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# N-flation: Non-Gaussianity in the horizon-crossing approximation

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We analyze the cosmic nongaussianity produced in inflation models with multiple uncoupled fields with monomial potentials, such as Nflation. Using the horizon-crossing approximation to compute the non-gaussianity, we show that when each field has the same form of potential, the prediction is *independent* the number of fields, their initial conditions, and the spectrum of masses/couplings. It depends only on the number of $e$-foldings after the horizon crossing of observable perturbations. We also provide a further generalization to the case where the fields can have monomial potentials with different powers. Unless the horizon-crossing approximation is substantially violated, the predicted nongaussianity is too small to ever be observed.

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## I. INTRODUCTION

There has been recent interest in models of inflation with multiple uncoupled fields, an example being the Nflation model of Dimopoulos *et al.* [1] which corresponds to a collection of massive fields. Inflation may proceed more efficiently in such scenarios due to the assisted inflation phenomenon [2], and the models may be well-motivated within the context of string theory or dimensional reduction [1,3–5].

Various observational predictions have been made for such scenarios. Alabidi and Lyth [6] made a comprehensive study of the case of massive fields, demonstrating that the tensor-to-scalar ratio $r$ always takes the single-field value $r = 8/N$, where $N$ is the number of $e$-foldings since horizon crossing, independently of the mass spectrum and the initial conditions. This result was extended to monomial potentials by Piao [7]. Alabidi and Lyth also showed that the spectral index $n_s$ was model dependent, but always less than the single-field value, as previously shown for two massive fields by Lyth and Riotto [8]. Actual values of $n_s$ were evaluated for a particular choice of mass spectrum and initial conditions by Easther and McAllister [5], and for various choices of the number of fields $N_f$ and mass spectrum with random field initial conditions by Kim and Liddle [9].

A key interest of multifield models is whether they can generate significant nongaussianity [6,10–14]. The emerging view is that the non-Gaussianity is always small in models of the type considered here (though see Ref. [13]). Alabidi and Lyth [6] computed the nonlinearity parameter $f_{\rm NL}$ using the separate Universes approach, obtaining a formula claimed to indicate that it is always less than unity though it was not explicitly calculated for any models. Vernizzi and Wands [14] did explicitly evaluate a similar formula for the case of two massive uncoupled fields, indicating that it is indeed suppressed by the values of slow-roll parameters at horizon exit and hence much less than unity.

In this article, we explicitly calculate the nonlinearity parameter $f_{\rm NL}$ for multiple uncoupled fields with monomial potentials, i.e.

$$V = \sum_{i=1}^{N_f} \lambda_i \phi_i^\alpha \tag{1}$$

where $\alpha$ is an even positive integer (the same for each field) and each of the $N_f$ fields may have a different mass/coupling $\lambda_i$. We use a simplified version of the formalism of Vernizzi and Wands [14] by adopting the horizon-crossing approximation, which in essence assumes that the field trajectory becomes straight by the end of inflation or soon after, and that isocurvature perturbations do not play a role subsequently. Our work extends that of Alabidi and Lyth [6] by explicitly evaluating the nongaussianity expression for these models, extends that of Vernizzi and Wands [14] by considering more than two fields, and extends both by evaluating the result for general (even) monomial potentials. We end by further generalizing to allow each potential to have a different power-law index $\alpha_i$.

## II. THE CALCULATION

We follow the notation of our earlier paper [9] and of Vernizzi and Wands [14]. The calculation is a straightforward implementation of those already in the literature. For a set of uncoupled fields, the equation for the number of $e$-foldings $N$, in the slow-roll approximation, is [8]

$$N \simeq -\frac{1}{M_{\rm Pl}^2}\sum_i \int_{\phi_i}^{\phi_i^{\rm end}} \frac{V_i}{V_i'} d\phi_i \simeq \frac{\sum_i \phi_i^2}{2\alpha M_{\rm Pl}^2}, \tag{2}$$

and the tensor-to-scalar ratio is [7]

$$r \simeq \frac{8M_{\rm Pl}^2}{\sum_i (V_i/V_i')^2} \simeq \frac{4\alpha}{N}. \tag{3}$$

Here $V_i$ is the potential of the $i$-th field $\phi_i$, $V_i' \equiv dV_i/d\phi_i$, $M_{\rm Pl}$ is the reduced Planck mass, and throughout there are no summations unless indicated explicitly. In the last expression for $N$ the lower limits of the integrals, corresponding to the end of inflation, can be neglected and have been.

An expression for the nongaussianity can be obtained using the separate Universes/$\delta N$ formalism [11,15–17]. The nonlinearity parameter $f_{\rm NL}$ is then given by [10,12,14]

$$-\frac{6}{5}f_{\rm NL}=\frac{r}{16}(1+f)+\frac{\sum_{i,j}N_{,i}N_{,j}N_{,ij}}{(\sum_k N_{,k}^2)^2}, \quad (4)$$

where '$,i$' indicates derivative with respect to $\phi_i$. Here $r$ is the tensor-to-scalar ratio, given by Eq. (3) for the models we are discussing, and $f$ is a geometric factor relating to the triangular bispectrum configuration being studied, lying in the range $0\leq f\leq 5/6$ [10]. The first term is thus guaranteed to be small by current observational limits on $r$ [18]. The second term is denoted $f_{\rm NL}^{(4)}$ and needs to be computed.

We evaluate the second term using the horizon-crossing approximation. This assumes that there will be a negligible correction when shifting from an initially spatially-flat hypersurface to a final uniform-density hypersurface. This is guaranteed if the trajectory becomes straight before inflation ends (or even somewhat after), which in multifield models of the type we are studying should be typical but cannot be absolutely generic.

In the two-field case, this was recently studied in detail by Vernizzi and Wands [14], who track the evolution of the perturbations during inflation. Their expression for the nongaussianity mostly features terms evaluated at horizon crossing, plus one additional term denoted $Z_c$. This term accounts for the contribution to the change in $e$-foldings at the final uniform-density hypersurface, and evolves during inflation driving evolution of $f_{\rm NL}$. If $Z_c$ is set to zero, the formula Eq. (5) we give below is recovered. We have reproduced their calculation, and find that while $Z_c$ is substantial at horizon crossing in the specific case they analyze, it becomes negligible by the end of inflation. Accordingly, our expression is an excellent approximation to the desired answer, being the one at the end of inflation, even though it is entirely evaluated at horizon crossing. We expect the horizon-crossing approximation to hold very well in typical situations (as already commented in Ref. [6]), though a more detailed analysis of this point is in progress.

Using the horizon-crossing approximation, the derivatives of the number of $e$-foldings can be written in terms of the potential as $M_{\rm Pl}^2N_{,i}\simeq V_i/V_i'$ [8], leading to

$$-\frac{6}{5}f_{\rm NL}^{(4)}\simeq M_{\rm Pl}^2\left(\sum_j\frac{V_j^2}{V_j'^2}\right)^{-2}\sum_i\frac{V_i^2}{V_i'^2}\left(1-\frac{V_iV_i''}{V_i'^2}\right) \quad (5)$$

$$\simeq\frac{\alpha M_{\rm Pl}^2}{\sum_i\phi_i^2}. \quad (6)$$

Using Eqs. (2) and (3) immediately yields a final answer

$$-\frac{6}{5}f_{\rm NL}=\frac{1}{2N}(2+f)=\frac{r}{8\alpha}(2+f), \quad (7)$$

which is the main result of this paper.

Equation (7) matches exactly the result found by Vernizzi and Wands [14], but their result was calculated only for two massive fields. We have shown that the same result holds for arbitrary numbers of fields and for general (even) monomial potentials. Such models are therefore highly predictive in their nongaussianity, but sadly the prediction is for a number so small that it is swamped by effects of nonlinear gravity and can never be detected.

In fact we can even generalize this calculation further, by allowing each field to have a different exponent $\alpha_i$:

$$V=\sum_{i=1}^{N_f}\lambda_i\phi_i^{\alpha_i}. \quad (8)$$

Using Eqs. (2) and (3), the number of $e$-foldings and the tensor-to-scalar ratio will be

$$N\simeq\frac{1}{2M_{\rm Pl}^2}\sum_i\frac{\phi_i^2}{\alpha_i}, \quad (9)$$

$$r\simeq\frac{8M_{\rm Pl}^2}{\sum_i(\phi_i/\alpha_i)^2}. \quad (10)$$

The first term of Eq. (4) is unchanged, but the second now reads

$$-\frac{6}{5}f_{\rm NL}^{(4)}\simeq M_{\rm Pl}^2\frac{\sum_i\phi_i^2/\alpha_i^3}{(\sum_j\phi_j^2/\alpha_j^2)^2}, \quad (11)$$

$$\simeq\frac{r^2}{64M_{\rm Pl}^2}\sum_i\frac{\phi_i^2}{\alpha_i^3}, \quad (12)$$

$$\simeq\frac{1}{2N}\left[\frac{\sum_i\phi_i^2/\alpha_i^3}{(\sum_k\phi_k^2/\alpha_k^2)^2}\sum_j\left(\frac{\phi_j^2}{\alpha_j}\right)\right], \quad (13)$$

where we have written it in various equivalent forms. If the $\alpha_i$ are all the same we recover the previous result Eq. (6). However if the $\alpha_i$ are different the result does depend on initial conditions and on the model parameters, while still being slow-roll suppressed. The easiest way to see this is to bear in mind that $\alpha_i\gtrsim 2$, and use the second of the above equations to obtain $|(6/5)f_{\rm NL}^{(4)}|\leq r/16$.

## III. CONCLUSIONS

We have computed the nonlinear parameter $f_{\rm NL}$ that measures primordial nongaussianity for models with multiple uncoupled fields, generalizing calculations in the literature. We focussed mainly on the case where the fields have monomial potentials with the same slope but different amplitudes, e.g. a set of massive fields with an arbitrary mass spectrum. We have shown that within the horizon-crossing approximation these models make a unique prediction for the amplitude of nongaussianity, independent of the field initial conditions, of the number of fields, and of their mass/coupling spectrum. The predicted nongaussianity is however too small be measured. We also generalized this result further to allow different power-laws for each field.

Our calculation gives the perturbations associated with the horizon-crossing epoch. There is also the question of whether further perturbations might be generated after inflation, for instance by a curvatonlike mechanism (see e.g. the discussion in Ref. [14]). Such effects would be absent if the late stages of inflation are driven by a single field, but otherwise would depend on the routes by which the scalar fields decay into conventional and dark matter. We have assumed such effects are absent.

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