect contribution), and the other the effect of primordial non-Gaussianity.

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It is not our intention in this paper to provide a detailed modelling of the reionization history, but rather to introduce a highly simplified model for reionization which allows us to assess how the epoch of reionization is modified if the cosmological model is altered. Our model is based on the Press–Schechter approach, following a strategy introduced by Tegmark et al. (1994). Despite its simplicity, it makes predictions for the reionization epoch which are in broad agreement with those from deeper investigations using *N*-body/smoothed particle hydrodynamics (SPH) simulations (Cen 2002, 2003; Ciardi et al. 2003) or more detailed semi-analytical modelling (Fukugita & Kawasaki 1994, 2003; Benson et al. 2001; Chiu, Fan & Ostriker 2003; Wyithe & Loeb 2003).

The model we use is by some way the simplest in the literature, and one might ask whether it is really adequate in the light of the high level of sophistication commonly seen in recent papers. One answer is that it certainly would not be adequate if we were aiming to predict the reionization epoch accurately *assuming* a particular cosmological model. For instance, attempts to assess the reionization history in the concordance model, for example, require more sophisticated approaches than ours. However that is not the purpose of our paper, which is to point out that there may be large effects coming from non-standard assumptions on the initial power spectrum, which are still permitted by present observational data. The effects from these variations can be much larger than those from accurate astrophysical modelling, as we will see. Further, one expects a good decoupling of the effects of power-spectrum modelling from the detailed implementation of astrophysics, so that our model allows us to isolate in a simple way the most important cosmological factors which affect cosmic reionization. The former tells us when the early structures begin to form, and to an acceptable approximation for our purposes the subsequent astrophysics is the same, however it is modelled. For the purpose of surveying cosmological models, we therefore believe that our simple approach is well justified, and whenever accurate calculations are available for the standard cosmology in future, our results will enable a scaling to models with different seed perturbations.

2 THE REDSHIFT OF REIONIZATION

In the context of cold dark matter (CDM) models the first stars form in low-mass haloes ($\sim 10^6\,\mathrm{M}_\odot$) at high redshift. The precise computation of the reionization redshift requires knowledge of the fraction $f_{\rm reion}$ of baryons that must be bound in those haloes in order to bring the fraction of ionized gas close to 100 per cent. This parameter is highly uncertain since it depends on several astrophysical quantities, some of which are poorly known. Here we shall use the estimates of $f_{\rm reion}$ derived by Tegmark et al. (1994). They computed the fraction of the intergalactic medium that is ionized as a product of several factors, namely: the fraction of baryons in non-linear structures, the number of ultraviolet (UV) photons emitted into the IGM per proton in non-linear structures and the net ionizations per emitted UV photon. They concluded that the most reasonable estimate for the collapsed fraction required to induce complete reionization is $f_{\rm reion} \sim 8 \times 10^{-3}$. This estimate seems to be consistent with recent more detailed investigations using large cosmological *N*-body/SPH simulations, where radiative transfer calculations were carried out by embedding massive Population III stars in gas clouds (Yoshida, Sokasian & Hernquist 2003). Tegmark et al. (1994) also computed conservative lower ($f_{\rm reion} \sim 4 \times 10^{-5}$) and upper ($f_{\rm reion} \sim 0.8$) limits for $f_{\rm reion}$ (in their terminology referred to as 'optimistic' and 'pessimistic') reflecting the large uncertainties associated with this parameter. We shall use these estimates in the following sections in order to estimate the redshift of reionization $z_{\rm reion}$ for various models.

The choice of $10^6\,\mathrm{M}_\odot$ as the threshold mass is clearly an order-of-magnitude one, motivated by cooling conditions from H and H_2 but not attempting to implement complex cooling criteria accurately. In order to confirm that this precise choice does not affect our results, we repeated all the calculations in this paper using a higher threshold mass of $10^7\,\mathrm{M}_\odot$. We found that the results shown in Figs 1–3 below are hardly changed at all, confirming that the predicted reionization redshift is weakly dependent on the threshold halo mass (and the effect of the change is even less if we are interested only in the shift in reionization epoch as we change other assumptions).

Despite the large uncertainties associated with the determination of the collapsed baryon fraction necessary to reionize the Universe, in the context of models with initial Gaussian fluctuations this fraction is exponentially sensitive to the dispersion of the density field and hence to the redshift. This means that a large uncertainty in $f_{\rm reion}$ translates into a smaller uncertainty in the redshift of reionization, $z_{\rm reion}$. However, we will see that this may not be the case when we consider non-Gaussian fluctuations.

3 THE MASS FRACTION

We shall use the Press–Schechter approximation (Press & Schechter 1974) to compute the mass fraction, $f(>M)$, associated with collapsed objects with mass larger than a given mass threshold M. This was originally proposed in the context of initial Gaussian density perturbations and was much later generalized to accommodate non-Gaussian initial conditions (Chiu, Ostriker & Strauss 1998). This generalization of the original Press–Schechter approximation has successfully reproduced the results obtained from *N*-body simulations with non-Gaussian initial conditions (Robinson & Baker 2000). However, it has been shown by Avelino & Viana (2000) that this generalization does not adequately solve the cloud-in-cloud problem of Press–Schechter theory and that deviations from the mass function predicted by this approach in the rare-events regime are expected (see also Inoue & Nagashima 2002). Nevertheless, given the large uncertainties in $f_{\rm reion}$, these deviations will have a small impact on our final results and we shall not consider them further in the following analysis.

In this context the mass fraction, $f(>M)$, is assumed to be proportional to the fraction of space in which the linear density contrast, smoothed on the scale M, exceeds a given threshold $\delta_{\rm c}$:

$$f(>M) = A_f \int_{\delta_{\rm c}}^{\infty} \mathcal{P}(\delta)\,{\rm d}\delta. \qquad (1)$$

Here $\mathcal{P}(\delta)$ is the one-point probability distribution function (PDF) of the linear density field δ and A_f is a constant which is computed by requiring that $f(>0) = 1$, thus taking into account the accretion of material initially present in underdense regions (note that $A_f = 2$ in the case of Gaussian initial conditions).

Next we need to specify the filter function used to perform the smoothing, and the threshold value $\delta_{\rm c}$. We shall use a top-hat filter

$$W(kR) = 3\left[\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2}\right] \qquad (2)$$

where $M = 4\pi\rho_{\rm b}R^3/3$ (here $\rho_{\rm b}$ is the background matter density) and take $\delta_{\rm c} = 1.7$, motivated by the spherical collapse model and *N*-body simulations. For spherical collapse $\delta_{\rm c}$ is almost independent of the background cosmology (e.g. Eke, Cole & Frenk 1996).

We consider an initial density field with a chi-squared one-point probability distribution function (PDF) with n degrees of freedom, the PDF having been shifted so that its mean is zero (such a PDF becomes Gaussian when $n \to \infty$). We further assume that the shape of the PDF is independent of the scale and redshift under consideration, that is $\mathcal{P}_R(\delta) \equiv \mathcal{P}(R, \delta/\sigma(R, z))/\sigma(R, z)$ is always the same function. Although this is not expected to be precisely true in realistic models, in particular those motivated by topological defects (Avelino et al. 1998a), it is reasonable to expect it to be a good approximation on the range of scales which are probed by reionization.

In order to compute the mass fraction, $f(>M, z)$, associated with collapsed objects with mass larger than a given mass threshold M, as a function of redshift, one needs to compute the dispersion of the density field

$$\sigma^2(R, 0) = \int_0^\infty k^2\, |\delta_k|^2\, W^2(kR)\, \mathrm{d}k \tag{3}$$

where $|\,\delta_k|^2$ is the power spectrum. For $z \gtrsim 1$ the dispersion of the density field is simply given by

$$\sigma(R, z) = \frac{\sigma(R, 0)}{g\left(\Omega_\mathrm{m}^0, \Omega_\Lambda^0\right)(1+z)}, \tag{4}$$

where the suppression factor

$$g\left(\Omega_\mathrm{m}^0, \Omega_\Lambda^0\right) = \frac{2.5\Omega_\mathrm{m}^0}{\left(\Omega_\mathrm{m}^0\right)^{4/7} - \Omega_\Lambda^0 + \left(1 + \Omega_\mathrm{m}^0/2\right)\left(1 + \Omega_\Lambda^0/70\right)}, \tag{5}$$

accounts for the dependence of the growth of density perturbations on the cosmological parameters Ω_m^0 and Ω_Λ^0 (Carroll, Press & Turner 1992). This has also been verified to be a good approximation in the context of generic models with topological defects (Avelino & de Carvalho 1999).

4 RESULTS

Throughout, we shall adopt a cosmological model motivated by the *WMAP* results (Bennett et al. 2003). As their mild evidence for a running of the spectral index is driven by Lyman-α data, whose interpretation has proven controversial (e.g. Seljak, McDonald & Makarov 2003), we shall take as our base cosmology throughout their preferred model assuming a power-law initial spectrum (Spergel et al. 2003). The parameters are a matter density $\Omega_\mathrm{m}^0 = 0.29$, dark energy density $\Omega_\Lambda^0 = 0.71$, baryon density $\Omega_\mathrm{B}^0 = 0.047$, Hubble parameter $h = 0.72$, normalization $\sigma_8 = 0.9$, and perturbation spectral index $n_\mathrm{s} = 0.99$.

To determine the amplitude of perturbations on the short scales relevant to reionization, for inflationary perturbations we use the transfer function from Bardeen et al. (1986):

$$|\delta_k|_I^2 \propto k^{n_s}\left[\frac{\ln(1+\epsilon_0 q)}{\epsilon_0 q}\left(\sum_{i=0}^{4}(\epsilon_i q)^i\right)^{-1/4}\right]^2, \tag{6}$$

where $q = k/h\,\Gamma$, $[k] = \mathrm{Mpc}^{-1}$,

$$\epsilon = [2.34, 3.89, 16.1, 5.46, 6.71], \tag{7}$$

and

$$\Gamma = \Omega_\mathrm{m}^0 h \exp\left[-\Omega_\mathrm{B}^0\left(1 + \sqrt{2h}/\Omega_\mathrm{m}^0\right)\right] \tag{8}$$

is the shape parameter (Sugiyama 1995). We note that this transfer function was shown to be a good approximation for wavelengths small enough to come into the horizon before matter–radiation equality if $\Omega_\mathrm{B} = 0$ (Weinberg 2002). Also, although the effect of a non-zero baryon fraction is more complex than a simple rescaling of the shape parameter (Weinberg 2002), the Sugiyama correction turns out to be reasonably accurate for the values of the cosmological parameters we shall consider here.

4.1 The standard cosmological model

We begin by considering the simplest case, where the initial perturbations are Gaussian. Computing the perturbations and mass fraction as described, following Tegmark et al. (1994) and Liddle & Lyth (1995), we find a central value of $z_\mathrm{reion} = 17$, with the range from highest to lowest plausible values being from 27 down to 10. This range is in excellent agreement with the *WMAP* determination of the reionization epoch from the temperature–polarization correlation function. Based on the assumption of instantaneous and complete reionization, Kogut et al. (2003) find $z_\mathrm{reion} = 17 \pm 3\ (1\sigma)$ based on the cross-correlation alone, while a global fit to *WMAP* and other data (Spergel et al. 2003) gives $z_\mathrm{reion} = 17 \pm 4$. However this model appears in conflict with the high-redshift quasar data, and Kogut et al. (2003) show that allowing more complex ionization histories can widen the allowed range considerably (see also Cen 2003; Ciardi et al. 2003; Haiman & Holdder 2003).

The predictions above are our fiducial result, against which we will be comparing variations under different assumptions about the initial perturbations. If more detailed calculations of the reionization epoch give a different value, our estimates of the fractional variation about this value in other cosmologies should remain valid to a good approximation. For instance, a recent detailed analysis by Fukugita & Kawasaki (2003) suggests a somewhat lower value of $z_\mathrm{reion} = 13$ as the earliest possible reionization, with the reionization optical depth receiving a significant contribution before reionization is complete.

4.2 Running spectral index

While our main focus is to examine the effects of primordial non-Gaussianities, we begin with a brief exploration of the effect of a possible running of the spectral index, motivated by *WMAP*. The *WMAP* preferred value $\mathrm{d}n_s/\mathrm{d}\ln\ k = -0.031$ (Spergel et al. 2003) is much larger than anticipated from simple inflation models (Kosowsky & Turner 1995; Copeland, Grivell & Liddle 1998), and is large enough to have significant effect. The *WMAP* running model has been studied in detail by Haiman & Holder (2003) and Somerville et al. (2003); we will consider a general running.

To determine the effect of running, we define it at the scale $k = 0.0072\ \mathrm{Mpc}^{-1}$ at which the *WMAP* running cosmology has the same spectral index as the power-law fit above. A negative running then progressively reduces short-scale power, delaying reionization, while a positive running has the opposite effect. The shift in the reionization epoch is shown in Fig. 1, and is well fitted (to around 5 per cent within the region plotted) by

$$\frac{1 + z_\mathrm{reion}^\mathrm{r}}{1 + z_\mathrm{reion}^\mathrm{nr}} = 1 + 14y + 138y^2; \quad y \equiv \frac{\mathrm{d}n_s}{\mathrm{d}\ln k}, \tag{9}$$

where ‘nr’ indicates the fiducial model without running, and ‘r’ the inclusion of running. We conclude that running can systematically shift the expected redshift of reionization. For $\mathrm{d}n_s/\mathrm{d}\ln\ k = -0.031$ the shift in $1 + z_\mathrm{reion}$ is about 30 per cent, which is consistent with the shift found by Somerville et al. (2003) for the *WMAP* running model, supporting the view that our simplified model can make good estimates of the shift in reionization epoch. Since running at that level is permitted by the current data, we conclude that uncertainty in the small-scale power spectrum contributes a fractional uncertainty

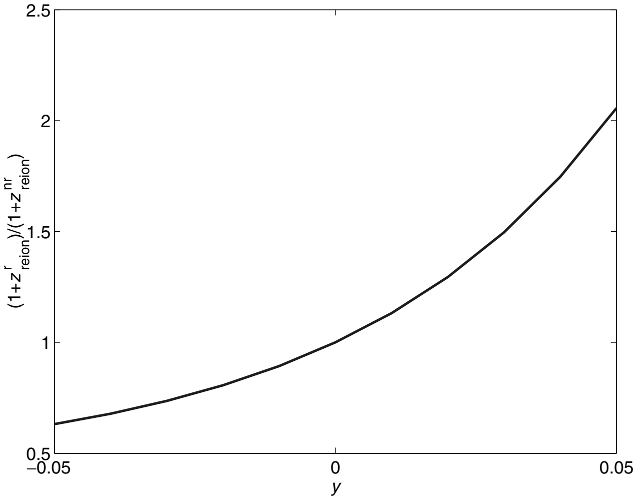

Figure 1. The shift in the expected epoch of reionization induced by the inclusion of running. Negative running, as mildly indicated by *WMAP*, reduces the redshift of reionization.

in predicting $z_{\rm reion}$ of at least that level, unless one imposes a prior restricting the effect of running.

4.3 Non-Gaussian inflation

We now consider two possible models of non-Gaussian primordial perturbations. The first is a model with primordial fluctuations which may have been generated during inflation, but featuring a chi-squared probability distribution as may appear in the context of some non-standard inflationary models (see for example Salopek 1992; Linde & Mukhanov 1997; Peebles 1999; Bartolo, Matarrese & Riotto 2001, 2002; Bernardeau & Uzan 2003).

In Fig. 2 we plot the evolution of the mass fraction $f(>10^6\ {\rm M}_\odot)$, as a function of redshift z, for chi-squared distributions with different numbers of degrees of freedom ($n = 1, 2, 4, 8, 16, 32, 64, 128,$

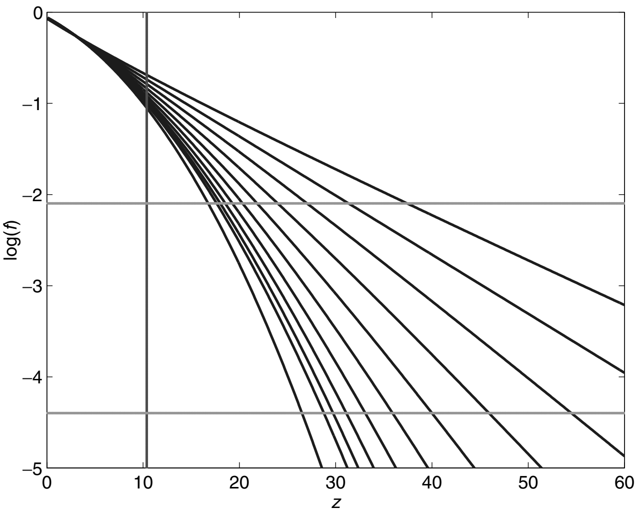

Figure 2. The evolution of the mass fraction, $f(>10^6\ {\rm M}_\odot)$, as a function of redshift z for non-Gaussian inflation, as a function of the number of chi-squared degrees of freedom ($n = 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, \infty$ from top to bottom). The intersection of mass fraction curves with the horizontal lines give the upper limit and best estimate for the redshift of reionization. The lower limit is given by the vertical line.

$256, 512, \infty$). The intersections of the mass fraction curves with the horizontal lines give the upper limit and best estimate for the redshift of reionization.

The lower limit is instead given by the vertical line. The reason is that in this case $f_{\rm reion} \sim 0.8$ is too large to be reliably predicted by Press–Schechter, so we follow Liddle & Lyth (1995) in replacing this condition by the criterion that a significant fraction of the mass has to have collapsed for reionization to occur. We quantify this assuming a crude lower estimate for the redshift of reionization given by $\sigma(10^6\ {\rm M}_\odot, z_{\rm reion}) = 1$ that leads to

$$1 + z_{\rm reion} = \sigma(10^6 {\rm M}_\odot, 0)/g\left(\Omega^0_{\rm m}, \Omega^0_\Lambda\right),$$

which is independent of the degree of non-Gaussianity of the initial density field.

We see that (except in the extreme case of the lower limit) the predicted redshift of reionization increases as the number of degrees of freedom of the chi-squared distributed PDF decreases, as does the uncertainty on the redshift of reionization. We also see that the evolution of $f(>10^6\ {\rm M}_\odot)$ with redshift is not as fast for smaller n, which indicates that reionization may be a slower process. This happens because in this case $f(>10^6\ {\rm M}_\odot)$ is less sensitive to the dispersion of the density field than in the Gaussian case.

We cannot exclude any of these models, as the lower estimate for the redshift is independent of the degree of non-Gaussianity. However, the best estimate increases as the number of degrees of freedom reduces, becoming inconsistent with *WMAP* at low n. If in the future $f_{\rm reion}$ can be accurately determined, our results will quickly indicate where the limit sets in.

We see that the non-Gaussian character of the cosmological fluctuations modifies the slope of the mass fraction curves, leading to a slower evolution of f with z in the case of density perturbations with a χ^2 (or similar) PDF, so that reionization is less well approximated as instantaneous. A simple parametrization of the reionization history of the Universe is given by

$$\chi = 10^{\beta(z_{\rm reion}-z)}, \qquad (10)$$

if $z \geqslant z_{\rm reion}$ and $\chi = 1$ if $z < z_{\rm reion}$. Here χ is the ionization fraction and β is closely related to the slope of the mass fraction curves. Preliminary results obtained using this simple model indicate that in the context of non-Gaussian fluctuations it might be possible to reconcile the high optical depth suggested by *WMAP* and a low redshift of (complete) reionization suggested by quasar data. However, a more detailed investigation using a more sophisticated model to study the reionization history of the Universe in the context of well-motivated non-Gaussian models will be made elsewhere.

4.4 Inflation plus defects

Our second non-Gaussian model is one in which both string and inflationary perturbations are present. Although standard topological defect models are completely excluded as the sole source of perturbations in the Universe, it remains plausible that they may play a subdominant role (see for example Durrer, Kunz & Melchiorri 2002 and references therein). In particular, the popular hybrid inflation scenario relies on a phase transition to end inflation, which would be predicted to produce defects at a subdominant level (Copeland et al. 1994; Contaldi, Hindmarsh & Magueijo 1999). As we shall see, reionization is a particularly powerful probe of the effects of defects. We will focus on inflation plus cosmic strings, assuming initially that the two perturbation types have equal amplitude at the standard 8 h^{-1} Mpc scale.

A good fit to the string-seeded CDM power spectrum is given by (Wu et al. 2002)

$$|\delta_k|^2_S \propto (0.7q)^{p(q)}, \tag{11}$$

where

$$p(q) = 0.9 - \frac{2.7}{1 + (2.8q)^{-0.44}}. \tag{12}$$

Given that the inflationary and string-induced perturbations are uncorrelated, we can write the combined power spectrum as

$$|\delta_k|^2_{\text{combined}} \propto \alpha\, |\delta_k|^2_I + (1-\alpha)\, |\delta_k|^2_S\,, \tag{13}$$

where α is such that inflationary and string-induced perturbations make an equal contribution to the dispersion of the density field, σ_8 on the standard 8 h^{-1} Mpc scale. On the much smaller scales relevant for reionization, the cosmic string-induced perturbations completely dominate over the inflationary ones.

The cosmic string-seeded density perturbations on a given scale R can be roughly divided into a nearly Gaussian component plus a strongly skewed non-Gaussian part generated when the string correlation length was smaller/larger than R respectively. Using simulation results for string-induced CDM perturbations, Avelino, Shellard & Wu (2000) found that the positive side of the one-point PDF was well approximated by a chi-squared distribution with the number of degrees of freedom being an increasing function of scale. Although these simulations did not have enough dynamical range to probe the scales relevant to reionization, they showed clearly that on length-scales smaller than $1.5(\Omega^0_{\rm m} h^2)^{-1}$ Mpc the perturbations seeded by cosmic strings have a strong non-Gaussian character (Avelino et al. 1998a).

Our results are shown in Fig. 3. Here, we separate the effect of the non-Gaussian nature of the string perturbations from that of the power spectrum. The power spectrum of string-induced CDM perturbations dominates on small scales due to the wake-like signature of cosmic strings leading to a small-scale power spectrum close to k^{-2}. Again the redshift of reionization increases as the number of degrees of freedom of the chi-squared distributed PDF decreases, the same effect as that obtained for non-Gaussian inflation. As previously discussed we expect the results obtained for small n to be representative of what is expected for the defect contribution (in particular cosmic strings).

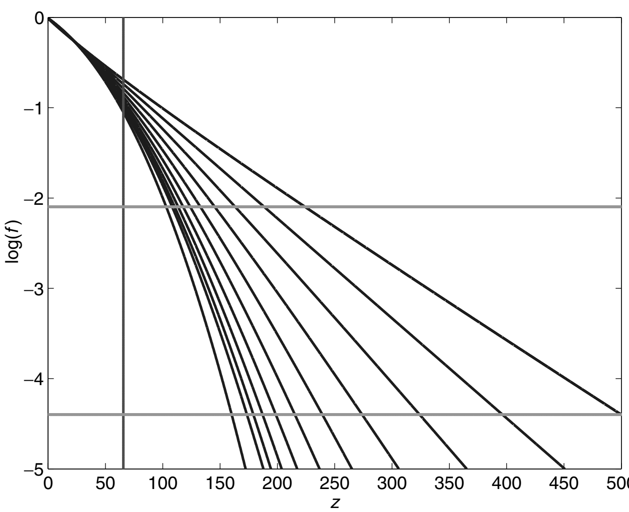

Figure 3. As Fig. 2, but for a hybrid model with both string and inflationary perturbations. The two perturbation types are assumed to have equal amplitude at the standard 8 h^{-1} Mpc scale. We treat the degree of non-Gaussianity of the combined perturbation field as a variable parametrized by $n = 1$, 2, 4, 8, 16, 32, 64, 128, 256, 512, ∞ (the number of degrees of freedom of the assumed chi-squared one-point PDF). Again, the intersection of mass fraction curves with the horizontal lines give the upper limit and best estimate for the redshift of reionization. The lower limit is given by the vertical line.

In this case we see that the much larger amplitude of the cosmic string contribution to the matter power spectrum (relative to the inflationary one) on scales of the order of 10^6 $M_\odot$ results in a much earlier reionization ($z \gtrsim 65$). Even if the perturbations were Gaussian (the lowest curve) we have very early reionization due to the different shape of the power spectrum, with non-Gaussianity further exacerbating this. The predicted reionization epoch is completely incompatible with the observed microwave anisotropies, easily excluding models where the defects produce half the power at 8 h^{-1} Mpc. We note that in this case the string perturbation spectrum has $\sigma^S_8 = \sigma_8/\sqrt{2} \sim 0.64$ which is close to the expected value for a string mass per unit length $G\,\mu \sim 10^{-6}$) (Avelino et al. 1998b; Wu et al. 2002). Assuming the best guess for $f_{\rm reion}$, reionization seems to occur about a factor of 10 earlier in redshift in the case of the hybrid model considered above (the exact number depending on the degree of non-Gaussianity on the scales relevant for reionization). Accordingly, in order to restore viability with the data the string contribution has to be significantly lowered, at least by a factor of 10. This indicates that the string contribution to the matter power spectrum on the standard 8 h^{-1} Mpc would have to be at most at the level of 1 per cent, in which case an upper limit to the string mass per unit length would be $G\mu \lesssim 10^{-7}$. However we note that this result is sensitive to the highly uncertain value of $f_{\rm reion}$. Still, we can clearly see that reionization imposes very strong constraints on any contribution from defects to the seed fluctuation spectrum. We expect our overall results to remain valid at some level in the context of other defect models, as the small-scale non-Gaussianity and excess power are generic predictions of models of this type (Pen, Spergel & Turok 1994).

5 CONCLUSIONS

We have shown that in a model with initial Gaussian fluctuations reionization is expected to occur at a redshift consistent with the recent *WMAP* results, while running of the spectral index within the range permitted by observations leads to a significant uncertainty in predicting the reionization redshift. Although this result does not leave much room for non-Gaussian perturbations, we have shown that even a small level of non-Gaussianity may have interesting consequences. In the case of an initial density field with a chi-squared PDF, motivated both by some non-standard models of inflation and topological defects, reionization is likely to be a slower process. We also show that the reionization history of the Universe imposes strong constraints on the energy scale of defects given their ability to influence it through the induced small-scale excess power and non-Gaussianity. We estimate that in the case of a hybrid model with cosmic strings the string contribution to the total power spectrum on the standard 8 h^{-1} Mpc is likely to be at most at the level of 1 per cent thus requiring a low value of the string mass per unit length ($G\mu \lesssim 10^{-7}$).

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