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Quintessence arising from exponential potentials

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We demonstrate how the properties of the attractor solutions of exponential potentials can lead to models of quintessence with the currently observationally favored equation of state. Moreover, we show that these properties hold for a wide range of initial conditions and for natural values of model parameters.

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I. INTRODUCTION

Measurements of the redshift-luminosity distance relation using high redshift type Ia supernovas combined with cosmic microwave background (CMB) and galaxy clusters data appear to suggest that the present Universe is flat and undergoing a period of Λ driven inflation, with the energy density split into two main contributions, $\Omega_{\text{matter}} \approx 1/3$ and $\Omega_{\Lambda} \approx 2/3$ [1–3]. Such a startling finding has naturally led theorists to propose explanations for such a phenomenon. One such possibility that has attracted a great deal of attention is the suggestion that a minimally coupled homogeneous scalar field Q (the ‘‘quintessence’’ field), slowly rolling down its potential, could provide the dominant contribution to the energy density today thanks to the special form of the potential [4,5]. Non-minimally coupled models have also been investigated [6–11]. The advantage of considering a more general component that evolves in time so as to dominate the energy density today, as opposed to simply inserting the familiar cosmological constant, is that the latter would require a term $\rho_{\Lambda} \approx 10^{-47} \text{ GeV}^4$ to be present at all epochs, a rather small value when compared to typical particle physics scales. On the other hand, quintessence models possess attractor solutions which allow for a wide range of initial conditions, all of which can correspond to the same energy density today simply by tuning one overall multiplicative parameter in the potential.

There is a long history to the study of scalar field cosmology especially related to time varying cosmological constants. Some of the most influential early work is to be found in Refs. [12–14]. One particular case which at first sight appears promising is the one involving exponential potentials of the form $V \propto \exp(\lambda \kappa Q)$, where $\kappa^2 = 8\pi G$ [12–19]. These have two possible late-time attractors in the presence of a barotropic fluid: a scaling regime where the scalar field mimics the dynamics of the background fluid present, with a constant ratio between both energy densities, or a solution dominated by the scalar field. The former regime cannot explain the observed values for the cosmological parameters discussed above; basically it does not allow for an accelerating expansion in the presence of a matter background fluid. However, the latter regime does not provide a feasible scenario either, as there is a tight constraint on the allowed magnitude of Ω_Q at nucleosynthesis [17,18]. It turns out that it must satisfy $\Omega_Q(1 \text{ MeV}) < 0.13$. On the other hand, we must allow time for formation of structure before the Universe starts accelerating. For this scenario to be possible we

would have to fine-tune the initial value of ρ_Q , but this is precisely the kind of thing we want to avoid.

A number of authors have proposed potentials which will lead to Λ dominance today. The initial suggestion was an inverse power law potential (‘‘tracker type’’) $V \propto Q^{-\alpha}$ [5,12,19], which can be found in models of supersymmetric QCD [20,21]. Here the ratio of energy densities is no longer a constant but ρ_Q scales slower than ρ_B (the background energy density) and will eventually dominate. This epoch can be set conveniently to be today by tuning the value of only one parameter in the potential. However, although appealing, these models suffer in that their predicted equation of state $w_Q = p_Q/\rho_Q$ is marginally compatible with the favored values emerging from observations using SNIa and CMB measurements, considering a flat universe [22–24]. For example, at the 2σ confidence level in the Ω_{M-w_Q} plane, the data prefer $w_Q < -0.6$ with a favored cosmological constant $w_Q = -1$ (see e.g. [24]), whereas the values permitted by these tracker potentials (without fine-tuning) have $w_Q > -0.7$ [25]. For an interpretation of the data which allows for $w_Q < -1$ see Ref. [26].

Since this initial proposal, a number of authors have made suggestions as to the form the quintessence potential could take [27–33]. In particular, Brax and Martin [28] constructed a simple positive scalar potential motivated from supergravity models, $V \propto \exp(Q^2)/Q^\alpha$, and showed that even with the condition $\alpha \geq 11$, the equation of state could be pushed to $w_Q \approx -0.82$, for $\Omega_Q = 0.7$. A different approach was followed by the authors of [30]. They investigated a class of scalar field potentials where the quintessence field scales through an exponential regime until it gets trapped in a minimum with a non-zero vacuum energy, leading to a period of de Sitter inflation with $w_Q \rightarrow -1$.

In this Brief Report we investigate a simple class of potentials which lead to striking results. Despite previous claims, exponential potentials by themselves are a promising fundamental tool to build quintessence potentials. In particular, we show that potentials consisting of sums of exponential terms can easily deliver acceptable models of quintessence in close agreement with observations for natural values of parameters.

II. MODEL

We first recall some of the results presented in [14,17,18]. Consider the dynamics of a scalar field Q , with an exponential potential $V \propto \exp(\lambda \kappa Q)$. The field is evolving in a spa-

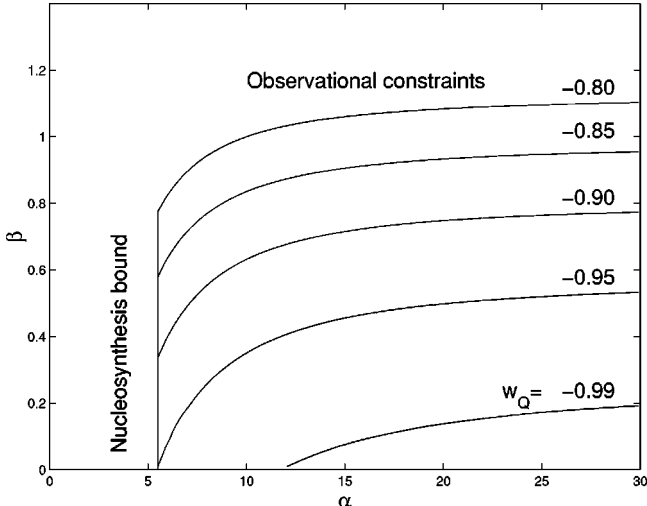


FIG. 1. Contour plot of $w_Q(\text{today})$ as a function of (α, β) , with the constraint $\Omega_Q(\text{today}) \approx 0.7$. The region $\alpha < 5.5$ is excluded because of the nucleosynthesis bound, $\Omega_Q(1 \text{ MeV}) < 0.13$, and the upper region due to 1σ observational constraints.

tially flat Friedmann-Robertson-Walker (FRW) universe with a background fluid which has an equation of state $p_B = w_B \rho_B$. There exists just two possible late time attractor solutions with quite different properties, depending on the values of λ and w_B :

(1) $\lambda^2 > 3(w_B + 1)$. The late time attractor is one where the scalar field mimics the evolution of the barotropic fluid with $w_Q = w_B$, and the relation $\Omega_Q = 3(w_B + 1)/\lambda^2$ holds.

(2) $\lambda^2 < 3(w_B + 1)$. The late time attractor is the scalar field dominated solution ($\Omega_Q = 1$) with $w_Q = -1 + \lambda^2/3$.

Given that single exponential terms can lead to one of the above scaling solutions, then it should follow that a combination of the above regimes should allow for a scenario where the universe can evolve through a radiation-matter regime (attractor 1) and at some recent epoch evolve into the scalar field dominated regime (attractor 2). We will show that this does in fact occur for a wide range of initial conditions. For a concrete example consider the following potential for a scalar field Q :

$$V(Q) = M^4(e^{\alpha\kappa Q} + e^{\beta\kappa Q}), \quad (1)$$

where we assume α to be positive (the case $\alpha < 0$ can always be obtained taking $Q \rightarrow -Q$). We also require $\alpha > 5.5$, a constraint coming from the nucleosynthesis bounds on Ω_Q mentioned earlier [17,18].

First, we assume that β is also positive. In order to have an idea of what the value of β should be, note that if today we were in the regime dominated by the scalar field (i.e. attractor 2), then in order to satisfy observational constraints for the quintessence equation of state (i.e. $w_Q < -0.8$), we must have $\beta < 0.8$. We are not obviously in the dominant regime today but in the transition between the two regimes so this is just a central value to be considered. In Fig. 1 we show that acceptable solutions to Einstein's equations in the presence of radiation, matter and the quintessence field can be accommodated for a large range of parameters (α, β) .

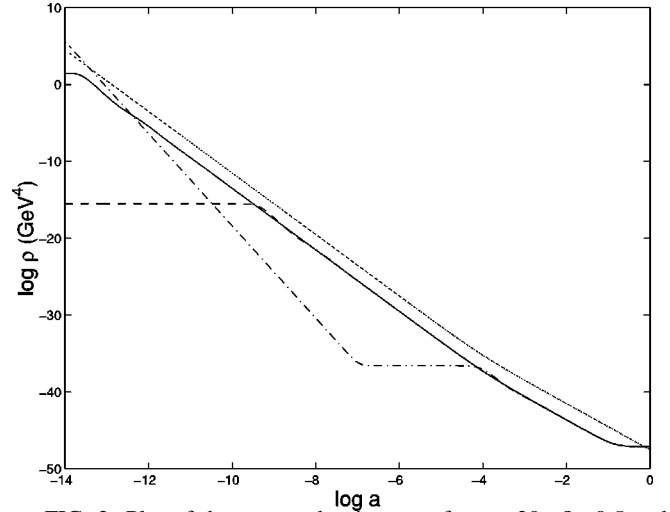


FIG. 2. Plot of the energy density, ρ_Q , for $\alpha=20$, $\beta=0.5$ and several initial conditions admitting an $\Omega_Q=0.7$ flat universe today. The solid line represents the evolution which emerges from equipartition at the end of inflation and the dotted line represents $\rho_{\text{matter}} + \rho_{\text{radiation}}$.

The value of M in Eq. (1) is chosen so that today $\rho_Q \approx \rho_c \approx 10^{-47} \text{ GeV}^4$. This then implies $M \approx 10^{-31} M_{\text{Pl}} \approx 10^{-3} \text{ eV}$. However, note that if we generalize the potential in Eq. (1) to

$$V(Q) = M_{\text{Pl}}^4(e^{\alpha\kappa(Q-A)} + e^{\beta\kappa(Q-B)}), \quad (2)$$

then all the parameters become of the order of the Planck scale. Since the scaling regime of exponential potentials does not depend upon its mass scale [i.e. M in Eq. (1)], A is actually a free parameter that can, for simplicity, be set to M_{Pl} or even to zero. On the other hand, as was the case for M , here B needs to be such that today we obtain the right value of ρ_Q . In other words, we require $M^4 \sim M_{\text{Pl}}^4 e^{-\beta\kappa B} \sim \rho_Q$. This turns out to be $B = \mathcal{O}(100)M_{\text{Pl}}$, depending on the precise values of α , β and A .

There is another important advantage to the potentials of the form in Eq. (1) or Eq. (2); namely, we obtain acceptable solutions for a wider range of initial energy densities of the quintessence field than we would with say the inverse power law potentials. For example, in Fig. 2 we show that it is perfectly acceptable to start with the energy density of the quintessence field above that of radiation, and still enter into a subdominant scaling regime at later times; however, this is an impossible feature in the context of inverse power law type potentials [25].

Another manifestation of this wider class of solutions can be seen by considering the case where the field evolution began at the end of an initial period of inflation. In that case, as discussed in Ref. [25], we could expect that the energy density of the system is equally divided among all the thousands of degrees of freedom in the cosmological fluid. This equipartition of energy would imply that just after inflation $\Omega_i \approx 10^{-3}$. If this were the case, for inverse power law potentials, the power could not be smaller than 5 if the field was to reach the attractor by matter domination. Otherwise, Q would freeze at some value and simply act as a cosmo-

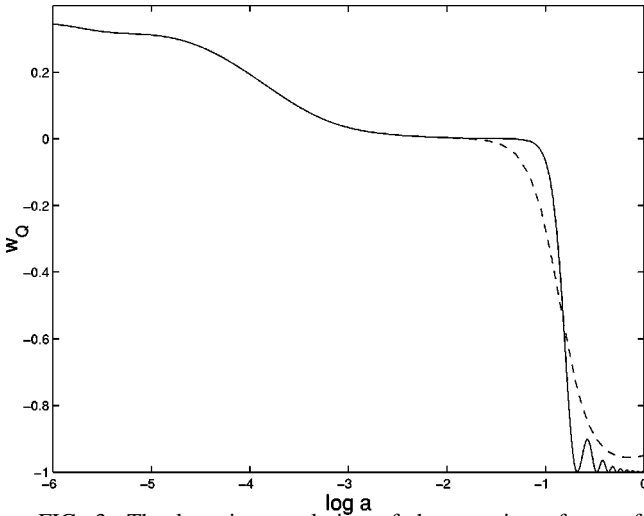


FIG. 3. The late time evolution of the equation of state for parameters (α, β) : dashed line $(20, 0.5)$; solid line $(20, -20)$ and $\Omega_Q \approx 0.7$ ($a_0 = 1$ today).

logical constant until the present (a perfectly acceptable scenario of course, but not as interesting). Such a bound on the power implies $w_Q > -0.44$ for $\Omega_Q = 0.7$. With an exponential term, this constraint is considerably weakened. Using the fact that the field is frozen at a value $Q_f \approx Q_i - \sqrt{6} \Omega_i / \kappa$, where Q_i is the initial value of the field [25], we can see that the equivalent problem only arises when

$$\alpha \sqrt{6\Omega_i} - 2 \ln \alpha \gtrsim \ln \left(\frac{\rho_{Q_i}}{2\rho_{eq}} \right), \quad (3)$$

where ρ_{Q_i} is the initial energy density of the scalar field and ρ_{eq} is the background energy density at radiation-matter equality. For instance, for our plots with $a_i = 10^{-14}$, $a_{eq} = 10^{-4}$, this results in a bound $\alpha \lesssim 10^3$.

A new feature arises when we consider potentials of the form given in Eq. (1) with the nucleosynthesis bound $\alpha > 5.5$ but taking this time $\beta < 0$. In this case the potential has a minimum at $\kappa Q_{\min} = \ln(-\beta/\alpha)/(\alpha - \beta)$ with a corresponding value

$$V_{\min} = M^4 \frac{\beta - \alpha}{\beta} \left(-\frac{\beta}{\alpha} \right)^{\alpha/(\alpha - \beta)}.$$

Far from the minimum, the scalar field scales as described above (attractor 1). However, when the field reaches the minimum, the effective cosmological constant V_{\min} will quickly take over the evolution as the oscillations are damped, driving the equation of state towards $w_Q = -1$. This scenario is illustrated in Fig. 3, where the evolution of the equation of state is shown and compared to the previous case with $\beta > 0$. In many ways this is the key result of the paper, as in this figure it is clearly seen that the field scales the radiation ($w = 1/3$) and matter ($w = 0$) evolutions before settling in an accelerating ($w < 0$) expansion. Once again, as a result of the scaling behavior of attractor 1, it is clear that there exists a wide range of initial conditions that provide realistic results. The feature resembles the recent suggestions

of Albrecht and Skordis [30]. The same mechanism can be used to stabilize the dilaton in string theories where the minimum of the potential is fine-tuned to be zero rather than the non-zero value it has in these models [33].

In [25], a quantity $\Gamma \equiv V''V/(V')^2$ is proposed as an indicator of how well a given model converges to a tracker solution. If it remains nearly constant, then the solutions can converge to a tracker solution. It is easy to see from Eq. (1) that apart from the transient regime where the solution evolves from attractor 1 to attractor 2, $\Gamma = 1$ to a high degree of accuracy.

It is important to note that for this mechanism to work, we are not limited to potentials containing only two exponential terms and one field. Indeed, all we require of the dynamics is to enter one period like regime 1, which can either be followed by one regime like 2, or by the field settling in a minimum with a non-zero vacuum energy. We can consider as an example the case of a potential depending on two fields of the form

$$V(Q_1, Q_2) = M^4 (e^{\alpha_1 \kappa Q_1 + \alpha_2 \kappa Q_2} + e^{\beta_1 \kappa Q_1 + \beta_2 \kappa Q_2}), \quad (4)$$

where all the coefficients are positive. This leads to similar results to Eq. (1) for a single field Q , with effective early and late slopes given by $\alpha_{\text{eff}}^2 = \alpha_1^2 + \alpha_2^2$ and $\beta_{\text{eff}}^2 = \beta_1^2 + \beta_2^2$, respectively. Such a result is not surprising and is caused by the assisted behavior that can occur for multiple fields [34]. Note that for this type of multiple field examples the effective slopes in the resulting effective potential are larger than the individual slopes, a useful feature since we require α_{eff} to be large.

III. DISCUSSION

So far, we have presented a series of potentials that can lead to the type of quintessence behavior capable of explaining the current data arising from high redshift type Ia supernovas, CMB and cluster measurements. The beautiful property of exponential potentials is that they lead to scaling solutions which can either mimic the background fluid or dominate the background dynamics depending on the slope of the potential. We have used this to develop a picture where by simply combining potentials of different slopes, it is easy to obtain solutions which first enter a period of scaling through the radiation and matter domination eras and then smoothly evolve to dominate the energy density today. We have been able to demonstrate that the quintessence behavior occurred for a wide range of initial conditions of the field, whether ρ_Q be initially higher or lower than $\rho_{\text{matter}} + \rho_{\text{radiation}}$. We have also shown that the favored observational values for the equation of state $w_Q(\text{today}) < -0.8$ can be easily reached for natural values of the parameters in the potential. This is a big improvement in respect to most quintessence models as they usually give either $w_Q \gtrsim -0.8$ or $w_Q = -1$.

We have to ask, how sensible are such potentials? Can they be found in nature and, if so, can we make use of them here? The answer to the first question seems to be, yes they do arise in realistic particle physics models [35–40], but the current models do not have the correct slopes. Unfortunately, the tight constraint emerging from nucleosynthesis, namely $\alpha > 5.5$, is difficult to satisfy in the models considered to date which generally have $\alpha \leq 1$. It remains a challenge to see if such potentials with the required slopes can arise out of particle physics. One possibility is that the desirable slopes will be obtained from the assisted behavior when several fields are present as mentioned above.

It is encouraging that the quintessence behavior required to match current observations occurs for such simple potentials.

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