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Power Spectra of the Cosmic Microwave Background and Density Fluctuations Seeded by Local Cosmic Strings

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We compute the power spectra in the cosmic microwave background and cold dark matter (CDM) fluctuations seeded by strings, using the largest string simulations performed so far. We find that local strings differ from global defects in that the scalar components of the stress-energy tensor dominate over vector and tensor components. This result has far reaching consequences. We find that cosmic strings exhibit a single Doppler peak of acceptable height at high ℓ. They also seem to have a less severe bias problem than global defects, although the CDM power spectrum in the "standard" cosmology is the wrong shape to fit large scale structure data. [S0031-9007(98)08249-0]

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Recent times have witnessed unprecedented progress in mapping the cosmic microwave background (CMB) temperature anisotropy and the large scale structure (LSS) of the Universe. Predicting these observables in topological defect scenarios [1] has become a major challenge. In these theories, as the Universe cools down, high temperature symmetries are spontaneously broken. Remnants of the unbroken phase, called topological defects, may survive the transition, and later seed fluctuations in the CMB and LSS. The defect evolution is highly nonlinear, thereby complicating computations. Last year saw a number of computational breakthroughs in defect theories, partly related to improvements in computer technology. Most strikingly, the method described in [2] showed how one could glean from defect simulations all the information required to compute accurately CMB and LSS power spectra. This method was applied to theories based on global symmetries. Work on cosmic strings associated with gauged (or local) symmetries appeared at about the same time [3,4], but making use of rather different methods.

In this Letter we report on a calculation of the local cosmic string power spectrum, using the method of [2] applied directly to local string simulations. In this method the simulations are used uniquely for evaluating the two-point functions (known as unequal time correlators, or UETCs) of the defects' stress-energy tensor. UETCs are all that are required for computing CMB and LSS power spectra. Furthermore, they are constrained by requirements of self-similarity (or scaling) and causality, which enable us to radically extend the dynamical range of simulations, a fact central to the success of the method.

We believe that our work has significant advantages over [3,4]. In [3] string simulations are used directly as sources for the cosmological perturbations. As the authors point out, this means that one is severely limited in dynamic range by the string simulation itself. The UETC method allows us to cover the full dynamic range required for CMB and LSS computations. In [4] one made use of an analytical model for strings, first proposed by one of the authors in [5]. Although the model has been shown to approximate some of the UETCs quite well [4,5], our direct use of string simulations is clearly an improvement. We show elsewhere [6] how the model misses some key features found in simulations.

Local strings have an extra complication over global defects, which stems from the fact that we are unable to simulate the underlying field theory. Instead, we approximate the true dynamics with linelike relativistic strings. This is thought to be reasonable for the large scale properties of the stress-energy tensor, but we do not have a good understanding of how the string network loses energy in order to maintain scaling. In any case, one must conserve the total energy momentum tensor, and so one is forced to make assumptions about which cosmological fluids pick up this deficit. It is often assumed that all the strings' energy and momentum is radiated into gravitational waves, approximated by a relativistic fluid. This is by no means certain, and it may well be that the energy and momentum is transferred to particles [7], and hence to the baryon, photon, and CDM components.

We explore these possibilities in this paper, and one of the main results is that the matter power spectrum is very sensitive to the assumptions made about string decay. In particular, it is possible to reduce the bias at $100h^{-1}$ Mpc scale to 1.6, which runs against the current orthodoxy, that defects necessarily have a large bias at this scale. The shape of the CDM power spectrum is still glaringly different from the data [8]. Regardless of assumptions made on string decay products, we see a fairly distinctive peak in the CMB power spectrum, differing from [3,4] and from global defect theories, albeit with no secondary oscillations as expected.

We proceed to describe in detail our calculation. The unequal time correlators are defined as $C_{\mu\nu,\alpha\beta}(k,\tau,\tau') \equiv \langle\Theta_{\mu\nu}(\mathbf{k},\tau)\Theta^{\star}_{\alpha\beta}(\mathbf{k},\tau')\rangle$ where $\Theta_{\mu\nu}$ is the stress energy tensor, $\mathbf{k}$ is the wave vector, and τ and τ' are any two (conformal) times. The UETCs determine all other two-point functions, most notably CMB and LSS power spectra C_ℓ

and $P(k)$. Realistic UETCs have to be measured from defect simulations, although analytical modeling [4,5] gives insights into the observed forms.

We performed flat space cosmic string simulations using the algorithm described in [9]. We used a previously developed code [10] implementing this algorithm. Flat space codes achieve great efficiency and accuracy by neglecting the effect of Hubble damping on the network, and by restricting the strings to lie on a cubic lattice. They are thought to give a good quantitative picture of a real string network on large scales, although the overall string density is probably too high (an effect which will only affect the normalization $G\mu$). Another weakness is that we cannot model the reduction in the string density at the radiation-matter transition. However, this is only a 25% effect in the *comoving* energy density (although it seems larger when quoted as physical density [1]), and from linearity we may expect an effect of the same order of magnitude in CMB and LSS. Should the effect be larger, some of our results may need revision.

We performed simulations in 128^3, 256^3, 450^3, and 600^3 boxes, with a cutoff on the loop size of four links. To evaluate the UETCs from the simulations we selected times in the range $0.1N < t < N/4$, where N is the box size. In this range the UETC themselves tell us that we have lost memory of the initial conditions. Also the equal time correlators may be used to check that indeed we have scaling (as documented in [5,11]). Boundary effects, in this time range, are excluded by causality. For each of these times we compute and fast Fourier transform the string stress-energy tensor $\Theta_{\mu\nu}$. We then decompose the $\Theta_{\mu\nu}(\mathbf{k},\tau)$ modes into scalar, vector, and tensor (SVT) components (e.g., [12]). In [6] we show that there are only 14 independent UETCs. We compute them by cross-correlating a target time in the middle of our time range with all other times, and averaging over several runs.

Scaling and causality impose powerful constraints on the correlators [13], and can be used to extend the dynamical range of the simulations. Scaling implies that $C_{\mu\nu,\alpha\beta}(k,\tau,\tau') = c_{\mu\nu,\alpha\beta}(k\tau,k\tau')/\sqrt{\tau\tau'}$. The scaling functions $c_{\mu\nu,\alpha\beta}(k\tau,k\tau')$ can be found from simulations, although they are noisy in the $k\tau, k\tau' \approx 0$ region. Causality may be used to reduce the noise, as it implies that the real space correlators of the fluctuating part of $\Theta_{\mu\nu}$ must be strictly zero for $r > \tau + \tau'$. Real space correlators $C(r,\tau,\tau')$ may then be truncated at $r > \tau + \tau'$, and inverted into $C(k,\tau,\tau')$. This procedure completes our knowledge of the functions $c_{\mu\nu,\alpha\beta}$ at $k\tau \approx 0$.

We will report details in [6], but a striking feature of our results is the dominance of Θ_{00} over all other components. The string anisotropic stresses are in the predicted [14] ratios $|\Theta^S|^2{:}|\Theta^V|^2{:}|\Theta^T|^2$ of 3:2:4, as $k\tau \to 0$. However $|\Theta_{00}|^2 \gg |\Theta^S|^2$, and so scalars dominate over vectors and tensors. Also the energy density power spectrum rises from a white noise tail at $k\tau \approx 0$ into a peak at $k\tau \approx 20$, after which it falls off. Subhorizon modes are therefore of great importance.

The UETCs $c_{\mu\nu,\alpha\beta}(k\tau,k\tau')$ may be diagonalized as

$$c_{\mu\nu,\alpha\beta}(k\tau,k\tau') = \sum_i \lambda^{(i)} v^{(i)}_{\mu\nu}(k\tau) v^{(i)}_{\alpha\beta}(k\tau'), \qquad (1)$$

where $\lambda^{(i)}$ are eigenvalues [1]. In general, defects are incoherent [15], which means that this matrix does not factorize into the product of two vectors $v_{\mu\nu}(k\tau)v_{\alpha\beta}(k\tau')$. Standard codes solving for CMB and LSS power spectra assume coherence. However, we see that an incoherent source may be represented as an incoherent sum of coherent sources. We may therefore feed each eigenmode into standard codes to find the $C_\ell^{(i)}$ and $P^{(i)}(k)$ associated with each mode. The series $\sum \lambda^{(i)} C_\ell^{(i)}$ and $\sum \lambda^{(i)} P^{(i)}(k)$ provide convergent approximations to the power spectra.

The response of radiation, neutrinos, CDM, and baryons to coherent sources may be computed with a Boltzmann code (see [16] for formalism). A popular and fast implementation is CMBFAST [17], which is accurate to about 1%. Sudden recombination approximation [18] codes are even faster, but only achieve about 5% accuracy. We experimented with an implementation of the sudden recombination approximation, a full Boltzmann code, and CMBFAST. We found that for all active perturbations tested our sudden recombination code never differs from the full Boltzmann code by more than 10% in C_ℓ and 5% in $P(k)$. We have also reproduced the results in [2] with this code. Since the uncertainties in the string UETCs lead to much larger errors, we felt that a sudden recombination approximation was good enough.

A significant difference between current codes for simulating local cosmic strings and global defects is the former's lack of energy and momentum conservation. Long strings lose energy and momentum to small loops which are excised from the simulation. These can also be thought of as representing the decay of the long strings into gravitational radiation or high energy particles. This feature introduces two novelties in the calculation. First, one cannot measure a reduced number of defect correlators (typically three scalars, one vector, one tensor) and determine the others by energy conservation (as in [2]). Instead, we must compute all 14 correlators, and from them infer the long string violations of energy and momentum conservation.

Second, we must model the real physics of the decay products of the long strings. To this end one can introduce an extra fluid, specified by two scalar equations of state (e.g., $p^X = w^X \Theta^X_{00}$ and $\Pi^{SX} = 0$), and a vector equation of state ($\Pi^{VX} = 0$). For gravitational radiation $w^X = 1/3$, while a fluid of loops has $w^l \approx v^2/3$, with v the rms center of mass velocity. We explore the range $0 < w^X < 1/3$, although $w^X = 0$ is probably unphysical, as it is difficult to envisage that all the energy of the strings could end up as the mass energy of nonrelativistic particles. As extensively explained in [19], these equations of state uniquely specify the dynamics of the extra fluid, and the energy transfer process on all scales.

Another possibility is that strings decay into very high energy particles [7,11], which must scatter and eventually thermalize with the background fluids. This process entails transfer of energy and momentum to radiation, baryons, and CDM. In such scenarios, in addition to being active perturbations, strings would also seed entropy fluctuations [20]. We parametrize such process by a single parameter on all scales, the percentage of energy coming out of the network permitted to go into these various channels.

String decay products are clearly the most uncertain aspect of cosmic string theory. By measuring the full 14 UETCs associated with long strings, we assume nothing about decay products when extracting information from simulations (unlike [4] where long strings and decay products are modeled together). The simulations will then also place constraints upon the decay products.

In Fig. 1 we plot $[\ell(\ell + 1)C_\ell/2\pi]^{1/2}$, setting the Hubble constant to $H_0 = 50$ km sec^{-1} Mpc^{-1}, the baryon fraction to $\Omega_b = 0.05$, and assuming a flat geometry, no cosmological constant, three massless neutrinos, standard recombination, and cold dark matter. We superimpose also current experimental points. The most interesting feature is the presence of a reasonably high Doppler peak at $\ell = 400$–600, following a pronouncedly tilted large angle plateau (cf. [21]). This feature sets local strings apart from global defects. It puts them in better shape to face the current data.

The CMB power spectrum is relatively insensitive to the equation of state of the extra fluid. We have plotted results for $w^X = 1/3, 0.1, 0.01$. Dumping some energy into CDM has negligible effect. Small dumps into baryon and radiation fluids, on the contrary, boost the Doppler peak very strongly. We plotted the effect of dumping 5% of the energy into the radiation fluid.

The LSS power spectra on the other hand is strongly dependent on w^X. In Fig. 2 we plotted the CDM power spectrum $P(k)$ together with experimental points as in [8]. The normalization has been fixed by COBE data points. We see that the peak of the spectrum is always at smaller scales than standard CDM predictions, or observations. However, the overall normalization of the spectrum increases considerably as w^X decreases.

The CDM rms fluctuation in $8h^{-1}$ Mpc spheres is $\sigma_8 = 0.4, 0.6, 1.8$ for $w^X = 1/3, 0.1, 0.01$. Hence, relativistic decay products match well the observed $\sigma_8 \approx 0.5$. On the other hand in $100h^{-1}$ Mpc spheres one requires bias $b_{100} = \sigma_{100}^{\rm data}/\sigma_{100} = 4.9, 3.7, 1.6$ to match observations.

Energy dumps into radiation have no effect on the CDM power spectrum. However, if there is energy transfer into CDM or baryons, even with $w^X = 1/3$, the CDM power spectrum is highly enhanced. This is due to the addition of small scale entropy fluctuations to the usual fluctuations gravitationally induced by the stings. We plot the result of a 5% transfer into CDM and a 20% transfer into baryons (with $w^X = 1/3$) for which $b_{100} = 2.0, 1.5$.

Hence in our calculations local strings have a bias problem at $100h^{-1}$ Mpc, although its magnitude is not as great as found in [4]. It depends sensitively on the decay products, being reduced if the strings have a channel into nonrelativistic particles, or if there is some energy transfer into

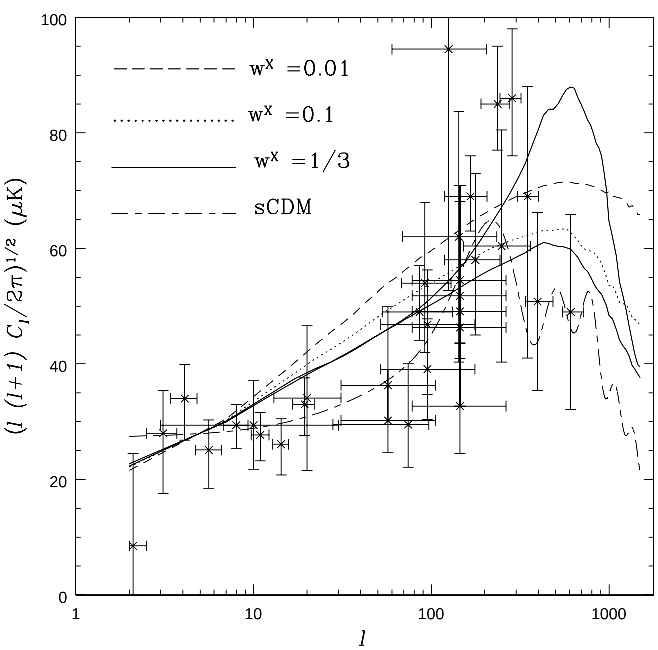

FIG. 1. The CMB power spectra predicted by cosmic strings decaying into loop and radiation fluids with $w^X = 1/3, 0.1, 0.01, 0$. We have plotted $[\ell(\ell + 1)C_\ell/2\pi]^{1/2}$ in μK, and superposed several experimental points. The higher curve corresponding to $w^X = 1/3$ shows what happens if 5% of the energy goes into the radiation fluid.

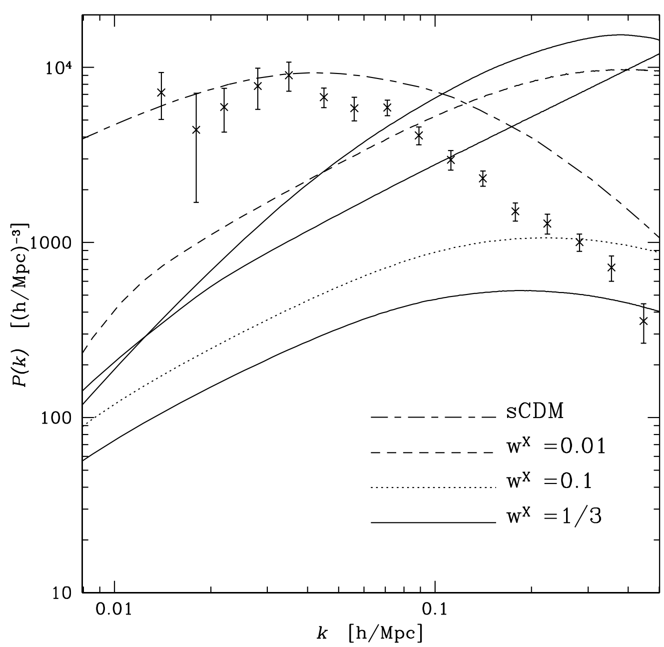

FIG. 2. The power spectrum in CDM fluctuations for cosmic strings, with $w^X = 0.01, 0.1, 1/3$. We plotted also the standard CDM scenario prediction and points inferred by Peacock and Dodds from galaxy surveys. The top 2 $w^X = 1/3$ curves correspond to a 5% transfer into CDM, and a 20% transfer into baryons (top).

the baryon and CDM fluid. The main problem with strings in an $\Omega = 1$, $\Omega_b = 0.05$, $\Omega_\Lambda = 0$ CDM universe is that the shape of $P(k)$ never seems to match observations. This may not be the case with other cosmological parameters.

We performed a large number of checks on our results. We found fast convergence with the box size. The COBE normalized C_ℓ vary by less than 2% as we go from 128^3 to 256^3, and by less than a percent from 256^3 to 450^3. The normalization itself varies significantly. $G\mu$ decreases by 25% as we go from 128^3 to 256^3, but hardly changes from 256^3 to 450^3. For a 256^3 box a COBE normalization at $\ell = 5$ produces $G\mu \approx 1.0 \times 10^{-6}$ (with $w^X = 1/3$). Because of the large tilt this normalization changes considerably with the value of ℓ where the fit is made. The COBE normalized LSS power spectrum changes by less than a percent with the box size. We also checked that different discretizations applied to $k\tau$ led to different eigenmode expansions, but the same final answer.

The results we have obtained are consistent with previous arguments on scalar, vector, and tensor modes in defect theories. In [14] rigorous arguments on the ratios $|\Theta^S|^2{:}|\Theta^V|^2{:}|\Theta^T|^2$ for modes at $k\tau \approx 0$ were derived. It was then shown how these translated into ratios $C_\ell^S{:}C_\ell^V{:}C_\ell^T$, under certain conditions. Two of the conditions were the subdominance of modes inside the horizon ($k\tau > 5$), and that anisotropic stresses should have a similar amplitude to the energy density. As we have pointed out, local strings violate both these conditions. Hence, although we have observed the predicted ratios for the anisotropic stresses, the argument need not apply to C_ℓ. We also checked our CMB code by using the UETCs of [2] as sources, and were able to reproduce the results. The conclusion is that the CMB and LSS predictions for local strings and global defects are different because their UETCs are indeed qualitatively different.

If we take $w^X \approx 1/3$ our results are close to those of [4] (a bias at $100h^{-1}$ Mpc of 4.9 instead of 5.4; a higher Doppler peak). In [4] the defect energy-momentum tensor is modeled as a gas of randomly oriented straight string segments, with random velocities, whose length and number density depend on time in the correct way to obtain scaling. In [6] we develop further this analytical model and show how the original model may miss some key features found in simulations. Also in [4] the extra energy-conserving fluid is relativistic and noninteracting. We stress that the results in [3] assume a rather different background cosmology ($H_0 = 80$ km sec^{-1} Mpc^{-1} and $\Omega_b = 0.02$).

In summary, we have computed the CMB and LSS power spectra for local cosmic strings, using extensive flat space string simulations to model the sources. We have explored the consequences of relaxing previous assumptions about the decay products of the strings. We find that the $100h^{-1}$ Mpc bias problem and the absence of a Doppler peak, thought to be generic features of defects, may not be as severe for local strings as they are for global defects. It appears that CMB and LSS power spectra depend on the details of the defect considered, and more seriously in the case of local strings, on the physics of the transfer of energy and momentum to matter and radiation. In an Einstein-de Sitter CDM Universe, with $\Omega_b = 0.05$ and $H_0 = 50$ km s^{-1} Mpc^{-1}, the shape of the CDM power spectrum cannot be made to fit the data [8] even with our relaxed assumptions. Other cosmological parameters remain to be explored [6].

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