

Anomaly mediation and cosmology

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Anomaly mediation and cosmology

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ABSTRACT: We consider an extension of the MSSM wherein anomaly mediation is the source of supersymmetry-breaking, and the tachyonic slepton problem is solved by a gauged $U(1)$ symmetry, which is broken at high energies in a manner preserving supersymmetry, thereby also facilitating the see-saw mechanism for neutrino masses and a natural source for the Higgs μ -term. We show that these favourable outcomes can occur both in the presence and the absence of a large Fayet-Iliopoulos (FI) D -term associated with the new $U(1)$. We explore the cosmological consequences of the model, showing that it naturally produces a period of hybrid inflation, terminating in the production of cosmic strings. In spite of the presence of a $U(1)$ (even with an FI term), inflation is effected by the F -term, with a D -flat tree potential (the FI term, if present, being cancelled by non-zero squark and slepton fields). Calculating the 1-loop corrections to the inflaton potential, we estimate the constraints on the parameters of the model from Cosmic Microwave Background data. We will see that a consequence of these constraints is that the Higgs μ -term necessarily small. We briefly discuss the mechanisms for baryogenesis via conventional leptogenesis, the out-of-equilibrium production of neutrinos from the cosmic strings, or the Affleck-Dine mechanism. Cosmic string decays also boost the relic density of dark matter above the low value normally obtained in AMSB scenarios.

KEYWORDS: Supersymmetric gauge theory, Supersymmetry Breaking, Cosmology of Theories beyond the SM

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1 Introduction

Low energy supersymmetry remains a popular possibility for physics Beyond the Standard Model awaiting discovery at the LHC. Much of the associated work has concentrated on the CMSSM scenario, where it is assumed that the unification of gauge couplings at high energies is accompanied by a corresponding unification in both the soft supersymmetry-breaking scalar masses and the gaugino masses; and also that the cubic scalar interactions are of the same form as the Yukawa couplings and related to them by a common constant of proportionality, the A -parameter. This paradigm is not, however, founded on a compelling underlying theory and therefore it is worthwhile exploring other possibilities.

In this paper we focus on Anomaly Mediation (AM) [1]–[3]. This is a framework in which a single mass parameter determines the $\phi^*\phi$, ϕ^3 and $\lambda\lambda$ supersymmetry-breaking terms in terms of calculable and, moreover, renormalisation group (RG) invariant functions of the dimensionless couplings, in an elegant and predictive way; too predictive, in fact, in that the theory in its simplest form leads to tachyonic sleptons and fails to accommodate the usual electroweak vacuum state. There is a natural solution to this, however, which restores the correct vacuum while retaining the RG invariance (and hence the ultra-violet insensitivity) of the predictions. This is achieved simply (and without introducing another source of explicit supersymmetry-breaking) by the introduction of an additional anomaly-free gauged $U(1)'$, broken at a high scale, so that the mass contributions from the $U(1)'$ D -term are naturally of the same order of magnitude as the soft breaking terms, and can eliminate the tachyonic slepton problem. This scenario was first explored in any detail in ref. [4], and subsequently by a number of authors [5]–[12].

In ref. [11] it was shown how the characteristic low energy theory can arise in a natural way from a $U(1)'$ with a Fayet-Iliopoulos (FI) D -term. The scale of the $U(1)'$ breaking was associated with the scale of the FI term, but, as indicated above, the resulting mass contributions to the MSSM scalars are naturally of order the supersymmetry-breaking scale.

In ref. [12], it was shown how it was possible to work with a superpotential which was more complicated but still purely cubic, and nevertheless dispense with the FI term. The $U(1)'$ breaking scale was then generated by dimensional transmutation in a way reminiscent of the geometric hierarchy model of Witten [13].

Here we return to an improved version of the original model of ref. [11], augmented by the introduction in the superpotential of a linear term for the gauge singlet field S . We will see that this incarnation of the model is natural, in the sense that the superpotential contains every possible term allowed both by the $U(1)'$ and a $U(1)_R$ global symmetry. The coefficient of the S linear term provides an alternative to the FI term for setting the scale of the $U(1)'$ breaking.

In this paper we explore some cosmological consequences which flow naturally from the introduction of $U(1)'$. We will see that the minimal acceptable form of the theory incorporates a natural mechanism for supersymmetric F -term inflation, although with a potential somewhat more complicated than the simple models in the literature. Inflation ends with a phase transition producing gauge cosmic strings, and hence tight constraints on the parameters from the Cosmic Microwave Background [14]. In common with other supersymmetry models with conserved R -parity, it naturally accommodates a Dark Matter candidate, which is mostly wino. Although it has been argued that in AMSB models the annihilation cross-section is too large for conventional freeze-out to generate the dark matter, there is another source of dark matter in the form of particles radiated by cosmic strings. The model also has the possibility of baryogenesis via leptogenesis or the Affleck-Dine mechanism, with CP violation supplied by the neutrino sector. Gravitinos are very massive, and so decay early enough not to be in conflict with nucleosynthesis.

The model has the same field content as the F_D hybrid inflation model [15, 16], but different charge assignments and couplings. F_D hybrid inflation also has a singlet which is a natural inflaton candidate, but differs in other ways: for example, right-handed

neutrinos have electroweak-scale masses, and the gravitino problem is countered by entropy generation.

Since we entertain the possibility of a Fayet-Iliopoulos term in our model, it behoves us to address issues raised by the recent papers on the connection of FI terms with supergravity, by Komargodski and Seiberg, and others [17]–[20]. The upshot of this work is the conclusion that a global theory with a FI term cannot be consistently embedded in supergravity. An exception would be allowed if the full supergravity theory has a certain exact continuous global symmetry; but no consistent theory of gravity is allowed to have such a symmetry. However, this latter assertion (that global symmetries are forbidden from theories with gravity) has the status of a (admittedly widely believed) conjecture rather than a proven theorem.¹ For a relevant and detailed analysis see ref. [21]. We propose to exploit this loophole to justify this aspect of our discussion, and argue that our model has enough interesting features to render this exercise worthwhile. One such feature is that, as we shall see, our model indeed has an exact global $U(1)_R$ symmetry; although it is not clear that this symmetry (or indeed the global version of $U(1)'$) is of the type required by ref. [17]. We will, however, also see that our model is perfectly viable if in fact it has no FI term.

2 The AMSB soft terms

We will assume that supersymmetry breaking arises via the renormalisation group invariant form characteristic of Anomaly Mediation, so that the soft parameters for the gaugino mass M , the ϕ^3 interaction h and the $\phi^*\phi$ and ϕ^2 mass terms m^2 and m_3^2 take the generic form

$$M_i = m_{\frac{3}{2}}\beta_{g_i}/g_i \tag{2.1}$$

$$h_{U,D,E,N} = -m_{\frac{3}{2}}\beta_{Y_{U,D,E,N}} \tag{2.2}$$

$$(m^2)^i_j = \frac{1}{2}m_{\frac{3}{2}}^2\mu\frac{d}{d\mu}\gamma^i_j + kY_i\delta^i_j, \tag{2.3}$$

$$m_3^2 = \kappa m_{\frac{3}{2}}\mu_h - m_{\frac{3}{2}}\beta_{\mu_h}. \tag{2.4}$$

Here μ is the renormalisation scale, $m_{\frac{3}{2}}$ is the gravitino mass, which to obtain a reasonable supersymmetric spectrum will be typically 40 – 50TeV. β_{g_i} are the gauge β -functions and γ the chiral supermultiplet anomalous dimension matrix. $Y_{U,D,E,N}$ are the 3×3 Yukawa matrices, while the Y_i are charges corresponding to a $U(1)$ symmetry of the theory with no mixed anomalies with the gauge group; the kY term corresponds in form to a FI D -term. As we shall see, we can generate this kY term naturally with k of $O(m_{\frac{3}{2}}^2)$, by breaking a $U(1)'$ symmetry at a large scale. The parameter κ is an arbitrary constant; its presence means that in sparticle spectrum calculations one is free to determine m_3^2 (and the value of the Higgs μ -term, μ_h) by minimising the Higgs potential at the electroweak scale in the usual way.

¹We thank Tom Banks for conversations on this topic.

| | | | | | | |
|-----|-------------------|-------------------------|------------------------|--------------|-------------|---------------|
| | Q | U | D | H_1 | H_2 | N |
| q | $-\frac{1}{3}q_L$ | $-q_E - \frac{2}{3}q_L$ | $q_E + \frac{4}{3}q_L$ | $-q_E - q_L$ | $q_E + q_L$ | $-2q_L - q_E$ |

Table 1. Anomaly free U(1) symmetry for arbitrary lepton doublet and singlet charges q_L and q_E respectively.

| U(1) | Q | U | D | L | E | H_1 | H_2 | N | Φ | $\bar{\Phi}$ | S |
|------|------|------|------|-----|-----|-------|-------|-----|--------|--------------|-----|
| Y | 1/3 | -4/3 | 2/3 | -1 | 2 | -1 | 1 | 0 | 0 | 0 | 0 |
| Y' | -2/3 | -7/3 | 11/3 | 2 | 1 | -3 | 3 | -5 | 10 | -10 | 0 |

Table 2. Hypercharges for U(1) $_Y$ and U(1)'.

3 The U(1)' symmetry

The MSSM (including right-handed neutrinos) admits two independent generation-blind anomaly-free U(1) symmetries.² The possible charge assignments are shown in table 1. The SM gauged U(1) $_Y$ is $q_L = 1, q_E = -2$; this U(1) is of course anomaly free even in the absence of N . U(1) $^{B-L}$ is $q_E = -q_L = 1$; in the absence of N this would have U(1)³ and U(1)-gravitational anomalies, but no mixed anomalies with the SM gauge group.

Our model will have, in addition, a pair of MSSM singlet fields $\Phi, \bar{\Phi}$ with U(1)' charges $q_{\phi, \bar{\phi}} = \pm(4q_L + 2q_E)$ and a gauge singlet S . In order to solve the tachyon slepton problem we will need that, for our new gauge symmetry U(1)', the charges q_L, q_E have the same sign; where numerical results are required we will use $q_L = 2, q_E = 1$. The resulting hypercharges are shown in table 2. Finally, note that with *any* charge assignment such that $q_{L,E} > 0$, only E, L, D, H_2, Φ have *positive* U(1)' charges. This will be important in what follows when we come to consider the inflationary regime.

4 The superpotential and spontaneous U(1)' breaking

The complete superpotential for our model is:

$$W = W_A + W_B \tag{4.1}$$

where W_A is the MSSM superpotential, omitting the Higgs μ -term, and augmented by Yukawa couplings for the right-handed neutrinos:

$$W_A = H_2 Q Y_U U + H_1 Q Y_D D + H_1 L Y_E E + H_2 L Y_N N \tag{4.2}$$

and

$$W_B = \lambda_1 \Phi \bar{\Phi} S + \frac{1}{2} \lambda_2 N N \Phi + \lambda_3 S H_1 H_2 + M^2 S, \tag{4.3}$$

²One of the attractive features of minimal Anomaly Mediation is that squark/slepton mediated flavour changing neutral currents are naturally small [22]; this feature is preserved by a generation-blind U(1) but not by a flavour-dependent U(1), so we stick to the former here.

where M, λ_1, λ_3 are real and positive and λ_2 is a symmetric 3×3 matrix. This model differs from the one first considered in ref. [11] by the inclusion of the λ_3 term and the linear term for S . Although the old model led to satisfactory $U(1)'$ breaking, it had several defects from a cosmological perspective; in particular a second stable particle in addition to the usual LSP, because its superpotential was invariant with respect to

$$\overline{\Phi} \rightarrow -\overline{\Phi}, S \rightarrow -S \quad (4.4)$$

with all other fields unchanged. Now in the AMSB context, an additional Dark Matter candidate would not be a bad thing per se, since (in AMSB) the LSP is generally the neutral wino, whose annihilation cross-section is generally considered to be too high for comfort. However, the additional stable particle in the original formulation of the model has a mass at the $U(1)'$ -breaking scale, which, as we will see, we are going to want to be rather large.

Note that the $U(1)'$ symmetry forbids the renormalisable B and L violating superpotential interaction terms of the form QLD, UDD, LLE, H_1H_2N and NS^2, N^2S and N^3 , as well as the mass terms NS, N^2 and LH_2 and the linear term N . Moreover W_B contains the only cubic term involving $\Phi, \overline{\Phi}$ that is allowed. Our new superpotential eq. (4.1) in fact is completely natural, in the sense that it is invariant under a global R -symmetry, with superfield charges

$$S = 2, L = E = N = U = D = Q = 1, H_1 = H_2 = \Phi = \overline{\Phi} = 0, \quad (4.5)$$

which forbids the remaining gauge invariant renormalisable terms ($S^2, S^3, \Phi\overline{\Phi}$ and H_1H_2). Moreover, this R -symmetry forbids the quartic superpotential terms $QQQL$ and $UUDE$, which are allowed by the $U(1)'$ symmetry, and give rise to dimension 5 operators capable of causing proton decay [23]–[25]. It is easy to see, in fact, that the charges in eq. (4.5) disallow B -violating operators in the superpotential of arbitrary dimension. Of course this R -symmetry is broken by the soft supersymmetry breaking.

Retaining for the moment only the scalar fields $\phi, \overline{\phi}, s$ (the scalar component of their upper case counterpart superfields) we write the scalar potential:

$$\begin{aligned} V = & \lambda_1^2(|\phi s|^2 + |\overline{\phi} s|^2) + |\lambda_1 \phi \overline{\phi} + M^2|^2 + \frac{g'^2}{2} (\xi - q_\phi |\phi|^2 + q_\phi |\overline{\phi}|^2)^2 \\ & + m_\phi^2 |\phi|^2 + m_{\overline{\phi}}^2 |\overline{\phi}|^2 + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) \\ & + h_{\lambda_1} \phi \overline{\phi} s + c.c. \dots \end{aligned} \quad (4.6)$$

Here the h_{λ_1} term is a soft breaking, determined in accordance with eq. (2.2):

$$h_{\lambda_1} = -m_{\frac{3}{2}} \frac{\lambda_1}{16\pi^2} \left(3\lambda_1^2 + \frac{1}{2} \text{Tr} \lambda_2^2 + 2\lambda_3^2 - 4q_\phi^2 g'^2 \right), \quad (4.7)$$

denoting the $U(1)'$ charge by g' . We also introduce a soft breaking term linear in s (see [26] for a discussion of linear terms in this context), and a Fayet-Iliopoulos term ξ for $U(1)'$. Our model thus has two explicit mass parameters $M, \sqrt{\xi}$ as well as the gravitino mass $m_{\frac{3}{2}}$. It is easy to show that if $M \gg m_{\frac{3}{2}}$, then V above is minimised with $\phi, \overline{\phi}$ acquiring

large vevs, while s acquires a vev of $O(m_{\frac{3}{2}})$. Thus the right-handed neutrinos acquire large masses, and an appropriate μ -term for the Higgs doublets is generated in the manner of the NMSSM (for a review of and references for the NMSSM see ref. [27]).

At the minimum of V we have

$$\lambda_1 v_\phi v_{\bar{\phi}} + M^2 \approx 0, \tag{4.8}$$

$$\xi - q_\phi v_\phi^2 + q_{\bar{\phi}} v_{\bar{\phi}}^2 \approx 0, \tag{4.9}$$

$$v_s \approx -\frac{h_{\lambda_1} v_\phi v_{\bar{\phi}} + \rho M^2 m_{\frac{3}{2}}}{v_\phi^2 + v_{\bar{\phi}}^2} \tag{4.10}$$

and we see that v_s is $O(\lambda_1 m_{\frac{3}{2}}/(16\pi^2))$ if $M \gg \sqrt{\lambda_1 \xi}$ or if $M \sim \sqrt{\lambda_1 \xi}$, and v_s is $O(m_{\frac{3}{2}} M^2/(16\pi^2 \xi))$ if $M \ll \sqrt{\lambda_1 \xi}$. (Here we assume that ρ , like h_{λ_1} , is suppressed by a loop factor, and that $q_\phi g' \sim O(1)$). For large M and/or ξ , (all trace of the $U(1)'$ in the effective low energy Lagrangian disappears, except for contributions to the masses of the matter fields, arising from the $U(1)'$ D -term, which are naturally of the same order as the AMSB ones. If we neglect terms of $O(m_{\frac{3}{2}})$, the breaking of $U(1)'$ preserves supersymmetry; thus the $U(1)'$ gauge boson, its gaugino (with one combination of $\psi_{\phi, \bar{\phi}}$) and the Higgs boson form a massive supermultiplet with mass $m \sim g' \sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$, while the remaining combination of ϕ and $\bar{\phi}$ and the other combination of $\psi_{\phi, \bar{\phi}}$ form a massive chiral supermultiplet, with mass $m \sim \lambda_1 \sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$.

Evidently s also gets a large supersymmetric mass, as does the N triplet, thus naturally implementing the see-saw mechanism. Moreover, the vev for s introduces a Higgs μ term, the magnitude of which is naturally related to the supersymmetry-breaking scale.

It is easy to show that the contribution to the slepton masses arising from the $U(1)'$ term which resolves the tachyonic slepton problem is given (after spontaneous breaking of the $U(1)'$) by [11]

$$\delta m^2 \sim -\frac{q_{L,e} m_\phi^2}{q_\phi}, \tag{4.11}$$

and also (using eqs. (8), (10) and figure 1 of ref. [11]) that we need

$$\delta m^2 \sim 0.1 \left(\frac{m_{\frac{3}{2}}}{40\text{TeV}} \right)^2. \tag{4.12}$$

So, if we assume that the one-loop γ_ϕ is dominated by its gauge contribution, then we have from eq. (2.3) that

$$m_\phi^2 \sim m_{\frac{3}{2}}^2 \beta'_g \frac{\partial}{\partial g'} \gamma_\phi \sim -m_{\frac{3}{2}}^2 Q q_\phi^2 g'^4 / (16\pi^2)^2 \tag{4.13}$$

where here Q is the sum of the squares of all the $U(1)'$ charges. So we would want

$$q_{L,e} Q q_\phi g'^4 \sim \left(\frac{m_{\frac{3}{2}}}{40\text{TeV}} \right)^2 \tag{4.14}$$

at the supersymmetry breaking scale.

Let us use $q_L = 2, q_E = 1$, as mentioned previously. Then, $Q = 516$, and it is easy to show using the RG equation for $\beta_{g'}$ that if $g' = 1$ at 10^{16}GeV , then at 10^3GeV , $g' \sim 0.1$ and that consequently eq. (4.14) is reasonably well satisfied (for $m_{\frac{3}{2}} = 40\text{TeV}$). Subsequently we will use $g' = 1$ at high energies.

It is quite interesting to contrast our model with the conventional versions of the NMSSM, which does not, in its basic form, contain an extra $U(1)$, but where a vev (of the scale of supersymmetry breaking) for the gauge singlet s generates a Higgs μ -term in much the same way, as is done here. However, while in the NMSSM case the s fields are very much part of the Higgs spectrum, here, in spite of the comparatively small s -vev, the s -quanta obtain large supersymmetric masses and are decoupled from the low energy physics, which becomes simply that of the MSSM. Another nice feature is the natural emergence of the seesaw mechanism via the spontaneous breaking of the $U(1)'$. These features of the model still hold for $\xi = 0$, as can easily be seen from eqs. (4.8)–(4.10). Moreover, the FI-type mass contributions of the form of eq. (4.11) are still present. This makes the model an interesting one from the particle physics point of view both with and without the FI term. In the latter case we of course have no problem with the conclusions of ref. [17]; but we also discuss the former case also because of the novel nature of the inflationary regime in that case.

5 The scalar potential at large s

As we have developed it, the theory naturally provides for F -term inflation [28]–[30]. Although we have a $U(1)'$, there is a crucial difference between our scenario and the original D -term inflation paradigm [31], which is that the SM fields are necessarily charged under $U(1)'$. Consequently, it is possible for the large s tree potential to be D -flat for the whole gauge group, including $U(1)'$; even in the presence of a Fayet-Iliopoulos term, which is one thing that makes this case of interest.

The tree potential is (each term below involves implicit summation over all indices, including generation, not involved in explicit manipulations, and d^c , for example, represents all three generations of SU_2 singlet (anti-)down squarks):

$$\begin{aligned}
 V_{\text{tree}} = & \lambda_1^2 |\phi s|^2 + |\lambda_1 \bar{\phi} s + \frac{1}{2} \lambda_2 \nu^c \nu^c|^2 \\
 & + |\lambda_2 \nu^c \phi + l Y_N h_2|^2 + |\lambda_3 h_1 h_2 + \lambda_1 \phi \bar{\phi} + M^2|^2 \\
 & + |\lambda_3 h_1 s + q Y_U u^c + l Y_N \nu^c|^2 + |\lambda_3 h_2 s + q Y_D d^c + l Y_E e^c|^2 \\
 & + |Y_U u^c h_2 + Y_D d^c h_1|^2 + |Y_E e^c h_1 + Y_N \nu^c h_2|^2 \\
 & + \frac{1}{2} g^2 \left(\xi - q_\Phi \phi^* \phi - q_{\bar{\Phi}} \bar{\phi}^* \bar{\phi} - q_Q q^\dagger q - q_D d^{c\dagger} d^c - q_U u^{c\dagger} u^c \right. \\
 & \left. - q_L l^\dagger l - q_E e^{c\dagger} e^c - q_N \nu^{c\dagger} \nu^c - q_{H_1} h_1^\dagger h_1 - q_{H_2} h_2^\dagger h_2 \right)^2 \\
 & + \frac{1}{8} g_3^2 \sum_a \left(q^\dagger \lambda^a q + d^{c\dagger} (-\lambda^a)^T d^c + u^{c\dagger} (-\lambda^a)^T u^c \right)^2 \\
 & + \frac{1}{8} g_2^2 \sum_a \left(q^\dagger \sigma^a q + l^\dagger \sigma^a l + h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2 \right)^2
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{8} g_1^2 \left(\frac{1}{3} q^\dagger q - \frac{4}{3} u^{c\dagger} u^c + \frac{2}{3} d^{c\dagger} d^c - l^\dagger l + 2e^{c\dagger} e^c - h_1^\dagger h_1 + h_2^\dagger h_2 \right)^2 \\
 & + V_{\text{soft}}.
 \end{aligned} \tag{5.1}$$

Here V_{soft} contains the AMSB soft terms, which are suppressed by at least one power of $m_{\frac{3}{2}}$, that is

$$V_{\text{soft}} = \rho M^2 m_{\frac{3}{2}} (s + s^*) + m_s^2 |s|^2 + m_\phi^2 |\phi|^2 + m_{\bar{\phi}}^2 |\bar{\phi}|^2 + m_l^2 |l|^2 + \dots \tag{5.2}$$

Note that in eq. (5.1) we have written the $U(1)_Y$ gauge coupling as g_1 , although its normalisation corresponds to the usual SM convention, not that appropriate for $SU(5)$ unification. This is to avoid confusion with the $U(1)'$ coupling, g' .

At large fixed s we see that there are mass terms proportional to $\lambda_1^2 |s|^2$ for $\phi, \bar{\phi}$ which will mean that in this region their vevs will be zero. (Actually this is a rather more subtle point than it might appear; we will discuss it in more detail in section (5.1).) In the presence of a $U(1)'$ FI term, we introduce s -independent vevs for some of the MSSM scalars in such a manner as to achieve D -flatness for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)'$.

5.1 Where in field space is the minimum?

The minimum of our tree potential is in fact very degenerate, like the well known case the MSSM. Let us begin by enumerating the degrees of freedom represented in the potential. A naive count would suggest that there are 55 complex F -terms; one for each complex degree of freedom (c.d.o.f.). In addition there are 13 real D -terms, and 13 (real) gauge choices, one of each for each generator of the symmetry. This should (naively) lift all degeneracy. However, as in the MSSM, choosing $\langle H_{1,2}^\alpha \rangle = 0$ kills the F -terms for all SM fields except the Higgs fields themselves, and thus the degeneracy arises. Now we have 51 c.d.o.f. with only 10 F -terms left, and 13 D -terms i.e. a 28 complex dimensional degenerate space.

Let us look at the cases of large s (inflation) and small s (today) separately. The latter case was addressed in section 4, but it is possible that there are other minima. For example, F_S could be made zero by giving the Higgs fields large vevs instead of $\phi, \bar{\phi}$. That would require the remaining MSSM scalars to have zero vevs. Alternatively, if the Higgs fields had zero vevs, one could have a vev in some combination of MSSM scalar fields, just as in the scenario we in fact pursue for inflation. We shall not investigate this further; we have a good local minimum in hand, which gives us the SM physics we know and an explanation for other things as well, like the Higgs μ -term, and neutrino masses and mixings. Whether our minimum is favoured after loop corrections, we shall not investigate. But it is not clear that we are in the global minimum anyway.

For $M, \sqrt{\xi} \ll \langle s \rangle$ i.e. during inflation, we shall focus on a field space region with $\langle \phi \rangle = \langle \bar{\phi} \rangle = 0$, so that $V_{\text{tree}} = M^4$, with an appropriate set of MSSM field vevs arranged to render all the D -terms (including the $U(1)'$ term, which for $\xi \neq 0$ must have some non-zero vevs) and F -terms (excluding F_S) zero. This is naturally motivated by the presence in V of $|\phi s|^2$ and $|s \bar{\phi}|^2$ terms. In fact, however, it does not represent an absolute minimum of V_{tree} ; it is easy to show that by choosing $\langle \bar{\phi} \rangle \sim s^N$, $\langle \phi \rangle \sim s^{-N}$ and $\nu^c \sim s^{\frac{N+1}{2}}$ for some $N > 1$, with other vevs chosen so as to achieve D -flatness, it is possible to make V_{tree}

arbitrarily small for s large. Nevertheless, $\langle \phi \rangle = \langle \bar{\phi} \rangle = 0$ does represent a local minimum of the tree potential, for $s^2 > s_c^2$ where

$$s_c^2 = M^2/\lambda_1. \quad (5.3)$$

As we shall see, this value for s corresponds to the appearance of a zero eigenvalue in the $\phi, \bar{\phi}$ mass system, and we believe the radiative corrections lead to decrease towards this critical point. (Certainly the afore-mentioned $|s\bar{\phi}|^2$ term in V will discourage evolution towards large $\bar{\phi}$).

The flat space (the large space with $V = V_{\min}$) in the MSSM is complicated because it is not additive in field space - ie. the sum of two position vectors each pointing to a place in the flat space is not (necessarily) a position vector pointing to a place in the flat space. But it is, of course, scalar multiplicative (in the MSSM), ie. a position vector pointing to a place in the flat space multiplied by any scalar is still a position vector pointing to a place in the flat space. In our model, the flat space loses this virtue when $\xi \neq 0$; then even the origin in field space is not part of the flat space, since it has $D' = g'\xi$. In any event, it is hopeless to parameterise a 28 complex dimensional space. In contrast to the MSSM, which has no mass scales (except the Higgs μ -term), our model has $\langle s \rangle, M, \sqrt{\xi}$ which contribute mass terms to scalars, fermions and vectors. Moreover, since $F_S \neq 0$, loop-corrections can lift the degeneracy, in contrast to in the MSSM.

It is not feasible to parameterise this large space, but we can get a taste of it, by showing a 4 dimensional subspace:

$$\begin{aligned} \langle u_2 \rangle &= \langle c_3 \rangle = \langle t_1 \rangle = \Delta_A \sqrt{\frac{\xi}{5}}, \\ \langle d_3 \rangle &= \langle s_1 \rangle = \langle b_2 \rangle = \Delta_B \sqrt{\frac{\xi}{5}}, \\ \langle u_1^c \rangle &= \langle c_2^c \rangle = \langle t_3^c \rangle = \Delta_C \sqrt{\frac{\xi}{5}}, \\ \langle d_1^c \rangle &= \langle s_2^c \rangle = \langle b_3^c \rangle = \Delta_D \sqrt{\frac{\xi}{5}}, \\ \langle \nu_e \rangle &= \sqrt{1 + 2\Delta_A^2 - \Delta_B^2 + \Delta_C^2 - 2\Delta_D^2} \sqrt{\frac{\xi}{5}}, \\ \langle \mu \rangle &= \sqrt{1 - \Delta_A^2 + 2\Delta_B^2 + \Delta_C^2 - 2\Delta_D^2} \sqrt{\frac{\xi}{5}}, \\ \langle \tau^c \rangle &= \sqrt{1 + 3\Delta_C^2 - 3\Delta_D^2} \sqrt{\frac{\xi}{5}}. \end{aligned} \quad (5.4)$$

We have now changed the notation for the fields so as to distinguish the generations by name, and explicitly indicate the colour index on the down squarks. Here we see we have given vevs to all superfields that can have one: Q, U, D, L, E We recognise $\Delta_A = \Delta_B = \Delta_C = \Delta_D = 0$ as LLE and $\Delta_A = \Delta_B = \Delta_C = 0, \Delta_D = 1/\sqrt{3}$ as $DDLL$. We can also see that SM flat directions $QQQL, UUUU$ are present, but the parameters cannot make either of these alone. That is because they have the wrong sign of $U(1)'$ charge, and thus cannot balance ξ .

Even this system is too complicated to analyse in general; one could choose some cases to try for specific values of the 4 parameters, but we shall not pursue this further here. Rather, we shall look at a specific subspace of the flat space with just 1 free parameter, and investigate how the the loop potential depends on it.

We put vevs in $DDDLL$ and LLE invariants, by having nonzero vevs only for $L_{1,2}, E_3, D_{1,2,3}$, where the label denotes generation. Specifically, we set

$$\begin{aligned} \langle \tau^c \rangle = v_E &= \Delta \sqrt{\frac{\xi}{15}}, \\ \langle \nu_e \rangle = \langle \mu \rangle = v_L &= \sqrt{\left(1 + \frac{2}{3}\Delta^2\right) \frac{\xi}{15}}, \\ \langle d_1^c \rangle = \langle s_2^c \rangle = \langle b_3^c \rangle = v_D &= \sqrt{\left(1 - \frac{1}{3}\Delta^2\right) \frac{\xi}{15}} \end{aligned} \quad (5.5)$$

where we introduce the more convenient parameter $\Delta = \sqrt{3 - 9\Delta_D^2}$ which is real and satisfies $0 \leq \Delta \leq \sqrt{3}$.

By working in a “basis” such that $Y_{U,D,E,N}$ are diagonal, we ensure that, (given eq. (5.5)) there is no contribution to V_{tree} from any F -term except F_S , and the CKM-matrix is the identity matrix. This is not an approximation as such, one can adjust for the influence of the mixings by choosing vevs that are rotated correspondingly. Note that the vevs are in “colour=generation” for D and “weak charge=generation” for L , and in the third generation for E . Since each superfield only has one nonzero entry, flatness is independent of the phases of the fields. One can use the gauge choices of the diagonal generators to remove one phase each from the vevs of the fields. The 5 gauge choices are made so as to remove all phase differences in the vevs. To make the result even simpler, we have chosen the common phase to be zero; all vevs real and positive (this corresponds to a choice for the global $U(1)$ symmetry of the SM). We have also taken the vev of S to be real - this defines the coordinate system for the couplings of S .

The combination of zero vevs for $\phi, \bar{\phi}$ with the set of vevs described in eq. (5.5) means that for large s we have (neglecting the tree soft terms)

$$V_{\text{tree}} = M^4. \quad (5.6)$$

With the inclusion of one loop corrections this becomes

$$V = M^4 + \Delta V, \quad (5.7)$$

where ΔV represents the one-loop corrections, given as usual by

$$\Delta V = \frac{1}{64\pi^2} \text{Str}(M^2(s))^2 \ln(M^2(s)/\mu^2). \quad (5.8)$$

Here

$$\text{Str} \equiv \sum_{\text{scalars}} -2 \sum_{\text{fermions}} + 3 \sum_{\text{vectors}}. \quad (5.9)$$

Contributions to ΔV from fields with large masses will be more significant than those from the neglected soft terms; but of course for fields which, although massive, form degenerate

supermultiplets, the contributions to ΔV will cancel exactly. It is easy to see that there are two relevant sets of contributions.

5.2 The $\Phi, \bar{\Phi}$ system

Let us consider the $\Phi, \bar{\Phi}$ subsystem, which in fact appears in the minimal F -term inflation model [28]. The scalar mass matrix eigenvalues at large fixed s are given by

$$M_{\phi, \bar{\phi}}^2 = \lambda_1^2 s^2 \pm \lambda_1 M^2, \quad (\text{twice each}). \quad (5.10)$$

(Note that there is no contribution to these mass terms from the $U(1)'$ D -term, because of the D -flatness engendered by eq. (5.5). But of course as a result of these vevs there will be further significant contributions to the one-loop potential beyond those considered in this subsection; these we will describe in the next one). The corresponding fermion masses are simply $\tilde{m}_{\phi, \bar{\phi}}^2 = \lambda_1^2 s^2$. The contribution to the one-loop scalar potential is

$$\begin{aligned} \Delta V_1 = & \frac{1}{32\pi^2} [(\lambda_1^2 s^2 + \lambda_1 M^2)^2 \ln \left(\frac{\lambda_1^2 s^2 + \lambda_1 M^2}{\mu^2} \right) + (\lambda_1^2 s^2 - \lambda_1 M^2)^2 \ln \left(\frac{\lambda_1^2 s^2 - \lambda_1 M^2}{\mu^2} \right) \\ & - 2\lambda_1^4 s^4 \ln \left(\frac{\lambda_1^2 s^2}{\mu^2} \right)]. \end{aligned} \quad (5.11)$$

If we assume that we are interested in values of s for which $\lambda_1 s^2 \gg M^2$, it is easy to show that this reduces to

$$\Delta V_1 = \frac{1}{16\pi^2} \lambda_1^2 M^4 \ln \left(\frac{\lambda_1^2 s^2}{\mu^2} \right). \quad (5.12)$$

5.3 The $(H_1, H_2, Q, E_{1,2}, L_3, N_{1,2})$ system

Apart from the $\Phi, \bar{\Phi}$ system already considered, the only other contributions to the one loop potential comes from the $(H_1, H_2, Q, E_{1,2}, L_3, N_{1,2})$ system, where the scalar mass matrix can be split into two separate 12×12 complex matrices. Note that it is one particular linear combination of the three doublets Q which is selected by the D -vevs; thus if we define

$$Q = \frac{y_d Q_1 + y_s Q_2 + y_b Q_3}{\sqrt{y_d^2 + y_s^2 + y_b^2}} \quad (5.13)$$

then the Higgs-squark doublet mixing term is

$$\lambda_3 y s v_D q^\dagger h_2 + c.c., \quad (5.14)$$

where $y = \sqrt{y_d^2 + y_s^2 + y_b^2}$. The first scalar matrix $(h_1^1, h_2^2, q, \mu^c, \tau, \nu_e^c)$ takes the form

$$\begin{pmatrix} M_S^2 + M_D^2 + M_{E_2}^2 + M_{E_3}^2 & M_M^2 \cdot \sigma_1 & \cdot & \cdot & \cdot & M_{\nu_1} M_S \\ M_M^2 \cdot \sigma_1 & M_S^2 + M_{\nu_1}^2 & M_S M_D & M_S M_{E_2} & M_S M_{E_3} & \cdot \\ \cdot & M_S M_D & M_D^2 & M_D M_{E_2} & M_D M_{E_3} & \cdot \\ \cdot & M_S M_{E_2} & M_D M_{E_2} & M_{E_2}^2 & M_{E_2} M_{E_3} & \cdot \\ \cdot & M_S M_{E_3} & M_D M_{E_3} & M_{E_2} M_{E_3} & M_{E_3}^2 & \cdot \\ M_{\nu_1} M_S & \cdot & \cdot & \cdot & \cdot & M_{\nu_1}^2 \end{pmatrix} \quad (5.15)$$

where

$$M_S = \lambda_3 s, M_D = y v_D, M_{E_2} = y_\mu v_L, M_M^2 = M^2 \lambda_3, M_{\nu_1} = y_{\nu_e} v_L, M_{E_3} = y_\tau v_E. \quad (5.16)$$

σ_1 is the usual Pauli matrix, and if no 2×2 matrix is indicated the identity matrix is to be assumed. A dot indicates the zero matrix. The second matrix is identical except that M_M^2 is replaced by $-M_M^2$, $y_{\nu_e} v_L$ is replaced by $y_{\nu_\mu} v_L$, and $y_\mu v_L$ is replaced by $y_e v_L$.

The eigenvalue equation for the matrix eq. (5.15) has four zero eigenvalues; the rest of it can be factorised into a product of two identical quartic equations of the form

$$x^4 - 2a_3 x^3 + (a_3^2 + 2a_2 a_1 - a_0^2) x^2 - (2a_3 a_2 a_1 - (a_2 + a_1) a_0^2) x + a_2 a_1 (a_2 a_1 - a_0^2) = 0, \quad (5.17)$$

where

$$\begin{aligned} a_0 &= M_M^2, \\ a_1 &= M_{E_2}^2 + M_D^2 + M_{E_3}^2, \\ a_2 &= M_{\nu_1}^2, \\ a_3 &= M_S^2 + a_2 + a_1. \end{aligned} \quad (5.18)$$

For a quartic equation with non-zero real coefficients, of the form $x^4 - ax^3 + bx^2 - cx + d = 0$, and all roots known to be real, the necessary and sufficient conditions that all its roots be positive are $a, b, c, d > 0$.³ We see from eq. (5.17) that these conditions are satisfied provided

$$a_2 a_1 > a_0^2. \quad (5.19)$$

Inserting the vevs, we require

$$\left(y^2 \left(1 - \frac{1}{3} \Delta^2 \right) + y_\mu^2 \left(1 + \frac{2}{3} \Delta^2 \right) + y_\tau^2 \Delta^2 \right) y_{\nu_e}^2 \left(1 + \frac{2}{3} \Delta^2 \right) \left(\frac{\xi}{15} \right)^2 > \lambda_3^2 M^4 \quad (5.20)$$

Given eq. (5.20) and $s^2 > M^2/\lambda_1$, we see that our tree potential has no tachyonic instabilities.

We now proceed to consider the effect of the one-loop corrections to the potential. Solving the quartic eq. (5.17) exactly yields rather unwieldy expressions for the eigenvalues. However if we expand the solutions as a series in a_0 we obtain manageable forms for them as follows:

$$\begin{aligned} x_{1,2} &= f_1 \pm \sqrt{d_1} a_0 + e_1 a_0^2 + \dots \\ x_{3,4} &= f_2 \pm \sqrt{d_2} a_0 - e_1 a_0^2 + \dots \end{aligned} \quad (5.21)$$

³Corollary of Descartes' rule of signs.

where

$$\begin{aligned}
 f_1 &= \frac{1}{2} \left(a_3 - \sqrt{a_3^2 - 4a_2a_1} \right), \\
 f_2 &= \frac{1}{2} \left(a_3 + \sqrt{a_3^2 - 4a_2a_1} \right), \\
 d_1 &= \frac{(a_3 - a_2 - a_1)f_1}{a_3^2 - 4a_2a_1}, \\
 d_2 &= \frac{(a_3 - a_2 - a_1)f_2}{a_3^2 - 4a_2a_1}, \\
 e_1 &= \frac{-a_2(a_3 - 4a_1) + a_3a_1}{2(a_3^2 - 4a_2a_1)^{\frac{3}{2}}}.
 \end{aligned} \tag{5.22}$$

For each set of four bosonic eigenvalues of the form above, we have eigenvalues of the corresponding fermion mass matrix of the form $f_{1,2}$. This is simply because in the absence of a_0 (that is to say, of M) the configuration would be supersymmetric.

The contribution to the one-loop potential from the matrix eq. (5.15) becomes (to $O(a_0^2)$), and retaining the leading contribution only in each logarithm):

$$16\pi^2 \Delta V_2 = a_0^2 [(d_1 + 2f_1e_1) \ln(f_1/\mu^2) + (d_2 - 2f_2e_1) \ln(f_2/\mu^2)]. \tag{5.23}$$

If we further assume that $a_1 \ll a_{2,3}$ we can simplify ΔV_2 by expanding to leading order in a_1 , when we obtain

$$\begin{aligned}
 16\pi^2 \Delta V_2 &= \lambda_3^2 M^4 \left[1 + M_{F_2}^2 \frac{M_{\nu_1}^2 (M_{\nu_1}^2 - M_S^2)}{(M_{\nu_1}^2 + M_S^2)^3} \right] \ln((M_S^2 + M_{\nu_1}^2)/\mu^2) \\
 &\quad + \lambda_3^2 M^4 M_{F_2}^2 \frac{(M_S^2 - M_{\nu_1}^2) M_{\nu_1}^2}{(M_{\nu_1}^2 + M_S^2)^3} \ln(M_{F_2}^2/\mu^2),
 \end{aligned} \tag{5.24}$$

where we have now written $a_1 \equiv M_{F_2}^2$. The contribution from the other $(H_1, H_2, Q, E_{1,2}, L_3, N_{1,2})$ matrix similar to eq. (5.15) is given by:

$$\begin{aligned}
 16\pi^2 \Delta V_3 &= \lambda_3^2 M^4 \left[1 + M_{F_1}^2 \frac{M_{\nu_2}^2 (M_{\nu_2}^2 - M_S^2)}{(M_{\nu_2}^2 + M_S^2)^3} \right] \ln((M_S^2 + M_{\nu_2}^2)/\mu^2) \\
 &\quad + \lambda_3^2 M^4 M_{F_1}^2 \frac{(M_S^2 - M_{\nu_2}^2) M_{\nu_2}^2}{(M_{\nu_2}^2 + M_S^2)^3} \ln(M_{F_1}^2/\mu^2),
 \end{aligned} \tag{5.25}$$

where

$$\begin{aligned}
 M_{F_1}^2 &= M_{E_1}^2 + M_D^2 + M_{E_3}^2, \\
 M_{\nu_2} &= y_{\nu\mu} v_L, \\
 M_{e_1} &= y_e v_L.
 \end{aligned} \tag{5.26}$$

So our analytic approximation to the scalar potential is finally

$$V = M^4 + \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{5.27}$$

where ΔV_1 was given in eq. (5.12).

5.4 The $\xi = 0$ case

In this special case the potential is D -flat without invoking the MSSM vevs introduced above. The one loop potential is dominated by the $\phi, \bar{\phi}$ system described in section (5.2), and similar contributions from $h_{1,2}$ as is easily seen from eq. (5.1). For both $\lambda_1 s^2 \gg M^2$ and $\lambda_3 s^2 \gg M^2$ the one-loop corrected potential becomes

$$V = M^4 + \frac{1}{16\pi^2} M^4 \left[\lambda_1^2 \ln \left(\frac{\lambda_1^2 s^2}{\mu^2} \right) + 2\lambda_3^2 \ln \left(\frac{\lambda_3^2 s^2}{\mu^2} \right) \right]. \quad (5.28)$$

This result is easily obtained from eq. (5.12) and by setting $\xi = 0$ in eq. (5.24) and eq. (5.25). (The inequality introduced in eq. (5.19) and eq. (5.20) is not applicable for $\xi = 0$, because this corresponds to $a_2 = a_1 = 0$.) In this case we would require $\lambda_3 > \lambda_1$, since otherwise we would find that $s_c^2 = M^2/\lambda_3$, rather than M^2/λ_1 , and it would be the Higgses that developed vevs rather than $\phi, \bar{\phi}$. Note however that we require $\lambda_3 > \lambda_1$ only if $\lambda_3 \neq 0$; $\lambda_3 = 0$ is allowed, since then the Higgs directions which are unstable for $\lambda_3^2 s^2 < \lambda_3 M^2$ become flat.

6 Inflation

In the limit $\lambda_3 \ll \lambda_1$ (with $\xi \neq 0$), ΔV_2 and ΔV_3 are negligible and the effective potential for the s field reduces to that of standard F -term inflation [28]–[30]. In this section we outline the basic features of this limit as a reference point, showing how one can estimate constraints from the CMB data. When we do our more detailed parameter search it will turn out that we are forced to this limit by other constraints.

The aim is to compute the principal inflationary observables, the scalar and tensor power spectra \mathcal{P}_s and \mathcal{P}_t , the scalar spectral index n_s . The importance of the tensor power spectrum is often parametrised by $r = 4\mathcal{P}_t/\mathcal{P}_s$. In slow-roll single-field inflation these are given by the standard formulae (see e.g. [32])

$$\mathcal{P}_s(k) \simeq \frac{1}{24\pi^2} \frac{V}{m_p^4} \frac{1}{\epsilon} \Big|_{N_k}, \quad \mathcal{P}_t(k) \simeq \frac{1}{6\pi^2} \frac{V}{m_p^4} \Big|_{N_k}, \quad (6.1)$$

$$n_s \simeq (1 - 6\epsilon + 2\eta) \Big|_{N_k}, \quad r = 16\epsilon \Big|_{N_k}, \quad (6.2)$$

where

$$\epsilon = \frac{m_p^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = m_p^2 \left(\frac{V''}{V} \right), \quad (6.3)$$

and N_k is the e -fold at which the co-moving scale k “crosses the horizon”, i.e. $aH = k$. In order to fix N_k we need a complete history of the universe, and in particular the temperature to which it reheats after inflation T_{rh} . When fitting to data we are generally interested in the scale $k_0 = 0.002 \text{ Mpc}^{-1}$, in which case $N_{k_0} \simeq 55 + \ln(T_{\text{rh}}/10^{15} \text{ GeV})$.

Inflation finishes when ϕ and $\bar{\phi}$ become unstable, at a critical value of s given by eq. (5.3). From eq. (5.10) we see that this value for s corresponds to the appearance of a zero eigenvalue in the $\phi, \bar{\phi}$ mass system.

Let us first consider the case where we approximate ΔV_1 by eq. (5.12). This is appropriate if $\xi \neq 0$, and $\lambda_3 \ll \lambda_1$. If we choose the renormalisation scale $\mu^2 = \lambda_1^2 s_c^2/2$, we obtain

$$V(s) \simeq M^4 \left[1 + \frac{\lambda_1^2}{16\pi^2} \ln \frac{2s^2}{s_c^2} \right] \simeq M^4 \left(\frac{s_R^2}{s_c^2} \right)^\alpha, \quad (6.4)$$

where $s_R = \sqrt{2}s$ is a canonically normalised real scalar field, and

$$\alpha = \frac{\lambda_1^2}{16\pi^2}. \quad (6.5)$$

One can express the solution to the slow-roll equations⁴ as

$$N(s_R) = \frac{1}{m_p^2} \int_{s_c}^s ds' \frac{V}{V_{,s'}}, \quad (6.6)$$

where $N = \ln(a_{\text{end}}/a(t))$ is the number of e-foldings before the end of inflation. Hence

$$s_R^2 = s_c^2 + 4\alpha N m_p^2, \quad (6.7)$$

with

$$\epsilon = 2\alpha^2 \frac{m_p^2}{s_R^2}, \quad \eta = -2\alpha(1 - 2\alpha) \frac{m_p^2}{s_R^2}. \quad (6.8)$$

The assumption that $s_R \gg s_c$ is valid provided

$$\lambda_1 \alpha \gg \frac{1}{4N} \frac{M^2}{m_p^2}. \quad (6.9)$$

We also want to work in an effective theory well below the Planck scale, ensuring $s_R^2 \ll m_p^2$, which is true provided

$$\alpha \ll \frac{1}{4N}. \quad (6.10)$$

This is easily satisfied if $\lambda_1 \ll O(1)$.

The solution (6.7) gives the potential and the slow-roll parameters in terms of N :

$$\frac{V}{m_p^4} \simeq \left(\frac{M}{m_p} \right)^{4-2\alpha} (4\alpha\lambda_1 N_k)^\alpha, \quad (6.11)$$

$$\epsilon \simeq \frac{\alpha}{2N_k}, \quad \eta \simeq -\frac{1}{2N_k}. \quad (6.12)$$

Hence

$$\mathcal{P}_s(k) \simeq \frac{1}{24\pi^2} \frac{2N_k}{\alpha} \left(\frac{M}{m_p} \right)^{4-2\alpha} (4\alpha\lambda_1 N_k)^\alpha, \quad (6.13)$$

$$\mathcal{P}_t(k) \simeq \frac{1}{6\pi^2} \left(\frac{M}{m_p} \right)^{4-2\alpha} (4\alpha\lambda_1 N_k)^\alpha, \quad (6.14)$$

$$n_s \simeq \left(1 - \frac{1}{N_k} \right), \quad (6.15)$$

⁴ $3H\dot{s}_R = -V_{,s_R}$, $H^2 = V(s_R)/3m_p^2$, where $H = \dot{a}/a$ is the Hubble parameter and a the cosmological scalar factor.

and we find

$$\mathcal{P}_s(k) \simeq 7.3 \times 10^{-9} \left(\frac{M^2}{10^{-5} \lambda_1 m_p^2} \right)^2, \quad n_s \simeq 0.982, \quad (6.16)$$

where we have taken $N_k = 55$, and neglected a factor raised to the power α , as $\alpha \sim 10^{-2}$.

The data we use consists of the WMAP7 best-fit values for $\mathcal{P}_s(k_0)$ and n_s in the standard Λ CDM model are [33]

$$\mathcal{P}_s(k_0) = (2.43 \pm 0.11) \times 10^{-9}, \quad n_s = 0.963 \pm 0.012(68\%CL) \quad (6.17)$$

with an upper limit on r of [34]

$$r < 0.36(95\%CL). \quad (6.18)$$

The string contribution is small, so we can equate the F -term prediction for the scalar power spectrum to the WMAP measured value to find

$$\frac{M^2}{\lambda_1 m_p^2} \simeq 6 \times 10^{-6}. \quad (6.19)$$

Assuming 55 e-foldings of inflation, the allowed range of λ_1 is therefore approximately

$$2.0 \times 10^{-3} \ll \lambda_1 \ll 1, \quad (6.20)$$

where the lower bound comes from the requirement that $s_R \gg s_c$ (eq. (6.9)), and the upper bound from $s_R^2 \ll m_p^2$ (eq. (6.10)).

Note that the tilt is about 2σ away from the best-fit value for single-field inflation. A small string contribution to the CMB power spectrum at the level of 5-10% restores or even slightly improves the CMB fit [14, 35, 36] although at the cost of a higher baryon fraction and a less steep dark matter power spectrum, putting the model into tension with other data [14].

6.1 The $\xi = 0$ case

We can perform a similar analysis to that presented above for the case $\xi = 0$, $\lambda_3 > \lambda_1$. By exploiting the freedom to add finite local counterterms to the one loop potential, we may write (from eq. (5.28)):

$$V(s) \simeq M^4 \left[1 + \frac{\lambda_1^2 + 2\lambda_3^2}{16\pi^2} \ln \frac{2s^2}{s_c^2} \right] \simeq M^4 \left(\frac{s_R^2}{s_c^2} \right)^\alpha, \quad (6.21)$$

where we still have $s_c^2 = M^2/\lambda_1$, but now

$$\alpha = \frac{\lambda_1^2 + 2\lambda_3^2}{16\pi^2}. \quad (6.22)$$

The analysis of eq. (6.4)-eq. (6.20) goes through essentially unchanged, except that in eq. (6.16)-eq. (6.20), λ_1 is replaced by $\sqrt{\lambda_1^2 + 2\lambda_3^2}$.

While we require $\lambda_3 > \lambda_1$, (unless $\lambda_3 = 0$, which is allowed, as explained above) we cannot have $\lambda_3 \gg \lambda_1$, since were this the case the amplitude of the inflation perturbations would be proportional to $(M^2/(\lambda_3 m_p^2))^2$ and dominated by the string perturbations, which are proportional to $(M^2/(\lambda_1 m_p^2))^2$. One sees this easily from eq. (6.16) with α defined by eq. (6.22), and the string tension by eq. (A.13).

7 Cosmic microwave background string constraints

The symmetry-breaking of the $U(1)'$ symmetry at the end of inflation produces cosmic strings [37, 38], which are a source of gravitational perturbations and contribute to the cosmic microwave background fluctuations. The exact constraint depends on details of the modelling of the strings, but simulations of the Abelian Higgs model compared to WMAP data [36] give

$$G\mu \lesssim 7 \times 10^{-7}. \tag{7.1}$$

where μ is the string tension (not to be confused with the renormalisation scale μ appearing in eq. (2.3) or eq. (5.8), for example). The Unconnected Segment Model of string perturbations [39] gives a similar upper bound [40]. The simple F -term hybrid inflation model is more tightly constrained [14], as the string tension is related to the inflation scale. However, the AMSB model has more freedom, and we shall use the more general string bound eq. (7.1)). For this we will need to calculate the string tension.

For our model, the string tension is a function of the parameters λ_1 , $q_\phi g'$, M^2 and ξ/q_ϕ . There are two limits where we can write analytic expressions (see appendix):

(a) $\xi \gg q_\phi M^2/\lambda_1$, for which the string tension is

$$\mu_a \simeq 2\pi\xi/q_\phi. \tag{7.2}$$

(b) $\xi \ll q_\phi M^2/\lambda_1$, for which the string tension is

$$\mu_b \simeq 2\pi B(\lambda_1^2/2q_\phi^2 g'^2) \frac{2M^2}{\lambda_1}. \tag{7.3}$$

where B is a slowly varying function of its argument, satisfying $B(1) = 1$. Recall that $q_\phi = 10$ in our model, so its presence is significant.

Case (a) is already ruled out. The string constraint eq. (7.1) can be rewritten as

$$\frac{\xi}{q_\phi m_p^2} \lesssim 3 \times 10^{-6}, \tag{7.4}$$

which together with eq. (6.19) is inconsistent with the assumption $\xi \gg q_\phi M^2/\lambda_1$.

Let us turn to case (b). Given that $G\mu \lesssim 7 \times 10^{-7}$ we can substitute the string tension eq. (7.3) and use the inflationary normalisation eq. (6.19) to derive an approximate upper bound on B ,

$$B \lesssim 7 \times 10^{-7} \left(\frac{2\lambda_1 m_p^2}{M^2} \right) \simeq 0.2. \tag{7.5}$$

Hence the value of $\lambda_1^2/2q_\phi^2 g'^2$ has to be small in order for strings not to exceed the CMB bound. Using the approximation [41] $B(\beta) \simeq 2.4/\ln(2/\beta)$ for $\beta \lesssim 10^{-2}$, we find

$$\frac{\lambda_1}{\sqrt{2}q_\phi g'} \lesssim 3 \times 10^{-3}. \tag{7.6}$$

A Monte-Carlo fit in the simple F -term inflation model, using the numerically determined string tension, and taking into account the degeneracies between $G\mu$ and the other

cosmological parameters, has been performed by Battye, Garbrecht & Moss [14]. One should note that their inflation superpotential is $W_{\text{BGM}} = \kappa S(\Phi\bar{\Phi} - M_{\text{BGM}}^2)$, so that $\kappa = \lambda_1$ and $M_{\text{BGM}} = M/\sqrt{\lambda_1}$, and that they take $g' = 0.7$ and $q_\phi = 1$. They take the string tension to be [42] eq. (A.4), with $v^2 = M_{\text{BGM}}^2$. The best fit models have $M_{\text{BGM}} \sim 5 \times 10^{15}$ GeV and $\kappa \sim 10^{-3}-10^{-2}$.

Comparison is not straightforward, as our model's string tension receives contributions from the D -term eq. (A.13).⁵ This difference in the string tension formulae, the accuracy of the fit, and the slowly varying nature of the function B mean that our estimate on the coupling ($\lambda_1 \lesssim 0.4$) is broadly compatible with the upper bound on their κ .

8 Numerical scan of parameter space

We have done some numerical testing of the parameter space of the model, looking for combinations which are consistent with our assumptions and the data. We have designated the SM parameters to their measured value, taken $\tan(\beta) = 60$, and, as mentioned, taken the Yukawa couplings to be real and diagonal. In the subspace we have investigated this leaves the following 6 variables: $M, \lambda_1, \lambda_3, y_{\nu_e}, y_{\nu_\mu}, \sqrt{\xi}$. In this analysis we used the approximations for the appropriate mass eigenvalues given by eq. (5.21). The following conditions must be satisfied for a successful model.

1. All vevs, masses and mass scales should be less than the Planck scale, otherwise our neglect of gravitational corrections becomes inconsistent. We check the value of the inflation field s and all masses between $s = s_{55}$, where s_{55} is the s field value 55 e-foldings before the end of inflation, and $s = s_c$.
2. The string tension should satisfy the CMB upper bound eq. (7.1), which we use in conjunction with the formulae of the appendix. This depends on g' through eq. (A.12); we have taken a weak limit, namely the one that arises from $g' = 1$. We have not treated g' as a variable since there is no dependence on g' other than here in the string tension.
3. There should be no tachyons during inflation, other than those which drive the vevs of ϕ and $\bar{\phi}$ at the end of inflation. This means obeying eq. (5.20).
4. The amplitude of scalar perturbations should be consistent with observations, eq. (6.17). We ignore the small string contribution to the power spectrum for the purpose of our approximate survey.
5. We require that the scalar spectral tilt n_s be within 3σ of its measure mean value, eq. (6.17).

⁵Note that the two formulae disagree by a factor 2 even in the limit $\xi \rightarrow 0$. This is the result of an incorrect application (in ref. [14]) of the standard formula for Abelian Higgs string tension (A.4), by neglecting the fact that F -term strings have two scalar fields.

6. We require that all couplings be perturbative, i.e. less than 1 at the inflation scale M . Applying this condition (approximately) to the elements of the matrix λ_2 puts an upper bound on the neutrino Dirac Yukawa couplings, via the seesaw formula

$$m_{\nu_i} = \frac{m_D^2}{m_N} = \frac{y_{\nu_i}^2 v^2 \sin^2 \beta}{\lambda_{2i,\text{eff}} v_\phi}. \quad (8.1)$$

Here $v = 174\text{GeV}$ and m_ν , m_D , m_N are the physical, Dirac and Majorana masses of the neutrino, and $\lambda_{2i,\text{eff}}$ is the effective parameter from λ_2 relevant for the i 'th neutrino state. The vev v_ϕ is determined by eqs. (4.8)–(4.10). The CMB upper limit on the sum of the neutrino masses of approximately 1.5 eV [33] can then be translated into an upper bound $y_{\nu_i}^2 < m_{\nu_i} v_\phi / v^2 \sin^2 \beta$. In practice this does not set an extra bound on y_{ν_i} above the perturbativity bound, as $m_{\nu_i} v_\phi / v^2 \lesssim 50$.

Some successful parameter combinations are placed in the left half of table 3. In the right half we put the resulting slow-roll parameters, scalar amplitude, tilt, string tension as a fraction of the μ_{max} (the maximal tension eq. 7.1 evaluated with $g' = 1$), and the value of the inflaton in Planck units at $N = 55$ e-folds of inflation.

The first line is the case when $\lambda_3 = 0$ and $\xi = 0$, which is the limit in which the radiative corrections from the MSSM field vanish, $\Delta V_2 = \Delta V_3 = 0$, and the system reverts to the simple F-term model of section 5.3. This case seems to have a high n_s compared to the single-field inflation mean value of 0.963 [33], but as pointed out in section 6, a high n_s is a feature in models with cosmic strings contributing 5-10% to the power spectrum at $\ell = 10$ [35, 36]. This scenario ($\lambda_3 = 0$) would mean we would require an alternative source for the Higgs μ -term. The second row is another case with vanishing FI term $\xi = 0$, but $\lambda_3 \neq 0$.

For the other cases, we have demanded $\lambda_1 \lambda_3 \geq 10^{-5}$, $\sqrt{\xi}/M \geq 0.2$ to reject values of the effective μ that are unnecessarily low and so that ξ, λ_3 actually play a role. We see that n_s is almost independent of ξ, λ_3 with the assumptions made.

These parameter values should only be taken as indicative of regions of parameter space where a proper Monte Carlo fit with more accurate formulae should be undertaken. This we leave for a future work.

The third row has the highest $\lambda_1 \lambda_3$ and λ_3 , the fourth has the highest n_s , the fifth has the lowest n_s and lowest M^2/λ_1 , the sixth has the highest M^2/λ_1 and the seventh has the highest $\sqrt{\xi}$.

In a supergravity extension of the model, with Kähler potential

$$K = |s|^2 + c|s|^4/m_{\text{P}}^2, \quad (8.2)$$

the tree-level potential would be modified to

$$V_{\text{sugra}} = M^4 \left(1 - 4c \frac{|s|^2}{m_{\text{P}}^2} + \left(\frac{1}{2} - 7c + 16c^2 \right) \frac{|s|^4}{m_{\text{P}}^4} \right), \quad (8.3)$$

which potentially gives rise to the well-known supergravity η -problem [28]. One can hope that some symmetry sets $c = 0$, but a quartic term in the scalar potential is unavoidable,

| $\frac{M}{10^{14}\text{GeV}}$ | $\frac{\lambda_1}{10^{-3}}$ | $\frac{M^2}{10^{-6}\lambda_1 m_{pl}^2}$ | $\frac{\sqrt{\xi}}{M}$ | $\frac{\lambda_3}{10^{-3}}$ | $\frac{\epsilon}{10^{-9}}$ | $\frac{\eta}{10^{-3}}$ | $\frac{\mathcal{P}_s}{10^{-9}}$ | n_s | $\frac{\mu}{\mu_{\max}}$ | $\frac{s_{55}}{10^{-3}m_P}$ |
|-------------------------------|-----------------------------|---|------------------------|-----------------------------|----------------------------|------------------------|---------------------------------|-------|--------------------------|-----------------------------|
| 23.1 | 161 | 5.62 | 0 | 0 | 5720 | -8.70 | 2.43 | 0.983 | 1.000 | 190 |
| 5.28 | 4.87 | 9.71 | 0 | 4.87 | 3550 | -7.89 | 2.43 | 0.984 | 1.000 | 10.7 |
| 7.98 | 18.3 | 5.89 | 0.284 | 4.33 | 20.3 | -8.61 | 2.41 | 0.983 | 0.993 | 23.3 |
| 4.12 | 4.31 | 6.67 | 0.287 | 2.61 | 1.42 | -5.66 | 2.45 | 0.989 | 0.981 | 6.54 |
| 13.0 | 51.0 | 5.61 | 0.200 | 0.196 | 1440 | -8.75 | 2.41 | 0.982 | 0.987 | 61.5 |
| 4.12 | 4.03 | 7.13 | 0.200 | 2.61 | 1.42 | -6.41 | 2.44 | 0.987 | 0.983 | 6.65 |
| 4.12 | 5.10 | 5.64 | 0.590 | 2.38 | 1.42 | -6.77 | 2.44 | 0.986 | 0.962 | 6.78 |

Table 3. Mass scale and coupling constant values (left half), with inflationary and cosmic string parameters (right half) for cases satisfying the numbered constraints in the text.

and a minimal requirement for success of the model is that $s_{55}/m_P \ll 1$. One can easily check which cases are afflicted by the η -problem by evaluating

$$\Delta\eta_{\text{sugra}} = m_P^2 \frac{V''_{\text{sugra}}}{V_{\text{sugra}}}, \tag{8.4}$$

which is an upper bound on the change in η due to the supergravity corrections. Recall that $s_R = (\Re s)/\sqrt{2}$, and without loss of generality we may suppose that s is real. In the $c = 0$ case we find that

$$\Delta\eta_{\text{sugra}} \simeq \frac{3}{2} \frac{s_R^2}{m_P^2} \left(1 + \frac{1}{8} \frac{s_R^4}{m_P^4} \right)^{-1}. \tag{8.5}$$

Using the values of s_{55}/m_P listed in the last column of the table, we can verify that only the first is afflicted, with $\Delta\eta_{\text{sugra}} = 0.122$.

A possible (albeit tuned) solution is to posit a small positive value of c , which reduces η by an amount $\Delta\eta \sim -4c$. The supergravity corrections can also be reduced by reducing s_{55} , and from eq. (6.7) one sees this can be effected with smaller values of λ_1 . If M^2 is also reduced, so as to keep M^2/λ_1 constant, then the string tension μ decreases approximately logarithmically.

The problem, however, lies in the requirement that, although $\lambda_3 \geq \lambda_1$, we cannot have $\lambda_3 \gg \lambda_1$, because then, as mentioned in section 6.1, the inflation CMB perturbations would be suppressed relative to the string ones. Inevitably, therefore, the Higgs μ -term $\mu_h = \lambda_3 v_s$ would be reduced. Here we thus encounter an interesting tension between the requirements of the theory in two quite different epochs, corresponding to inflation and electro-weak symmetry breaking.

Note that values of $\lambda_{1,3}$ as small as those obtained above have consequences for the Higgs μ term. This was given by $\mu = \lambda_3 v_s$; if we assume that ρ is, unlike the other soft terms, unsuppressed by loop factors, we find from eq. (4.10) that $v_s \sim \rho \lambda_1 m_{\frac{3}{2}}$ suggesting that $\mu \sim O(\text{GeV})$ rather than $O(100\text{GeV})$. This will impact the electroweak vacuum minimisation and the associated sparticle spectrum, which we will explore elsewhere.⁶

⁶A constraint on μ_h might, for example, lead to a prediction for $\tan\beta$ from the minimisation.

9 Other cosmological constraints

While the main topic of this paper is the possibility for inflation in AMSB, there are other cosmological phenomena to consider. They come both as bounds and as possibilities to explain observed phenomena, e.g. the presence of baryonic and dark matter.

9.1 Gravitational wave and cosmic ray constraints

Cosmic strings have a property known as *scaling* [37, 38], which means that they maintain a constant density parameter $\Omega_s = \rho_s/\rho_c$, where ρ_s is the string energy density and $\rho_c = 3m_p^2 H^2$ is the critical density. Numerical simulations [43]–[45] indicate that there are $O(1)$ or a few Hubble lengths of string per Hubble volume, so that $\rho_s \sim \mu/t^2$ and $\Omega_s \sim G\mu$.

The string energy density is therefore decaying at a rate $\dot{\rho}_s \sim \mu/t^3$. There are two scenarios for the products of this decay: the primary channel is either via closed loops of string into gravitational radiation or into high energy particles of the fields from which the string is made. The first scenario is constrained by bounds on the stochastic background of gravitational radiation (see e.g. [46]), and the second by the flux of cosmic rays [47]. The first depends on the typical loop size relative to the Hubble length, α , and the second on the complex decay processes of the massive scalars, gauge bosons, and fermions of the string fields, here the ϕ , $\bar{\phi}$, $U(1)'$ gauge field, and the neutrino zero modes. It is also possible that the Higgs field h_2 has a vev in the string core (see appendix).

In the first scenario, with the assumption about the average loop size $\alpha \ll G\mu$, the upper bound on the string tension is [40]

$$G\mu \lesssim 7 \times 10^{-7}. \tag{9.1}$$

In the second scenario a detailed modelling of the decay cascades is required as the bounds on the mass scale are sensitive to the primary Standard Model decay products of the “X” particles into which the string decays [48]. Cosmic strings constitute a $p = 1$ TD (topological defect or top-down) [48] model, for which there is an upper bound on the energy injection rate from the low energy diffuse γ -ray background [48]

$$Q_0 \lesssim 4.4 \times 10^{-23} h \text{ eV cm}^{-3} \text{ s}^{-1}. \tag{9.2}$$

Assuming the massive particles decay into Standard Model particles with a non-zero branching fraction f , one can derive a bound [49]–[52]

$$G\mu \lesssim 10^{-9} x_*^2 f^{-1}, \tag{9.3}$$

where $x_*^2 = \mu/\rho_s t^2$ parametrises the Hubble lengths of string per Hubble volume, with $x_* \sim 0.3$ [43]–[45]. The string gauge field couples to all Standard Model particles, so the cosmic ray constraints are potentially strong in this scenario. In view of the uncertainty about which is correct, we have adopted the weaker constraint (9.1), which is no stronger than the CMB bound (7.1).

9.2 Dark matter

Low energy supersymmetry (with the imposition of R -parity) has the attractive feature that the LSP is stable, and is thus a dark matter candidate. For a recent general review of the situation, see [53]. For an agnostic, the parameter space of low energy supersymmetry allows many candidates for the LSP, including the gravitino; in conventional AMSB the gravitino is certainly too heavy. Since a charged LSP would surely have been detected in terrestrial studies, a framework which automatically excludes them is to be welcomed. This is true of the version of AMSB we discuss here, except very close to the boundaries of (q_L, q_E) parameter space, where the LSP can in fact be a charged lepton. Generally, however, the LSP is a neutralino with a dominant neutral wino component [11]. This has been argued to disfavour it as a dark matter candidate in AMSB, because of the (comparatively) large annihilation cross-section of such a neutralino [2]. However, AMSB models in general, and our model in particular, could produce the required neutralino abundance from the decays of thermally produced gravitinos, provided that the reheat temperature is high enough [54]–[56]. For $m_{\frac{3}{2}} \simeq 40$ TeV this is around 2×10^{10} GeV. (For a Bayesian analysis of how the minimal AMSB scenario is constrained by other observables, see [57]).

Our model also has cosmic strings, and neutralinos will generically be produced by decays of particles radiated from them [58]. We note that this and other works (see for example [59, 60]) studying dark matter production from strings make conservative assumptions about the amount of particle production by assuming that gravitational radiation dominates, so a re-calculation of the dark matter density as a function of the string tension μ and the branching fraction of string decays into neutralinos would be extremely useful.

Finally, one might also entertain the possibility that the AMSB pattern of supersymmetry breaking is associated with a mass scale other than the gravitino mass; we would then be free to consider a gravitino light enough to be the LSP, with the wino-dominated neutralino now the NLSP and metastable. That would however, not be consistent with the leptogenesis scenario described in the next section.

9.3 Baryogenesis

Creation of the observed baryon asymmetry requires baryon number violation, departure from thermal equilibrium and C and CP-violation [61]. Our model has no conserved lepton numbers, and so it is natural to explore creation of the observed baryon number via leptogenesis.

Now in AMSB, non-CKM CP-violating phases do not exist in the soft-breaking sector, apart from a possible phase associated with the μ and B terms, for which we do not have a complete theory; they are simply constrained so as to produce the SM vacuum. To put it another way, κ in eq. (2.4) could be complex. An interesting potential source (of CP violation) is, however, the Yukawa sector for right-handed neutrinos, which (with the standard see-saw mechanism for generating neutrino masses) is relevant for leptogenesis [62]. Successful supersymmetric leptogenesis requires that the lightest right-handed neutrino (and the post-inflation reheating temperature) be greater than 10^9 GeV; note that because of the large gravitino mass (around 40 TeV) associated with AMSB there is no danger that the

decay of gravitinos produced in this reheating will pose a problem for nucleosynthesis [63]. (For a recent discussion of some other ways of evading the gravitino bound see ref. [64]).

There is also a source of leptogenesis through out-of-equilibrium decays of particles radiated by the strings, along the lines of the scenario investigated in refs. [65]–[67] for $B - L$ cosmic strings. It would be interesting to investigate this further in conjunction with the dark matter and cosmic ray constraints on strings.

Finally, in the $\xi \neq 0$ case, our AMSB model also has the right conditions for Affleck-Dine baryogenesis [68], in that inflation naturally generates large vevs for fields with baryon and lepton number through the minimisation of the D -term. A detailed investigation would involve numerical simulations of the dynamics of the fields at the end of inflation, which is beyond the scope of this paper.

10 Conclusions

We have shown how a theory with low energy supersymmetry, constructed so as to produce a viable sparticle spectrum based on anomaly mediation, also has significant cosmological consequences. The AMSB scenario is an attractive alternative to (and easily distinguished from) the CMSSM. (It was believed that AMSB was disfavoured in terms of accommodating the existing discrepancy between theory and experiment for the anomalous magnetic moment of the muon; but this conclusion has been challenged recently [22]). We have shown how a $U(1)'$ gauge symmetry originally introduced to solve the AMSB tachyonic slepton problem leads to interesting cosmological possibilities.

In the minimal form presented here, the $U(1)'$ gauge symmetry requires three extra chiral Standard Model singlets, two of which are charged under the $U(1)'$. From this new structure we obtain a μ -term, Majorana masses for the right-handed neutrinos, and potentially CP-violating mixings.

The model naturally realises F -term hybrid inflation, terminating with the production of cosmic strings. CMB data put strong constraints on the extra parameters introduced, principally the F -term and D -term mass scales M and $\sqrt{\xi}$, and the inflaton couplings λ_1 and λ_3 . If $\xi \neq 0$, the D -term induces squark and slepton vevs during inflation, which allows Affleck-Dine baryogenesis to take place, using CP-violation in the neutrino sector. If we set $\xi = 0$, as argued for in ref. [17], then there are other sources of baryogenesis include conventional leptogenesis and non-thermal leptogenesis from cosmic string decays. Cosmic string and gravitino decays also boost the dark matter density, which is normally low in the conventional freeze-out scenario.

We have seen that choosing parameters so as to avoid the η -problem has the surprising consequence that the prediction for the Higgs μ -term is reduced. If it proves nevertheless possible to implement electro-weak breaking in a satisfactory way this will count as a success for the model, providing as it does a potential solution for the “little hierarchy” problem.

In conclusion, our AMSB model can satisfy the principal cosmological constraints, and provide an acceptable particle physics phenomenology, in the framework of a renormalisable quantum field theory with few extra parameters above those of the Standard Model. There are the neutrino coupling matrices λ_2 and Y_N (which are common to models incorporating

neutrinos), two coupling constants λ_1 and λ_3 and the leptonic $U(1)'$ charges (q_L, q_E) . There are in general two mass scales M and ξ , and two parameters associated with the supersymmetry breaking, $m_{\frac{3}{2}}$ and κ . This economy makes this a model worthy of more detailed investigation on all fronts.

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A String tension in the AMSB model

For the ordinary Abelian Higgs model the string tension is given in terms of the quartic coupling λ , the gauge coupling which we will call g' , and the expectation value of the complex scalar field v . With the ansatz

$$\phi(r, \theta) = vR(r)e^{i\theta}, \quad A_i = \hat{\theta}_i \frac{a(r)}{g'r}, \quad (\text{A.1})$$

and boundary conditions $a, R \rightarrow 0$ as $r \rightarrow 0$ and $a, R \rightarrow 1$ as $r \rightarrow \infty$ we obtain a solution for which the string tension is

$$\mu = 2\pi \int_0^\infty dr r \left(\frac{1}{2} \left(\frac{a'}{g'r} \right)^2 + v^2 (R')^2 + \frac{(1-a)^2}{r^2} R^2 v^2 + \frac{1}{2} \lambda v^4 (R^2 - 1)^2 \right) \quad (\text{A.2})$$

Defining a dimensionless radial coordinate $z = rv g'$, we have

$$\mu = 2\pi v^2 \int_0^\infty dz z \left(\frac{1}{2} \left(\frac{a'}{z} \right)^2 + (R')^2 + \frac{(1-a)^2}{r^2} R^2 + \frac{1}{2} \beta (1 - R^2)^2 \right) \quad (\text{A.3})$$

where $\beta = \lambda/g'^2$. In the special case $\beta = 1$, the string tension is $2\pi v^2$. More generally

$$\mu = 2\pi B(\beta) v^2, \quad (\text{A.4})$$

where B is a slowly varying function of its argument, satisfying $B(1) = 1$. For low β the function can be approximated by [41]

$$B(\beta) \simeq \begin{cases} 1.04\beta^{0.195}, & 10^{-2} < \beta \ll 1 \\ 2.4/\ln(2/\beta), & \beta < 10^{-2} \end{cases} \quad (\text{A.5})$$

For the AMSB model, the string tension is a function of the parameters $\lambda_1, q_\phi g', M^2$ and ξ/q_ϕ . The ansatz is

$$\phi(r, \theta) = v_\phi R(r) e^{i\theta}, \quad \bar{\phi}(r, \theta) = -v_{\bar{\phi}} \bar{R}(r) e^{-i\theta}, \quad A_i = \hat{\theta}_i \frac{a(r)}{q_\phi g' r}, \quad (\text{A.6})$$

with the vacuum expectation values of the fields ϕ and $\bar{\phi}$ given by (see eqs. (4.8), (4.9))

$$v_\phi^2 = \frac{1}{2} \left[\sqrt{\left(\frac{\xi}{q_\phi}\right)^2 + \left(\frac{2M^2}{\lambda_1}\right)^2} + \frac{\xi}{q_\phi} \right], \quad (\text{A.7})$$

$$v_{\bar{\phi}}^2 = \frac{1}{2} \left[\sqrt{\left(\frac{\xi}{q_\phi}\right)^2 + \left(\frac{2M^2}{\lambda_1}\right)^2} - \frac{\xi}{q_\phi} \right]. \quad (\text{A.8})$$

The string tension is

$$\begin{aligned} \mu = 2\pi \int_0^\infty dr r \left(\frac{1}{2} \left(\frac{a'}{q_\phi g' r} \right)^2 + [v_\phi^2 (R')^2 + v_{\bar{\phi}}^2 (\bar{R}')^2] + \frac{(1-a)^2}{r^2} [v_\phi^2 R^2 + v_{\bar{\phi}}^2 \bar{R}^2] \right. \\ \left. + \frac{1}{2} g'^2 (\xi - q_\phi v_\phi^2 R^2 + q_\phi v_{\bar{\phi}}^2 \bar{R}^2)^2 + (\lambda_1 v_\phi v_{\bar{\phi}} R \bar{R} - M^2)^2 \right). \end{aligned} \quad (\text{A.9})$$

We can get an upper bound and a reasonable approximation by assuming that $\bar{R} = R$, in which case the string tension can be written

$$\mu \simeq 2\pi (v_\phi^2 + v_{\bar{\phi}}^2) \int_0^\infty dz z \left[\frac{1}{2} \left(\frac{a'}{z} \right)^2 + (R')^2 + \frac{(1-a)^2}{z^2} R^2 + \frac{\beta_{\text{eff}}}{2} (1-R^2)^2 \right] \quad (\text{A.10})$$

where

$$z^2 = q_\phi^2 g'^2 r^2 (v_\phi^2 + v_{\bar{\phi}}^2), \quad \beta_{\text{eff}} = \frac{1 + \psi\beta}{1 + \psi}. \quad (\text{A.11})$$

Here we have defined

$$\psi = \frac{2M^2 q_\phi}{\lambda_1 \xi}, \quad \beta = \frac{\lambda_1^2}{2q_\phi^2 g'^2}. \quad (\text{A.12})$$

Thus we see that the string tension in the AMSB model is approximately

$$\mu \lesssim 2\pi B(\beta_{\text{eff}}) \sqrt{\left(\frac{\xi}{q_\phi}\right)^2 + \left(\frac{2M^2}{\lambda_1}\right)^2}. \quad (\text{A.13})$$

The approximation becomes an equality in the limits $\psi \rightarrow 0, \infty$. In the first case the assumed symmetry between ϕ and $\bar{\phi}$ becomes exact as the D -term becomes negligible, and in the second case $\bar{\phi}$ vanishes as the F -term becomes negligible. The expressions for the string tension in these two limits is

$$\mu_a = 2\pi \frac{\xi}{q_\phi}, \quad \mu_b = 2\pi B(\beta) \frac{2M^2}{\lambda_1}. \quad (\text{A.14})$$

A more accurate solution can be obtained by a numerical minimisation of the string tension function (A.9). One should also allow for the possibility of the MSSM scalars with positive q_ϕ gaining an expectation value in the core of the string, as this reduces the D -term potential energy density which would otherwise be g'^2/ξ at the core of the string. A prime candidate is the Higgs field h_2 , as it already has a vev. The other candidates are l and e^c , but they have lower q_ϕ and are therefore less unstable in the string core.

Finally, we note that the string will have fermionic zero modes from two sources: from the neutrinos thanks to the $\frac{1}{2}\lambda_2 NN\Phi$ coupling, and from mixtures of the superpartners of s , ϕ and $\bar{\phi}$, thanks to the $S\Phi\bar{\Phi}$ coupling [69].

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