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Planck and re-ionization history: a model selection view

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ABSTRACT

We use Bayesian model selection tools to forecast the *Planck* satellite’s ability to distinguish between different models for the re-ionization history of the Universe, using the large angular scale signal in the cosmic microwave background polarization spectrum. We find that *Planck* is not expected to be able to distinguish between an instantaneous re-ionization model and a two-parameter smooth re-ionization model, except for extreme values of the additional re-ionization parameter. If it cannot, then it will be unable to distinguish between different two-parameter models either. However, Bayesian model averaging will be needed to obtain unbiased estimates of the optical depth to re-ionization. We also generalize our results to a hypothetical future cosmic variance limited microwave anisotropy survey, where the outlook is more optimistic.

Key words: methods: data analysis – methods: statistical – cosmology: theory.

1 INTRODUCTION

The five-year data from the *Wilkinson Microwave Anisotropy Probe* (*WMAP*; Dunkley et al. 2008; Hinshaw et al. 2008; Komatsu et al. 2008) have given reasonably tight constraints on the optical depth to Thomson scattering from the last-scattering surface, $\tau = 0.09 \pm 0.02$ with modest dependence on inclusion of additional data sets and changes to model assumptions. It has not, however, had the accuracy needed to go beyond this one-parameter description of the ionization history of the Universe to give a more detailed view of how re-ionization took place and to distinguish between the various models in the literature (though combined with tentative indication of a change in Lyman α optical depth around redshift 7, it does give some indication that re-ionization is an extended process).

Theoretical studies suggest that the process of re-ionization can be quite complex (e.g. Barkana & Loeb 2001; Cen 2003; Haiman & Holder 2003; for reviews see Barkana & Loeb 2007a; Meiksin 2007). The *Planck* satellite may have the sensitivity to go beyond a one-parameter description of the process. For instance, Lewis, Weller & Battye (2006) considered three specific re-ionization histories (with other cosmological parameters held fixed), and assessed whether *Planck* would be able to distinguish amongst them, finding that it did indeed have some ability to do so.

However, the true data analysis problem is more complicated than in their study. Future experiments will not be trying to distinguish between a small set of specific re-ionization histories. Rather, there will be competing *models* for re-ionization each of which feature parameters that need to be determined from the data. That is, the problem is one of *model selection* (see Gregory 2005; Liddle, Mukherjee & Parkinson 2006a; Trota 2008, and references therein).

In this paper we use Bayesian model selection tools to forecast the ability of the *Planck* satellite, and a putative cosmic variance limited future survey, to distinguish between two re-ionization models, instantaneous re-ionization (parametrized solely by the optical depth to re-ionization τ or equivalently the redshift of re-ionization) and a smooth transition to the ionized state, parametrized by a further parameter d_η which measures the rapidity of the transition (in conformal time η).

We consider only the large-scale bump in the cosmic microwave background (CMB) polarization spectrum generated by Thomson scattering of the CMB quadrupolar anisotropy during re-ionization. The detailed shape of the bump is related to the evolution of the globally averaged ionized fraction during re-ionization (Hu & Holder 2003; Kaplinghat et al. 2003; Colombo et al. 2005). The power on scales smaller than the horizon size at re-ionization is uniformly damped by $e^{-2\tau}$; this then cannot be used to constrain the details of re-ionization beyond τ , or even to constrain τ itself which would be almost completely degenerate with the amplitude of perturbations. Other degeneracies are discussed in Martins et al. (2004) and Trota & Hansen (2004). Re-ionization affects the CMB spectrum again on much smaller scales via secondary effects due to inhomogeneous or patchy re-ionization and the Ostriker–Vishniac effects (Ostriker & Vishniac 1986; Weller 1999; Hu 2000). We do not consider these effects which are beyond multipole $\ell \sim 2000$, modelling only uniform re-ionization. In the future, 21-cm emission from neutral hydrogen is expected to provide a good tracer of the details of re-ionization (e.g. see Barkana & Loeb 2007b), and there are experiments that will focus on mapping this emission.

2 THE MODELS

Our cosmological model is the usual spatially flat Λ cold dark matter (Λ CDM) cosmology, seeded by power-law adiabatic

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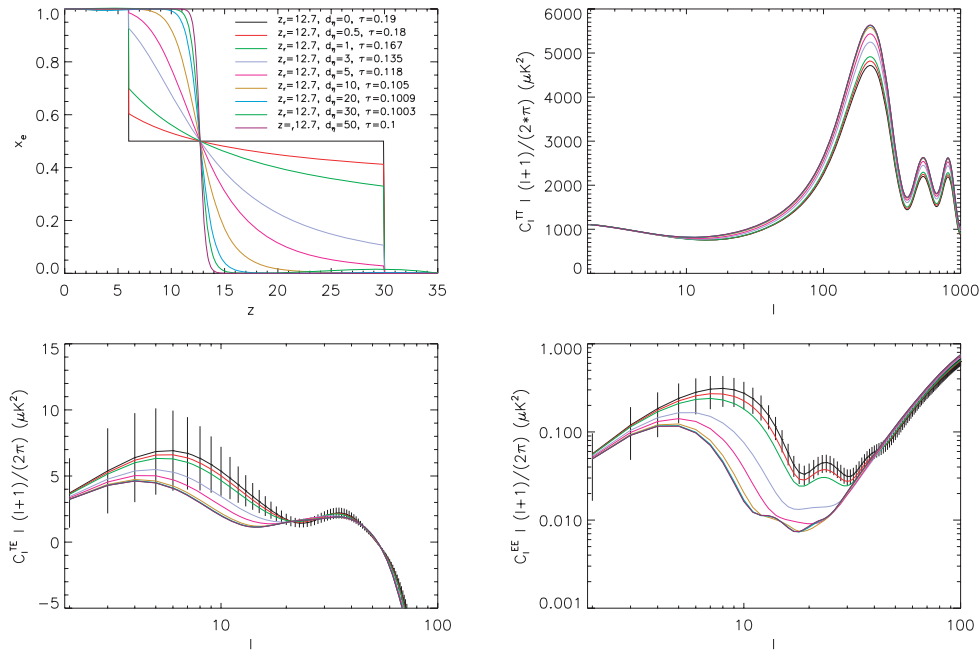


Figure 1. The upper left-hand panel shows a series of re-ionization histories, in each case with the re-ionization redshift fixed to $z_r = 12.7$. The remaining panels show the TT (top right), TE (bottom left) and EE (bottom right) spectra generated by these models. The error bars indicate the uncertainty due to cosmic variance.

density perturbations. Its adjustable parameters are the dark matter and baryon densities Ω_c and Ω_b , the Hubble parameter h and the perturbation amplitude A_s and spectral index n_s . These are fixed to *WMAP3* best-fitting values¹ (Spergel et al. 2007) for $\Omega_b h^2$, $\Omega_c h^2$, the projected sound horizon θ , $A_s \exp(-2\tau)$ and n_s . We then study the re-ionization signal from the TE and EE spectra out to ℓ of 100. It is possible to use such an analysis procedure because the non-reionization parameters are very well determined by the TT spectrum, and because the large-scale signal in CMB polarization is independent of the other parameters. A similar procedure has been followed in works including Holder et al. (2003), Kaplinghat et al. (2003) and Mortonson & Hu (2008a,b). The uncertainty on τ derived holding these parameters fixed is expected to be an underestimate by about 10 per cent (Mortonson & Hu 2008a).

We assume standard recombination. If the recombination model eventually needs to be modified to account for two-photon decays (Dubrovich & Grachev 2005; Wong & Scott 2007; Chluba & Sunyaev 2008; Hirata 2008), this should not affect the model comparisons we present here because it would be common to all the models. In addition, the spectrum changes on intermediate to small scales while we are using only the large scales here.

We mainly consider a two-parameter re-ionization model defined by the ionization fraction history:

$$x_e(\eta_i) = [1 - x_e(\eta_{i-1})] \frac{\tanh\{[(\eta/\eta_{z_r}) - 1]d_\eta + 1\}}{2} + x_e(\eta_{i-1}), \quad (1)$$

where x_e refers to the ionization fraction, η_i and η_{i-1} refer to consecutive time steps, η to the conformal time at the i th time step, z_r is the redshift at which the ionization fraction is 0.5, η_{z_r} is the conformal time corresponding to that redshift and d_η gives the (inverse) width of the transition. Such a transition is implemented

in CAMB (Lewis, Challinor & Lasenby 2000), and the commonly used instantaneous re-ionization scenario corresponds to d_η having a large enough value, such as 50, that z_r is effectively the redshift of instantaneous re-ionization.²

We additionally force the ionization fraction to unity for $z < 6$, to avoid conflict with quasar absorption spectrum data, and to zero for $z < 30$ as no ionizing sources are expected so early. The optical depth τ is computed numerically for any such re-ionization history. Given the re-ionization history, we compute the CMB power spectra using a version of CAMB with minor modifications. Our assumed cosmological model has only scalar initial perturbations, and so we do not compute the BB polarization spectra.

Figs 1 and 2 show some predicted power spectra for these models, showing in particular that τ is indeed mainly responsible for variations in the predictions and hence the most readily measured parameter. Furthermore, at fixed τ it is clear from Fig. 2 that all the discriminating power is in the polarization spectra rather than temperature.

Through most of the following analysis we take a fiducial z_r - d_η model with $d_\eta = 3$ and $z_r = 8.9$, corresponding to $\tau = 0.1$ as already well determined by *WMAP5*.

Flat priors are assumed on z_r and d_η over ranges of 6–30 and 0–10, respectively (the figures show that values of d_η larger than this result in scenarios very close to instantaneous). Fig. 1 also motivates us to try a logarithmic prior on d_η , for which we take the range 0.3–30. However, a $\tau < 0.3$ prior is also imposed, so that the one additional parameter as compared to instant re-ionization does

¹ Our calculations predated the *WMAP* five-year data release (Hinshaw et al. 2008), which however left the numbers almost unchanged.

² We use a version of CAMB prior to 2008 April that includes only hydrogen re-ionization and thus a final ionization fraction of unity. Including helium re-ionization the final ionization fraction would be ~ 1.08 . See section XIII of CAMB notes, via a link from <http://camb.info/readme.html>, for further details. The model selection results presented in this paper are not expected to change following this inclusion of helium re-ionization.

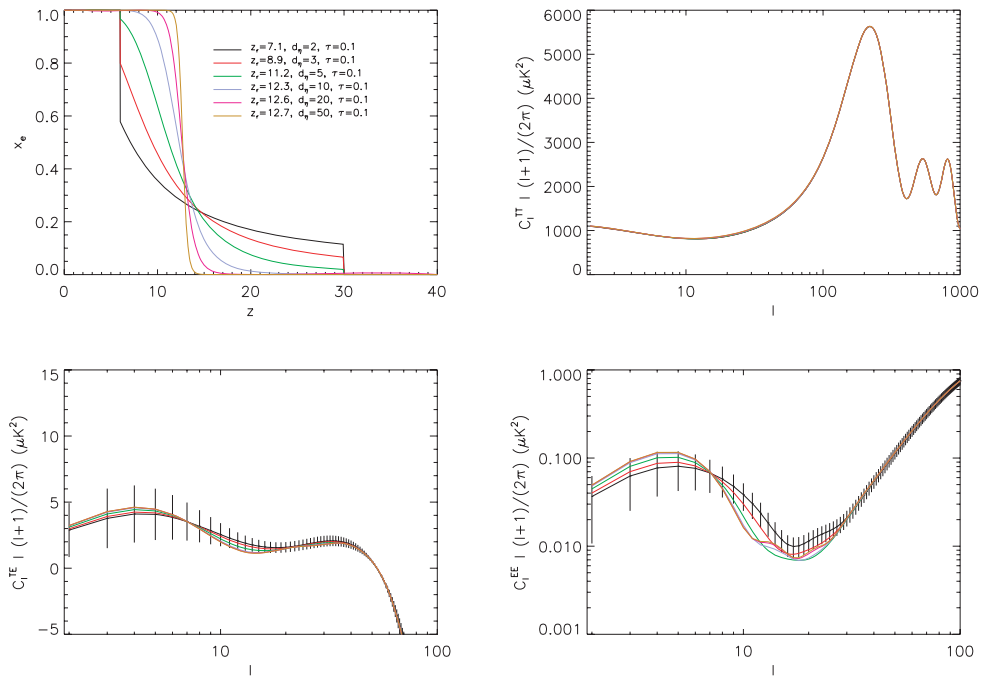


Figure 2. As Fig. 1, but for models each with τ fixed at 0.1. At fixed τ the spectra, especially TT, have only a weak dependence on d_η .

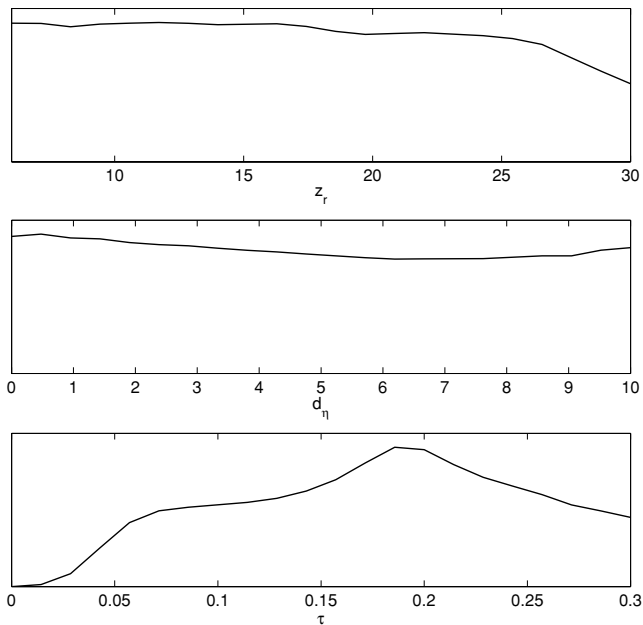


Figure 3. Uniform priors on z_r between 6 and 30, and on d_η between 0 and 10, result in a non-uniform prior on τ (solid curves). We can work with such a prior on τ because expected uncertainties from a *Planck*-like experiment are $\Delta\tau = 0.01$ and over such a range the prior is fairly flat.

not correspond to one additional degree of freedom. For this reason regular likelihood ratio tests, of the kind performed in Holder et al. (2003) and Kaplinghat et al. (2003), will not be valid. Fig. 3 shows the induced prior on τ resulting from linear priors on z_r and d_η (their own priors are not perfectly flat due to the extra imposition of the $\tau < 0.3$ prior). The uncertainty on τ from *Planck* is of order 0.01, and over such widths the prior on τ is roughly uniform; it is even

more so around the fiducial value of τ . The same applies to the case of a log prior on d_η .

Other models of re-ionization have been proposed (Cen 2003; Haiman & Holder 2003), for instance the double re-ionization scenario considered in Lewis et al. (2006), which would in general require a third parameter. From a model selection point of view (i.e. taking into account parameter uncertainties within each model) results in this paper indicate that discriminating such a model from a smooth transition model would be beyond the scope of *Planck*, though perhaps within the scope of a closer to cosmic variance limited experiment. Here our focus is mainly on clarifying what *Planck* can learn about re-ionization.

3 MODEL SELECTION FORECASTING METHODOLOGY

Model selection forecasting assesses a given experiment's ability to distinguish between different cosmological models. This ability necessarily depends on the true model and on its parameter values (and of course on the overriding assumption that one of the models we will be considering is, if not the actual true model, at least representative of its predictions for the experiment under consideration). We call this true model and its parameter values the *fiducial model*. Trotta (2007) introduced an approach to model selection forecasting, Predictive Posterior Odds Distribution (PPOD), which averaged the model selection forecast over the current knowledge of model parameters, so as to give the probability of the future experiment giving different model selection outcomes. Mukherjee et al. (2006b) adopted a different approach where the model selection outcome was forecasted as a function of the fiducial model parameters, so as to assess where in the parameter space the experiment could strongly distinguish the models, and defined some experimental figures of merit based on this notion.

Computational restrictions prevent us from making an extensive investigation of how the model selection outcome will depend on the fiducial model chosen. Instead, we consider a single fiducial z_r - d_η model with $d_\eta = 3$ and $z_r = 8.9$, corresponding to $\tau = 0.1$,

Table 1. Analysing TE and EE spectra of *Planck* specifications (first panel), with a fiducial model of $z_r = 8.9$ and $d_\eta = 3$ (implying $\tau = 0.1$) using three test models. The second panel shows the same for a cosmic variance limited experiment. In Evidences are based on four estimates of the evidence for each model.

	Model, priors	Parameter estimates	ln Evidence	$\Delta \ln E$
<i>Planck</i> satellite	Instantaneous re-ionization z_r : 6–30, $d_\eta = 50$	$z_r = 12.9 \pm 0.5$ $\tau = 0.108 \pm 0.006$	-6.3 ± 0.1	0.0
	Linear d_η model z_r : 6–30, d_η : 0–10	$z_r = 10.1 \pm 1.7$, $d_\eta = 4.4 \pm 1.9$ $\tau = 0.103 \pm 0.006$	-4.4 ± 0.2	1.9
	log d_η model z_r : 6–30, log d_η : -1.2 – 3.4	$z_r = 9.9 \pm 1.9$, $d_\eta = 4.1 \pm 1.9$ $\tau = 0.103 \pm 0.006$	-4.7 ± 0.2	1.6
Cosmic variance limited	Instantaneous re-ionization z_r : 6–30, $d_\eta = 50$	$z_r = 12.7 \pm 0.2$ $\tau = 0.106 \pm 0.003$	-15.8 ± 0.1	0.0
	Linear d_η model z_r : 6–30, d_η : 0–10	$z_r = 9.4 \pm 1.0$, $d_\eta = 3.3 \pm 0.5$ $\tau = 0.102 \pm 0.005$	-6.6 ± 0.1	9.2
	log d_η model z_r : 6–30, log d_η : -1.2 – 3.4	$z_r = 9.2 \pm 1.0$, $d_\eta = 3.2 \pm 0.4$ $\tau = 0.101 \pm 0.005$	-6.8 ± 0.3	9.0

and simulate *Planck* quality data for it. Fig. 2 shows that this model is quite different from an instantaneous re-ionization model with the same τ (though models can be constructed that are even more drastically different, up to the step model seen in Fig. 1). Our goal is to determine whether *Planck* could distinguish the two-parameter model from an instantaneous re-ionization model, for these fiducial parameters with a certain level of strength of evidence. If it can, then the threshold for detection lies between this model and the instantaneous re-ionization model, otherwise it lies further away.

Throughout we use the nested sampling algorithm for computing model evidences, first implemented for cosmological applications in Mukherjee, Parkinson & Liddle (2006a) and subsequently developed in Parkinson, Mukherjee & Liddle (2006). It was used to make forecasts for *Planck*'s ability to determine inflationary parameters in Pahud et al. (2006, 2007). This algorithm, due to Skilling (2006), computes the Bayesian evidence for any given model, as well as providing parameter estimates within that model. The evidence is the probability of the data given the model, hence can be used to determine how likely each model is to have given rise to the data. A difference of 2.5 in log evidence can be taken to be significant, and 5 decisive, evidence in favour of the model with larger evidence (Jeffreys 1961).

4 RESULTS

4.1 The *Planck* satellite

We model *Planck* TE and EE data using just the 143-GHz polarization channel, following for its specifications the current *Planck* documentation.³ The full likelihood is constructed in the manner of Lewis (2005) and Pahud et al. (2006, 2007), without creating noisy data realizations. This ensures that the bias issues we discuss below are not a result of realization noise, but instead the forecast is equivalent to averaging over many data realizations and is thus itself effectively unbiased (see the appendix of Sahlén et al. 2008). We assume a sky coverage of 0.8, and take the likelihood up to a maximum multipole of $\ell = 100$.

The first entry in Table 1 shows that the result of analysing *Planck* data based on this chosen fiducial model with a one-parameter

instantaneous re-ionization model (e.g. with d_η held fixed at 50, varying z_r over the 6–30 prior range). The second and third entries show the data analysed with the (correct) z_r – d_η model. Linear and log priors on d_η are assumed to check for the dependence of results on such assumptions.

Our main result is the relative evidences of these models, where the instantaneous re-ionization model has a ln evidence which is less than 2 smaller than the two-parameter models. Accordingly, *Planck* will not be good enough to exclude the instantaneous re-ionization model, even though the true model appears to have a quite different ionization history. Put another way, *Planck* is not powerful enough to explore two-parameter models of re-ionization (at least unless the true model is even further from instantaneous re-ionization than our fiducial model).

Besides the evidence, one can also compute the Bayesian complexity of *Planck* data for the chosen fiducial model in the manner of Kunz, Trotta & Parkinson (2006). Using such an analysis Kunz et al. found that τ is already a required parameter with *WMAP* 3-yr data (and a similar analysis would likely show that another re-ionization parameter is required when the evidence supports it).

This is, however, not quite the end of the story. The parameter distributions given from the nested sampling algorithm are shown in Fig. 4. It is apparent from this that the estimated τ (and z_r) are biased high in the instantaneous re-ionization model. The bias goes away in the two-parameter model, as it should since that model can describe the true behaviour of the data. Accordingly, to avoid a possible bias in measuring τ one should consider both the one-parameter and two-parameter models, and Bayesian model average as in Liddle et al. (2006b) to obtain constraints on τ (the cost being a slightly increased uncertainty in τ). Similar conclusions have been reported in other papers (e.g. Holder et al. 2003; Kaplinghat et al. 2003).

We have made some assumptions about the true (fiducial) model that we are not certain about. In practice we do not know the fiducial z_{\max} when re-ionization started. This has been assumed to be 30 in the fiducial model. A different z_{\max} corresponds to a different re-ionization history, hence z_{\max} could be treated as an additional re-ionization parameter, but we do not go into a third re-ionization parameter here. Instead we ask what outcome arises if we analyse data so simulated (with a z_{\max} of 30) with a z_r – d_η model with $z_{\max} = 20$. Such an ‘incorrect’ model would not be distinguishable from the true model by *Planck*. Further, if our incorrect model was

³ http://www.rssd.esa.int/index.php?project=PLANCK&page=perf_top

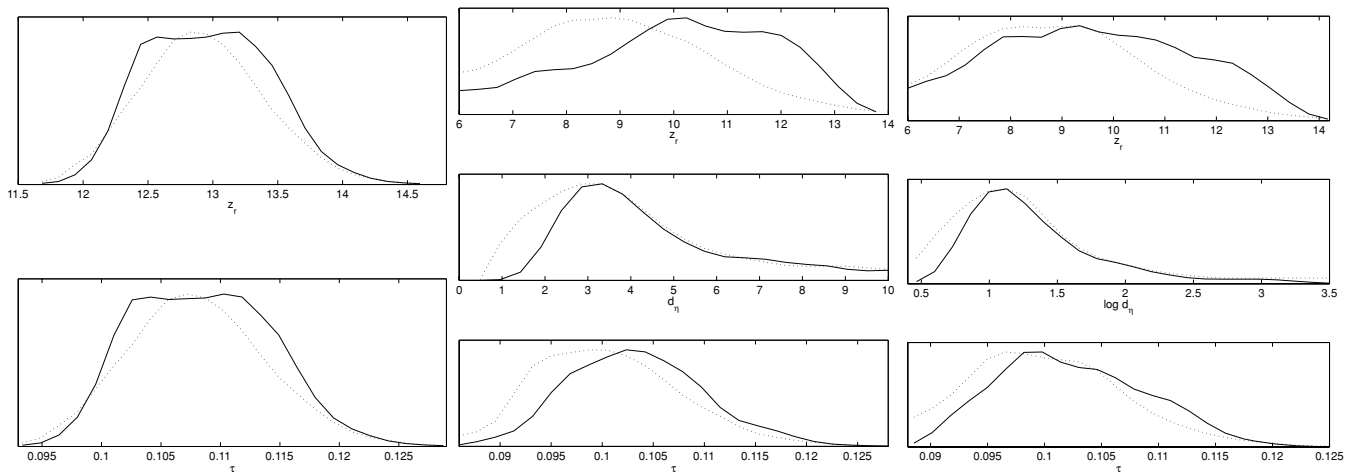


Figure 4. A fiducial model with $z_r = 8.9$ and $d_\eta = 3$ treated with an instantaneous re-ionization model (d_η fixed at 50, z_r allowed to vary between 6 and 30; left-hand panel), a z_r - d_η re-ionization model with z_r between 6 and 30 and d_η between 0 and 10 (centre panel) and a z_r - d_η re-ionization model with log prior in d_η (z_r between 6 and 30, $\log d_\eta$ between -1.2 and 3.4 corresponding to 0.3 and 30; right-hand panel).

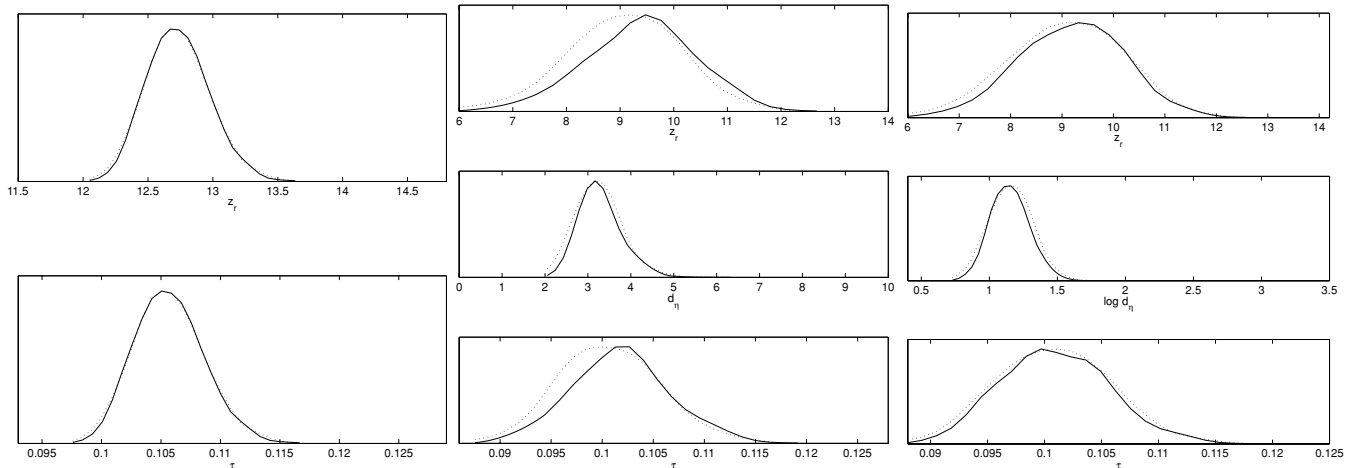


Figure 5. As Fig. 4, for a cosmic variance limited experiment. The axes scales are the same as in Fig. 4.

not a smooth transition model but one involving a step function, again with only two parameters, corresponding to z_{\max} (prior range 7–30), with re-ionization ending at redshift 6, and with a constant re-ionization fraction in between these two redshifts of x_e (prior range 0–1), such a model would again not be distinguishable from the assumed true model by *Planck*. These results are borne out of numbers presented in the next subsection for a cosmic variance limited hypothetical experiment.

4.2 Cosmic variance limited case

For a cosmic variance limited hypothetical experiment, the corresponding results are shown in the lower panel of Table 1 and in Fig. 5. Again only TE and EE spectra are considered out to a maximum multipole of 100. This time the evidence favours the smooth and gradual transition z_r - d_η model decisively over the instantaneous re-ionization model.

These results also show that, as before, a simpler model leads to a biased τ , a bias that disappears upon using a complicated enough

model for re-ionization. The choice of prior on d_η (log or linear) does not make much difference.

Table 2 shows an incorrect assumption regarding z_{\max} of the z_r - d_η model does not significantly affect the evidence, e.g. $z_{\max} = 20$ is indistinguishable from $z_{\max} = 30$. d_η is underestimated to make up for the difference in z_{\max} , and τ is not misestimated. It also shows that the smooth and gradual transition model is favoured strongly, almost decisively, over the incorrect step model based on the difference in log evidence, while τ is biased low under this incorrect model assumption. Both incorrect models are clearly distinguishable from (and favoured over) the instantaneous re-ionization model. However, the fact that different choices of z_{\max} are not distinguishable indicates that even cosmic variance limited experiments cannot probe very fine details of the re-ionization history.

5 CONCLUSIONS

We find that *Planck* is not expected to be able to distinguish significantly between a single-parameter and a two-parameter model of re-ionization in the model comparison sense, though it will mildly

Table 2. As Table 1, but analysing the same fiducial model with an incorrect test model for the cosmic variance limited case. The $\Delta \ln E$ are with respect to the instantaneous model in Table 1.

Model, priors	Parameter estimates	ln Evidence	$\Delta \ln E$
(Incorrect) $z_{\max} = 20$ $z_r: 6-20, d_\eta: 0-10$	$z_r = 8.8 \pm 1.4, d_\eta = 1.7 \pm 0.6$ $\tau = 0.100 \pm 0.004$	-8.0 ± 0.1	7.8
(Incorrect) $z_{\max}-x_e$ model $z_{\max}: 7-30, x_e: 0-1$	$z_{\max} = 18.9 \pm 0.8, x_e = 0.39 \pm 0.05$ $\tau = 0.094 \pm 0.003$	-10.8 ± 0.1	5.0

favour the two-parameter model for our chosen fiducial values. If the parameter values of the two-parameter true model were more extreme, then *Planck* might favour it significantly. However, Bayesian model averaging the parameters over the two models will eliminate any bias in the optical depth to rescattering.

A cosmic variance limited hypothetical experiment will be able to decisively distinguish between the one- and two-parameter models, to distinguish between some two-parameter models, and may be able to go on to a third parameter.

The model comparison approach advocated here should if possible be applied to models parametrized by describing physically relevant quantities for the re-ionization history model; the phenomenological quantities employed here are an intermediate step.

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