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Vacuum fluctuations in axion-dilaton cosmologies

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We study axion-dilaton cosmologies derived from the low-energy string effective action. We present the classical homogeneous Friedmann-Robertson-Walker solutions and derive the semiclassical perturbation spectra in the dilaton, axion, and moduli fields in the pre-big-bang scenario. By constructing the unique S-duality-invariant field perturbations for the axion and dilaton fields we derive S-duality-invariant solutions, valid when the axion field is time dependent as well as in a dilaton-vacuum cosmology. Whereas the dilaton and moduli fields have steep blue perturbation spectra (with spectral index $n=4$) we find that the axion spectrum depends upon the expansion rate of the internal dimensions ($0.54<n\leqslant 4$) which allows scale-invariant ($n=1$) spectra. We note that for $n\leqslant 1$ the metric is nonsingular in the conformal frame in which the axion is minimally coupled. [S0556-2821(97)00514-6]

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I. INTRODUCTION

The dramatic progress that has been claimed in understanding black holes in the context of string theory [1,2] in the past year focuses attention upon the implications that strings might have for cosmology. The early universe provides a natural arena in which to seek observational evidence for string theory as a fundamental theory. General relativity describes all currently observed gravitational physics with remarkable accuracy [3], but its description of the very early universe is expected to break down near the Planck scale, and possibly at lower energies as well.

Most investigations of the cosmological consequences of string theory have focused on the role of the dilaton field φ, which provides a varying effective gravitational constant. In the absence of other matter, the low-energy action [4] leads to an effective theory of gravity of the form proposed by Brans and Dicke [5]. Although the predicted value of the Brans-Dicke parameter $\omega=-1$ is incompatible with present day post-Newtonian tests [3], it may be that loop corrections yield an experimentally acceptable general relativistic limit [6,7], or that the dilaton's present day value may be fixed by it acquiring a mass (for a detailed list of references see [8]).

The pre-big-bang scenario [9] is a specific example of a cosmological model based on string theory which differs radically from that predicted by general relativity because of the presence of the dilaton field. The weakly coupled, expanding, ''pre-big-bang'' solutions have many similarities with conventional inflation models [10], most notably the approach to flatness and homogeneity on large scales by stretching the quantum vacuum state on small scales up to large (superhorizon) scales. Solutions derived from the low-energy effective action run into a ''big bang'' singularity [11–13] but the hope is that in the full string theory, including higher-order terms, the pre-big-bang solution may be smoothly connected to a solution describing an expanding, ''post-big-bang'' universe [14,15] and, ultimately, general relativistic evolution.

Our intention in this paper is to draw attention to the crucial role that the antisymmetric tensor field, $H_{abc}\equiv\partial_{[a}B_{bc]}$, may have in cosmological scenarios based on the low-energy limit of the superstring action. An antisymmetric tensor inevitably appears, along with the graviton and dilaton, in the Neveu-Schwarz bosonic sector of the low-energy string effective action:

$$S=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g_D}e^{-\varphi}\left\{R_D+(\nabla\varphi)^2-\frac{1}{12}H^2\right\}, \quad (1)$$

where $\kappa_D^2\equiv 8\pi G_D$, and G_D is Newton's constant in D dimensions. Previous studies [16–25] have considered the classical evolution of the antisymmetric tensor field in simple cosmologies. The role of the additional tensor fields derived from the Ramond-Ramond sector of type-IIA and type-IIB string theory has recently been considered in [23,25]. In this paper we calculate the spectrum of perturbations about the classical background field that may be produced due to vacuum fluctuations in the antisymmetric tensor field.

We will consider spacetimes which contain a four-dimensional homogeneous and isotropic external metric. In four spacetime dimensions the antisymmetric tensor field has only one degree of freedom which may be represented by a pseudoscalar axion field, σ. For a homogeneous and isotropic metric we must have a homogeneous axion field, $\sigma=\sigma(t)$. This has been referred to as the solitonic anstaz due to the close connection with solitonic p-brane solutions [23]. An alternative ansatz, analogous to elementary p-brane solutions [23], taking the tensor potential $B_{\mu\nu}(t)$ to be time dependent leads to an inhomogeneous axion field and hence will be incompatible with a $D=4$ Friedmann-Robertson-Walker (FRW) cosmology [20].

In Sec. II we review the classical evolution of FRW cosmologies both with and without an evolving axion, stressing the important role of the axion when the scale factor in the string frame becomes small. Incorporating the evolution of the axion leads to a cosmology that is very different from the classical pre-big-bang scenario where the axion field is fixed. We explore the differences between the cosmological evolution as seen in different conformally related metrics in Sec. III, drawing attention to the evolution in the axion frame (where the axion field is minimally coupled). Specifically, we find that the singular evolution of the metric in the string frame can be nonsingular in the axion frame. These axion-dilaton solutions of the low-energy action can be related by a duality transformation which reduces to a scale factor duality in the absence of the axion field, as described in Sec. IV.

In Sec. V we set out our formalism for describing inhomogeneous linear perturbations about the homogeneous and isotropic four-dimensional (4D) background solutions. Even when the background axion field is set to zero, there will inevitably be quantum fluctuations in the field. In Sec. VI we calculate the spectrum of semiclassical axion perturbations as well as dilaton and moduli perturbation spectra produced in the pre-big-bang scenario. In Sec. VII we extend this calculation to more general axion-dilaton cosmologies by constructing S-duality-invariant combinations of the field perturbations that enable us to derive S-duality-invariant solutions. Moreover we demonstrate that the late-time dilaton and axion spectra turn out to be independent of the preceding evolution along different but S-duality related classical solutions. Importantly, the tilt of the axion spectrum can be significantly different from the steep ''blue'' spectra of dilatons and gravitons predicted by the pre-big-bang scenario.

II. CLASSICAL AXION-DILATON COSMOLOGY

Here we consider cosmological solutions derived from the low-energy string action where we take the full D-dimensional spacetime to have a metric of the form

$$ds_D^2=-dt^2+g_{ij}dx^i dx^j+\gamma_{IJ}dX^I dX^J. \tag{2}$$

i, j run from 1 to 3, and I, J run from 1 to $n=D-4$. We will allow for the variation of the n compactified dimensions by including a single modulus field, $\exp(n\beta)$, proportional to the volume of this internal space, but neglect any curvature or anisotropy so that $\gamma_{IJ}=e^{2\beta}\delta_{IJ}$.

The effective dilaton in the four-dimensional external spacetime is then

$$\phi\equiv\varphi-n\beta, \tag{3}$$

and the antisymmetric tensor field in four dimensions can be written in terms of the pseudoscalar axion field σ as

$$H^{abc}\equiv e^{\phi}\epsilon^{abcd}\nabla_d\sigma, \tag{4}$$

where ϵ^{abcd} is the covariant antisymmetric four-form such that $\nabla_e\epsilon^{abcd}=0$.

The low-energy string effective action, given in Eq. (1), then becomes

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-\phi}\left\{R+(\nabla\phi)^2-n(\nabla\beta)^2-\frac{1}{2}e^{2\phi}(\nabla\sigma)^2\right\}, \tag{5}$$

where $\kappa^2\equiv 8\pi/M_{\rm Pl}^2$ determines the effective value of the Planck mass when $\phi=0$, and R is the Ricci scalar of the four-dimensional external spacetime.

We assume the external four-dimensional spacetime is described by a flat FRW metric with the line element

$$ds^2=a^2(\eta)\{-d\eta^2+\delta_{ij}dx^i dx^j\}, \tag{6}$$

where $a(\eta)$ is the scale factor. In addition, FRW solutions with nonzero spatial curvature can also be found [19]. To be compatible with a homogeneous and isotropic metric, all the fields must be homogeneous and the action then reduces to (up to a total derivative)

$$S=\frac{1}{2\kappa^2}\int d^3x\int d\eta e^{-\phi}\left\{-6a'^2+6aa'\phi'-a^2\phi'^2+na^2\beta'^2+\frac{1}{2}e^{2\phi}a^2\sigma'^2\right\}. \tag{7}$$

We refer to models with a constant axion field ($\sigma'=0$) as *dilaton-vacuum* solutions. These are the well-known monotonic power-law solutions[1]

$$e^{\phi}=e^{\phi_*}\left|\frac{\eta}{\eta_*}\right|^{r_\pm}, \tag{8}$$

$$a=a_*\left|\frac{\eta}{\eta_*}\right|^{(1+r_\pm)/2}, \tag{9}$$

$$e^{\beta}=e^{\beta_*}\left|\frac{\eta}{\eta_*}\right|^{s}, \tag{10}$$

where the integration constants r and s determine the rate of change of the effective dilaton and internal volume, respectively. Note that there is a constraint equation[2] which requires

$$r_\pm=\pm\sqrt{3-2ns^2}. \tag{11}$$

The dilaton-vacuum solutions are shown in Figs. 1–3. If stable compactification has occurred and the volume of the internal spaces is fixed ($s=0$, or $D=4$) we have $r_\pm=\pm\sqrt{3}$.

[1]We shall not consider here the trivial flat spacetime solution $\phi'=\beta'=a'=0$.

[2]See Eq. (29) in the next section.

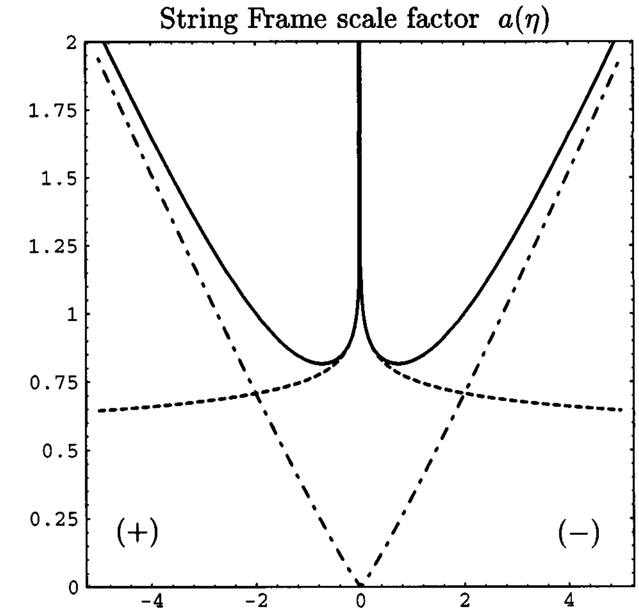

FIG. 1. The string frame scale factor for the axion-dilaton solution (solid line) given in Eq. (18). It approaches the $r_- = -r$ dilaton-vacuum solution given in Eq. (9) (dashed line) as $\eta \to 0$ and the $r_+ = +r$ dilaton-vacuum solution (dot-dashed line) as $\eta \to \pm\infty$. The $(+)$ and $(-)$ branches are also labeled. The specific parameters for the axion-dilaton solution are $a_* = 1$, $r = 1.2$, and $\eta_* = 2$.

These dilaton-vacuum solutions can be expressed in terms of the proper time, $t \equiv \int a\, d\eta$, giving

$$e^{\phi} = e^{\phi_*} \left| \frac{t}{t_*} \right|^{2r_\pm/(3+r_\pm)}, \tag{12}$$

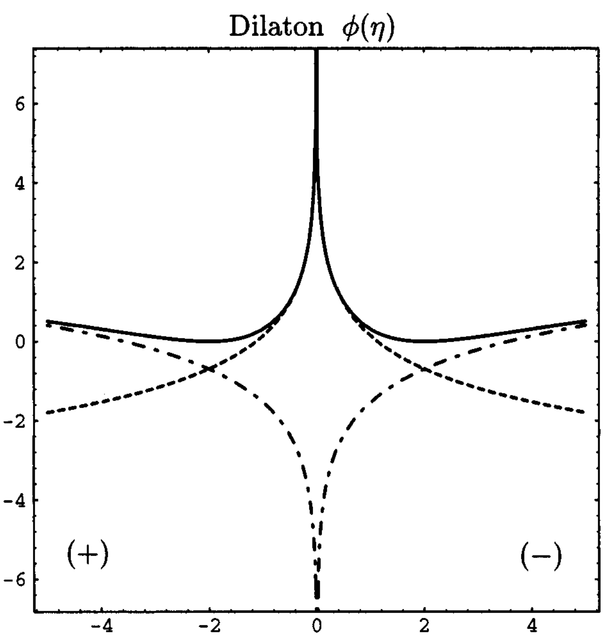

FIG. 2. The dilaton, ϕ, with $\phi_* = 0$ and the same notation and parameter values as in Fig. 1. The axion-dilaton solution [Eq. (17)] approaches the r_- dilaton-vacuum solution [Eq. (8)] at small $|\eta|$ and the r_+ solution at large $|\eta|$.

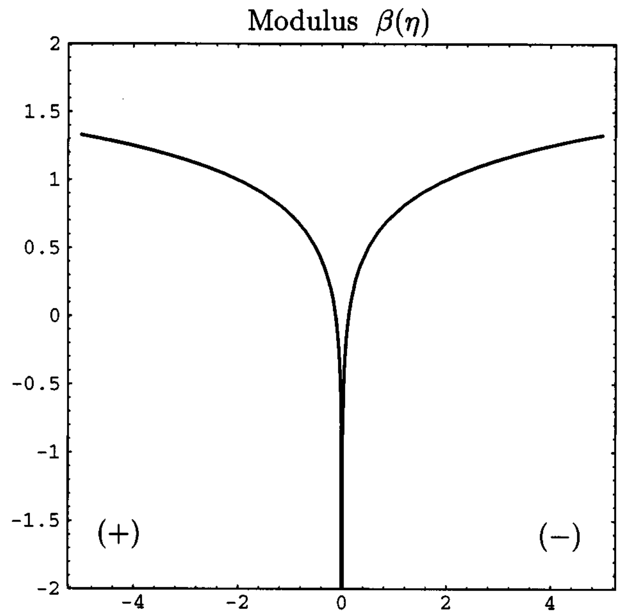

FIG. 3. The modulus, β, with $n = 6$ and the same parameter values as in Fig. 1 and $s = \sqrt{13}/10$. The evolution is the same, for both the dilaton-vacuum in Eq. (10) and axion-dilaton solutions in Eq. (19).

$$a = a_* \left| \frac{t}{t_*} \right|^{(1+r_\pm)/(3+r_\pm)}, \tag{13}$$

$$e^{\beta} = e^{\beta_*} \left| \frac{t}{t_*} \right|^{2s/(3+r_\pm)}. \tag{14}$$

All these solutions have semi-infinite proper lifetimes. Those starting from a singularity at $t = 0$ for $t \geqslant 0$, are denoted the $(-)$ branch in Ref. [11], while those which approach a singularity at $t = 0$ for $t \leqslant 0$ are referred to as the $(+)$ branch.

Our choice of origin for the time coordinate is arbitrary. A more fundamental definition of the $(+/-)$ branches may be given by considering the evolution of the shifted dilaton [26,17,13]

$$\overline{\phi} \equiv \phi - 3\ln(a). \tag{15}$$

Its time derivative,

$$\overline{\phi'} = -\frac{3 + r_\pm}{2\eta}, \tag{16}$$

is always positive on the $(+)$ branch (or $\eta < 0$) due to the constraint on the value of $r_\pm$ [Eq. (11)], and always negative on the $(-)$ branch (or $\eta > 0$).

These $(+/-)$ branches do *not* refer to the choice of sign for $r_\pm$ in Eq. (11). On either the $(+)$ or $(-)$ branches of the dilaton-vacuum cosmologies we still have two distinct solutions corresponding to the choice of the exponent r_+ or r_-, which determines whether ϕ goes to negative or positive infinity, respectively, as $\eta \to 0$.

The generalization of Eqs. (8)–(10) to the more general axion-dilaton cosmology where $\sigma' \neq 0$ is particularly simple [19]:

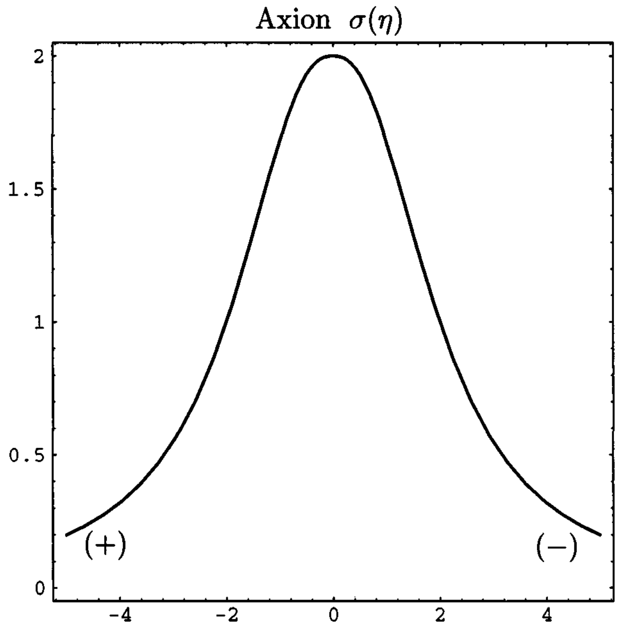

FIG. 4. The axion, σ, of the axion-dilaton solution given in Eq. (20), with the same parameter values as in Fig. 1 and $\sigma_*=1$.

$$e^{\phi}=\frac{e^{\phi_*}}{2}\left\{\left|\frac{\eta}{\eta_*}\right|^{-r}+\left|\frac{\eta}{\eta_*}\right|^{r}\right\}, \tag{17}$$

$$a^2=\frac{a_*^2}{2}\left\{\left|\frac{\eta}{\eta_*}\right|^{1-r}+\left|\frac{\eta}{\eta_*}\right|^{1+r}\right\}, \tag{18}$$

$$e^{\beta}=e^{\beta_*}\left|\frac{\eta}{\eta_*}\right|^{s}, \tag{19}$$

$$\sigma=\sigma_*\pm e^{-\phi_*}\left\{\frac{|\eta/\eta_*|^{-r}-|\eta/\eta_*|^{r}}{|\eta/\eta_*|^{-r}+|\eta/\eta_*|^{r}}\right\}, \tag{20}$$

where the exponents are related via $r=\sqrt{3-2ns^2}$. The time-dependent axion solutions are plotted in Figs. 1–4. The presence of the axion places a lower bound on the value of the dilaton, $\phi\geqslant\phi_*$. In doing so it interpolates between two dilaton-vacuum solutions with an asymptotically constant axion field. When $\eta\to\pm\infty$ the solutions approach the $r_+=+r$ dilaton-vacuum solution and as $\eta\to 0$ the solution approaches the $r_-=-r$ dilaton-vacuum solution. We shall see that the asymptotic approach to dilaton-vacuum solutions at early and late times leads to a particularly simple form for the semiclassical perturbation spectra, independent of the intermediate evolution.

The dynamical effect of the axion field is negligible except near $\eta\sim\eta_*$, when it leads to a bounce in the dilaton, $\phi'=0$. Lukas *et al.* [23] have recently drawn attention to the connection between these cosmological solutions and solitonic p-brane solutions. If $r>1$ then this also leads to a bounce of the scale factor, $a'=0$. However we still have the two disconnected branches, as defined by Eq. (16), corresponding to an increasing shifted dilaton, approaching a singularity on the $(+)$ branch, or a decreasing shifted dilaton, on the $(-)$ branch.

III. CONFORMAL FRAMES

Thus far we have written all the solutions in terms of the string frame. If stringy matter is minimally coupled in this frame then stringy test particles will follow geodesics with respect to this metric. However, in order to understand the physical evolution in these models it is revealing to look at conformally related metrics, $g_{ab}\to\Omega^2 g_{ab}$. If the conformal factor Ω^2 is itself homogeneous then the transformed metric remains a FRW metric but with scale factor $a\to\Omega a$.

This transformation of the scale factor can lead to some ambiguities in the interpretation of the cosmological solutions [10]. For instance inflation is often defined as accelerated expansion $(\ddot{a}>0)$. However such a definition is dependent upon the choice of conformal frame in which one chooses to evaluate the acceleration.

Note that the proper time also changes under a conformal transformation, $t\to\int\Omega dt$. One must not assume that a finite proper time interval in one frame necessarily coincides with a finite time in another frame and, in particular, we shall see that what looks like a singular evolution in one frame may appear nonsingular in another frame.

A. The Einstein frame

By choosing a conformal factor $\Omega^2=e^{-\phi}$ we can work in a frame[3] $\tilde{g}_{ab}=\Omega^2 g_{ab}$ where the dilaton is minimally coupled to the external metric. Thus the gravitational part of the action in Eq. (5) reduces to the Einstein-Hilbert action [10,27]

$$S=\frac{1}{2\kappa^2}\int d^4\sqrt{-\tilde{g}}\left\{\tilde{R}-\frac{1}{2}(\tilde{\nabla}\phi)^2-n(\tilde{\nabla}\beta)^2-\frac{1}{2}e^{2\phi}(\tilde{\nabla}\sigma)^2\right\}, \tag{21}$$

and hence this is known as the Einstein frame.[4] Both the dilaton and the moduli fields have standard kinetic terms in this frame and thus moduli particles and dilatons (in a constant axion field) would therefore follow geodesics in the four-dimensional external spacetime of the Einstein frame.

The familiar field equations of general relativity

$$\tilde{R}_{ab}-\frac{1}{2}\tilde{g}_{ab}\tilde{R}=\kappa^2\tilde{T}_{ab} \tag{22}$$

apply in this frame. This may help one's cosmological intuition, which is rooted in four-dimensional general relativity, but also assists mathematically by decoupling the equations for the evolution of the metric from the value of ϕ. The stress-energy tensor

[3]All quantities calculated with respect to the Einstein metric will carry a tilde.

[4]Some authors refer to the frame where the full D-dimensional metric is minimally coupled to the dilaton as the Einstein frame. This corresponds to a Kaluza-Klein gravity theory, which will not in general coincide with Einstein gravity in the four-dimensional external metric.

$$\kappa^2\tilde{T}_{ab}=\frac{1}{2}\left(\tilde{g}^c_a\tilde{g}^d_b-\frac{1}{2}\tilde{g}_{ab}\tilde{g}^{cd}\right)\times(\phi_{,c}\phi_{,d}+2n\beta_{,c}\beta_{,d}+e^{2\phi}\sigma_{,c}\sigma_{,d})\qquad(23)$$

for homogeneous fields reduces to that for a perfect fluid with a stiff equation of state [28,29], i.e., pressure equal to density,

$$\tilde{p}=\tilde{\rho}=\frac{1}{4\kappa^2}(\phi'^2+2n\beta'^2+e^{2\phi}\sigma'^2).\qquad(24)$$

The evolution equations for homogeneous fields in an FRW metric are then

$$\phi''+2\tilde{h}\phi'=e^{2\phi}\sigma'^2,\qquad(25)$$

$$\sigma''+2\tilde{h}\sigma'=-2\phi'\sigma',\qquad(26)$$

$$\beta''+2\tilde{h}\beta'=0,\qquad(27)$$

$$\tilde{h}'=-\frac{1}{6}(\phi'^2+2n\beta'^2+e^{2\phi}\sigma'^2),\qquad(28)$$

plus the constraint

$$\tilde{h}^2=\frac{1}{12}(\phi'^2+2n\beta'^2+e^{2\phi}\sigma'^2),\qquad(29)$$

where $\tilde{h}\equiv\tilde{a}'/\tilde{a}$.

From Eqs. (28) and (29) we see that in the general axion-dilaton case where we allow the dilaton, axion, and/or moduli fields to evolve, the expansion in the Einstein frame always obeys $\tilde{h}'+2\tilde{h}^2=0$ leading to the simple solution for the scale factor

$$\tilde{a}\equiv ae^{-\phi/2}=\tilde{a}_*\left(\frac{\eta}{\eta_*}\right)^{1/2}.\qquad(30)$$

The scalar field equations of motion, Eqs. (25)–(27), can then be integrated to give the solutions presented in Eqs. (17)–(20). Even in spatially curved FRW models the equations of motion remain integrable [19] despite the apparently nontrivial couplings between the fields, because we can make a conformal transformation to the Einstein frame where all the fields are minimally coupled to the metric and, so long as they are all homogeneous, their combined dynamical effect is no different from a single massless field [29].

Because there is no interaction potential for the fields, the strong energy condition ($\tilde{p}>0$) is always satisfied [see Eq. (24)] and the general relativistic singularity theorems must hold. Thus we know there is no way to construct a nonsingular evolution in the Einstein frame with these massless fields.

In the string frame the usual general relativistic results do not hold. Even without an interaction potential we can obtain an accelerated expansion in Eq. (13) for the (+) dilaton-vacuum branch ($t<0$) with $r_-<-1$ (or $r>1$ in the axion-dilaton solution). However unlike conventional power-law inflation, $a\propto t^p$ with $p>1$, we have ''pole inflation'' with $p<0$, and we approach a curvature singularity with $a\to\infty$ and $R\sim t^{-2}$ as $t\to 0$.

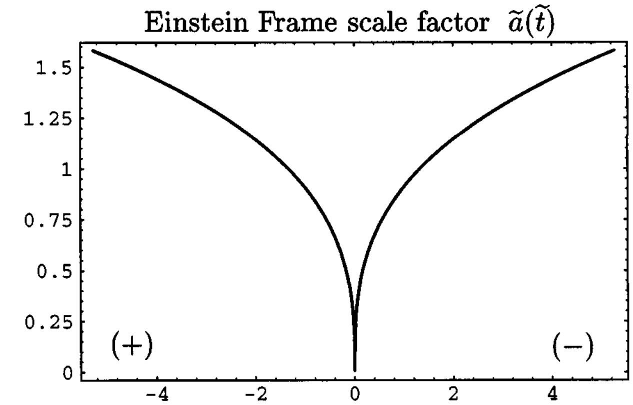

FIG. 5. The Einstein frame scale factor, $\tilde{a}$, plotted against proper time in the Einstein frame for the dilaton-vacuum and axion-dilaton solutions displayed in the previous figures.

In the Einstein frame we see that $\eta\to 0$ on the (+) branch always corresponds to a collapsing universe with $\tilde{a}\to 0$ (see Fig. 5). However this still fulfills one definition of inflation, namely, that the comoving Hubble length ($|d\tilde{a}/d\tilde{t}|^{-1}=|\tilde{a}/\tilde{a}'|=2|\eta|$) decreases with time [10]. Thus a given comoving scale that starts arbitrarily far within the Hubble scale in either conformal frame at $\eta\to-\infty$ inevitably becomes larger than the Hubble scale in that frame as $\eta\to 0$. This allows one to produce perturbations in the dilaton, moduli, and graviton fields on scales much larger than the present Hubble scale from quantum fluctuations in flat spacetime at earlier times, as we shall discuss in more detail later.

In both the string frame and the Einstein frame we either reach a curvature singularity in a finite proper time in the future for $\eta<0$ or emerge from a curvature singularity at a finite time in the past for $\eta>0$. The only exception to this is the axion-dilaton solutions in the string frame when $r=1$ ($s=\pm\sqrt{1/n}$) in Eqs. (17)–(20) which bounce at exactly $\eta=0$ [30]. However even in this case the dilaton and moduli become infinite at a finite proper time.

B. The axion frame

Both the dilaton and moduli fields are minimally coupled in the Einstein frame (i.e., they have standard kinetic terms). However the axion's kinetic term retains a nonminimal coupling to the dilaton. This can be removed by a conformal transformation to another conformally related metric, the ''axion frame,'' given by $\bar{g}_{ab}=e^{-2\phi}g_{ab}$ and, hence,

$$\bar{a}\equiv e^{\phi/2}a\equiv e^{\phi}\tilde{a}.\qquad(31)$$

The axion field is a minimally coupled massless scalar field in this frame and thus axionic test particles would follow geodesics with respect to this metric. Although conformally related to the string and Einstein frames, the metric the axions ''see'' may behave very differently from the metrics in the string or Einstein frames.

In terms of conformal time, the axionic scale factor for the dilaton-vacuum solutions given by Eqs. (8) and (9) when $\sigma'=0$, evolves as

$$\overline{a}=\overline{a}_{*}\left(\frac{\eta}{\eta_{*}}\right)^{r_{\pm}+(1/2)}. \tag{32}$$

We see that the proper time in the axion frame is given by

$$\overline{t}\equiv\int \overline{a}\,d\eta \sim |\eta|^{r_{\pm}+(3/2)}, \tag{33}$$

so it takes an infinite proper time to reach $\eta=0$ for $r_{-}<-3/2$ (and thus $ns^2<3/8$) and the scalar curvature for the axion metric, $\overline{R}\sim\overline{t}^{-2}$, vanishes as $\eta\to 0$. However, these same dilaton-vacuum solutions then reach $\eta\to\pm\infty$ in a finite proper time where $\overline{R}$ diverges. Because the conformal factor diverges as $\eta\to 0$, it stretches out the curvature singularity in the string metric into a nonsingular evolution in the axion frame. But as $\Omega^2=e^{\phi}\to 0$ as $\eta\to\pm\infty$ the nonsingular evolution in the string frame gets compressed into a curvature singularity in the axion frame.

Similar behavior has previously been noted in the case of black holes in the low-energy limit of string theory [31]. Astronauts made of axionic matter falling into an axion-dilaton black hole in $D=4$ would take an infinite proper time (measured by their axionic clocks) to fall into what appears, in the Einstein frame, to be the singularity.

In terms of the proper time $\overline{t}$ in the axion frame we have

$$\overline{a}=\overline{a}_{*}\left(\frac{\overline{t}}{\overline{t}_{*}}\right)^{(1+2r_{\pm})/(3+2r_{\pm})}. \tag{34}$$

For $r_{-}<-3/2$ we have conventional power-law inflation (not pole inflation) with $\overline{a}\sim\overline{t}^{\,\overline{p}}$ with $\overline{p}=1+[2/(-2r_{-}-3)]>1$. We shall see that this has important consequences for the tilt of the power spectrum of semiclassical perturbations in the axion field produced on large scales.

These dilaton-vacuum solutions still have a curvature singularity as $\overline{t}\to 0$ so the solutions still have only a semi-infinite lifetime in the axion frame,[5] but for $r_{-}<-3/2$ this now coincides with $\eta\to\pm\infty$, so the identification of the $(+)$ and $(-)$ branches as solutions approaching or leaving a singularity is interchanged for the r_{-} solution in the axion frame when $r_{-}<-3/2$.

However this implies that the axion-dilaton solutions with a time-dependent axion field will be nonsingular in the axion frame if $r>3/2$. Remember that the axion-dilaton solutions with $\sigma'\neq 0$ match $r_{-}=-r$ dilaton-vacuum solutions at $\eta\to 0$ onto $r_{+}=+r$ solutions as $|\eta|\to\infty$. Thus for $r>3/2$, the axion-dilaton solutions are nonsingular in the axion frame as $\eta\to 0$ (because $-r<-3/2$) *and* nonsingular as $\eta\to\infty$ (because $+r>-3/2$).

The general evolution for the axion-dilaton system, Eqs. (17)–(20) is given in the axion frame by Eq. (31), so

$$\overline{a}=\frac{\overline{a}_{*}}{2}\left\{\left|\frac{\eta}{\eta_{*}}\right|^{(1/2)-r}+\left|\frac{\eta}{\eta_{*}}\right|^{(1/2)+r}\right\}. \tag{35}$$

From this we can extract the proper time $\overline{t}$,

$$\frac{\overline{t}}{\overline{t}_{*}}=\left(\frac{9-4r^2}{12}\right)\left[\frac{2}{3-2r}\left|\frac{\eta}{\eta_{*}}\right|^{(3/2)-r}+\frac{2}{3+2r}\left|\frac{\eta}{\eta_{*}}\right|^{(3/2)+r}\right] \tag{36}$$

which, as shown above, is semi-infinite ($0\leqslant\overline{t}/\overline{t}_{*}<\infty$) for $r<3/2$ but unbounded ($-\infty<\overline{t}/\overline{t}_{*}<\infty$) for $r\geqslant 3/2$. Equations (35) and (36) give us a parametric solution for the axion frame scale factor in terms of the proper time in that frame.

Representative examples showing the behavior of $\overline{a}(\overline{t})$ for different values of r are given in Fig. 6. Note that the scale factor $\overline{a}$ has a nonzero minimum value (i.e., a bounce) whenever $r>1/2$. When $r>3/2$, $\overline{a}$ becomes infinite as $\overline{t}\to-\infty$, passes through a nonzero minimum value and then expands indefinitely as $\overline{t}\to\infty$. In particular, if stable compactification has occured so that the moduli field is fixed ($s=0$), or if $D=4$ (so that $n=0$), then $r=\sqrt{3}$ and the axion-dilaton solution is *always* nonsingular in the axion frame. When $1/2<r<3/2$, $\overline{a}$ does have a bounce but is singular, since it becomes infinitely large in a finite proper time, as $\overline{t}\to 0$. Finally, when $r<1/2$, the solutions are monotonic and there is a singularity when $\overline{a}$ vanishes at $\overline{t}=0$.

IV. DUALITY

A. Scale-factor duality

The constant axion solutions given in Eqs. (12)–(14) are related by the scale factor duality transformation [26,32]

$$a\to\frac{1}{a},\quad e^{\phi}\to\frac{e^{\phi}}{a^6} \tag{37}$$

which corresponds to a change in the parameters

$$a_{*}\to\frac{1}{a_{*}},\quad e^{\phi*}\to\frac{e^{\phi*}}{a_{*}^6},\quad r_{\pm}\to-\frac{3+2r_{\pm}}{2+r_{\pm}}, \tag{38}$$

in Eqs. (12)–(14). This is a particular case of a more general $O(d,d)$ duality [32] where the axion field remains constant. When we can neglect the evolution of the moduli fields ($s=0$ and hence $r_{\pm}=\pm\sqrt{3}$) this coincides with $r_{\pm}\to r_{\mp}$. Note that this scale factor duality does *not* take one from the $(+)$ to $(-)$ branch or vice versa. This would require a time reversal.

The pre-big-bang scenario postulates a nonsingular universe by linking the semi-infinite lifetime expanding $(+)$ branch, $a\sim(-\eta)^{-p}$, starting at $\eta=-\infty$ to the semi-infinite expanding $(-)$ branch, $a\sim\eta^{p}$, traveling off to $\eta=+\infty$ via a scale factor duality transformation, plus time reversal near the singularity at $\eta\approx 0$.

The presence of a time-dependent axion field $\sigma(t)$ (the solitonic ansatz) breaks the $O(d,d)$ invariance which requires that it is the antisymmetric potential B_{ab} which is

[5]The case $r_{-}=-3/2$ is an exception as it corresponds to de Sitter expansion with $\overline{a}\sim\exp(H\overline{t})$.

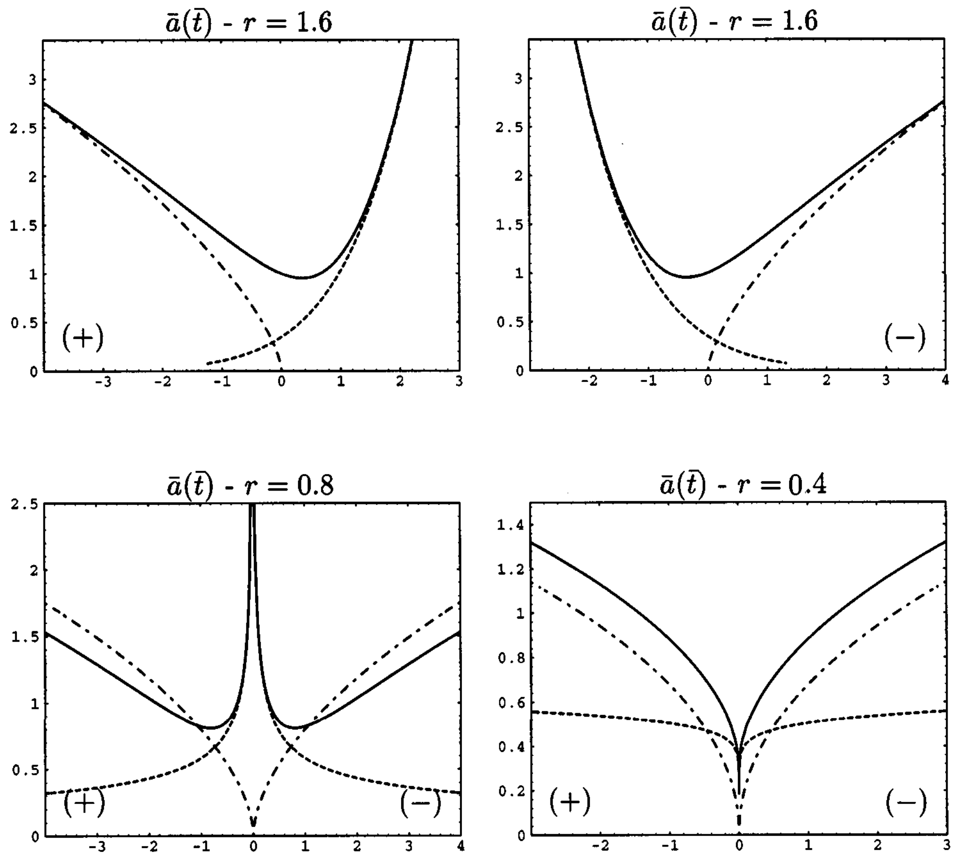

FIG. 6. The axion frame scale factor, $\overline{a}$, plotted against proper time in the axion frame for axion-dilaton cosmologies (solid lines) with three different values of r, while the other constants are the same as in Fig. 1. The asymptotic dilaton-vacuum solutions with $r_{-}=-r$ (dashed lines) and $r_{+}=+r$ (dot-dashed lines) are also shown. When $r\geqslant 1.5$ the semi-infinite interval in η is mapped onto an unbounded interval for the proper time, so the $(+)$ and $(-)$ branch solutions are displayed separately in the two top figures.

homogeneous (the elementary ansatz). The elementary ansatz is only compatible for a restricted class of metrics in anisotropic 4D spacetimes [20,22], or if the axion is constant.

B. *S* duality

Solutions with a time-dependent axion field do respect the invariance of the low-energy string action under the SL$(2,R)$ transformation:

$$\lambda \rightarrow \frac{\alpha\lambda+\beta}{\gamma\lambda+\delta}, \tag{39}$$

where α, β, γ, and δ are real constants subject to $\alpha\delta-\beta\gamma=1$, and λ is the complex dilaton field

$$\lambda=\sigma+ie^{-\phi}. \tag{40}$$

This leads to

$$e^{\phi}\rightarrow\gamma^2 e^{-\phi}+(\delta+\gamma\sigma)^2 e^{\phi}, \tag{41}$$

$$e^{\phi}\sigma\rightarrow(\beta+\alpha\sigma)(\delta+\gamma\sigma)e^{\phi}+\alpha\gamma e^{-\phi}. \tag{42}$$

In the underlying string theory this represents the modular invariance of the complex dilaton [8], but working only with the classical fields in the low-energy action, it represents a transformation between the dilaton and axion fields which leaves invariant

$$dS^2\equiv e^{2\phi}d\lambda d\lambda^{*}=d\phi^2+e^{2\phi}d\sigma^2. \tag{43}$$

In terms of ϕ and σ, or indeed λ, it is not immediately apparent that dS^2 should remain invariant under the transformation given in Eq. (39). It is rather more transparent if we define the matrix

$$N=\begin{pmatrix} e^{\phi} & e^{\phi}\sigma \\ e^{\phi}\sigma & e^{-\phi}+e^{\phi}\sigma^2 \end{pmatrix}, \tag{44}$$

which obeys $N^T J N=J$, where

$$J=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{45}$$

and thus is a member of SL$(2,R)$. The particular SL$(2,R)$ transformation given in Eq. (39) is given by

$$N\rightarrow\Theta N\Theta^T, \tag{46}$$

where

$$\Theta=\begin{pmatrix} \delta & \gamma \\ \beta & \alpha \end{pmatrix}, \tag{47}$$

is also a member of SL$(2,R)$. Then we can write $dS^2 = \mathrm{tr}(JdNJdN)/2$ and, noting that $\Theta^T J\Theta = J$, it is straightforward to verify that dS^2 is invariant. We will also find this notation particularly convenient later to construct explicitly S-duality invariant dilaton and axion field perturbations.

The Lagrange density of the axion and dilaton fields in the Einstein frame

$$-\frac{1}{4}\mathrm{tr}(J\nabla_\mu NJ\nabla^\mu N) = -\frac{1}{2}(\tilde{\nabla}\phi)^2 - \frac{1}{2}e^{2\phi}(\tilde{\nabla}\sigma)^2 \quad (48)$$

is S-duality invariant and, because the dilaton and axion are minimally coupled to the other fields in the Einstein frame, the evolution of the moduli field, β, and scale factor, $\tilde{a}$, are unaffected by the S-duality transformation. However the scale factor in the original string frame must transform and will not remain invariant under a nontrivial transformation.

If we choose $\gamma/\delta = -1/\sigma_*$, when $\sigma = \sigma_* = \mathrm{const}$, the solutions given in Eqs. (8)–(10) are mapped by the transformation in Eq. (39) to

$$e^\phi \to \gamma^2 e^{-\phi}, \quad (49)$$

$$\sigma \to \frac{\alpha}{\gamma}, \quad (50)$$

$$a \to \gamma e^{-\phi} a, \quad (51)$$

which leaves σ constant. In particular, for $\gamma^2 = 1$ we have $\phi \to -\phi$ and hence this is a transformation between strong and weak coupling. The form of the solutions given in Eqs. (8)–(10) are unchanged, but the parameters

$$e^{\phi_*} \to e^{-\phi_*}, \quad a_* \to e^{-\phi_*} a_*, \quad r_\pm \to r_\mp . \quad (52)$$

Comparing with Eq. (38), we find that in the particular case when $ns^2 = 0$, and hence $r_\pm = \pm\sqrt{3}$, this coincides with the scale factor duality given in Eq. (37).

The more general S-duality transformations of e^ϕ and σ given in Eqs. (41) and (42) can be shown to relate the dilaton-vacuum cosmologies, given in Eqs. (8)–(10), to the more general axion-dilaton cosmologies with a time-dependent axion field, given in Eqs. (17)–(20), with a fixed value of $r = |r_\pm|$. Thus the S-duality transformation allows one to generate the general axion-dilaton solutions with a given value of r starting from only with the dilaton-vacuum solution with $r_\pm = \pm r$.

V. LINEAR PERTURBATIONS

Thus far we have considered only homogeneous classical solutions to the equations of motion. In the next section we will consider inhomogeneous perturbations that may be generated due to vacuum fluctuations. In order to follow their evolution we will set up in this section the formalism required to describe linear perturbations about the homogeneous background metric.

We shall consider perturbations of the four-dimensional metric in the spatially flat gauge[6] (or in more general FRW models, the uniform spatial curvature gauge [34]), using the Einstein frame, so that to first order the perturbed line element can be written as

$$d\tilde{s}^2 = \tilde{a}^2(\eta)\{-(1+2\tilde{A})d\eta^2 + 2\tilde{B}_{,i}d\eta dx^i + [\delta_{ij} + h_{ij}]dx^i dx^j\}, \quad (53)$$

where $\tilde{A}$ and $\tilde{B}$ are the scalar metric perturbations (in the notation of Ref. [35]) and h_{ij} represents a transverse and traceless tensor perturbation. Linear perturbations about the homogeneous background fields can be decomposed as a sum of Fourier modes with comoving wave number k (and in the case of the tensor perturbations two independent polarizations) which evolve independently of other wave numbers.

A. Scalar metric perturbations

The advantage of splitting the metric perturbations into scalar and tensor parts is that the scalar and tensor modes evolve independently to first order with only the scalar perturbations being coupled to scalar field fluctuations [35]. In the spatially flat gauge we have the added simplification that the evolution equations for linear perturbations about homogeneous scalar fields are decoupled from the metric perturbations, although they are still related by a constraint equation.

The field equations for the linearized scalar perturbations are

$$\delta\phi'' + 2\tilde{h}\delta\phi' + k^2\delta\phi = 2e^{2\phi}\sigma'^2\delta\phi + 2e^{2\phi}\sigma'\delta\sigma', \quad (54)$$

$$\delta\sigma'' + 2\tilde{h}\delta\sigma' + k^2\delta\sigma = -2(\sigma'\delta\phi' + \phi'\delta\sigma'), \quad (55)$$

$$\delta\beta'' + 2\tilde{h}\delta\beta' + k^2\delta\beta = 0, \quad (56)$$

$$\tilde{A}'' + 2\tilde{h}\tilde{A}' + k^2\tilde{A} = 0, \quad (57)$$

plus the constraints

$$\tilde{A} = -(\tilde{B}' + 2\tilde{h}\tilde{B}) \quad (58)$$

$$= \frac{\phi'}{4\tilde{h}}\delta\phi + \frac{e^{2\phi}\sigma'}{4\tilde{h}}\delta\sigma + \frac{n\beta'}{2\tilde{h}}\delta\beta. \quad (59)$$

Note that the scalar metric perturbations are not invariant under a conformal transformation. Even the spatially flat nature of the line element in Eq. (53) is not preserved under a conformal transformation back to the string frame due to the first-order perturbation in the conformal factor $e^\phi = e^{\phi_0}(1+\delta\phi)$. However the tensor perturbation remains invariant under both conformal transformations and gauge transformations $\eta \to \eta + \delta\eta$.

The evolution equation for the scalar metric perturbations, Eq. (57), is independent of the evolution of the different scalar fields and is dependent only on the evolution of the Einstein frame scale factor $\tilde{a}(\eta)$ given by Eq. (30). This in turn is determined solely by the stiff fluid equation of state

[6] Called the ''off-diagonal gauge'' in Ref. [33].

for the homogeneous fields in the Einstein frame, regardless of the time dependence of the axion field. Equation (57) can be integrated to give the general solution

$$\tilde{A}=[A_+H_0^{(1)}(-k\eta)+A_-H_0^{(2)}(-k\eta)], \tag{60}$$

where $H_\nu^{(1)}(z)\equiv J_\nu(z)+iY_\nu(z)$ and $H_\nu^{(2)}\equiv J_\nu(z)-iY_\nu(z)$ are Hankel functions of the first and second kind. Using the recurrence relation between Bessel functions, we obtain from Eqs. (58) and (60)

$$\tilde{B}=\frac{1}{k}[A_+H_1^{(1)}(-k\eta)+A_-H_1^{(2)}(-k\eta)]. \tag{61}$$

Our scalar metric perturbations can be written in terms of the gauge-invariant metric potentials [35,36]

$$\tilde{A}\equiv\tilde{\Phi}+\tilde{\Psi}+\left(\frac{\tilde{\Psi}}{\tilde{h}}\right)', \tag{62}$$

$$\tilde{B}\equiv-\frac{\tilde{\Psi}}{\tilde{h}}. \tag{63}$$

Note that the gauge transformation

$$\eta\rightarrow\eta-\frac{\tilde{\Psi}}{\tilde{h}} \tag{64}$$

brings the metric of Eq. (53) into the more commonly used longitudinal gauge [36] where

$$d\tilde{s}^2\rightarrow\tilde{a}^2(\eta)\{-(1+2\tilde{\Phi})d\eta^2+[(1-2\tilde{\Psi})\delta_{ij}+h_{ij}]dx^idx^j\}. \tag{65}$$

The curvature perturbation on uniform energy density hypersurfaces (as $k\eta\rightarrow 0$) is commonly denoted by ζ [36] and is given by

$$\zeta\equiv\tilde{\Phi}-\frac{\tilde{h}^2}{\tilde{h}'-\tilde{h}^2}(\tilde{\Phi}+\tilde{h}^{-1}\tilde{\Phi}'), \tag{66}$$

and hence with $\tilde{h}$ given by Eq. (30) for the scale factor in the Einstein frame, we have

$$\zeta=\frac{\tilde{A}}{3}, \tag{67}$$

in any dilaton-vacuum or axion-dilaton cosmology.

ζ is a particularly useful quantity to calculate as it becomes constant on scales much larger than the Hubble scale ($|k\eta|\ll 1$) for purely adiabatic perturbations, even through changes in the equation of state. In single-field inflation models this allows one to compute the density perturbation at late times, during the matter or radiation dominated eras, by equating ζ at ''reentry'' ($k=\tilde{a}\tilde{H}$) with that at horizon crossing during inflation. Thus previous studies have calculated the spectrum of $\tilde{A}$, and hence ζ, in order to predict the density perturbations induced in the pre-big-bang scenario [33,34]. However, the situation is not really so straightforward in the pre-big-bang scenario as in single-field inflation, because in the full low-energy string effective action there will be many fields present which can lead to nonadiabatic perturbations. We must be aware of the fact that density perturbations at late times may not be simply related to ζ alone, but may also be dependent upon fluctuations in other fields. One such field is the axion field, and we shall see that it may have a markedly different spectrum from ζ.

The scalar field perturbations themselves transform under the gauge transformation $\eta\rightarrow\eta+\delta\eta$ giving $\delta x\rightarrow\delta x-x'\delta\eta$. Thus the scalar field perturbations in the longitudinal gauge (δx_l) are related to those in the spatially flat gauge (δx) under the gauge transformation in Eq. (64) as

$$\delta x\rightarrow\delta x_l=\delta x+x'\frac{\tilde{\Psi}}{\tilde{h}}. \tag{68}$$

B. Tensor metric perturbations

Fortunately, the gravitational wave perturbations h_{ij} are both gauge and conformally invariant. They decouple from the scalar perturbations in the Einstein frame to give a simple evolution equation for each Fourier mode

$$h_k''+2\tilde{h}h_{k'}+k^2h_k=0. \tag{69}$$

The growing mode in the long wavelength ($|k\eta|\rightarrow 0$) limit is $h_k\sim\ln|k\eta|$. (We have not considered gravitational waves propagating in the n internal dimensions. See Ref. [37].) The spectrum depends solely on the dynamics of the scale factor in the Einstein frame given in Eq. (30), which as we have seen is the same regardless of the time dependence of the moduli or axion fields. It leads to a spectrum of primordial gravitational waves steeply growing on short scales, with a spectral index $n_T=3$ [9,33]. This is in contrast to conventional inflation models which require $n_T<0$ [38]. The graviton spectrum appears to be a robust and distinctive prediction of any pre-big-bang type evolution based upon the low-order string effective action. This has been discussed extensively elsewhere [9,33,39], so we now turn to discuss in more detail the spectra corresponding to scalar perturbations.

VI. PRE-BIG-BANG SPECTRA

While the solutions for the homogeneous dilaton, axion, and scale factor in the different frames may lead to interesting behavior in the early universe, the success of the standard big-bang model suggests that the evolution should closely approach the conventional general relativistic evolution at least by the time of nucleosynthesis. If we are to see any trace of the earlier evolution it will be in the primordial spectrum of inhomogeneities present on large scales that we observe today. Such a large-scale structure can only be generated by some unconventional physics, such as inflation, topological defects, or a pre-big-bang epoch. During a period of accelerated expansion the comoving Hubble length $|a/a'|$ decreases and vacuum fluctuations which are assumed to start in the flat-spacetime vacuum state may be stretched up to exponentially large scales. The precise form of the spectrum depends on the expansion of the homogeneous background and the couplings between the fields.

We have seen that the comoving Hubble length does indeed decrease in the Einstein frame during the contracting phase when $\eta<0$. Because the dilaton, moduli fields and graviton are minimally coupled to this metric, this ensures that small-scale vacuum fluctuations will eventually be stretched beyond the comoving Hubble scale during this epoch.

The production of scalar and tensor metric perturbations in the pre-big-bang scenario has been studied by various authors (see, for example, [33,34]). As we remarked earlier, the axion field is taken to be a constant in these solutions. However, while a constant axion field may be a consistent particular solution when describing the background classical field, one cannot necessarily neglect quantum fluctuations in this field. In this section we will consider the production of axions during a pre-big-bang type evolution (where the background axion field is constant) and then go on to discuss the perturbation spectrum in the more general case with $\sigma'\neq 0$. We will also analyze the behavior of these cosmological vacuum states to first order under S-duality transformations.

First of all, let us consider the perturbation spectra produced when the background axion field remains constant, $\sigma'=0$. The evolution of the homogeneous background fields is given in Eqs. (8)–(10) and the dilaton and moduli fields both evolve as minimally coupled massless fields in the Einstein frame. In particular, the dilaton perturbations are decoupled from the axion perturbations and the equations of motion in the spatially flat gauge, Eqs. (54)–(56), become

$$\delta\phi''+2\widetilde{h}\,\delta\phi'+k^2\delta\phi=0, \tag{70}$$

$$\delta\sigma''+2\widetilde{h}\,\delta\sigma'+k^2\delta\sigma=-2\phi'\,\delta\sigma', \tag{71}$$

$$\delta\beta''+2\widetilde{h}\,\delta\beta'+k^2\delta\beta=0, \tag{72}$$

plus we have the constraint, Eq. (58),

$$\widetilde{A}=\frac{\phi'}{4\widetilde{h}}\delta\phi+\frac{n\beta'}{2\widetilde{h}}\delta\beta. \tag{73}$$

A. Dilaton and moduli perturbations

From Eq. (73) we see that, to first order, the metric perturbation $\widetilde{A}$ is determined solely by the dilaton and moduli field perturbations. The canonically normalized field perturbations are [40,33,37]

$$u\equiv\frac{1}{\sqrt{2}\kappa}\widetilde{a}\,\delta\phi, \tag{74}$$

$$w\equiv\frac{\sqrt{n}}{\kappa}\widetilde{a}\,\delta\beta, \tag{75}$$

which, from Eqs. (70) and (72), obey the wave equations

$$u''+\left(k^2-\frac{\widetilde{a}''}{\widetilde{a}}\right)u=0, \tag{76}$$

$$w''+\left(k^2-\frac{\widetilde{a}''}{\widetilde{a}}\right)w=0. \tag{77}$$

After inserting the simple solution for the Einstein frame scale factor given in Eqs. (30) we find that these equations give the general solutions

$$u=|k\eta|^{1/2}[u_+H_0^{(1)}(|k\eta|)+u_-H_0^{(2)}(|k\eta|)], \tag{78}$$

$$w=|k\eta|^{1/2}[w_+H_0^{(1)}(|k\eta|)+w_-H_0^{(2)}(|k\eta|)]. \tag{79}$$

On the $(+)$ branch, i.e., when $\eta<0$, we can normalize modes at early times, $\eta\to-\infty$, where all the modes are far inside the Hubble scale, $k\gg|\eta|^{-1}$, and can be assumed to be in a flat-spacetime vacuum.[7] Just as in conventional inflation, this produces perturbations on scales far outside the horizon, $k\ll|\eta|^{-1}$, at late times, $\eta\to 0$.

Conversely, the solution for the $(-)$ branch with $\eta>0$ is dependent upon the initial state of modes far outside the horizon, $k\ll|\eta|^{-1}$, at early times where $\eta\to 0$. The role of a period of inflation, or of the pre-big-bang $(+)$ branch, is precisely to set up this initial state which otherwise appears as a mysterious initial condition in the conventional (noninflationary) big-bang model.

Allowing only positive frequency modes in the flat-spacetime vacuum state at early times for the pre-big-bang $(+)$ branch requires [41] that, as $k\eta\to-\infty$,

$$u\to\frac{e^{-ik\eta}}{\sqrt{2k}}, \tag{80}$$

and similarly for w, giving

$$u_+=w_+=e^{i\pi/4}\frac{\sqrt{\pi}}{2\sqrt{k}},\quad u_-=w_-=0. \tag{81}$$

The power spectrum for perturbations is commonly denoted by

$$\mathcal{P}_{\delta x}\equiv\frac{k^3}{2\pi^2}|\delta x|^2, \tag{82}$$

and thus for modes far outside the horizon $(k\eta\to 0)$ we have

$$\mathcal{P}_{\delta\phi}\to\frac{4}{\pi^3}\kappa^2\widetilde{H}^2(-k\eta)^3[\ln(-k\eta)]^2, \tag{83}$$

$$\mathcal{P}_{\delta\beta}\to\frac{2}{n\pi^3}\kappa^2\widetilde{H}^2(-k\eta)^3[\ln(-k\eta)]^2, \tag{84}$$

where $\widetilde{H}\equiv\widetilde{a}'/\widetilde{a}^2$ is the Hubble rate in the Einstein frame, and recall n is the number of compact dimensions. The amplitude of the perturbations grows towards small scales, but only becomes large for modes outside the horizon $(|k\eta|<1)$ when $\kappa^2\widetilde{H}^2\sim 1$, i.e., the Planck scale in the Einstein frame. The spectral tilt of the perturbation spectra is given by

[7] It is interesting to note that in conventional inflation we have to assume that this result for a quantum field in a classical background holds at the Planck scale. Here, however, the normalization is done in the zero-curvature limit in the infinite past.

$$n_x - 1 \equiv \frac{d\ln\mathcal{P}_{\delta x}}{d\ln k} \tag{85}$$

which from Eqs. (83) and (84) gives $n_\phi = n_\beta = 4$ (where we neglect the logarithmic dependence).

We need also to compute the amplitude of the scalar metric perturbations, to check the validity of our linear perturbation analysis. Normalizing the amplitude of the spectrum for the metric perturbation $\widetilde{A}$ in Eq. (60) from the constraint Eq. (59), using Eqs. (9) and (10) for the background fields and Eqs. (83) and (84) for their perturbations, we have

$$\mathcal{P}_{\widetilde{A}} = \frac{3}{\pi^3}\kappa^2 \widetilde{H}^2(-k\eta)^3[\ln(-k\eta)]^2. \tag{86}$$

(Remember that we are adding independent random variables. The 3 comes from $r_\pm^2 + 2ns^2 = 3$.) Note that this spectrum of scalar metric perturbations in entirely independent of the integration constants that parameterize the solutions given in Eqs. (9) and (10). The scalar spectrum, just like the spectrum of tensor perturbations, is a robust prediction of any pre-big-bang scenario where the universe collapses in the Einstein frame, and becomes dominated by homogeneous scalar fields.

Just like the field perturbations, the scalar metric perturbations have a steep blue spectrum, $n_{\widetilde{A}} = 4$, which becomes large on superhorizon scales $|k\eta| < 1$ only near the Planck scale, $\kappa^2\widetilde{H}^2 \sim 1$. Note that Bardeen's gauge-invariant perturbations $\widetilde{\Phi}$ and $\widetilde{\Psi}$, defined in Eqs. (62) and (63), actually become large much earlier [33], but the fact that the perturbations remain small in our choice of gauge implies that our linear calculation is in fact valid up until the Planck epoch [33].

Unfortunately this leaves us with such a steeply tilted spectrum of metric perturbations that there would be effectively no primordial metric perturbations on large (supergalactic) scales in our present universe if the post-big-bang era began close to the Planck scale. The metric fluctuations are of order unity on the Planck scale (10^{-33} cm) when $T \sim 10^{32}$ K in the standard post-big-bang model. This corresponds to a comoving scale of about 0.1 cm today (when $T = 2.7$ K), about 10^{-29} times the scale of perturbations observed on the microwave background sky. Thus the microwave background temperature anisotropies should be of order 10^{-87} rather than the observed 10^{-5}. However, it turns out that the presence of the axion field could provide an alternative spectrum of perturbations more suitable as a source of large-scale structure.

B. Axion perturbations

While the dilaton and moduli fields evolve as massless minimally coupled fields in the Einstein frame, the axion evolves as a massless minimally coupled field in the axion frame and the canonically normalized field perturbation is

$$v \equiv \frac{1}{\sqrt{2}\kappa}\bar{a}\,\delta\sigma. \tag{87}$$

In this section we are considering the axion spectrum in the pre-big-bang scenario where the background axion field is constant. As a result density perturbations are only second order in the axion perturbation and so we can neglect the backreaction from the metric to linear order. The field perturbation $\delta\sigma$ is gauge invariant when $\sigma' = 0$ [see Eq. (68)] and in any gauge, the axion perturbation obeys the decoupled wave equation given in Eq. (71) which can be rewritten in terms of v as

$$v'' + \left(k^2 - \frac{\bar{a}''}{\bar{a}}\right)v = 0. \tag{88}$$

As we have just mentioned, whereas the dilaton and moduli evolve as massless minimally coupled fields in the Einstein frame, the axion is minimally coupled in the axion frame, whose evolution given in Eq. (32) is significantly different. In fact, substituting Eq. (32) in Eq. (88) we have

$$v = |k\eta|^{1/2}[v_+ H_r^{(1)}(|k\eta|) + v_- H_r^{(2)}(|k\eta|)], \tag{89}$$

where we have used $r \equiv |r_\pm|$. Once again, we can only normalize this using the flat spacetime vacuum state at early times as $-k\eta \to \infty$ on the $(+)$ branch, as in Eq. (80), which gives

$$v_+ = e^{i(2r+1)\pi/4}\frac{\sqrt{\pi}}{2\sqrt{k}}, \quad v_- = 0. \tag{90}$$

and hence we have

$$\delta\sigma = \kappa\sqrt{\frac{\pi}{2k}}e^{i(2r+1)\pi/4}\frac{\sqrt{-k\eta}}{\bar{a}}H_r^{(1)}(-k\eta). \tag{91}$$

At late times, as $-k\eta \to 0$, we find[8]

$$\mathcal{P}_{\delta\sigma} = 2\kappa^2\left(\frac{C(r)}{2\pi}\right)^2\frac{k^2}{\bar{a}^2}(-k\eta)^{1-2r}, \tag{92}$$

where the numerical coefficient

$$C(r) \equiv \frac{2^r\Gamma(r)}{2^{3/2}\Gamma(3/2)} \tag{93}$$

approaches unity for $r = 3/2$.

The expression for the axion power spectrum can be written in terms of the field perturbation when each mode crosses outside the horizon

$$\mathcal{P}_{\delta\sigma_c} = 2\kappa^2\left[\frac{C(r)}{r_\pm + (1/2)}\right]^2\left(\frac{\bar{H}_c}{2\pi}\right)^2, \tag{94}$$

where $\bar{H}_c$ is the Hubble rate when $|k\eta| = 1$. This is the power spectrum for a massless scalar field during power-law infla-

[8]When $2ns^2 = 3$ and $r = 0$ the dilaton remains constant and the axion frame and Einstein frame coincide, up to a constant factor. Thus the axion spectrum behaves like that for the dilaton and moduli fields and the late time evolution in this case is that logarithmic with respect to $-k\eta$, as given in Eqs. (83) and (84).

tion which approaches the famous result $\mathcal{P}_{\delta\sigma}/2\kappa^2=(\bar{H}_c/2\pi)^2$ as $r_-\to-3/2$, and the expansion in the axion frame becomes exponential.[9]

More importantly, the spectral index

$$n_\sigma=4-2r=4-2\sqrt{3-2ns^2} \tag{95}$$

depends crucially upon the value of $r=|r_\pm|$. The spectrum becomes the classic scale-invariant Harrison-Zel'dovich spectrum as $r\to3/2$. The lowest possible value of the spectral tilt n_σ is $4-2\sqrt{3}\simeq0.54$ which is obtained when stable compactification has occurred and the moduli field β is fixed. The more rapidly the internal dimensions evolve, the steeper the resulting axion spectrum until for $2ns^2=3$ and $r=0$ we have $n_\sigma=4$ like the dilaton and moduli spectra. Note that the condition for a negatively tilted spectrum coincides exactly with the requirement for conventional power-law inflation, rather than pole inflation, in the axion frame.

Of course, when the background axion field is constant these perturbations, unlike the dilaton or moduli perturbations, do not affect the scalar metric perturbations (i.e., these are isocurvature perturbations). However, if the axion field does affect the energy density at late times (for instance, by the axion field acquiring a mass) then the spectrum of density perturbations need not have a steeply tilted blue spectrum like the dilaton perturbations, but rather could have a nearly scale-invariant spectrum as required for large-scale structure formation [38].

VII. PERTURBATION SPECTRA IN GENERAL AXION-DILATON COSMOLOGIES

When we allow the background homogeneous axion field to be time dependent we must allow for the interaction between the dilaton and axion field and the metric to first order.

In fact we have seen that in the spatially flat gauge the evolution equations for both the scalar and tensor metric perturbations [Eqs. (57) and (69)] are independent of the evolution of the different scalar fields and are determined solely by the evolution of the Einstein scale factor given in Eq. (30). Because the moduli field perturbations remain decoupled from both the axion and dilaton, their evolution equation, Eq. (72), is also unaffected. Thus the spectral tilts of the scalar and tensor metric perturbations and the moduli spectrum, Eq. (84), remain the same as in the pre-big-bang scenario.

We can understand this in terms of the S-duality transformations that relate the general axion-dilaton solutions to the dilaton-vacuum solutions. These transformations leave the Einstein frame metric and moduli field invariant and thus not only the homogeneous fields, but also their perturbations, are identical in S-duality related cosmologies. However the dilaton and axion fields and their perturbations will in general be affected by S-duality transformations.

A. Axion and dilaton perturbations

The dilaton and axion perturbation field equations (54) and (55) become coupled to first order when $\sigma'\neq0$, and the chances of obtaining analytic solutions might appear to be remote. However, we can exploit the S-duality symmetry which relates the general axion-dilaton cosmologies to the much simpler dilaton-vacuum cosmologies in order to find linear combinations of the axion and dilaton perturbations which remain straightforward to integrate even in the more general case.

We define two new S-duality invariant variables:

$$x\equiv e^{\phi}\left(\frac{\phi'}{\tilde{h}}\delta\sigma-\frac{\sigma'}{\tilde{h}}\delta\phi\right), \tag{96}$$

$$y\equiv\frac{\phi'}{\tilde{h}}\delta\phi+\frac{e^{2\phi}\sigma'}{\tilde{h}}\delta\sigma. \tag{97}$$

In terms of x and y the perturbation equations decouple and the field equations (54) and (55) then become

$$x''+2\tilde{h}x'+[k^2-(\phi'^2+e^{2\phi}\sigma'^2)]x=0, \tag{98}$$

$$y''+2\tilde{h}y'+k^2y=0. \tag{99}$$

It is far from obvious on first inspection that these variables should be invariant under the S-duality transformation given in Eq. (39). However written in terms of the matrix N defined in Eq. (44) we have

$$2\tilde{h}x=-\mathrm{tr}(JNJN'J\delta N), \tag{100}$$

$$2\tilde{h}y=\mathrm{tr}(JN'J\delta N), \tag{101}$$

and we can see that these variables are the unique S-duality invariant linear combinations of the axion and dilaton perturbations. They reduce to the (decoupled) axion and dilaton perturbations in the pure dilaton-vacuum background, as $\sigma'\to0$, where we have

$$x\to\frac{\phi'}{\tilde{h}}e^{\phi}\delta\sigma=2r_\pm e^{\phi}\delta\sigma, \tag{102}$$

$$y\to\frac{\phi'}{\tilde{h}}\delta\phi=2r_\pm\delta\phi. \tag{103}$$

Note that x is not only S-duality invariant, but also gauge invariant. That is, it does not matter which gauge we choose to calculate $\delta\sigma$ and $\delta\phi$, the combination which defines x remains unchanged. It is proportional to the axion perturbation on uniform-dilaton hypersurfaces, $\delta\sigma|_\phi\equiv\delta\sigma-\sigma'(\delta\phi/\phi')$. By symmetry, it is also proportional to the dilaton perturbation on constant axion hypersurfaces, though this perturbation diverges in the limit that the background axion field becomes constant.

Having found S-duality-invariant variables, one can verify that the evolution equations for these variables, Eqs. (98) and (99) are themselves invariant under S duality. Remembering that the general axion-dilaton cosmological solutions can always be related to the dilaton-vacuum solutions by an S-duality transform, we see that the evolution equations for x and y in an arbitrary axion-dilaton cosmology are exactly the same as those for the axion and dilaton perturba-

[9]The factor $2\kappa^2$ arises due to our dimensionless definition of σ.

tions in the dilaton-vacuum case. Just as in the constant axion case, we can define canonically normalized variables:

$$u \equiv \frac{1}{2r\sqrt{2}\kappa}\widetilde{a}y, \tag{104}$$

$$v \equiv \frac{1}{2r\sqrt{2}\kappa}\widetilde{a}x, \tag{105}$$

which coincide with the definitions given in Eqs. (74) and (87) in the dilaton-vacuum case. In general, u obeys the S-duality-invariant equation of motion given in Eq. (76) and whose general solution is given by Eq. (78). The equation of motion for v given in Eq. (88), however, is not invariant under an S-duality transformation. Instead the S-duality-invariant version of the equation of motion is

$$v'' + \left(k^2 - \frac{r^2 - 1/4}{\eta^2}\right)v = 0, \tag{106}$$

which coincides with Eq. (88) when $\sigma' = 0$. The general solution for v is thus still given by Eq. (89).

We can still normalize cosmological vacuum perturbations at early times on the $(+)$ branch as $\eta \to -\infty$ because we have seen that in this limit the general axion-dilaton solution given in Eqs. (17)–(20) approach the constant axion solutions with $r_+ = +r$. Thus the constants $u_\pm$ and $v_\pm$ are given by Eqs. (81) and (90). By picking S-duality-invariant field perturbations we have been able to calculate the general axion-dilaton cosmological perturbation spectra using the pure dilaton-vacuum cosmological vacuum states. We have

$$\mathcal{P}_y \to \frac{16r^2}{\pi^3}\kappa^2\widetilde{H}^2(-k\eta)^3[\ln(-k\eta)]^2, \tag{107}$$

and the generalized axion perturbation spectrum is given by

$$\mathcal{P}_x \to 8r^2\kappa^2\left(\frac{C(r)}{2\pi}\right)^2\frac{k^2}{\widetilde{a}^2}(-k\eta)^{1-2r}. \tag{108}$$

To recover the actual (though gauge and S-duality dependent) axion and dilaton perturbations we can invert Eqs. (96) and (97) to give

$$\delta\sigma = \frac{e^{-\phi}}{4r^2}\left(\frac{\phi'}{\widetilde{h}}x + \frac{e^{\phi}\sigma'}{\widetilde{h}}y\right), \tag{109}$$

$$\delta\phi = \frac{1}{4r^2}\left(\frac{\phi'}{\widetilde{h}}y - \frac{e^{\phi}\sigma'}{\widetilde{h}}x\right). \tag{110}$$

However, at late times on the $(+)$ branch, as $\eta \to 0$ the general axion-dilaton solutions approach dilaton-vacuum solutions with $r_- = -r$, and hence $\delta\phi \to y/2r_-$ and $\delta\sigma \to e^{-\phi}x/2r_-$. Note that the change of sign from $r_+ = +r$ to $r_- = -r$ between the early- and late-time dilaton-vacuum solutions leads to a phase shift $e^{i\pi}$ with respect to the late-time behavior of the pure dilaton-vacuum solutions. But the final power spectrum for the dilaton and axion perturbations as $\eta \to 0$ in the general axion-dilaton cosmologies is identical to that given in Eqs. (83) and (92) for the S-duality related dilaton-vacuum case. The tilt and amplitude of the spectra are determined solely by the parameter $r = |r_\pm|$ and are insensitive to the specific time dependence of the axion field in different, but S-duality related, solutions.

The constraint equation for $\widetilde{A}$, Eq. (59), includes only y and $\delta\beta$;

$$\widetilde{A} = \frac{1}{4}y + \frac{n\beta'}{2\widetilde{h}}\delta\beta. \tag{111}$$

From Eqs. (107) and (84) we see that the spectrum of scalar metric perturbations is unaffected by the time dependence of the axion field and is the same as that obtained in the constant axion case, given in Eq. (86).

VIII. DISCUSSION

The low-energy limit of string theory, or M theory, contains many different degrees of freedom. In this paper we have considered a very simple model containing only a $D = 4$ spatially flat FRW metric with a dilaton, a single massless modulus field (representing the volume of n internal dimensions) and a pseudoscalar axion field derived from the Neveu-Schwarz antisymmetric tensor potential. The axion-dilaton solutions can be generated from the dilaton-vacuum solutions by an S-duality transformation. They generalize the power-law dilaton-vacuum solutions in a particularly simple way, interpolating between two asymptotically dilaton-vacuum regimes, which are themselves related by an S-duality transformation.

Although the general axion-dilaton solutions do not alter the singular nature of the cosmological solutions in the string or Einstein frame, we draw attention to the fact that the evolution in the conformally related axion frame (in which the axion is minimally coupled) can become nonsingular when the axion field is allowed to be time dependent. The world lines of axionic observers can have an infinite proper lifetime in this frame. There is no graceful exit [11] from the pre-big-bang $(+)$ branch to the post-big-bang $(-)$ branch, but the $(+)$ or $(-)$ branches themselves can have an infinite proper lifetime.

The shrinking comoving Hubble length during a pre-big-bang era generates a spectrum of perturbations about the homogeneous background fields from quantum fluctuations. We have calculated the spectrum of large-scale perturbations produced in the axion field. The axion spectral index can lie anywhere in the range 0.54 to 4, which includes the possibility of the nearly scale-invariant $(n \sim 1)$ spectrum required for structure formation. This is in contrast to the dilaton and moduli perturbations which have a steep blue spectrum with an index of $n = 4$, making them incapable of seeding large-scale structure in our present universe. The actual value of the axion spectral index depends on the rate of expansion of the internal dimensions. If stable compactification has already occured, leading to an effective four-dimensional spacetime, the spectral index is $n = 0.54$.

In the simplest case where the background axion field is constant, the axion perturbations are isocurvature perturbations during the pre-big-bang epoch. Whether these axion perturbations are able to seed large-scale structure in the post-big-bang universe depends crucially on the coupling between the axion and the matter which dominates the universe today. Nonetheless, it is intriguing that, in principle, the axion could give rise to a nearly scale-invariant spectrum and that the tilt of that spectrum is dependent on the compactification of the internal dimensions.

We have seen that S duality is a powerful tool for calculating not only the classical background solutions in general axion-dilaton cosmologies but also the semiclassical perturbation spectra. By constructing explicitly S-duality- invariant field perturbations we are able to calculate the perturbation spectra in the more general axion-dilaton cosmologies as well as the dilaton-vacuum case. It is not surprising that by taking S-duality-invariant field perturbations we can derive S-duality-invariant solutions. More remarkably, however, the late-time dilaton and axion spectra turn out to be independent of the preceding evolution along different, but S-duality related, classical solutions. This results from the fact that S-duality related axion-dilaton solutions all approach the same dilaton-vacuum solution at late times. By contrast, other symmetries present in the low-energy string action, such as the symmetry which mixes the moduli field with the dilaton and axion [24], do not relate solutions with the same late-time behavior and so will not leave the perturbation spectra invariant.

Despite the well-known problems associated with achieving a graceful exit from the pre-big-bang era, it is worth noting a couple of advantages that the pre-big-bang predictions have over conventional (potential-dominated) inflation models. First, the perturbations originate as vacuum fluctuations at early times, where their amplitude is normalized in a low-curvature, weakly coupled regime in the infinite past, and not at arbitrarily small scales during the Planck epoch when the correct vacuum state may be uncertain. Second, one can give analytic expressions for the asymptotic perturbations on large scales without having to invoke any slow-roll-type approximations as must usually be done in conventional inflation models. This is possible not only in the presence of the dilaton alone, but also when one incorporates the moduli and axion fields.

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[1] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[2] J. M. Maldacena, Ph.D. thesis, Princeton University, 1996.
[3] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993).
[4] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. **158B**, 316 (1985); C. G. Callan, D. Friedan, E. J. Martinec, and M. J. Perry, Nucl. Phys. **B262**, 593 (1985); C. Lovelace, *ibid.* **B273**, 413 (1986).
[5] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
[6] J. A. Casas, J. García-Bellido, and M. Quirós, Nucl. Phys. **B361**, 713 (1991); J. García-Bellido and M. Quirós, *ibid.* **B385**, 558 (1992).
[7] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994).
[8] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987), Vol. 1.
[9] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993).
[10] M. Gasperini and G. Veneziano, Mod. Phys. Lett. A **8**, 3701 (1993).
[11] R. Brustein and G. Veneziano, Phys. Lett. B **329**, 429 (1994).
[12] N. Kaloper, R. Madden, and K. A. Olive, Nucl. Phys. B **452**, 677 (1995); Phys. Lett. B **371,** 34 (1996).
[13] R. Easther, K. Maeda, and D. Wands, Phys. Rev. D **53**, 4247 (1996).
[14] R. Easther and K. Maeda, Phys. Rev. D **54**, 7252 (1996).
[15] M. Gasperini, M. Maggiore, and G. Veneziano, CERN TH/96-267, hep-th/9611039 , 1996 (unpublished) [Nucl. Phys. B (to be published)].
[16] A. R. Liddle, R. G. Moorehouse, and A.B. Henriques, Nucl. Phys. **B311**, 719 (1989).
[17] A. A. Tseytlin, Int. J. Mod. Phys. D **1**, 223 (1992).
[18] D. S. Goldwirth and M. J. Perry, Phys. Rev. D **49**, 5019 (1994).
[19] E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D **50**, 4868 (1994).
[20] E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. D **51**, 1569 (1995).
[21] N. A. Batakis, Phys. Lett. B **353**, 39 (1995); N. A. Batakis and A. A. Kehagias, Nucl. Phys. **B449**, 248 (1995).
[22] J. D. Barrow and K. Kunze, Phys. Rev. D **55**, 623 (1997); J. D. Barrow and M. P. Dąbrowski, *ibid.* **55**, 630 (1997).
[23] A. Lukas, B. A. Ovrut, and D. Waldram, Phys. Lett. B **393**, 65 (1997); ''String and M-theory cosmological solutions with Ramond forms,'' University of Pennsylvania UPR-723T, hep-th/9610238, 1996 (unpublished).
[24] J. E. Lidsey, ''On the cosmology and symmetry of dilaton-axion gravity,'' Queen Mary Westfield Report No. gr-qc/9609063, 1996 (unpublished).
[25] H. Lü, S. Mukherji, C. N. Pope, and K.-W. Xu, Phys. Rev. D **55**, 7926 (1997); R. Poppe and S. Schwager, Phys. Lett. B **393**, 51 (1997); N. KIaloper, Phys. Rev. D **55,** 3394 (1997).
[26] G. Veneziano, Phys. Lett. B **265**, 287 (1991).
[27] D. Wands, Class. Quantum Grav. **11**, 269 (1994).
[28] R. Tabensky and A. H. Taub, Commun. Math. Phys. **29**, 61 (1973).
[29] J. P. Mimoso and D. Wands, Phys. Rev. D **51**, 477 (1995); **52**, 5612 (1995).
[30] K. Behrndt and S. Förste, Nucl. Phys. **B430**, 441 (1994).

[31] A. Shapere, S. Trivedi, and F. Wilczek, Mod. Phys. Lett. A **6**, 2677 (1991); A. Sen, *ibid.* **8**, 2023 (1993).
[32] K. Meissner and G. Veneziano, Phys. Lett. B **267**, 33 (1991); A. Sen, *ibid.* **271**, 295 (1991).
[33] R. Brustein, M. Gasperini, M. Giovannini, V. F. Mukhanov, and G. Veneziano, Phys. Rev. D **51**, 6744 (1995).
[34] J. Hwang, ''Structure formation in string cosmology,'' Report No. hep-th/9608041, 1996 (unpublished); ''Density spectrums from kinetic inflations,'' Kyungpook National University Report No. gr-qc/9612065, 1996 (unpublished).
[35] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980).
[36] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[37] M. Gasperini and M. Giovannini, Class. Quantum Grav. **14,** 735 (1997); M. Giovannini, Phys. Rev. D **55**, 595 (1997).
[38] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
[39] R. Brustein, M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Lett. B **361**, 45 (1995); M. Galluccio, M. Litterio, and F. Occhionero, ''Graviton spectra in string cosmology,'' Osservatorio Astronomico di Roma Report No. gr-qc/9608007, 1996 (unpublished).
[40] V. F. Mukhanov, Sov. Phys. JETP **68**, 1297 (1988).
[41] N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).