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Supersymmetric Higgs sector and $B - \bar{B}$ mixing for large $\tan \beta$

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(Received 10 May 2011; published 18 August 2011)

We match the Higgs sector of the most general flavor-breaking and $CP$-violating minimal supersymmetric standard model (MSSM) onto a generic two-Higgs-doublet model, paying special attention to the definition of $\tan \beta$ in the effective theory. In particular, no $\tan \beta$-enhanced loop corrections appear in the relation to $\tan \beta$ defined in the $\overline{\text{DR}}$ scheme in the MSSM. The corrections to the Higgs-mediated flavor-changing amplitudes, which result from this matching, are especially relevant for the $B_d$ and $B_s$ mass differences $\Delta M_{d,s}$ for minimal flavor violation, where the superficially leading contribution vanishes. We give a symmetry argument to explain this cancellation and perform a systematic study of all Higgs-mediated effects, including Higgs loops. The corrections to $\Delta M_{q_i}$ are at most 7% for $\mu > 0$ and $M_A < 600$ GeV if constraints from other observables are taken into account. For $\mu < 0$ they can be larger, but are always less than about 20%. Contrary to recent claims, we do not find numerically large contributions here, nor do we find any $\tan \beta$-enhanced contributions from loop corrections to the Higgs potential in $B^+ \to \tau^+ \nu$ or $B \to X \gamma$. We further update supersymmetric loop corrections to the Yukawa couplings, where we include all possible $CP$-violating phases and correct errors in the literature. The possible presence of $CP$-violating phases generated by Higgs exchange diagrams is briefly discussed as well. Finally, we provide improved values for the bag factors $P_1^{\text{VLL}}, P_2^{\text{LR}},$ and $P_1^{\text{SLL}}$ at the electroweak scale.

I. INTRODUCTION

Supersymmetry constrains the structure of the Yukawa couplings of the minimal supersymmetric standard-model (MSSM) onto those of a special two-Higgs-doublet model (2HDM). In this 2HDM of type II one Higgs doublet, $H_u$, only couples to up-type fermions, while the other one, $H_d$, only couples to down-type fermions. As a consequence, there are no dangerous tree-level flavor-changing neutral current (FCNC) couplings of the neutral-Higgs bosons. However, the presence of supersymmetry-breaking terms destroys this pattern at the one-loop level, permitting couplings of both Higgs doublets to all fermions. Thus the resulting Higgs sector is that of a general 2HDM, often called 2HDM of type III. As pointed out first by Hall, Rattazzi and Sarid, the loop-induced Yukawa couplings can compete with the tree-level ones in the limit of a large $\tan \beta = v_u/v_d$, which is the ratio of the vacuum expectation values (vevs) of $H_u$ and $H_d$ [1]: in the relationship between $H_{u,d}$-couplings and observed masses of the down-type fermions the loop-suppression factor $\sim 0.02$ is offset by a factor of $\tan \beta$, so that $O(1)$ corrections to the type II 2HDM are possible for $\tan \beta \sim 50$. In such scenarios, also $O(1)$ loop-induced FCNC couplings of neutral-Higgs bosons appear [2], which allow the branching fractions of (yet unobserved) leptonic $B$ decays to exceed their standard model values by more than 2 orders of magnitude [3]. This observation has stimulated a large activity in flavor physics and powerful constraints on the MSSM Higgs sector in scenarios with large $\tan \beta$ have been derived from $B$ factory data [3–6]. These Higgs-induced effects in flavor physics are very transparent in the limit

$$M_{\text{SUSY}} \gg M_A \sim v,$$  \hspace{1cm} (1)

where $M_{\text{SUSY}}$ denotes the generic mass scale of the superpartners and the masses $M_A$, $M_H$, $M_H^*$ of the five physical Higgs bosons are taken to be of the order of the electroweak scale $v = \sqrt{v_u^2 + v_d^2} = 246$ GeV. All low-energy observables can be computed in the type III 2HDM, which emerges as the effective theory in the limit of Eq. (1). The new couplings can be calculated from finite one-loop diagrams with supersymmetric particles and thus become functions of the MSSM parameters, so that the desired constraints on the supersymmetric parameter space can be derived. The effective 2HDM Lagrangian efficiently incorporates all large-$\tan \beta$ effects, equivalent to a perturbative all-order resummation of those radiative corrections which are enhanced by a factor of $\tan \beta$ [7].

$B_q - \bar{B}_q$ mixing (with $q = d$ or $s$) plays a special role among the FCNC transitions of $B$ mesons. Here the leading new effect stems from effective tree-level diagrams with neutral-Higgs bosons (see Fig. 1). \textit{A priori} the dominant contribution is expected from Yukawa couplings to right-handed $b$ quarks, generating the effective $\Delta B = 2$ operator

$$Q_1^{\text{SLL}} = (\tilde{b}_R q_L) (\tilde{b}_R q_L).$$  \hspace{1cm} (2)
FIG. 1. Leading contributions to $B_q - \bar{B}_q$ mixing from supersymmetric Higgs bosons. The FCNC couplings are induced by supersymmetric loops. The coefficient of $Q_1^{\text{SLL}} = (\bar{b}_R q_L)(\bar{b}_L q_R)$ vanishes, if the tree-level relations between Higgs masses and mixing angles are used.

However, the corresponding coefficient $C_1^{\text{SLL}}$ vanishes exactly if one employs the tree-level relations between the Higgs masses and mixing angles [2]. Nevertheless, sizeable effects in $B_q - \bar{B}_q$ mixing are possible even in scenarios with minimal flavor-violation (MFV) [8–16], in which the Cabibbo-Kobayashi-Maskawa (CKM) matrix [17] is the only source of flavor-violation: keeping the strange Yukawa coupling nonzero one finds a nonvanishing contribution to the coefficient of

$$Q_2^{\text{LR}} = (\bar{b}_R q_L)(\bar{b}_L q_R),$$

which depletes the $B_q - \bar{B}_q$ mass difference $\Delta M_q$ [5]. The tree-level vanishing of $C_1^{\text{SLL}}$ calls for a systematic analysis of all subleading effects. In particular, the contribution that stems from $Q_1^{\text{SLL}}$ can a priori compete with the contribution of the operator $Q_2^{\text{LR}}$ above if the one-loop corrections to the MSSM Higgs potential [18–24] are taken into account. While a lot of work has been devoted to the analysis of the Yukawa sector [2,3,5–7,25], little attention has been given to effects from the Higgs potential. An exception is Ref. [26], which finds large contributions. We revisit these effects in the present paper and perform a systematic matching of the MSSM Higgs sector onto the type III 2HDM. The result is not only relevant for the calculation of $C_1^{\text{SLL}}$, but also clarifies the relationship between the definitions of $\tan \beta$ in the MSSM and the effective 2HDM. This is important to link the constraints from flavor physics to other fields of MSSM phenomenology, in particular, Higgs physics. Our paper is organized as follows. We derive the corrected $B - \bar{B}$ mixing amplitude in Sec. II, including all relevant subleading contributions. The renormalization of $\tan \beta$ and some further technical issues are the subject of Sec. III. In Sec. IV, we apply our new formulas to the phenomenology of $B - \bar{B}$ mixing, analyzing the mass differences $\Delta M_d$ and $\Delta M_s$ as well as CP violation. Our results are summarized in Sec. V. We list our notation and our technical results in four appendices. Parts of our results were previously presented by one of us at a conference [27].

II. HIGGS-MEDIATED EFFECTS IN $B - \bar{B}$ MIXING

The quantity governing the $B_q - \bar{B}_q$ mass difference is the off-diagonal element of the $B_q - \bar{B}_q$ meson mass matrix: $\Delta M_q = 2 |M_{21}^q|$, with

$$M_{21}^q = \frac{\langle \bar{B}_q | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q \rangle}{2M_{B_q}}.$$  (4)

The $\Delta B = 2$ effective weak Hamiltonian $\mathcal{H}_{\text{eff}}^{\Delta B=2}$ consists in general of eight dimension-six operators:

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \frac{G_F^2 m_b^2}{16 \pi^2} \sum_{i=1}^{8} C_i(\mu_b) Q_i(\mu_b),$$  (5)

with $\lambda_{qib} = V_{iq} V^*_ib$. The set of operators in Eq. (5) comprises the standard-model operator,

$$Q_1^{\text{VLL}} = (\bar{b}_L \gamma \mu q_L)(\bar{b}_L \gamma \mu q_L),$$  (6)

the two scalar operators defined in Eqs. (2) and (3), the operator

$$Q_1^{\text{SRR}} = (\bar{b}_L q)(\bar{b}_L q_R),$$  (7)

and four other operators. The complete list of operators plus the relevant evanescent operators is given in Eq. (C4) and (C5) of Appendix C. We express our results in terms of matrix elements at the high-scale $\mu_h$, which we choose equal to the top mass $m_t(m_t) = 164$ GeV. In this way, the other four operators do not appear in our formulas. However, some of them are needed to connect $Q_i(\mu_h)$ with $Q_i(\mu_b)$ at the low scale $\mu_b \sim m_b$, at which their matrix elements are computed, because they mix with $C_1^{\text{SLL}}, Q_1^{\text{SRR}},$ or $Q_2^{\text{LR}}$ under renormalization. We follow the conventions of Refs. [28,29] for operators and matrix elements. In particular, we parametrize the hadronic matrix elements as

$$\langle \bar{B}_q | Q_i(\mu_b) | B_q \rangle = \frac{2}{3} M_{B_q}^2 f_{B_q} P_i.$$  (8)

The $P_i$’s are obtained [29] by renormalization-group evolution from the conventional bag factors $f_i$ computed at the low scale $\mu_b$. We calculate the $P_i$’s from up-to-date lattice QCD results in Appendix C, where we fully exploit constraints from heavy-quark relations. This is a new feature of our analysis compared to previous studies of new-physics effects in $B - \bar{B}$ mixing.

A. Effective tree-level Higgs exchange

The Higgs sector of the MSSM contains two $SU(2)$ doublets $H_u$ and $H_d$.

$$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} h_d^+ \\ h_d^0 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$  (9)

of hypercharge $+1/2$ and $-1/2$, respectively, with vacuum expectation values (vevs) $\langle h_{u,d}^0 \rangle = v_{u,d} / \sqrt{2}$ of relative size $\tan \beta = v_u / v_d$. Integrating out supersymmetric particles, the Lagrangian of the resulting effective 2HDM is no longer restricted to be of type II, and is constrained only by the electroweak symmetry. Neither will it be renormalizable, with operators of dimension greater than four.
encoding effects that decouple at least as $v/M_{\text{SUSY}}$ for heavy superpartners. We begin with a short review of some pertinent aspects of the general 2HDM.

Defining

$$\begin{pmatrix} \Phi \\ \Phi' \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} H_d^* \\ H_u \end{pmatrix},$$

(10)

the most general fermion-Higgs interactions up to dimension-four read

$$L_y = -\sqrt{2} v \bar{d}_{R_i} M_{d_{ij}} \Phi^\dagger Q_{L_j} - \bar{\Phi} K_{ij} \Phi^\dagger Q_{L_j}$$

$$-\sqrt{2} v \bar{u}_{R_i} M_{u_{ij}} Q_{L_j} + \bar{\Phi} K_{ij} \Phi^\dagger Q_{L_j} + \text{h.c.},$$

(11)

where we have employed the notation $a \cdot b \equiv a^T e b$. By construction, the vev of $\Phi'$ vanishes, whereas $\Phi$ has $\langle \Phi \rangle = (0, v/\sqrt{3} e)^T$ and contains all three Goldstone bosons. Hence, only $\Phi$ can contribute to the fermion masses and only $\Phi'$ can have flavor-violating neutral couplings. The flavor basis is defined such that the down-quark mass matrix $M_d$ is diagonal. In this basis, the FCNC Higgs couplings to $b$-quarks are governed by $\kappa_{bq}$ or $\kappa_{qb}$ ($q = d$ or $s$).

The renormalizable Higgs self-interactions are comprised in the most general gauge-invariant dimension-four two-Higgs-doublet potential [30],

$$V = m_{11}^2 H_1^2 H_d^2 + m_{22}^2 H_2^2 H_u^2 + \{ m_{12}^2 H_2 H_u + H_d + \text{H.c.} \}$$

$$+ \frac{\lambda_1}{2} (H_1^2 H_d)^2 + \frac{\lambda_2}{2} (H_2^2 H_u)^2 + \lambda_3 (H_1^2 H_u^2) (H_2^2 H_d)$$

$$+ \lambda_4 (H_1^2 H_d^2) (H_2^2 H_u) + \left\{ \frac{\lambda_5}{2} (H_d^2 H_u) \right\}^2$$

$$- \lambda_6 (H_2^2 H_d) (H_u^2 H_d) - \lambda_7 (H_1^2 H_u) (H_u^2 H_d) + \text{H.c.},$$

(12)

The couplings $m_{12}^2$, $\lambda_5$, $\lambda_6$, and $\lambda_7$ are in general complex, yet the vevs $v_{u,d}$ can be made real by a $U(1)$ transformation on the Higgs fields. The definitions of $m_{ij}^2$ and $\lambda_i$ in Eq. (12) coincide with Ref. [30], except for $\lambda_3$ and $\lambda_4$: we associate a different operator with $\lambda_4$ to eliminate it from tree-level neutral-Higgs phenomenology and have instead $\lambda_3 = \lambda_3^{[30]} + \lambda_3^{[30]}$ and $\lambda_4 = -\lambda_4^{[30]}$.

Shifting the fields in Eq. (12) by their vevs, which minimize $V$ at tree-level,\footnote{“Tree-level” here refers to the 2HDM. We defer a discussion of quantum corrections to $v_u$ and $v_d$ to Sec. III.}

$$h_{u,d} = \frac{1}{\sqrt{2}} (v_{u,d} + \phi_{u,d} + i \chi_{u,d}),$$

(13)

determines the physical Higgs-boson mass matrices and interactions. We write the neutral-Higgs mass matrix in the basis $(\phi_d, \phi_u, \chi_d, \chi_u)$ in terms of 2 $\times$ 2 blocks,

$$M_0^2 = \begin{pmatrix} M_R^2 & M_{RI}^2 \\ M_{RI}^* & M_I^2 \end{pmatrix},$$

(14)

with $M_R^2$, $M_{RI}^2$, and $M_I^2$ given in Eqs. (23)–(26) below. In the $CP$-conserving case, $M_{RI}^2 = 0$, and $M_R^2$ and $M_I^2$ are diagonalized by rotating the $CP$-even and $CP$-odd Higgs fields through angles $\alpha$ and $\beta$, respectively:

$$\begin{pmatrix} \phi_d \\ \phi_u \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix},$$

$$\begin{pmatrix} \chi_d \\ \chi_u \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}.$$

(15)

The same angle $\beta = \arctan v_u/v_d$ as defined above appears because (and only when) $v_u, v_d$ minimize $V$. If $CP$ violation is present, four physical mixing angles $\alpha_{1,2,3}$ and $\beta$ are required to diagonalize $M_0^2$. The charged-Higgs mass matrix $M_{\pm}^2$ is always diagonalized by $\beta$,

$$\begin{pmatrix} h_d^+ \\ h_u^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}.$$

(16)

The nonstandard $\Delta B = 2$ effective operators $Q_{1,2,3}^{1,2,3}$, and $Q_4$ are generated at tree-level via the exchange of neutral-Higgs bosons (see Fig. 1) with the Wilson coefficients

$$C_{2,3,4}^{1,2,3} = -\frac{8 \pi^2}{G_F M_W^2} \kappa_{hq}^2 (\kappa_{bq}^2 - \kappa_{gb}^2) F^+,$$

$$C_{1,2,3}^{1,2,3} = -\frac{8 \pi^2}{G_F M_W^2} \kappa_{hq}^2 (\kappa_{bq}^2 - \kappa_{gb}^2) F^-,$$

(17)

and $C_4^{1,2,3}$ obtained from $C_1^{1,2,3}$ through the replacement of $\kappa_{hq}^2$ by $\kappa_{gb}^2$ and $\kappa_{hq}^2$. We find that, in the general case, the Higgs propagation factors can be expressed as follows:

$$\mathcal{F}^+ = \frac{\text{det}(M_R^2 + M_I^2 + i M_{RI}^2 - i M_{RI}^2)}{m_1^2 m_2^2 m_3^2},$$

$$\mathcal{F}^- = \frac{\text{det}(M_R^2 - M_I^2 + i M_{RI}^2 - i M_{RI}^2)}{m_1^2 m_2^2 m_3^2},$$

(18)

where the denominators contain the product of the three nonzero eigenvalues of $M_0^2$. In the $CP$-conserving case, Eqs. (18) and (19) reduce to the well-known expressions

$$\mathcal{F}^\pm = \frac{\cos^2(\alpha - \beta)}{M_H^2} \pm \frac{\sin^2(\alpha - \beta)}{M_A^2} \pm \frac{1}{M_A^2},$$

(20)

where $M_{H,h}$ and $M_A$ denote the $CP$-even and $CP$-odd Higgs-boson masses, respectively.

The discussion so far has been completely general. Particularizing to the MSSM, a perturbative matching calculation relates the two theories. At tree-level, this trivially results in
At this order, $\kappa^{(0)}$ and $\tilde{\kappa}^{(0)}$ are aligned with $M_d^{(0)}$ and $M_u^{(0)}$, respectively, so that no FCNC are induced, as it must be in a model II. At one-loop, all couplings in Eq. (12) are determined by dimensionless couplings. Cf. Sec. III for a discussion of field renormalization. Our $\Delta K$ and $\Delta Y_d$ correspond to $\Delta_{\alpha} Y_d$ and $-\Delta_{\beta} Y_d$, respectively, in the first paper of Ref. [5].
angle $\alpha'$, determined by the relative phase of $\lambda_5$ and $\lambda_7$. Finally, the charged-Higgs mass matrix is given by

$$M^2_+ = \left(1 + \frac{v^2 (\lambda_5 + \lambda_7^2)}{2M_h^2}\right) M^2_f,$$

(27)

Here, no $\tan\beta$ enhancement due to loop-induced couplings occurs.

Unlike the case of the fermion mass matrix, the typical momentum flowing through the effective Lagrangian Eq. (12) for an on-shell Higgs is itself of $O(v)$ or $O(M_h)$. Hence Higgs-loop contributions to the Higgs masses cannot be included in Eq. (12), but rather the full effective action would be needed. Higgs-loop effects in $\kappa_{\nu}$ and $\kappa_{\nu b}$ multiplying $F^\pm$ could, however, be included via Eq. (11), since again the momenta flowing through the vertices are much smaller than $v, M_h$. This is not possible in Higgs boxes, where large momenta flow through the FCNC vertices. We will present a systematic method to include all Higgs-loop contributions in Sec. IIC.

It is instructive to consider the explicit form of the numerator in Eq. (19), which is

$$\det A = v^4 (\lambda_2 \lambda_5 - \lambda_7^2) c_\beta^2 + 2 (\lambda_2 \lambda_6^* - \lambda_1 \lambda_5^* + \lambda_4 \lambda_7^*) s_\beta c_\beta + (\lambda_1 \lambda_2 - \lambda_4^2 + 2 \lambda_6 \lambda_7^* + 4 \lambda_5 \lambda_7^*) s_\beta^2 c_\beta + 2 (\lambda_3 \lambda_6^* - \lambda_5 \lambda_1 + \lambda_4 \lambda_7^*) s_\beta c_\beta + (\lambda_1 \lambda_5 - \lambda_5^2) e_\beta^2 |c_\beta|.$$  

(28)

With Eq. (21), $\det A = v^4 (\lambda_2 - \lambda_5^2) c_\beta^2 = 0$, reproducing the known vanishing of $F^-$ employing the tree-level MSSM Higgs sector. The cancellation is removed already at the leading-logarithmic level. For instance, $\lambda_2$ alone receives a large additive correction $\propto y_t^4$ due to top-quark loops, which is also responsible for the most important correction to the tree-level mass of $h$. The corresponding corrections could be computed by RG-evolving the tree-level couplings in the effective 2HDM. However, as we are considering large $\tan\beta$, we expect (and find below) the most important effect to be due to $\lambda_5$ and $\lambda_7$, which remove the $O(c_\beta^3)$ suppression of the leading-log result, as anticipated above.

### B. The case of minimal flavor-violation

From the discussion so far, it follows that $|F^+| = O(1/M_h^2) \gg |F^-| = O(1/(16\pi^2 M_h^4))$, implying $|C_{SLL}^G| \gg |C_{SLL}^{gl}|$ for generic $\kappa_{\nu b}$, such that the motivation to consider $F^-$ at all is not very strong. The situation is fundamentally different for MFV because then the contribution proportion to $F^+$ turns out to be suppressed by a light quark mass, introducing a further small parameter $m_q/m_b$ comparable to $1/(16\pi^2)$ or $1/\tan\beta$ for $q = s$ (and negligible for $q = d$). For simplicity, in this paper we consider the simplest version of MFV, assuming flavor-universal soft-breaking terms $m^2_0$, $m^2_\mu$, and $m^2_d$ and trilinear SUSY-breaking terms $T_{u_{ij}}$, $T_{d_{ij}}$ which are proportional to the Yukawa matrices and therefore diagonal in the superCKM basis (denoted with a hat): $\hat{T}_{u_{ij}} = a_i y_u \delta_{ij}$ and $\hat{T}_{d_{ij}} = a_i y_d \delta_{ij}$, see Appendix A for details of our notation.

The structure of our results, however, does not depend on these additional assumptions. The $\tan\beta$-enhanced loop-induced FCNC couplings of the neutral-Higgs bosons in Eq. (11) can be expressed as:

$$\kappa_{\nu b} = e_1 y_t^2 \lambda_{\nu b} \sqrt{2 m_b\over v \cos \beta} \left(1 + \tilde{e}_1 \tan \beta \right) \left(1 + \epsilon_1 \tan \beta \right),$$  

(29)

$$\kappa_{\nu b} = e_1 y_t^2 \lambda_{\nu b} \sqrt{2 m_d\over v \cos \beta} \left(1 + \tilde{e}_1 \tan \beta \right) \left(1 + \epsilon_1 \tan \beta \right),$$  

(30)

with $y_t = \sqrt{2 m_t/(v \sin \beta)}$ and $\lambda_{\nu b} = V_{tb} V_{ub}^*$. The effective couplings $e_1$, $\epsilon_1$, and $\tilde{e}_1$, which depend on the MSSM parameters, have been analyzed in the decoupling limit $M_{SUSY} \gg v$ in the limit $g = g' = 0$ in Refs. [2–4] for the case that $e_1$, $\epsilon_1$, and $\tilde{e}_1$ are real and in Ref. [25] for the maximally CP-violating MFV scenario. We consider the CP-violating case allowing $\mu$, the universal trilinear term $a_t$, and the gaugino mass parameters to be complex. Effects from nonzero $g$, $g'$ have been taken into account in Ref. [5], where also effects beyond the decoupling limit were considered. The corresponding expressions for $M_{SUSY} \gg v$, suited for our analysis, were derived in Ref. [26]. We have recalculated the FCNC couplings of neutral-Higgs bosons for $g$, $g' \neq 0$ including all CP-violating phases and found agreement with the results for the FCNC self-energies given in Ref. [5], but encountered a significant discrepancy with Ref. [26]. In our results, the phase conventions of $\mu$, $a_t$, and $M_2$ can be inferred from Eqs. (A1) and (A3) of Appendix A. The phase convention for $M_{13}$ complies with that of $M_2$ and the gluino mass equals $M_{\tilde{g}} = |M_1|$. Of course, one can choose one of these parameters (e.g. $M_3$) real. Now the effective couplings of Eqs. (29) and (30) read:

$$e_0 = -\frac{2 a_t}{3 \pi M_3} \mu^* H_z \left(\frac{M^2_{bl}}{|M_3|^2} \frac{M^2_{hr}}{|M_1|^2} + \frac{g'^2}{96 \pi^2 M_1} \mu^* \right),$$

$$\times \left[ H_2 \left(\frac{M^2_{bl}}{|M_3|^2} |\mu|^2 \right) + 2 H_2 \left(\frac{M^2_{hr}}{|M_1|^2} |\mu|^2 \right) \right]$$

$$+ \frac{g'^2}{144 \pi^2 M_1} H_2 \left(\frac{M^2_{bl}}{|M_3|^2} |\mu|^2 \right)$$

$$+ \frac{3 g'^2}{32 \pi^2 M_2} H_2 \left(\frac{M^2_{bl}}{|M_2|^2} |\mu|^2 \right)$$

(31)
\[ \epsilon_Y = -\frac{1}{16\pi^2} \frac{a_i^*}{\mu} H_2 \left( \frac{M_{\hat{g}}^2}{|\mu|^2}, \frac{M_{\tilde{q}}^2}{|\mu|^2} \right) + \epsilon_{Y,v/M^2}. \]  

(32)

\[ \tilde{\epsilon}_3 = \epsilon_0 + y_\gamma^2 \epsilon_Y. \]  

(33)

Here

\[ H_2(x,y) = \frac{x \log x}{(1-x)(x-y)} + \frac{y \log y}{(1-y)(y-x)}. \]  

(34)

Numerically, the electroweak corrections in \( \epsilon_0 \) can be of \( O(10\%) \). They improve the comparison with the results computed with full chargino and squark mass matrices (see Eq. (5.1) in the second paper in Ref. [5]).

Ref. [5] also discusses threshold corrections to the fermion kinetic operators (wave-function renormalizations). While these terms are not \( \tan \beta \)-enhanced, the flavor-diagonal quark wave-function-renormalization constants receive sizable contributions from squark-gluino loops. One can parametrize these loops in terms of a new quantity \( \epsilon_{0,\text{kin}} \) which will add to \( \epsilon_0 \) in the relation between the MSSM Yukawa coupling \( y_{d,i} \) and the physical quark mass \( m_{d,i} \) (see Eq. (A6) for the case of the bottom Yukawa coupling). \( \epsilon_{0,\text{kin}} \) will likewise appear in the relation between \( \kappa_{ij} \) and \( y_{d,i} \), but it drops out once \( \kappa_{ij} \) is expressed in terms of \( m_{d,i} \) so that it does not appear in Eqs. (29) and (30). This cancellation of the flavor-diagonal quark wave-function-renormalization can be verified by inserting Eq. (2.29) into Eq. (2.26) of the second paper in Ref. [5]. This feature can be traced back to the fact that the wave-function-renormalization affects both the tree-level and the loop-induced Yukawa couplings with the same multiplicative factor.

Comparing our result with Ref. [26], we find different results for \( \epsilon_0 \) and \( \epsilon_Y \). In Ref. [26], the chargino-stop contribution proportional to \( g^2 \) is erroneously assigned to \( \epsilon_Y \) rather than to \( \epsilon_0 \). Since this piece does not contain any up-type Yukawa couplings, all three generations contribute in the same way and the resulting overall CKM structure takes the form

\[ \sum_{ij} C_{ij}^{\text{ll}} = \frac{1}{\sqrt{2}} \left( \frac{m_{q_i} m_{q_j}}{m_{\tilde{u}} m_{\tilde{d}}} \right) C_{ij}^{\text{RR}}, \]  

(35)

(Note that \( \sum_{ij} C_{ij}^{\text{ll}} \approx y_b \)) and be aware of the different sign conventions for \( y_b \) in Eq. (A6) and Ref. [5]. We stress that Eq. (35) must be evaluated for \( i \neq 3 \), so that the GIM cancellation of the above-mentioned wino-stop loop takes place. Numerically, one finds a marginal impact of \( \epsilon_{Y,v/M^2} \). Setting all supersymmetric massive parameters equal to a common value \( M_{\text{SUSY}} \), one finds that \( \epsilon_{Y,v/M^2} \) amounts to a mere 1.4% correction to \( \epsilon_Y \) for \( M_{\text{SUSY}} = 400 \) GeV. Even for \( M_{\text{SUSY}} = 150 \) GeV, for which the expansion in \( \nu/M_{\text{SUSY}} \) formally breaks down, \( \epsilon_{Y,v/M^2} \) depletes \( \epsilon_Y \) by at least 8%. \( \epsilon_{Y,v/M^2} \) also enters \( \tilde{\epsilon}_3 \) through Eq. (33). It can be inferred from Ref. [7] that this procedure indeed leads to the correct all-order resummation of the \( \tan \beta \)-enhanced corrections involving \( y_i \). Corrections to \( \tilde{\epsilon}_3 \) beyond the \( M_{\text{SUSY}} \gg \nu \) limit from \( \alpha_x, g, g', \) and \( y_b \) are considered in Refs. [5,7]. We remark that no terms proportional to \( y_b^2 \) occur in Eqs. (31)–(33), because the corresponding loops violate hypercharge and involve a suppression factor of \( \nu^2/M_{\text{SUSY}}^2 \).

We verify from Eqs. (29) and (30) that \( \kappa_{\text{eff}} \) multiplying \( \mathcal{F}^+ \) in Eq. (17) is suppressed by a factor \( m_{\tilde{q}}/m_b \) relative to \( \kappa_{\text{eff}}^2 \), which multiplies \( \mathcal{F}^- \). Hence, \( C_{ij}^{\text{LR}} \) is naively leading (over \( C_{ij}^{\text{RR}} \)) from the point of view of MFV alone, and a meaningful analysis of \( B_q - \bar{B}_q \) mixing requires a systematic investigation of all leading corrections to its vanishing “tree” value. (The coefficient \( C_{ij}^{\text{RR}} \) both undergoes a strong \( m_{\tilde{q}}^2/m_b^2 \) suppression and involves \( \mathcal{F}^{+-} \), and can thus be disregarded.) It is then useful to think of the \( \Delta B = 2 \) amplitude as being a function of the four small parameters identified so far:

\[ l = \frac{1}{(4\pi)^2}, \quad \omega = \frac{m_q}{m_b}, \quad \tan \beta, \quad \nu = \frac{\nu}{M_{\text{SUSY}}}. \]  

(36)

The vanishing 2HDM tree diagram for \( \mathcal{F}^- \) is (superficially) \( O((\cot \beta)^{-2} \mathcal{F}^0 \nu^2 \tan^2 \beta) \), i.e. \( O(1) \) when treating all expansion parameters on the same footing. Conversely, \( \mathcal{F}^+ \) is nonzero at the tree-level but is suppressed by one power of \( \omega \), which is non-negligible only for \( q = s \).

We have already seen that \( \mathcal{F}^{--} \) vanishes exactly for tree-level matching (or up to \( O(1/\tan^2 \beta) \) when including leading logs), so there are no \( O(1/\tan \beta) \) corrections at first subleading order. This leaves loop corrections (via sparticle correction to the \( \lambda_i \) as well as loops in the effective 2HDM) and possible corrections due to higher-dimensional operators, not written in Eqs. (11) and (12). We now discuss these contributions in turn.
SUPERSYMMETRIC HIGGS SECTOR AND $B - B \ldots$

Physical Review D 84, 034030 (2011)

Sparticle loops One-loop contributions from higgsinos, gauginos, and sfermions correct the values of $\lambda_{1,2,3,4}$ in Eq. (12) and induce nonzero couplings $\lambda_{5,6,7}$. As a technical result of our paper, we have computed the $\lambda_i$ for general sparticle masses and flavor structure. These results are reported in Appendix B. At tree-level in the effective theory and in the leading-order of $1/\tan \beta$ the quantity $\mathcal{F}^-$ receives only contributions from $\lambda_5$, $\lambda_6$, and $\lambda_7$, cf. Equation (28). The general results of Eqs. (71), (B1), (B5), (B6), (B8), (B10), (B11), and (B13), for the MFV case read

$$\lambda_7 = \tilde{\lambda}_7$$

$$= \frac{1}{16\pi^2} \left\{ \frac{1}{4} \mu \alpha_\ell |y_\ell|^2 \left( (3g^2 + \delta^2) C_0(\bar{m}_d, \bar{m}_Q, \bar{m}_ \bar{Q}) + 2g^2 C_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_ \bar{Q}) \right) + \mu_\alpha \frac{|y_\ell|^4(3|a_\ell|^2 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u)) + \mu \alpha \frac{|y_\ell|^4(3|a_\ell|^2 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u))}{\mu} \right) + \frac{3|C_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u) + C_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u)| + 3\mu \alpha \frac{|\mu|^2 |y_\ell|^4 D_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q)}{\mu} \right) + \frac{1}{4} \mu \alpha \frac{|y_\ell|^4(2g^2 C_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u))}{\mu} + (g^2 - \delta^2) C_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u)) + \mu \alpha \frac{|y_\ell|^4(3|a_\ell|^2 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u))}{\mu} \right) + \frac{3\mu_\alpha y_\ell^2 B'_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q))}{\mu} \right) + \frac{1}{4} \mu_\alpha \frac{(3|a_\ell|^2 y_\ell^2 B'_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q))}{\mu} \right) + \frac{3\mu_\alpha y_\ell^2 B'_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q))}{\mu} \right) + \frac{1}{4} \mu_\alpha \frac{(3|a_\ell|^2 y_\ell^2 B'_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q))}{\mu} \right) \right).$$

$$\lambda_5 = \tilde{\lambda}_5$$

$$= \frac{-1}{16\pi^2} \mu \cdot \left\{ \frac{3\alpha_\ell}{4} y_\ell^4 D_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q) + 3a_\ell y_\ell^4 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u) + \alpha_\ell y_\ell^4 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u) \right) + 3g^4 M_2^2 D_0(|\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2) - \frac{3}{4} \mu M_2 B'_0(|\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2)) \right).$$

$$\lambda_2 = \tilde{\lambda}_2$$

$$= \frac{-1}{16\pi^2} \left\{ \frac{3\alpha_\ell}{4} y_\ell^4 D_0(\bar{m}_d, \bar{m}_d, \bar{m}_Q, \bar{m}_Q) + 3a_\ell y_\ell^4 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u) + \alpha_\ell y_\ell^4 D_0(\bar{m}_Q, \bar{m}_Q, \bar{m}_u, \bar{m}_u) \right) + 3g^4 M_2^2 D_0(|\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2) - \frac{3}{4} \mu M_2 B'_0(|\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2, |\mathcal{M}|_2)) \right).$$
where the loop functions $B_0, C_0, D_0, B'_0, D_2, 	ilde{D}_4$, and $W$ are defined in Appendix Bd, and the notation $\tilde{\lambda}_j$ refers to the matching scheme as explained in Sec. III. Inspecting Eq. (28), $\lambda_7$ enters quadratically, which formally is of higher loop order. Nevertheless, it can be seen that $\tilde{\lambda}_3 \propto y_8^2$, as opposed to $\lambda_3 \propto \tilde{g}^2 y_4^2$, which can partly offset the additional loop-suppression. Indeed we find that, numerically, neglecting $\lambda_7$ is not always a good approximation (Sec. IV).

The form of the matching result depends on the renormalization schemes of both the full theory, i.e., the MSSM, and the effective theory, i.e., the 2HDM. The latter cancels in physical quantities, while explicit MSSM scheme dependence cancels against the one implicit in the MSSM parameters, to ensure that the couplings in the effective theory are independent of the renormalization of the MSSM at any given order of perturbation theory. The residual scheme dependence in both cases may, however, be important as we are considering a leading effect. We will discuss scheme issues in Sec. IIIa, paying special attention to the definitions of $\tan\beta$.

**Higgs loops** There is a considerable number of one-loop diagrams in the effective 2HDM that can contribute to $B - \bar{B}$ mixing amplitudes (Fig. 2, upper row). These give the following contributions to the Wilson coefficients multiplying $Q^{\text{ULL}}_1$ and $Q^{\text{VRR}}_1$:

$$C^{\text{ULL}}_{1\text{Higgsloops}} = -\frac{1}{4} \frac{m_b^2}{v^2 \cos^2 \beta (1+\tilde{\epsilon}_3 \tan\beta)^2} \times \frac{\kappa^{2}_{qb}}{G_{F}^2 M_{W}^2 \Lambda_{qb}^2} C_0(M_{A_1}^2, M_{A_2}^2, 0). \quad (40)$$

In these expressions, we have neglected the small Yukawa coupling $y_q$ and employed tree-level MSSM mass relations, in agreement with our approximation of working to leading-order in small parameters (in the present case, the loop factor $1/(16\pi^2)$). $C^{\text{VRR}}_1$ is suppressed by two powers of $m_q/m_b$ inside $\kappa^{2}_{qb}$ in the MFV case, hence beyond our accuracy. The results Eqs. (40) and (41) involve a great deal of cancellations, which can be understood in terms of symmetry arguments, as explained in Sec. IIc below. We note the absence of charged-Higgs contributions in the approximation considered here.

**$v/M$-suppressed effects** All of the couplings given in Eq. (11) correspond to the zeroth order in the $v/M_{\text{SUSY}}$ expansion, or equivalently to the level of dimension-four operators. Gauge invariance forbids dimension-five operators built from quark and Higgs fields, so the leading higher-dimensional operators have dimension six. This can lead to more general Higgs-fermion couplings than those deriving from Eq. (11) and, in consequence, the cancellation leading to $C^{\text{ULL}}_{1}=0$ might be broken. To see that this is indeed the case, consider the operator

$$Q^{(6)} = \frac{1}{M_{\text{SUSY}}^2} (H_d^\dagger H_u)(\tilde{b}_R H_d^\dagger Q_L), \quad (42)$$

which gives rise, *inter alia*, to effective dimension-three and -four couplings.

![Diagram](image-url)

FIG. 2. Upper row: A subset of one-loop diagrams for $B_q - \bar{B}_q$ mixing in the effective two-Higgs-doublet model. Lower row: Tree and one-loop diagrams contributing at large $\tan\beta$ when employing the Lagrangian $L_{\text{un}}$ and tree-level couplings. The crosses denote the flavor-changing neutral-Higgs couplings and [in diagrams (f) and (g)] loop-suppressed Higgs mass terms. On the lower row, arrows designate the flow of the conserved $U(1)$ charge discussed in Sec. IIc.

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GORBAHN et al. PHYSICAL REVIEW D 84, 034030 (2011)
\[
\frac{2\sqrt{2}v^2}{M_{\text{SUSY}}^2} \bar{b}_R s_L + \frac{2u^2}{M_{\text{SUSY}}^2} (\bar{b}_R s_L h^0_u + 2 \bar{b}_R s_L h^0_u^*) .
\] (43)

The first term is removed by a rediagonalization of the quark mass matrices, but the two remaining terms, in general, are not. The appearance of \( h^0_u \) in addition to \( h^0_d \) leads to a contribution to \( C^{\text{LL}}_{\text{SUSY}} \) proportional to \( \kappa_{h_d} C^{(6)} \). However, because of \( R \)-parity, SUSY particles do not contribute to tree graphs with external standard particles only, such that \( Q^{(6)} \) (or any other higher-dimension operator) is only induced at the loop level, and this loop-suppression factor is not compensated by factors of \( \tan \beta \). (Recall that the \( \mathcal{O}(1) \) FCNC couplings at dimension-four are nothing but rotated tree-level Yukawa couplings.) Hence any \( v/M_{\text{SUSY}} \) corrections that break the cancellation in \( \mathcal{F}^- \) involve an additional loop-suppression, and can be neglected for the present analysis. On the other hand, as Eq. (43) shows, the higher-dimensional operators do have an impact on the rediagonalization of the quark mass matrices and, consequently, on the size of the FCNC couplings \( \kappa_{h_d} \). These effects preserve the cancellations in \( \mathcal{F}^- \) discussed above but have a mild impact on the FCNC couplings multiplying \( \mathcal{F}^- \) in \( C^{\text{LR}} \) (cf. Eq. (35) and the discussion around it).

### C. \( U(1)_{\text{PQ}} \) and effective Lagrangian for large \( \tan \beta \)

To better understand the various types of cancellations in \( \mathcal{F}^- \) and in the Higgs-loop contributions to \( C^{\text{LL}}_{\text{SUSY}} \), as well as the suppression of the \( \mathcal{F}^+ \) contribution, we now introduce an effective 2HDM Lagrangian at large \( \tan \beta \). This will allow us, on the basis of simple symmetry arguments, to clarify the role of the parameters \( \lambda_\mu \) and \( \lambda_7 \), the structure of Eqs. (18), (19), and (28), as well as the vanishing of \( \mathcal{F}^- \) for tree-level Higgs couplings at leading-order in \( 1/\tan \beta \). It also provides a tool for computing loop diagrams involving Higgs bosons efficiently and consistently, which may be useful in other contexts such as collider processes with Higgses in the initial or final state.

As before, we eliminate \( m^2_{11}, m^2_{22}, \) and \( m^2_{12} \) by the minimization conditions and trade \( (m^2_{12})^\prime \) for \( M^2_\Lambda \) via Eq. (25). We then take the limit

\[
v_d \rightarrow 0, \quad v_u \rightarrow v, \quad M^2_\Lambda \text{ fixed, } \quad \lambda_i \text{ fixed}, \quad (44)
\]

of the Lagrangian (12) in the broken phase.\(^4\) Also we keep the Yukawa couplings fixed when considering the couplings to fermions. In this limit, we have \( \Phi = H_u, \Phi^\prime = eH^+_u \), and

\[
h^0_u = \frac{1}{\sqrt{2}} (v + \phi_u + iG^0), \quad h^0_d = \frac{1}{\sqrt{2}} (\phi_d - iA^0),
\]

\[
h^+_u = G^+, \quad h^+_d = H^+. \quad (45)
\]

If there were no mixing among neutral Higgses, we would have \( \phi_u = h^0 \) and \( \phi_d = H^0 \), and \( A^0 \) would be a mass eigenstate. The mass matrices are compactly expressed by the quadratic potential

\[
V^{(2)}_{\text{hh}} = \begin{bmatrix}
\begin{array}{cc}
\frac{\lambda_5}{2} v^2 & \frac{\lambda_4}{2} v^2 |h^+_d|^2 + \frac{\lambda_7}{2} v^2 \phi_u^2 \\
\frac{\lambda_4}{2} v^2 |h^+_d|^2 + \frac{\lambda_7}{2} v^2 \phi_u^2 & \frac{\lambda_4}{4} (h^0_d)^2 + \frac{\lambda_7}{\sqrt{2}} \phi_u h^0_d + \text{H.c.}
\end{array}
\end{bmatrix} v^2 , \quad (46)
\]

valid up to corrections of order \( \cos \beta \sim 1/\tan \beta \ll 1 \). The trilinear terms are given in Appendix D; the quartic terms follow trivially from those in the symmetric Lagrangian Eq. (12). Note that the first line of Eq. (46) is symmetric under the \( U(1) \) Peccei-Quinn (PQ) transformation

\[
h^0_d \rightarrow e^{-i\delta} h^0_d \quad \text{or equivalently,} \quad H_d \rightarrow e^{i\delta} H_d . \quad (47)
\]

while the second line is not. In the MSSM, the noninvariant terms appear only at the loop level. We note that the \( U(1) \) symmetry is not spontaneously broken in the large-\( \tan \beta \) limit, so there is no massless boson, in agreement with our keeping \( M^2_\Lambda \) fixed.\(^5\) Next, a PQ transformation makes \( \lambda_i \) real, such that the first term on the second line of Eq. (46) contributes with opposite sign to the mass terms for \( \phi_d \) and \( \chi_d = -A^0 + O(\cos \beta) \), splitting the two. There are only two independent mixing angles that do not vanish: they can be identified with the \( CP \)-conserving angle \( \alpha = O(\lambda_i^7) \) and a \( CP \)-violating \( \alpha' = O(\lambda_i^7) \); a third angle present in the general 2HDM is suppressed by \( O(\cot \beta; v/M) \). All of these are symmetry-breaking effects. To lowest order in the PQ-breaking couplings, the mass matrices are diagonalized by

\[
\begin{pmatrix}
H^1 \\
H^2 \\
H^3
\end{pmatrix} = \begin{bmatrix}
1 & -\frac{\lambda_5^2 v^2}{M_{\Lambda}^2 - \lambda_3 v^2} & \frac{\lambda_5^2 v^2}{M_{\Lambda}^2 - \lambda_3 v^2} \\
\frac{\lambda_4^2 v^2}{M_{\Lambda}^2 - \lambda_3 v^2} & 1 & 0 \\
-\frac{\lambda_4^2 v^2}{M_{\Lambda}^2 - \lambda_3 v^2} & 0 & 1
\end{bmatrix} \begin{pmatrix}
\phi_u \\
\phi_d \\
A^0
\end{pmatrix} , \quad (48)
\]

\[
m^2_1 = \lambda_2 v^2, \quad m^2_2 = M^2_\Lambda + |\lambda_5| v^2, \quad m^3 = M^2_\Lambda . \quad (49)
\]

In a general basis, \( CP \)-violating Higgs mixing is present if and only if \( \lambda_i^7 / \lambda_3 \) is complex. Note that there is no mixing for the charged scalars according to Eq. (46), i.e. no mixing between charged-Higgs and Goldstone bosons due to particles in the large-\( \tan \beta \) limit.

These considerations can be extended to the Higgs-fermion interactions. The operators up to dimension-four follow from (11), which, in the limit of infinite \( \tan \beta \), becomes

\(^5\)Also at finite (but large) \( \tan \beta \), there is no (pseudo-) Goldstone boson, as \( m^2_{11} \sim M^2_\Lambda > 0 \) contributes to the mass terms of both \( \phi_d \) and \( A^0 \) (see also Sec. IIIb).
\[ Z^{V} = \frac{\sqrt{2}}{v} d_{R|i} M_{d|i} H_{u}^{\dagger} Q_{L|i} - \bar{d}_{R|i} \kappa_{ij} Q_{L|i} \cdot H_{d} - \frac{\sqrt{2}}{v} \bar{\mu}_{R|i} M_{s|i} Q_{L|i} + \bar{\mu}_{R|i} \bar{\kappa}_{ij} H_{d}^{\dagger} Q_{L|i} + \text{H.c.}. \] 

(50)

This can be made approximately invariant by extending the symmetry transformation \((47)\) to fermions. One judicious PQ charge assignment is

\[ d_{R|i} \rightarrow e^{i\delta} d_{R|i}, \quad Q_{L|i} \rightarrow Q_{L|i}, \quad u_{Rk} \rightarrow u_{Rk}, \] 

(51)

which commutes with the SM gauge group, implying that neutral and charged gauge-boson couplings respect the symmetry. It has been previously used in \([13]\) to classify the Higgs-fermion couplings in MFV. However, since for MFV one has one more small parameter \(\kappa_{gb}/\kappa_{bq} \approx m_{q}/m_{b}\) for \(q = s \text{ or } d\), it is useful to consider the following variant of Eq. (51):

\[ b_{R} \rightarrow e^{i\delta} b_{R}, \quad q_{R} \rightarrow q_{R}, \quad L_{Q} \rightarrow Q_{L}, \quad u_{Rk} \rightarrow u_{Rk}. \] 

(52)

Now \(\kappa_{ij} d_{R|i} Q_{L|i} \cdot H_{d}\) in Eq. (50) breaks the symmetry unless \(d_{R|i} = d_{R|i} = b_{R}\). However, all \(U(1)_{PQ}\) breaking is still proportional to one of the small parameters of Eq. (36):

\[ \mathcal{F}^{+} = \frac{2\lambda_{2} M_{A}^{2} + (\lambda_{2}^{2} - |\lambda_{1}|^{2}) v^{2}}{\lambda_{2} M_{A}^{2} + (\lambda_{2}^{2} - |\lambda_{1}|^{2}) v^{2} M_{A}^{2} \lambda_{2}^{4} + (\lambda_{2}^{4} - |\lambda_{1}|^{2}) v^{2} M_{A}^{4} - (\lambda_{2}^{4} \lambda_{2}^{4} + \frac{1}{4} \lambda_{2}^{4} \lambda_{2}^{4}) v^{4}} \approx \frac{2}{M_{A}^{2}}, \] 

(53)

\[ \mathcal{F}^{-} = \frac{(\lambda_{2}^{2} - |\lambda_{1}|^{2}) v^{2}}{\lambda_{2} M_{A}^{4} + (\lambda_{2}^{2} - |\lambda_{1}|^{2}) v^{2} M_{A}^{4} - (\lambda_{2}^{4} \lambda_{2}^{4} + \frac{1}{4} \lambda_{2}^{4} \lambda_{2}^{4}) v^{4}} \approx -\frac{(\lambda_{2}^{2} - |\lambda_{1}|^{2}) v^{2}}{\lambda_{2} M_{A}^{2}}. \] 

(54)

where the rightmost expressions hold up to higher orders of small couplings. For \(\mathcal{F}^{-}\), this is identical to the sum of the two leading diagrams in a “mass-insertion approximation,” where the PQ-breaking contributions to the Higgs mass terms are treated as interactions [Fig. 2(f) and 2(g)].

At the loop level (in the 2HDM), up to doubly suppressed contributions one can employ the PQ-conserving parts of Eqs. (50) and (46), i.e. set \(\lambda_{5} = \lambda_{6} = \lambda_{7} = 0\), as well as ignore \(\kappa_{gb}\) and \(\kappa_{bq}\). The matching onto the weak Hamiltonian can be organized according to one-light-particle-irreducible chirality amplitudes. There are three amplitudes:

\[ \mathcal{A}_{RR} = \langle T(b_{R}(x_{1}) b_{R}(x_{2}) \bar{s}_{L}(x_{3}) \bar{s}_{L}(x_{4})) \rangle, \] 

(55)

\[ \mathcal{A}_{KL} = \langle T(b_{K}(x_{1}) b_{L}(x_{2}) \bar{s}_{L}(x_{3}) \bar{s}_{L}(x_{4})) \rangle, \] 

(56)

\[ \mathcal{A}_{VLL} = \langle T(b_{L}(x_{1}) b_{L}(x_{2}) \bar{s}_{L}(x_{3}) \bar{s}_{L}(x_{4})) \rangle, \] 

(57)

plus the parity conjugates of \(\mathcal{A}_{RR}\) and \(\mathcal{A}_{VLL}\). (We have omitted amplitudes that cannot match onto Lorentz-invariant local dimension-six operators.) Only \(\mathcal{A}_{VLL}\) is invariant under \(U(1)_{PQ}\) (both versions) and can be generated from a symmetric Lagrangian. It matches onto the standard-model operator \(Q^{VLL}_{1}\). There is a single diagram contributing, see Fig. 2(b). (Diagram (i) matches onto \(Q^{VRR}_{1}\) and would be allowed for the unmodified PQ assignment of Eq. (51.).)

The present discussion could be extended to other processes, and to higher loop-orders, by systematically treating the PQ-breaking couplings as interactions and working to a fixed total order in the small parameters; in practice, at such higher precision, one might want to extend the effective 2HDM by higher-dimensional operators to account for \(v/M_{\text{SUSY}}\) corrections.

Finally, let us remark that because our choice of shift parameters \(v_{u}\) and \(v_{d}\) minimize the potential \(V\) in the potential of our effective theory and not necessarily the full effective potential, the one-point functions for the (shifted) Higgs fields \(0/h_{i}(0) (h_{i} = \phi_{u}, \phi_{d}, \phi^{0})\) will, in general, not vanish. Hence, also “tadpole” diagrams involving quark or Higgs loops would have to be considered at the outset [Fig. 2(e)]. That they cancel in \(B - \bar{B}\) mixing in our approximation follows from the fact that no such diagrams are present when working with a complex \(h_{0}^{0}\) field and the Lagrangian \(V_{lh}\). Tadpoles may, however, be relevant in other contexts. We discuss our renormalization of \(v_{u}, v_{d}, \text{ and } \tan\beta\) in detail in the following section.
III. SYSTEMATICS OF THE LARGE- \( \tan \beta \) MSSM

The present section is devoted to certain technical aspects of the large \( \tan \beta \) limit. The first concerns the definition (i.e., renormalization) of \( \tan \beta \) in the MSSM and in the effective two-Higgs-doublet-model description of low-energy (i.e., Higgs, electroweak, and flavor) phenomenology, and the matching between the two. This is of phenomenological importance, as \( \tan \beta \) parameterizations are employed in the study of radiative corrections to the MSSM Higgs sector [31].

Having clarified the connection between our “full” and “effective” \( \tan \beta \), we justify the systematic expansion in \( 1/\tan \beta \) at the Lagrangian level employed in Sec. IIc.

A. Renormalization of \( \tan \beta \)

In the MSSM, \( \tan \beta = v_u/v_d \) is defined as a ratio of vacuum expectation values. This is an unambiguous notion at tree-level, because a preferred basis is provided by the chiral Higgs supermultiplets of definite hypercharges \( \pm 1/2 \). Beyond tree-level, a scheme dependence arises as the bare parameters \( p_i^0 \) \((p_i = m_i^0, m_1^0, B \mu, g, g', \text{etc.})\) are renormalized, \( p_i^0 = \frac{1}{2} p_i + \delta p_i \), as well as in the normalization of the fields and in defining renormalized shift parameters \( v_d, v_u \). To formalize the renormalization program, we first define bare shifts that minimize the bare effective potential including radiative corrections, which is equivalent to requiring vanishing one-point functions for the shifted fields, i.e.,

\[
\langle \left( h^0_{1/2}, \frac{1}{\sqrt{2}} v^0_i \right) \rangle_{\text{bare}} = 0,
\]

such that the \( v^0_i \) are indeed vacuum expectation values. Identifying (for any definition of renormalized shift parameters)

\[
v^0_i = Z_i^{1/2}(v_i - \delta v_i), \quad i = d, u,
\]

schemes dependence arises through, and only through, field renormalization and the counterterms \( \delta v_i \). Reference [32] argued that for a stable perturbation expansion, it is desirable to define the renormalized \( v_i \) such as to minimize the renormalized effective potential, i.e. \( \delta v_i = 0 \), and implemented this proposal for \( \text{DR} \) field renormalization and Landau gauge. The same condition and gauge fixing was imposed in the computation of one-loop corrections to the MSSM Higgs masses in [18–21].

References [22, 23] chose to work with on-shell fields and in \( \text{R}_\xi \) gauge instead, and their shifts do not strictly minimize the one-loop effective potential. In fact, in general gauges, for \( \delta v_i = 0 \) the effective action is not finite and the \( v_i \) are both divergent [22, 23] and gauge-dependent [33, 34] (as are the bare vevs \( v^0_i \)). Hence, to have finite renormalized \( v_i \) and \( \tan \beta \), \( \delta v_i \neq 0 \), containing a gauge-dependent divergence, is required. For \( \tan \beta \), we have

\[
\tan \beta^0 = \frac{v^0_u}{v^0_d} = \frac{1}{\tan \beta + \delta \tan \beta}
\]

Minimal subtraction for \( Z_u, Z_d, \delta v_u/v_u, \delta v_d/v_d \) defines \( \tan \beta^0 \). It also follows from Eq. (60) that a change between two schemes \( R \) and \( R' \) can be calculated from

\[
\tan \beta^R - \tan \beta^{R'} = \delta \tan \beta^R - \delta \tan \beta^{R'}
\]

hence any scheme where \( \delta \tan \beta \) is a pure divergence has

\[
\tan \beta^{\text{DR}} \text{ regardless of any nonminimal field renormalizations as those employed in [31]}. \text{ In the latter case, however, } \delta v_u, \delta v_d \text{ are nonminimal and the counterterm for } \tan \beta \text{ has no simple relation to the field renormalization constants.}
\]

\( \tan \beta^{\text{DR}} \) is gauge-dependent [36], but to one-loop order, the gauge-dependence drops out for the \( \text{R}_\xi \) gauges. In spite of its gauge-dependence, the \( \text{DR} \) scheme for \( \tan \beta \) has been shown to lead to a well-behaved perturbation expansion [31] and is also used in the most recent version of the

\[\text{References [35]}\]
publicly available computer programs FeynHiggs [37] and CPSuperH [38].

A second issue is that a fully minimal subtraction scheme, where in particular $\delta v^\text{finite} = 0$, generally entails $v_t$ that do not minimize the (renormalized) tree potential, such that the renormalized Lagrangian contains linear terms

$$\mathcal{L} \supset t_d \phi_d + t_u \phi_u$$

for the shifted (real parts of the) Higgs fields. On the other hand, from Eq. (58) and (59) it follows that

$$\Gamma^\text{ren} = t_i + \Gamma^{(1)}_i + \delta t_i = 0$$

always holds, if only $\delta v_u$ and $\delta v_d$ are included in $\delta t_i$. The presence of $t_u$, $t_d$ is perfectly fine, but tadpole diagrams then have to be retained in the calculation. (In particular, they appear in the expressions relating Higgs and gauge-boson mass parameters to the Lagrangian parameters. If all renormalization constants are minimal, Eq. (62) determines $t_i$ in terms of the bare proper one-point functions $\Gamma^{(1)}_i$ [211].) Yet it may be more convenient to perform additional finite renormalizations to work in a scheme where $t_i = 0$. This can be achieved either by suitable finite terms in $\delta v_i$ or by finite renormalizations of the mass and coupling parameters. The former shifts $\tan \beta$ from its $\overline{\text{DR}}$ value according to

$$\tan \beta^\text{eff} = \tan \beta^\overline{\text{DR}} \left( 1 - \frac{\delta v_u^{\text{ind}}}{v_d} + \frac{\delta v_d^{\text{ind}}}{v_u} \right).$$

The latter option does not modify $\tan \beta$.

Going from the MSSM to a general 2HDM, $\tan \beta$ becomes—strictly speaking—an ill-defined notion, as there is no preferred basis. Identifying $H_1 = -\epsilon H_d^\text{eff}$ and $H_2 = H_u$, an $SU(2)$ rotation $H_i \rightarrow U_i H_i$ removes the vacuum expectation value of one doublet; this corresponds to the $(\Phi, \Phi')$ basis introduced in Sec. II. Only $\Phi$ receives a vev, provides for the Higgs mechanism, and has flavor-conserving couplings, while $\Phi'$ is an ordinary scalar with FCNC couplings. To make contact with MSSM phenomenology, however, it is useful to keep the notion of $\tan \beta$ in the effective theory. In principle, we could fix a basis to enforce $\tan \beta^\text{eff} = \tan \beta^\overline{\text{DR}}$, but find it technically simpler to allow for a parametrically small (i.e. not $\tan \beta$-enhanced) shift, as we discuss in the following.

In complete analogy with the MSSM case discussed above, if we employ a general gauge and $\overline{\text{MS}}$ everywhere in the effective theory, $v_1$ and $v_2$ will not minimize the tree-level (nor the effective) potential. This would require a modification of the formalism in Sec. II. In particular, in writing the mass matrices Eqs. (23)–(25) and the flavor structure of the scalar-fermion couplings in Eq. (11) we assumed the minimization conditions $t_1 = t_2 = 0$. To avoid such modifications, as well as changed expressions for neutral meson mixing, we can either perform renormalizations on the parameters $m_{H_1}^2$ and $m_{H_2}^2$, such that $v_1$ and $v_2$ minimize the 2HDM potential, or achieve this through nonminimal $\delta v_{1,2}$. We pursue the latter option, keeping the symmetric parameters of the 2HDM minimally subtracted. This has the added virtue that the $\tan \beta$ such defined is gauge-independent at the order considered, as it is fully determined by $\overline{\text{MS}}$ mass and coupling parameters. These are gauge-invariant at one-loop, which is clear from our explicit matching calculation. We presume this to hold also at higher orders, at least if the appropriate wave-function renormalization is employed. The $\delta v_i$ are determined entirely in terms of “light”-particle loops and, at least at one-loop, do not lead to parametrically large shifts $\propto \tan^2 \beta \frac{1}{16\pi^2}$, as can be verified from the explicit expressions for the tadpoles in [23] or by considering tadpole diagrams in the large-$\tan \beta$ effective Lagrangian.

To find the precise connection between $\tan \beta^\text{DR}$ and our effective $\tan \beta$, consider the total tree plus one-loop contribution of the superpartners to the (MSSM) effective action for the gauge and Higgs fields,

$$S_{gh} = \int d^4 x \left[ (1 + \Delta Z_w) \left( -\frac{1}{4} W_{\mu \nu}^A W^{\mu \nu A} + (1 + \Delta Z_B) \right) \right. \times \left. \left( -\frac{1}{4} B_{\mu \nu} B^{\mu \nu} + (\delta_{ij} + \Delta Z_{ij}) (D_\mu H_i)^\dagger (D^\mu H_j) \right) \right. \left. - \frac{7}{2} m_{\tilde{t}}^2 H_i^\dagger H_j - \sum_{k=1}^7 \lambda_k O_k + \ldots \right].$$

Here, $O_i$ are the quartic terms constructed from the Higgs fields appearing in Eq. (12), and the dots denote higher-dimensional local terms, and $H_1, H_2$ are related to $H_i, H_j$ as above. The precise values for the coefficients depend on the MSSM renormalization scheme. We assume the MSSM has been regularized by dimensional reduction while the couplings, Higgs fields and $\tan \beta$ are minimally subtracted ($\overline{\text{DR}}$). The corresponding expressions $\lambda_k$ are reported in Eq. (21) (tree-level) and in Appendix B2 and 3 (one-loop).

Equation (64) can be identified with the classical action (ignoring 2HDM loops) for an effective two-Higgs–doublet model with noncanonically normalized fields. To obtain this from the $\overline{\text{MS}}$-renormalized Lagrangian in the presence of light-particle loops, one simply has to add the contributions (which are local) due to loops of 2 $\epsilon$ scalars present in DRED$^8$ and subsequently rescale the fields,

$$\begin{pmatrix} -\epsilon H_d^\text{eff} \\ H_u^\text{eff} \end{pmatrix} = \begin{pmatrix} Z_{dd} & Z_{du} \\ Z_{ud} & Z_{uu} \end{pmatrix} \begin{pmatrix} H_d^\text{eff} \\ H_u^\text{eff} \end{pmatrix}.$$

Integrating over the 2 $\epsilon$ scalars leaves a path integral over light fields that is identical to that in the DRED–regularized effective theory, including the $1/\epsilon$ divergence structure. We recall that the 2 $\epsilon$ scalars should be thought of as having a nonzero mass of $\mathcal{O}(M_{\text{SUSY}})$ [39].

---

In this section, the renormalized 2HDM vevs are denoted by $v_{1,2}$ instead of $v_t$ as in Sec. II in order to avoid confusion with the renormalized MSSM vevs. Correspondingly, the 2HDM tadpoles are denoted by $t_{1,2}$. 

---

8Integrating over the 2 $\epsilon$ scalars leaves a path integral over light fields that is identical to that in the DRED–regularized effective theory, including the $1/\epsilon$ divergence structure. We recall that the 2 $\epsilon$ scalars should be thought of as having a nonzero mass of $\mathcal{O}(M_{\text{SUSY}})$ [39].
subject to the condition $Z(1 + \Delta Z) = 1$. This provides the relation between the $\overline{\text{DR}}$ fields of the MSSM and one out of an infinite choice of $\overline{\text{MS}}$ fields in the effective theory, labeled “eff”. We fix the freedom to choose the Higgs basis in the effective theory by setting $Z_{uu} = 0$ and $Z_{uu} = Z_{uu} = 0$. The relation between the shifts and $\tan(\beta)$ for $\overline{\text{MSSM}}$ and of the 2HDM are now determined according to

\[
\delta v_{2}^{\overline{\text{DR}}} = \delta v_{2}^{\overline{\text{DR}}} - \delta Z_{u} v_{d} - \delta Z_{u} v_{u} + \delta v_{2}^{\text{eff}}
\]

\[
\delta v_{1}^{\overline{\text{DR}}} = \delta v_{1}^{\overline{\text{DR}}} - \delta Z_{d} v_{d} + \delta v_{1}^{\text{eff}}
\]

\[
\tan(\beta)^{\text{eff}} = \tan(\beta)^{\overline{\text{DR}}}(1 - \frac{\delta v_{1}^{\text{eff}}}{v_{1}} - \frac{\delta v_{2}^{\text{eff}}}{v_{2}} + \delta Z_{dd})
\]

Here, we have expanded $Z_{uu} = 1 + \delta Z_{u} / \delta u$ and $Z_{u} = \delta Z_{ud}$, and the $\delta Z_{ij}$ are related to the $\Delta Z_{ij}$ via $\Delta Z_{ij} = -2\delta Z_{u}/\delta u$, with the explicit expressions given in Appendix B1. The shifts $\delta v_{2}^{\text{eff}}$ are defined implicitly as discussed above. In summary, we have constructed a $\tan(\beta)$ which is appropriate for effective weak interactions, gauge-independent and, up to an ordinary (i.e., not $\tan(\beta)$-enhanced) loop correction, coincides with the widely used $\tan(\beta)^{\overline{\text{DR}}}$. It means that the $\tan(\beta)$ measured in flavor physics, for instance through $\mathcal{B}(B_{s} \to \mu^{+} \mu^{-})$, and employed in our analysis, can be identified with the corresponding $\overline{\text{DR}}$ parameter at large $\tan(\beta)$, up to small corrections.

We note that our framework leads to a transparent expression for the relation between the $\overline{\text{DR}}$ scheme and the so-called DCPR scheme employed in [22,23] in the limit $\nu \ll M_{\overline{\text{MSSM}}}$. In the latter scheme, finite but, unlike in our effective 2HDM, “diagonal” wave-function renormalizations of $H_{u}, H_{d}$ are performed, i.e., in our notation, $\delta Z_{uu} = \delta Z_{u} = 0$, as well as $\delta Z_{uu} = 2\delta Z_{u}^{\text{finite}}$. Moreover, the renormalization conditions include

\[
\delta v_{u} = \delta v_{d}, \quad \text{Re}\Sigma^{\overline{\text{DR}}}(M_{A}) = 0,
\]

(67)

where $\Sigma^{\overline{\text{DR}}}(k^{2})$ parameterizes the $A^{0} - Z^{0}$ mixing according to $\Sigma^{\mu}_{A^{0}-Z^{0}}(k) = k^{2}\Sigma^{A^{0}}(k^{2})$. Now, the particle contribution to $\Sigma^{\overline{\text{DR}}}(k^{2})$ reads

\[
\Sigma^{\overline{\text{DR}}}(k^{2}) = M_{Z} \sin^{2}\beta \text{Re}\Delta Z_{12}
\]

\[
+ M_{Z} \sin \beta \cos(\delta Z_{d}^{\text{finite}} - \delta Z_{u}^{\text{finite}}) + \ldots,
\]

(68)

where the dots denote terms proportional to $\cos(\beta)$ but not involving the wave-function renormalization constants, or terms suppressed in the limit where $\nu$ and $k \ll M_{\overline{\text{MSSM}}}$. This follows either by considering the mixed gauge-boson-Higgs-boson bilinear terms resulting from the covariant kinetic operator for the Higgs fields in Eq. (64), or via the Ward identity

\[
k_{\mu} \Sigma^{\overline{\text{DR}}}(k^{2}) + M_{Z} \Sigma^{A^{0}}(k^{2}) = O(k^{2} - M_{A}^{2})
\]

(69)

which is trivially satisfied in our $SU(2)$-invariant formalism from the terms bilinear in the gauge fields in the same term. The two conditions in Eq. (67) then determine

\[
(\delta \tan(\beta)^{\text{DCPR}})^{\text{finite}} = \frac{\tan(\beta)^{\overline{\text{DR}}}}{2} (\delta Z_{u}^{\text{finite}} - \delta Z_{d}^{\text{finite}})
\]

\[
= \frac{\tan^{2}(\beta)^{\overline{\text{DR}}}}{2} \text{Re}\Delta Z_{12} + \ldots,
\]

(70)

where the omitted terms are not $\tan(\beta)$-enhanced. This explains the large numerical differences between $\tan(\beta)^{\overline{\text{DR}}}$ and $\tan(\beta)^{\text{DCPR}}$ found in [31] as a parametrically large effect. Hence, $\tan(\beta)$ measured in flavor physics should not be identified with the corresponding DCPR parameter at large $\tan(\beta)^{\overline{\text{DR}}}$. As with the Higgs fields, we explicitly decouple the contributions of heavy particles to the gauge field wave

\[\text{Physical Review D 84, 034030 (2011)}\]

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functions (hence to \(g^{(0)}\)) by a finite renormalization \(g^{(0)}_{\text{DR}} = g^{(0)}_{\text{eff}} + \delta g^{(0)}\), cancelling the terms \(\Delta Z_B\) and \(\Delta Z_W\) in Eq. (64) of the gauge fields, \(B^\mu_{\text{eff}} = Z_B^\mu B^\mu_{\text{eff}}\) and \(W^\mu_{\text{eff}} = Z_W^\mu W^\mu_{\text{eff}}\). For DR-subtracted MSSM couplings, this gives MS-renormalized 2HDM gauge couplings.

We denote the quartic couplings in our 2HDM scheme by \(\lambda_i\). The finite renormalizations leave \(\lambda_5\) invariant, \(\lambda_5 = \tilde{\lambda}_5\), while the other quartic coupling constants transform like

\[
\tilde{\lambda}_1 = \lambda_1 + g^2 \delta Z_{\tilde{d}d} + \left(\frac{1}{2} (g^2 \delta g + g^\beta \delta g') \right),
\]

\[
\tilde{\lambda}_2 = \lambda_2 + g^2 \delta Z_{\tilde{u}u} + \left(\frac{1}{2} (g^2 \delta g + g^\beta \delta g') \right),
\]

\[
\tilde{\lambda}_3 = \lambda_3 - \frac{g^2}{2} (\delta Z_{\tilde{d}d} + \delta Z_{\tilde{u}u}) - \frac{1}{2} (g^2 \delta g + g^\beta \delta g'),
\]

\[
\tilde{\lambda}_4 = \lambda_4 + g^2 (\delta Z_{\tilde{d}d} + \delta Z_{\tilde{u}u}) + g^2 \delta g,
\]

\[
\tilde{\lambda}_6 = \lambda_6 - \frac{g^2}{4} \delta Z_{\tilde{u}d}, \quad \tilde{\lambda}_7 = \lambda_7 + \frac{g^2}{4} \delta Z_{\tilde{d}u},
\]

where \(x'\) and \(x\) denote the real and imaginary part of \(x\), respectively. The couplings \(\tilde{\lambda}_i\) and \(g^{(0)}_{\text{eff}} = g^{(0)}\) are MS couplings from the viewpoint of the effective theory.

The modification of the dimensionless couplings by the finite wave-function renormalizations affects the \(B - \bar{B}\) mixing amplitudes as a formally higher-order effect, as does the scheme dependence of \(\tan \beta\). Unlike the latter, however, the former is never \(\tan \beta\) enhanced unless the wave-function-renormalization constants themselves are.

### Invariance of \(B - \bar{B}\) mixing under field renormalization

The effects of an arbitrary linear field redefinition in the 2HDM such as Eq. (65) on the Higgs-mediated FCNC Eq. (11) are twofold: (i) the values for \(\cos \beta\) and \(\sin \beta\) in Eq. (10) are modified. This cancels the contributions to \(F^\pm\) from the redefinition of the mass matrices up to a global factor:

\[
F^+(\lambda_i, \nu_{u,d}, \lambda_A) \rightarrow F^+(\lambda'_i, \nu_{u,d}', \lambda'_A)
\]

\[
= \frac{v^2}{v'^2} |\det Z|^2 F^+(\lambda_i, \nu_{u,d}, \lambda_A),
\]

\[
F^-(\lambda_i, \nu_{u,d}, \lambda_A) \rightarrow F^-(\lambda'_i, \nu_{u,d}', \lambda'_A)
\]

\[
= \frac{v^2}{v'^2} (\det Z)^2 F^-(\lambda_i, \nu_{u,d}, \lambda_A).
\]

(ii) the \(\nu_{u,d}\) redefinition in (i) comes with a modification of \(\kappa_{ij}\):

\[
\kappa_{ij} \rightarrow \kappa'_{ij} = \frac{v'}{v} (\det Z)^{-1} \kappa_{ij}.
\]

The above factors cancel each other out in the products \(\kappa_{bd}'\) \(F^+\) and \(\kappa_{gb}'\kappa_{bq} F^+\), as they should. In particular, our choice of wave-function renormalization acting on the leading FCNC coupling Eq. (29) produces an extra term:

\[
\delta \kappa_{bq} = -\kappa_{bq} (s_\beta^2 \delta Z_{\tilde{d}d} + s_\beta c_\beta \delta Z_{\tilde{u}d} + c_\beta \delta Z_{\tilde{d}u}).
\]

Considering Eqs. (29), (30), (71), and (75), gives the same Wilson coefficients \(C^R\) and \(C^\text{SLL}\) as does considering Eqs. (29), (30), and (71), with the finite parts of \(\delta Z_{ij}\) set to zero. While in practice, wave-function renormalization has to be performed to relate the parameter \(M_A\) to the physical Higgs-boson masses and to take \(v = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}\) beyond leading-order precision, such renormalizations are not the source of a nonvanishing of the \(Q^{\text{SLL}}\) amplitude, to be found instead in the corrections to Higgs masses and mixings (via the self-couplings \(\lambda_i\), in particular \(\lambda_5\)); wave-function-renormalization effects enter that amplitude only at higher orders (as might have been expected). In this regard our findings disagree with the conclusions of [26].

### B. Health of the large-\(\tan \beta\) limit and fine-tuning

In Sec. Iic we took the limit \(\tan \beta \rightarrow \infty (\nu_d \rightarrow 0, M_A^2 = \text{const}, v_u^2 + v_d^2 = \text{const}, \lambda_i = \text{const}, \nu_u \text{ and } \nu_d \text{ defined as minima of the tree potential})\) at the Lagrangian level. One might wonder whether this procedure is valid at the quantum level. To justify it, we show that the \(\nu_d = 0\) case and the \(\nu_d \neq 0\) case are analytically connected, i.e. one can be reached from the other without a phase transition. It then follows that amplitudes are (in some neighborhood of a parameter point with \(\nu_d = 0\)) analytic functions of the parameters (either symmetric or broken). The renormalizability of the effective potential \(V_{\text{eff}}\) then follows by standard arguments from the fact that it is equivalent to the symmetric potential Eq. (12) (for a certain choice of parameters), which is renormalizable.

We first note that the number of independent minimization conditions is unchanged in the \(\nu_d = 0\) limit. First, for general values of the parameters, out of the four real (two complex) minimization conditions, at most three are independent. This follows from the \(U(1)_Y\) invariance but is easy to verify explicitly. Fixing \(\nu_u\) to be real and positive, three polynomials of degree three determining three unknowns \(\nu_u, \nu'_u, \nu'_d\) remain. The system has a solution \(\nu_d = 0\) if

\[
\lambda_2 m_{12}^2 + \lambda_4 m_{22}^2 = 0, \quad \nu_u^2 = -\frac{2m_{12}^2}{\lambda_2}
\]

Here, the second equation determines \(\nu'_d = v\) as a function of \(m_{12}^2\) and \(\lambda_2\) similarly to the case of a single doublet, while the first equation can be viewed as a fine-tuning condition between \(m_{12}^2\) and \(\lambda_4\). The dimensionless complex parameter

\[
\epsilon = \frac{m_{12}^2}{m_{11}^2} + \frac{\lambda_4 m_{22}^2}{\lambda_2 m_{11}^2}
\]

parameterizes the deviation from the fine-tuning limit; we may trade \(m_{12}^2\) in favor of \(\epsilon\). Clearly, at the limiting point

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034030-14
\]

PHYSICAL REVIEW D 84, 034030 (2011)

GORBAHN et al.
\[ f_d(\lambda, m^2_{1,2}, v_u, v_d) = 0, \] 

(78)

the Jacobian

\[ \frac{\partial (f_1, f_2, f_3, f_4)}{\partial (v_u, v^\prime_d, v^\prime_u)} \] 

(79)

has maximal rank (3) at any point with \( v_d = 0 \). (Physically, this just means that the neutral-Higgs mass matrix has three nonzero eigenvalues there.) Hence, by the implicit function theorem, we may solve for \((v_u, v_d)\) in a neighborhood of it, where the solutions will be (real-)analytic functions of \( \epsilon \). In particular, \( v_u \) behaves analytically (and is strictly positive) around \( \epsilon = 0 \), i.e. no phase boundary is encountered. Explicitly and to linear order, the real and imaginary parts of \( v_d \) are determined by

\[
\begin{pmatrix}
1 + \frac{\lambda_{1+} + \lambda_{1-}'}{2} \frac{v^2_d}{m_{11}} & \frac{\lambda_{1-}'}{2} \frac{v^2_d}{m_{11}} \\
\frac{\lambda_{1-}'}{2} \frac{v^2_d}{m_{11}} & 1 + \frac{\lambda_{1-} - \lambda_{1-}'}{2} \frac{v^2_d}{m_{11}}
\end{pmatrix}
\begin{pmatrix}
v^\prime_d \\
v^\prime_d
\end{pmatrix}
= v_u \begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix} + O(\epsilon^2),
\]

(80)

such that \( \tan \beta = O(1/|\epsilon|) \). The nonsingular linear term allows us to change variables from \( m^2_{1,2} \) to a complex \( v_d \). Of course, we may always perform a field redefinition of \( H_d \) such that \( v_d \) is real. Then, the mass parameters besides \( m^2_{1,1} \) are power series in \( 1/\tan \beta \), which read

\[
m^2_{12} = \frac{m^2_{11}}{2} + \frac{1}{2} v_u \lambda_{1+} v_d + \frac{1}{2} v_d^2 \lambda_{1-},
\]

\[
m^2_{22} = \frac{m^2_{11}}{2} + \frac{1}{2} v_u v_d (\lambda_3 + \lambda_{1+}) + \frac{1}{2} v_d^2 \lambda_{1-} + O(v_d^3/v_u^3),
\]

\[
M^2_A = m^2_{11} + \frac{\lambda_{1+} - \lambda_{1-}'}{2} v_u^2 + O(v_d/v_u),
\]

\[
M^2_H = \frac{\lambda_{1+} + \lambda_{1-}'}{2} v^2_u + O(\lambda_{1+}^2;\epsilon),
\]

\[
M^2_{H^+} = m^2_{11} + \frac{\lambda_{1+} + \lambda_{1-}'}{2} v^2 + O(v_d/v_u).
\]

(81)

We see explicitly that we can continuously change the dimensionful parameters in the Higgs potential from a situation where \( v_d \neq 0 \) to one where \( v_d = 0 \), keeping \( M^2_A \) (and the dimensionless couplings) fixed, as was assumed in Sec. IIc. The last three equations illustrate that the large-\( \tan \beta \) case is characterized by a “primary” doublet \( H_u \) which receives a large ew \( v_u \) and a “secondary” doublet \( H_d \) with a positive gauge-invariant mass \( m^2_{1,1} \) that receives corrections of \( O(v^2) \) and \( O(\epsilon) \), respectively, due to its dimensionless and dimensionful couplings to \( H_u \). Those corrections differ among the physical components of the doublet, approximately to be identified with \( H^0, \lambda^0, H^\pm \), due to electroweak symmetry-breaking. In principle, \( m^2_{1,1} \) could be negative, but in that case, \( v_d = 0 \) will typically not be the global minimum of the potential.

We close this section by considering the fine-tuning which is necessary to obtain a large \( \tan \beta \) while keeping the mass \( M_A \) fixed.\(^{11}\) For \( v_d \) real, Eq. (80) implies

\[
m^2_{12} = -\frac{\lambda_{1+}}{\lambda_{1-}'} m^2_{22} + \cot \beta \left( \frac{m^2_{11} + \lambda_3 + \lambda_{1+}}{\lambda_{1-}'} v^2_u \right)
\]

(82)

which illustrates the tuning that is known to be necessary to have large \( \tan \beta \) in the MSSM. For the generic situation \( m^2_{11} \sim M^2_{SUSY} \gg M^2_z \), the right-hand side is dominated by the \( m^2_{11} \) term: \( \lambda_{1+}' \) is down by a loop factor relative to \( \lambda_{1+} \), and \( m^2_{22} \sim v^2 \ll M^2_{SUSY} \) (the little hierarchy). Hence,

\[
m^2_{12}/M^2_{SUSY} \approx 1/\tan \beta,
\]

(83)

which implies an extra tuning beyond the one to achieve the correct weak scale. For smaller \( m^2_{11} \sim M^2_A \sim M^2_Z \), which is interesting from the point of view of \( B \)-physics phenomenology, the required tuning gets even worse—unless, of course, the whole SUSY scale is lowered to the weak scale, which is, however, problematic since then \( M^2_H \approx \lambda_2 v^2 \) is generally below the experimental lower limit. On the other hand, as we have seen, \( M^2_A \sim m^2_{11} \), such that no extra tuning is required to keep \( M^2_A \) finite, while one might have expected otherwise from the well-known tree-level formula

\[
M^2_A = (\tan \beta + \cot \beta) m^2_{12},
\]

(84)

which is generalized by Eq. (25). Also, while a small \( m^2_{12} \) is indeed sensitive to radiative corrections, those are automatically correlated with shifts of \( v_d \) and in consequence of \( \tan \beta \) in such a way that \( M^2_A \) receives only mild corrections.

### IV. PHENOMENOLOGY

In Sec. II, we performed a detailed study of the supersymmetric contributions to \( \Delta M_d \) and \( \Delta M_s \) in the generic framework of an effective 2HDM. The corresponding matching coefficients were computed at the one-loop level in Sec. III and Appendix B. In this section, we assess the maximal size of the various types of effects identified in the MFV case taking into account the existing constraints on the supersymmetric parameter space, in particular, from the \( B_s \rightarrow \mu^+ \mu^- \), \( B^+ \rightarrow \tau^+ \nu \), and \( b \rightarrow s \gamma \) branching fractions. For convenience, we start with a compendium of the formulas derived in Sec. II:

\[
\Delta M_q = |\Delta M^{SM}_q + \Delta M^{LR}_q + \Delta M^{LL}_q + \Delta M^{1L}_q| \equiv |1 + h_q|\Delta M^{SM}_q,
\]

(85)

where the standard-model, the left-right and left-left Higgs-pole (cf. Equation (17)), and the neutral-Higgs-loop (cf. Equation (40)) contributions read

\(^{11}\)This aspect has been considered before, and recently in an EFT framework in Ref. [40].
Explicit expressions for which drop out in physical observables, as discussed at large $\alpha_s$. The function-renormalization terms in Eqs. (87) and (88), for simplicity, we have omitted the Higgs wavefunction-renormalization terms in Eqs. (87) and (88), drop out in physical observables, as discussed at the end of Sec. IIIa. Explicit expressions for $\lambda_S$, $\lambda_I$, and $\lambda_2$ in the effective couplings were given in Eqs. (31)–(33) and (18), with the effective couplings $\lambda_I$ entering the effective mass matrix computed in Sec. IIIa and Appendix B. For large $\tan\beta$, we have in very good approximation:

$$ \mathcal{F}^+ \simeq \frac{2}{M_A^2}, \quad \mathcal{F}^- \simeq \frac{-\lambda^2_s + \lambda^2_I/\alpha_s}{M_A^4} u^2 $$

(88)

where $m_b$ is in GeV and $\phi_\chi = \text{arg}(\tilde{\kappa})$. $h_d$ is given by the same expression with $m_s$ replaced by $m_d$, so that the first term becomes subleading.

A first obvious remark is that $\Delta M_{qy}^{\text{LL}}$ cannot compete with $\Delta M_{qy}^{\text{LR}}$ or $\Delta M_{qy}^{\text{LL}}$ unless $y_b$ becomes nonperturbative. This is rather accidental (notice the small loop factor in Eq. (86) as well as the smallness of $P_{1y}^\text{VLL}$ with respect to $P_2^\text{LR}$ and $P_1^\text{LL}$). Further, the contribution of $\Delta M_{qy}^{\text{LR}}$ seems somewhat limited. However, the loop functions $\lambda_3$ and $\lambda_7$ could be enhanced for large $\mu$ or $a_{1b}$, see Eqs. (38) and (37). A more quantitative analysis is thus desirable. In the next two sections, we perform a random scan of the MFV-MSSM parameter space to find the maximal $\Delta M_{qy}^{\text{LL}}$ and $\Delta M_{qy}^{\text{LR}}$ values allowed by current experimental data. Equations (85)–(89) do allow for new CP-violating phases, yet these will be set to zero in the scan. CP-violating effects within the MFV scenario will be shortly discussed in Sec. IVc.

### A. Scan of the parameter space

The values of the various input parameters used in the scan are collected in Table II. Note that only the products $P_1^\text{LR} m_s$ and $P_2^\text{LR} m_d$, or alternatively $P_1^\text{LR} m_s$ and $m_d/m_s$, are needed, see Eq. (86). We scan over $P_1^\text{LR} m_s$ but keep $m_d/m_s$ fixed as $\Delta M_{qy}^{\text{LR}}$ is deemed to be small anyway. The decay constants $f_{B_q}$ and CKM factors $|V_{qy}^\text{VLL}|^2$ are not specified.
Instead, outputs are formulated in terms of ratios free from these rather poorly known parameters. Finally, we take \( \alpha = 1/127.9, \sin^2 \theta_W = 0.231, \) and \( M_Z = 91.1876 \) GeV.

For simplicity, the gaugino mass parameters are assumed to have the same sign (which we can choose positive), as well as the trilinear terms (positive or negative). Note that the absolute scale of \( M_{\text{SUSY}} \) plays no role as supersymmetric parameters enter \( \epsilon_{0,y} \) and \( \lambda_i \) by means of ratios. Only the spread of the interval chosen for \( M_{\text{SUSY}} \) matters. Still, \( M_{\text{SUSY}} \) should not be taken too large to help satisfy the \( b \rightarrow s \gamma \) constraint in the case \( \mu < 0 \). We will come back to this point later. We allow for rather large values of \( M_A \), close to the lower end of the interval chosen for \( M_{\text{SUSY}} \). Still, the matching performed in Sec. III and Appendix B remains valid as the corrections from higher-dimension operators at the electroweak scale are ruled by the ratio \( \nu/M_{\text{SUSY}} \) and not \( M_A/M_{\text{SUSY}} \). The formulas for the various observables at the \( B \) mass scale are thus unaffected.

The constraints imposed on the points generated inside the above ranges are summarized in Table III. We now discuss them in turn:

(i) The bottom Yukawa coupling \( y_b \) is maintained small enough, say, \( y_b < 2 \), to guarantee the validity of perturbation theory. This condition removes possible fine-tuned points in parameter space for which the denominators in Eq. (87) are close to zero.

(ii) The lightest Higgs-boson mass \( M_h \) has to come out large enough to comply with the LEP II experimental lower bound. \( M_h \) is obtained from the \( CP \)-even Higgs mass matrix in Eq. (23), with the effective couplings \( \lambda_i \) computed at the one-loop level. Higher-order corrections to \( \lambda_2 \) are known to be important [18–20]. However, \( h^1 \) comes up in the FCNC vertices \( \kappa_{ij} \) of Eq. (11), along with a cot\( \beta \) suppression factor. The \( \tan \beta \)-enhanced effects considered here are thus largely uncorrelated with \( M_h \). For this reason we do not correct the one-loop formulas and simply impose \( M_h > 115 \) GeV.

(iii) The following bounds are imposed on \( a_i \) and \( a_b \) to avoid the occurrence of color symmetry-breaking vacua at tree-level [41]:

\[
[a_i]^2 < 3(M_{\tilde{t}}^2 + M_{\tilde{b}}^2 + m_{\nu_\tau}^2),
\]

\[
[a_b]^2 < 3(M_{\tilde{L}}^2 + M_{\tilde{b}}^2 + m_{\nu_\tau}^2).
\]  

The corresponding bound for \( a_s \) is not imposed as slepton parameters have very little impact on the quark FCNC considered here anyway.

(iv) The most stringent constraint on the FCNC coupling \( \kappa \) comes from the \( B_s \rightarrow \mu^+ \mu^- \) branching fraction [3–5,42], which we normalize to \( \Delta M_t \) to avoid the occurrence of the parameters \( f_B \) and \( V_{ts}V_{tb}^* \). This time the Higgs-pole contribution overcomes the standard-model and Higgs-loop pieces.

In addition, these last two interfere destructively, so we will neglect them. The counterpart of Eq. (86) then reads (with \( m_{\mu}^2/M^2_{B_\mu} = 0 \) for simplicity):

\[
\mathcal{B}(B_q \rightarrow \mu^+ \mu^-) = \tau_{B_q} |V_{q\mu}V_{\mu}^*|^2 f_B^2 M_{B_q}^2 \frac{m_{\mu}^2}{M^2_{B_\mu}} \frac{|\kappa|^2}{16 \pi^2} \frac{[|\mathcal{F}_p|^2 + |\mathcal{F}_S|^2]}{\cos^2 \theta + \epsilon_{i\mu}^2 f_B^2}
\]

\[
= R_q \mathcal{B}(B_q \rightarrow \mu^+ \mu^-) \sin^2 \theta
\]  

where \( \mathcal{F}_p \) and \( \mathcal{F}_S \) refer to the Wilson coefficients of the effective operators \( Q_P = (\bar{b}_R s_L)(\bar{\ell} y_{S\ell}) \) and \( Q_S = (\bar{b}_R s_L)(\bar{\ell} \ell) \) arising from neutral-Higgs exchanges and

\[
\epsilon_{i\mu} = \frac{g_i^2}{16 \pi^2} \frac{\mu^*}{M_1} \left[ -\frac{1}{2} H_2 \left( \frac{M_{\tilde{L}}^2}{|M_1^2|} - \frac{|\mu|^2}{|M_1^2|} \right) + \frac{g_i^2}{16 \pi^2} \frac{\mu^*}{M_1} \right.
\]

\[
\times H_2 \left( \frac{M_{\tilde{L}}^2}{|M_1^2|} - \frac{|\mu|^2}{|M_1^2|} \right) + \frac{3 \epsilon_{i\mu}^2}{32 \pi^2} \frac{\mu^*}{M_2}
\]

\[
\times H_2 \left( \frac{M_{\tilde{L}}^2}{|M_2^2|} - \frac{|\mu|^2}{|M_2^2|} \right) \right]
\]

This result agrees with [43] but disagrees with [44]. The loop function \( H_2 \) was defined in Eq. (34) and \( M_{\tilde{L}}(R) = M_{\tilde{L}}(R) \) in our MFP scenario. In the large \( \tan \beta \) limit and at tree-level in the Higgs potential, we have: \( \mathcal{F}_p = -\mathcal{F}_S = \mathcal{F}_+^2 / 2 = 1/M_{\tilde{A}} \), so that \( \mathcal{B}(B_q \rightarrow \mu^+ \mu^-) \) is tightly correlated with \( \Delta M_t \) [5]. Going beyond the tree-level and large \( \tan \beta \) approximations we obtain: \( \mathcal{F}_p = s_\beta (\mathcal{F}_+ + \mathcal{F}_-) / 2 \), with \( \mathcal{F}_\pm \) given in Eqs. (18) and (19). This formula is actually valid in any 2HDM, including arbitrary \( CP \)-violating phases (in the \( CP \)-conserving case it reduces to the usual identity \( \mathcal{F}_p = s_\beta M_{\tilde{A}} \)). We did not find such a general and simple form for \( \mathcal{F}_S \), yet it is straightforward to write it in terms of \( M_A, \tan \beta \), and the \( \lambda_i \)'s (alternatively one can of course express it in terms of the neutral-Higgs masses and mixing angles). Note that one still has \( \mathcal{F}_S = \mathcal{F}_- / 2 \) up to \( \cot \beta \)-suppressed terms. Sparticle loop corrections to the Higgs self-energies turn out to be relevant in the case of \( \mathcal{F}_S \): they can be as large as 15% for small \( M_A \) after all constraints are taken into account, as we will see in Sec. IVb. Numerically, Higgs-mediated effects can easily be very large:

\[
R_s = 9930 \left[ 1 + \left( \frac{\lambda_2^2 + |\lambda_2|^2/\lambda_2^2}{M^2_\tilde{A}} \frac{1}{1 + \epsilon_5 t_\beta^2 + 1 + \epsilon_4 t_\beta^2 + M^2_\tilde{A}} \right) \frac{t_\beta}{40} \right]^6
\]

\[
\times \frac{[16 \pi^2 \epsilon_5^2 V^2(120 \text{GeV})]}{1 + \epsilon_5 t_\beta^2 + 1 + \epsilon_4 t_\beta^2 + M^2_\tilde{A}} \frac{t_\beta}{40}
\]

\[
(93)
\]
and $R_d = R_s$. The first correction factor above captures the bulk of the effects from the Higgs self-energies, yet the exact formula for $\mathcal{F}_s$ should be used for better precision. In practice, the looser constraint $\mathcal{B}(B_s \to \mu^+ \mu^-)/\Delta M_s < 5.7 \times 10^{-9}$ ps, obtained from $\mathcal{B}(B_s \to \mu^+ \mu^-)_{\text{exp}} < 10^{-7}$ [45] and $\Delta M_s^{\text{exp}} = 17.77 \pm 0.1 \pm 0.07$ ps$^{-1}$ [46], is built-in in the scan procedure, then the current 95% C.L. bound $\mathcal{B}(B_s \to \mu^+ \mu^-)/\Delta M_s < 3.3 \times 10^{-9}$ ps corresponding to $\mathcal{B}(B_s \to \mu^+ \mu^-)_{\text{exp}} < 5.8 \times 10^{-8}$ [47] is imposed. We also checked the bound $\mathcal{B}(B_d \to \mu^+ \mu^-)/\Delta M_d < 3.6 \times 10^{-8}$ ps, corresponding to $\mathcal{B}(B_d \to \mu^+ \mu^-)_{\text{exp}} < 1.8 \times 10^{-8}$ [47] and $\Delta M_d^{\text{exp}} = 0.507 \pm 0.005$ ps$^{-1}$ [49]. This provides no additional constraint. Neither do $\mathcal{B}(B_{s,d} \to \mu^+ \mu^-)$ and $\Delta M_{s,d}$ taken separately due to the large parametric uncertainties from $f_{B_{s,d}}$.

(vi) The $b \to s \gamma$ branching fraction with the energy cut $E_\gamma > 1.6$ GeV is computed using the fortran code SusyBSG [50]. Higgs-mediated effects now appear at loop level with smaller powers of $\tan \beta$, so that purely supersymmetric loop corrections (scaling as $1/M_{\text{SUSY}}$) are comparatively more important. For $a, \mu < 0$ and relatively light $M_{\text{SUSY}}$, chargino and charged-Higgs loops can interfere destructively and more room is left for New Physics. This interplay is welcome when $\mu < 0$ as the charged-Higgs contribution then tends to overshift the experimental branching fraction. On the other hand, in that case, the discrepancy between the $(g-2)_\mu$ standard-model prediction and its present measurement [51] increases (for a recent discussion, see e.g. [43] and references therein). The significance of this discrepancy, however, is questioned by the new $e^+e^- \to \tau^+\tau^-\gamma$ BABAR data [52]. We, therefore, still include the situation $\mu < 0$ in our considerations. The $\mathcal{B}(b \to s \gamma)$ experimental world average reads: $\mathcal{B}(b \to s \gamma)^{\text{exp}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$ [53]. The standard-model central value of the SusyBSG program agrees well with the next-to-next-to-leading-order prediction $\mathcal{B}(b \to s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$ [54]. We combine the experimental error with the uncertainties discussed in Ref. [50] and obtain the following two-sigma range: $2.71 \times 10^{-4} < \mathcal{B}(b \to s \gamma) < 4.33 \times 10^{-4}$.

(vi) The $B^+ \to \tau^+\nu$ branching fraction is given by

$$\mathcal{B}(B^+ \to \tau^+\nu) = \frac{G_F^2}{8\pi} |V_{ub}|^2 f_{B_d}^2 	imes M_{B^+} m_{\tau}^2 \left(1 - \frac{m_{\tau}^2}{M_{B^+}^2}\right)^2 |1 - g_{\rho}|^2. \quad (94)$$

where

$$g_{\rho} = \frac{M_{B^+}^2 r_{B_d}}{(1 + \epsilon_{B_d}^\rho)(1 + \epsilon_{B_d}^\rho) M_{H^+}^2} \quad (95)$$

parametrizes Higgs-mediated effects. $\epsilon_{\rho}$ is obtained from $\epsilon_{\mu}$ in Eq. (92) by the replacement $M_{\tilde{b}L(R)} \to M_{\tilde{H}L(R)}$. Corrections to the Higgs potential merely change the value of $M_{H^+}$, which becomes a function of $M_A$, $\tan \beta$, and the various supersymmetric parameters. Again, we include these corrections in our numerical analysis. Given the large theoretical and experimental uncertainties, we impose: $g_{\rho} < 0.36 \cup 1.64 < g_{\rho} < 2.73$. The constraint from $\mathcal{B}(B \to D\tau\nu)$ allows to reduce the second interval, and we end up with $g_{\rho} < 0.36 \cup 1.64 < g_{\rho} < 1.79$ [55].

B. Size of the new contributions

The various Higgs-mediated contributions $\Delta M_{sLR}$, $\Delta M_{qLR}$, $\Delta M_{qLL}$, and $\Delta M_{qUL}$ normalized to the standard-model prediction $\Delta M_{qSM}$ are displayed in Fig. 3(a)–3(c) as a function of the FCNC coupling $\kappa$. As expected from Eq. (89), Higgs-loop effects are very small (the bottom Yukawa coupling actually does not reach its upper bound $\kappa_B = 2$ in the presence of the other constraints, see Fig. 3(d). The upper and lower branches correspond to $\mu < 0$ and $\mu > 0$, respectively). Further, the contribution of $\Delta M_{qLL}$ appears to be much smaller than that of $\Delta M_{sLR}$ despite the fact that $m_b \mathcal{F}^-$ can compete with $m_t \mathcal{F}^+$, see Fig. 3(e) and 3(f). This suppression is a consequence of the $B_s \to \mu^+ \mu^-$ constraint. Indeed, large values of $\mathcal{F}^-$ are obtained for small values of $M_A$, to which $\mathcal{B}(B_s \to \mu^+ \mu^-)$ is particularly sensitive. As a result, the recent CDF bound [47] only leaves room for very small $\kappa$ couplings, killing practically all effects in $\Delta M_{qLL}$ (and actually also in $\Delta M_{qUL}$ for such small $M_A$ values). In Fig. 3(g), we illustrate this decrease of the maximal $\kappa$ value allowed by the $B_s \to \mu^+ \mu^-$ constraint with $M_A$. Blue/magenta/red (dark grey/light grey) points correspond to $M_A = 550/350/150$ GeV (the constraints in the right column of Table III were not imposed to keep the focus on $B_s \to \mu^+ \mu^-$). As one can see, for $M_A$ fixed, the largest possible $\kappa$ first increases with $\tan \beta$, as expected from Eq. (87), saturates the $\mathcal{B}(B_s \to \mu^+ \mu^-)$ experimental upper bound for some $\tan \beta$ value, and is then forced to decrease. For smaller $M_A$, the $B_s \to \mu^+ \mu^-$ constraint is more stringent and only a smaller $\kappa_{\text{max}}$ can be achieved. This growth of $\kappa_{\text{max}}$ with $M_A$ is sufficient to overcome the $1/M_{BR}^2$ suppression factor in $\Delta M_{s}$ but not the $1/M_{A}^2$ one in $\Delta M_{q}$ [56]. Overall, Higgs-mediated effects in $\Delta M_q$ are of the LR type and the room for such effects increases with $M_A$. The correlation between $\Delta M_s$ and $\mathcal{B}(B_s \to \mu^+ \mu^-)$ pointed out in Ref. [5] is thus preserved, up to the relatively small Higgs self-energy corrections to $B_s \to \mu^+ \mu^-$ mentioned above Eq. (93). These are only

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\textsuperscript{13}A newer preliminary result is $\mathcal{B}(B_s \to \mu^+ \mu^-)_{\text{exp}} < 4.3 \times 10^{-8}$ at 95% C.L. [48].
relevant for $\mu > 0$, $\tan\beta \leq 25$, small $M_A$, and large $\lambda_5$, though (see Fig. 4). The mass difference in the $B_d$ system, on the other hand, remains unaffected. These results seem to contradict those of Ref. [56], where large LL-type effects were claimed. Without attempting a close numerical comparison (the sign of $\Delta M^{LL}_d/\Delta M^{LR}_d$ in [56] is actually reversed with respect to ours), let us point out that, as shown by Figs. 3(e) and 3(f), a large $\Delta M^{LL}_d/\Delta M^{LR}_d$ ratio...
does not automatically lead to large nonstandard effects in \( \Delta M_s \) due to the \( \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \) constraint.

Being of the LR type, the maximal effect allowed in \( \Delta M_s \) is essentially determined by the current \( \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \) experimental upper bound for a fixed (but large enough) value of the ratio \( \tan \beta/M_A \). This is illustrated in Fig. 3(h) for a slightly larger bound (cf. left-hand side column in Table III). The correlation itself is displayed in Fig. 5, where each diagonal corresponds to a fixed value of the ratio \( \tan \beta/M_A \). We distinguish the cases \( \mu > 0 \) and \( \mu < 0 \) as the latter leads to larger effects due to smaller denominators in Eq. (87) but is disfavored by the measurement of \((g - 2)_\mu\), as mentioned previously. The sign of the various \( a \)-terms on the other hand, has only little impact. Still, in the case \( \mu < 0, a_i > 0 \) helps satisfy the \( b \rightarrow s \gamma \) constraint. Note that the effect of the \( B^+ \rightarrow \tau^+ \nu \) constraint is particularly transparent on Fig. 5: it removes the points with large \( \tan \beta/M_A \) ratios, i.e., the steepest diagonals. Altogether, for \( M_A < 600 \text{GeV} \), Higgs-mediated effects in \( \Delta M_s \) can reach \(-7\% \) (\(-20\% \)) for \( \mu > 0 \) (\( \mu < 0 \)). These findings agree with those of Ref. [57]. They merely follow from the \( \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \) constraint, as one can see from Figs. 3(g), 3(h), and 5.

Finally, for completeness (or out of curiosity), we display in Fig. 6 the typical size of various quantities and their dependence on effective couplings or supersymmetric parameters. In particular, in the last four plots, we illustrate how the loop functions \( e_0, e_s, e_\tau \), and \( \lambda_3 \) increase with the range chosen for \( M_{\text{SuN0}} \) (more precisely, they increase with the trilinear and \( \mu \) terms and decrease with the squark and slepton mass parameters \( M_{f_L} \) and \( M_{f_R} \) with \( f = t, b, \tau \).)

### C. CP-violating effects

The Higgs-mediated \( B - \bar{B} \) mixing amplitudes studied here can in principle generate new contributions to the \( CP \)-violating phases measured in the \( B_d \rightarrow J/\psi K_S \) time-dependent \( CP \) asymmetry and the \( B_s \rightarrow J/\psi \phi \) time-dependent angular distribution. The coefficients of the \( \sin(\Delta M_q t) \) terms are

\[
S_{J/\phi K_S} = \sin(2\beta + \phi_3),
\]

\[
S_{J/\phi \phi} = -\sin(-2\beta_s + \phi_3^s),
\]

where

\[
\beta = \arg\left(-\left(V_{td}^* V_{tb}\right)\left(V_{cd}^* V_{cb}\right)\right),
\]

\[
\beta_s = -\arg\left(-\left(V_{td}^* V_{tb}\right)\left(V_{cd}^* V_{cb}\right)\right),
\]

\[
\phi_3^s = \arg\left(M_{f_L}^2/M_{f_R}^{2,SM}\right) = \arg(1 + h_q^s).
\]

In \( B_s \rightarrow J/\psi \phi \), an angular analysis separates the different \( CP \) components, the sign quoted for \( S_{J/\phi K_S} \) in Eq. (96) refers to the dominant \( CP \)-even component. These phases have received a lot of attention recently. In particular, the new measurements of \(-2\beta_s + \phi_3^s \) by the CDF and D0 collaborations [58], both more than 1.5 sigma above its SM prediction [59], have triggered speculations about the validity of the SM [60]. A possible tension between the value of \( \sin 2\beta \) obtained from \( S_{J/\phi K_S} \) and the amount of \( CP \) violation in the kaon system was also pointed out [61].

Looking back at (17)–(19), it is clear that the new phases \( \phi_3^s \), when associated with the \( Q_{11}^{LR} \) effective operator, have to be brought up by the quark-Higgs couplings \( \kappa_{ij} \), as \( F' \) cannot develop an imaginary part. When associated with \( Q_1^{SLL} \) or \( Q_1^{SRR} \), on the other hand, they can arise from both
the Yukawa sector and the Higgs potential via $f^-$. Within MFV, $\kappa_{q_b}^{-}\kappa_{b_q}=|\tilde{\kappa}|^2 \lambda_{q_b}^2 m_{q_b} m_b / v^2$ and only $C_{12}^{SLL}$ can produce a new phase (via $\epsilon_{0, Y}$ or $\lambda_{S, Y}$). However, the $B_s \rightarrow \mu^+ \mu^-$ branching fraction is barely affected by $CP$-violating effects, so that its constraints on $|\tilde{\kappa}|$ are still very well approximated by the plain lines in Fig. 3(g) (for some representative $M_A$ values). As a result, just like in the $CP$-conserving case, the net effect of the suppression of $|\tilde{\kappa}|$ and enhancement of $f^-$ for small $M_A$ is quite small. The MSSM with large $\tan \beta$ and MFV is thus not able to account for a large nonstandard phase in $B_s - \bar{B}_s$, (or $B_d - \bar{B}_d$) mixing, if the evidence for such a phase were confirmed. Let us emphasize, however, that the formulation of MFV adopted here does not coincide exactly with the full symmetry-based definition of Ref. [13]. In the formalism of Ref. [13], it was shown recently that new phases could appear in the $\delta_{L}^{12, 23}$ sector, in addition to those in the $\delta_{L}^{13, 23}$ sector [16]. The possible impact of these MFV phases via $\kappa_{q_b}^{-}\kappa_{b_q}$ in $C_{23}^{LR}$ is $a$ priori rather limited due to the $B_s \rightarrow \mu^+ \mu^-$ constraint, yet a more quantitative analysis is desirable.

Beyond MFV, the $C_{23}^{LR}$ contribution is expected to dominate. As indicated before, supersymmetric loop corrections to the Higgs propagator $f^+$ do not bring in any new phases. These can only enter via the quark-Higgs couplings $\kappa_{q_b}$ and $\kappa_{b_q}$. The possible size of $CP$-violating effects generated in this way without violating the existing constraints deserves a study on its own. We will not discuss this further here.

V. CONCLUSIONS

We have studied supersymmetric loop corrections to the MSSM Higgs sector. While the tree-level Higgs sector of the MSSM is a 2HDM of type II, the soft supersymmetry-breaking terms lead to new loop-induced couplings which result in a generic 2HDM with FCNC couplings of neutral-Higgs bosons to quarks, even if the supersymmetry-breaking sector is minimally flavor-violating. The strength of these couplings to $d$-type quarks grows with $\tan \beta$ and precision observables of flavor physics are known to severely constrain large-$\tan \beta$ scenarios of the MSSM. The appropriate tool for such studies is an effective Lagrangian, which is derived by integrating out the heavy supersymmetric particles. The abundant literature on the subject has primarily focused on the flavor-changing Yukawa couplings [1–7]. Among the FCNC quantities, $B - \bar{B}$ mixing plays a special role, because the apparently dominant contribution of Fig. 1 vanishes. Therefore, $B - \bar{B}$ mixing is sensitive to subleading effects, whose systematic study was the main motivation for this paper. Pursuing this goal we
have derived several conceptual and analytic results which can be applied well beyond this topic. They can be classified into three categories:

1. **MSSM Higgs sector.** We have matched the complete MSSM Higgs sector, i.e., both the Yukawa interactions and the Higgs potential, onto an effective 2HDM. Our results for the effective Yukawa couplings are valid for arbitrary $CP$ phases of $\mu$, $a_t$, and the gaugino masses; and (31) and (32) correct the gaugino contributions to $\epsilon_0$ and $\epsilon_Y$ quoted in Ref. [26]. The complete one-loop matching corrections for the quartic Higgs couplings for the most general MSSM are explicitly listed in one place for the first time. This result goes beyond minimal flavor-violation and beyond the large-$\tan\beta$ limit. It is well-known that improper

FIG. 6. Dependence of various quantities on effective couplings or supersymmetric parameters.
choices of the MSSM renormalization scheme can lead to radiative corrections which grow with tan\(\beta\) rendering perturbative results unreliable [31]. At the heart of this problem is the feature that tan\(\beta\) is an ill-defined parameter in the general 2HDM, which permits arbitrary rotations among the two Higgs doublets. In the matching of the MSSM onto the 2HDM this feature enters through the wave-function-renormalization, and we propose an MS renormalization of tan\(\beta\) in the 2HDM which is stable in the limit of large tan\(\beta\). The relation to a DR-renormalized tan\(\beta\) in the MSSM is discussed including electroweak corrections. We identify the places in the effective Higgs potential where physical tan\(\beta\)-enhanced effects occur. The coefficients \(\lambda_2\), \(\lambda_5\), and \(\lambda_7\), which are important for \(B \rightarrow \bar{B}\) mixing, are explicitly specified for the MFV case in (39), (38), and (37). Certain loop corrections to the Higgs potential (\(\lambda_5\), \(\lambda_6\), \(\lambda_7\)) and their impact on tan\(\beta\) and the Higgs-fermion couplings have also been considered in an effective-field-theory framework in Ref. [40], which appeared during completion of this paper. Their results for the \(\lambda\)'s agree with ours for the parameter values considered. As far as the renormalization of tan\(\beta\) is concerned, in Sec. III of the present paper we critically compared with Ref. [40]. In particular, we do find a tan\(\beta\)-enhanced term in the relation of the \(\overline{\text{DR}}\) and DCPR tan\(\beta\) parameters. We stress that, in general, only the former is numerically close to the tan\(\beta\) parameter extracted from \(B\)-physics observables.

2. Large tan\(\beta\) phenomenology. The prime application of our results is \(B \rightarrow \bar{B}\) mixing. We have identified a global \(U(1)\) symmetry of the \(b_{r,q}l_l\) Higgs-mediated FCNC transitions and the tree-level Higgs potential in the large-tan\(\beta\) limit which suppresses the superficially leading contribution of Fig. 1. A systematic study of \(B \rightarrow \bar{B}\) mixing has required the analyses of four subleading contributions, which are governed by the small parameters \(m_{d,s}/m_b\), \(1/\tan\beta\), \(\nu/M_{\text{SUSY}}\) and the loop factor \(1/(4\pi)^2\). These parameters either provide a breaking of the \(U(1)\) symmetry or allow for a contribution proportional to the \(U(1)\)-conserving standard-model effective operator. Prior to this work, only corrections involving \(m_{d,s}/m_b\) had been studied [5] (with the exception of Ref. [26]). The \(\nu/M_{\text{SUSY}}\) corrections are found to be suppressed. The new loop contributions include all nondecoupling SUSY corrections to the quartic Higgs interactions \(\lambda_1-\lambda_7\) and the contribution of neutral-Higgs box diagrams in the effective theory. In the complex MSSM the results for \(\mathcal{F}^\pm\) comprising the neutral-Higgs propagators become cumbersome. We have expressed \(\mathcal{F}^\pm\) in terms of subdeterminants of the neutral-Higgs mass matrix. These expressions are easy to implement and clearly reveal the invariance of the Higgs-mediated amplitudes under rotations of the basis (\(-e\mathbf{H}_H^\dagger H_\nu\)). The results for the Higgs sector are also used to refine the MSSM predictions for the \(B^+ \rightarrow \tau^+\nu\) and the \(B_{s,d} \rightarrow \mu^+\mu^-\) branching ratios. In this context we stress that loop corrections to the Higgs potential do not give rise to additional tan\(\beta\)-enhanced contributions to the charged-Higgs-fermion couplings beyond those known before Ref. [26] appeared. Hence no parametrically large modification of the charged-Higgs contributions to \(B^+ \rightarrow \tau^+\nu\) or \(B \rightarrow X_s\gamma\) relative to Ref. [5] occurs.

While the MSSM corrections to \(B \rightarrow \bar{B}\) mixing in the large tan\(\beta\) scenario could be dominated by the contribution of \(\lambda_5\) and \(\lambda_7\), the size of this piece is limited by the experimental upper bound on \(\mathcal{B}(B_d \rightarrow \mu^+\mu^-)\). After performing an exhaustive analysis of this quantity, \(\mathcal{B}(B \rightarrow X_s\gamma), \mathcal{B}(B^+ \rightarrow \tau^+\nu)\) and the mass of the lightest neutral-Higgs boson \(M_h\), we find that the impact of the corrections to the Higgs potential on \(\Delta M_s\) is always weaker than that of the \(m_t/m_b\) correction identified in Ref. [5]. Assessing the total Higgs-mediated MSSM corrections to \(\Delta M_s\) we find an upper limit of 7% of the SM contribution for \(\mu > 0\) and \(M_A < 600\) GeV. If \(\mu\) is negative, the upper bound is around 20%. This is in contrast with Refs. [26,56], which show large effects of the Higgs potential on \(B \rightarrow \bar{B}\) mixing. The corrections to \(\mathcal{B}(B_q \rightarrow \mu^+\mu^-)\) from the Higgs potential are typically also small, but can reach 15% in some corners of the parameter space. In summary, the correlation between an enhancement of \(\mathcal{B}(B_q \rightarrow \mu^+\mu^-)\) and a (moderate) depletion of \(\Delta M_s\) found in Ref. [5] remains essentially intact.

We finally note that our new contributions can alter the CP phase of the \(B \rightarrow \bar{B}\) mixing amplitude, while the previously known Higgs contribution proportional to \(m_sm_b\mathcal{F}^+\) has the same phase as the SM term (in MFV scenarios). While the maximal possible CP phase is clearly below the sensitivity of the current Tevatron experiments, it is an open question whether future \(B \rightarrow \bar{B}\) mixing experiments can help to unravel the CP structure of the MSSM Higgs potential.

3. Heavy-quark relations and bag parameters. We have transformed the NLO anomalous dimensions computed in Ref. [28] to an operator basis and a renormalization scheme typically used in lattice calculations. The corresponding anomalous dimensions are needed to evaluate the “bag” parameters, which parametrise the hadronic matrix elements, at the electroweak scale. We further employed a heavy-quark relation to improve the numerical prediction of the bag parameter \(B_{1,\text{LL}}^{\text{SLL}}\) entering the SUSY contributions to \(B \rightarrow \bar{B}\) mixing. The heavy-quark relation essentially determines \(B_{1,\text{LL}}^{\text{SLL}}\) in terms of the bag parameter \(B_{1,\text{LL}}\), which is needed for the SM prediction [59]. We found \(\rho_{1,\text{LL}}^{\text{SLL}} = -\frac{9}{5}B_{1,\text{LL}}'(m_t) = -1.36 \pm 0.12\).

Acknowledgments

M.G. and S.J. appreciate helpful conversations with Dominik Stöckinger, Martin Beneke, John Ellis, Gian Giudice, Uli Haisch, Janusz Rosiek, Pietro Slavich, Jure Zupan, and other members of the CERN theory group on
GORBAHN et al.

Various aspects of this work. U.N. appreciates fruitful discussions with Lars Hofer und Dominik Scherer. S.J. acknowledges the hospitality of the University of Karlsruhe. We are grateful to A.J. Buras for comments on the manuscript, and to J.S. Lee and A. Pilafis for communications. This work was supported by the DFG Grant No. NI 1105/1–1, by project C6 of the DFG Research Unit SFB–TR 9 Computergestützte Theoretische Teilchenphysik by the European Union Contract No. MRTN-CT-2006-035482, “FLAVIAnet”, the DFG cluster of excellence “Origin and Structure of the Universe”, and the RTN European Program Grant No. MRTN-CT-2004-503369.

APPENDIX A: NOTATIONS AND CONVENTIONS

To state our phase conventions for $\mu$ and $M_2$ we quote the chargino mass matrix:

$$\mathcal{M}_{\chi^+} = \begin{pmatrix} M_2 & \frac{v \sin \beta}{\sqrt{2}} \\ \frac{v \cos \beta}{\sqrt{2}} & \mu \end{pmatrix}$$  \hspace{1cm} (A1)

with $v = 246$ GeV and the chargino mass term in the Lagrangian

$$\mathcal{L}_{\chi^+}^{\text{mass}} = - (\lambda^+ , \tilde{\eta}_d^0) \mathcal{M}_{\chi^+} (\lambda^+ , \tilde{\eta}_u^0)^T.$$  \hspace{1cm} (A2)

For the case of a general flavor structure of the squark mass matrices we define the trilinear couplings $\tilde{T}_{u,i}$ and $\tilde{T}_{d,i}$ (with flavor indices $i, j$) such that the squark mass matrices read

$$\tilde{M}_{\tilde{q}}^2 = \begin{pmatrix} \hat{M}_{\tilde{u}}^2 & \frac{v \sin \beta}{\sqrt{2}} [\tilde{T}_u - \mu \tilde{Y}_u \cot \beta] \\ \frac{v \cos \beta}{\sqrt{2}} [\tilde{T}_d - \mu^* \tilde{Y}_d \tan \beta] & \hat{M}_{\tilde{d}}^{2*} \end{pmatrix},$$  \hspace{1cm} (A3)

in the super-CKM basis, where the (DR-renormalized) Yukawa matrices are diagonal: $\tilde{Y}_q = \text{diag}(y_{q_1}, y_{q_2}, y_{q_3})$. The mass matrices in (A3) correspond to the squark mass term

$$\mathcal{L}_q^{\text{mass}} = - \Phi_{\tilde{u}}^T \hat{M}_{\tilde{u}}^2 \Phi_{\tilde{u}} - \Phi_{\tilde{d}}^T \hat{M}_{\tilde{d}}^2 \Phi_{\tilde{d}}.$$  \hspace{1cm} (A4)

with $\Phi_{\tilde{u}} = (\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R)^T$ and $\Phi_{\tilde{d}} = (\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R)^T$.

Our sign convention for the MSSM Yukawa couplings implies the following relations between 2HDM quark masses and MSSM Yukawa couplings in the up sector:

$$y_{u_i} = \frac{\sqrt{2}}{v \sin \beta} m_{u_i}.$$  \hspace{1cm} (A5)

In general, the analogous relations in the down sector involve complex phases associated with the $\tan \beta$-enhanced threshold corrections in (22). In particular, Ref. [24], which discusses the complex MSSM for the case without flavor mixing, relates the $b$ Yukawa coupling to the $b$ quark mass as

$$y_{b} = \frac{\sqrt{2}}{v \cos \beta} \frac{m_{b}}{1 + \tilde{c}_3 \tan \beta}.$$  \hspace{1cm} (A6)

rendering $y_{b}$ complex for complex $\tilde{c}_3$ in (33). Our approach of matching the MSSM to an effective 2HDM permits different phase conventions, because the quark fields in the MSSM and the 2HDM can be chosen to differ by a phase factor. We can rephase the $b_R$ superfield of the MSSM in such a way that $y_b$ is real and positive and $1 + \tilde{c}_3 \tan \beta$ in (A6) is replaced by $|1 + \tilde{c}_3 \tan \beta|$. The (physical) phase of $1 + \tilde{c}_3 \tan \beta$ will then, however, appear explicitly in the Higgs and higgsino couplings to bottom (s)quarks. Introducing $3 \times 3$ flavor mixing, the relation between $\tilde{Y}_q$ and $m_d, m_u, m_b$ is found from (22). Now the quark fields in the 2HDM differ from those in the MSSM by a complex rotation in flavor space and a particular choice for the phases of MSSM fields appears less obvious. In particular, one could render all $y_d$ real and positive by suitable rephasings of the right-handed superfields. Note that within MFV $y_b$ is still related to $m_b$ via (A6) in good approximation without such rephasings. The analogous relation for the first two generations reads

$$y_{d,s} = \frac{\sqrt{2}}{v \cos \beta} \frac{m_{d,s}}{1 + \epsilon_0 \tan \beta},$$  \hspace{1cm} (A7)

while in the lepton sector, we have

$$y_{\ell} = \frac{\sqrt{2}}{v \cos \beta} \frac{m_{\ell}}{1 + \epsilon_\ell \tan \beta} \text{ with } \ell = e, \mu, \tau.$$  \hspace{1cm} (A8)

In (most of) the paper we express our results in terms of fermion masses (i.e. avoiding $y_{d,i}$) to achieve formulas which are independent of such phase conventions. Note that the phases of $\epsilon_{d,i}$ and $\epsilon_{y}$ are physical and no phase convention other than that of the CKM matrix matters for $\kappa_{ij}$ in (29) and (30). While the phase convention of $y_{q_i}$ enters the phases in $\tilde{T}_q$, it drops out from the MFV parameters $a_q$ in (A12).

Finally, the quadratic squark soft-breaking terms are defined as follows:

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\[ (\hat{M}^2_{ul})_{ij} = (V^0_{CKM}\hat{m}^2_Q V^0_{CKM})_{ij} + \frac{v^2\sin^2\beta}{2}\delta_{ij}|y_u|^2 + \delta_{ij}M^2_Z\cos2\beta(1/2 - 2\sin^2\theta_W/3), \]
\[ (\hat{M}^2_{dl})_{ij} = (\hat{m}^2_Q)_{ij} + \frac{v^2\cos^2\beta}{2}\delta_{ij}|y_d|^2 + \delta_{ij}M^2_Z\cos2\beta(-1/2 + 2\sin^2\theta_W/3), \]
\[ (\hat{M}^2_{ub})_{ij} = (\hat{m}^2_Q)_{ij} + \frac{v^2\sin^2\beta}{2}\delta_{ij}|y_u|^2 + 2\delta_{ij}M^2_Z\cos2\beta\sin^2\theta_W/3, \]
\[ (\hat{M}^2_{db})_{ij} = (\hat{m}^2_Q)_{ij} + \frac{v^2\cos^2\beta}{2}\delta_{ij}|y_d|^2 - \delta_{ij}M^2_Z\cos2\beta\sin^2\theta_W/3, \]
where \( V^0_{CKM} \) corresponds to the relative rotation of left-handed \( u \)-type and \( d \)-type quark fields performed when diagonalizing the Yukawa matrices. It differs from the actual CKM matrix, defined by the rotations that diagonalize the 2HDM mass matrices rather than the MSSM Yukawa couplings, by loop-suppressed (but \( \tan\beta \)-enhanced) corrections. In particular, within MFV, we have:
\[ V_{CKM,ij} = \begin{cases} 
\frac{1 + \epsilon_i\tan\beta}{1 + \epsilon_j\tan\beta} V^0_{CKM,ij} & \text{for } (i, j) = (u, b), (c, b), (t, d), (t, s), \\
V^0_{CKM,ij} & \text{otherwise.} 
\end{cases} \]  
(A10)

The relations (A10) take a particularly compact form in the (exact) Wolfenstein parametrization, where one has
\[ A = \frac{1 + \epsilon_i\tan\beta}{1 + \epsilon_j\tan\beta} A^0, \quad \lambda = \lambda^0, \quad \rho = \rho^0, \quad \eta = \eta^0. \]  
(A11)

Whenever we consider the case of MFV we write
\[ \hat{T}_{uij} = a_iy_u\delta_{ij}, \quad \hat{T}_{dij} = a_ib\delta_{ij}, \quad \hat{m}^2_Q = \hat{m}^2_Q\delta_{ij}. \]
\[ \hat{m}^2_{uij} = \hat{m}^2_u\delta_{ij}, \quad \hat{m}^2_{dij} = \hat{m}^2_d\delta_{ij}. \]  
(A12)

The \( SU(2) \) relation between \( \hat{M}^2_{ul} \) and \( \hat{M}^2_{dl} \) then implies for the third generation:
\[ M^2_{li} = M^2_{bi} + m^2 - \frac{m^2}{|1 + \epsilon_i\tan\beta|^2} + M^2_Z\cos2\beta(1 - \sin^2\theta_W). \]  
(A13)

In the strict \( SU(2) \) limit (i.e., \( v/M_{SUSY} \to 0 \)) one has \( M^2_{li} = M^2_{bi} \), but for small \( M^2_{bi} \), the term involving \( m^2 \) can be relevant. Also, FCNC \( W-\tilde{u}_L \) loops vanish (for universal \( M^2_{bi} \)) by the GIM mechanism up to the \( m^2 \) term in (A13).

Finally, it is convenient to define the so-called superflavor basis, obtained from a generic electroweak interaction eigenstate basis by rotating the supermultiplets \( Q_L, u_R \) and \( d_R \) such that the quadratic squark soft-breaking terms are diagonal. We denote the corresponding entries by \( \hat{m}^2_{Q_L}, \) \( \hat{m}^2_u, \) and \( \hat{m}^2_d. \) For \( M_{SUSY} \gg v, \) these are just the squark masses, and the computation of the effective couplings \( \lambda_i \) induced by heavy squark loops for arbitrary flavor and \( CP \) structure is greatly simplified. The Yukawa matrices and trilinear terms in this basis are simply written \( Y_{u,d} \) and \( T_{u,d}, \) respectively. They are given in terms of \( \hat{Y}_{u,d} \) and \( \hat{T}_{u,d} \) as follows:

\[ Y^T_{u} = U_a\hat{Y}_{u}V^0_{CKM}V^d_{u}, \quad Y^T_{d} = U_d\hat{Y}_{d}V^d_{d}, \]  
(A14)
\[ T^T_{u} = U_a\hat{T}_{u}V^0_{CKM}V^d_{u}, \quad T^T_{d} = U_d\hat{T}_{d}V^d_{d}, \]  
(A15)

Assuming MFV, one is allowed to choose \( V_d = U_u = U_d = 1. \)

Our conventions comply with the Les Houches accord [62]. In particular, our \( Y_{u,d} \) and \( T_{u,d} \) matrices correspond to a particular choice of the generic \( Y_{u,d} \) and \( T_{u,d} \) matrices of Ref. [62]. Our conventions also agree with those of Ref. [63], except that the sign convention of our \( \hat{Y}_d \) in (A6) is opposite. Besides, our \( \hat{T}_u \) equals \(-A_u \) and our \( \hat{T}_d \) equals \( A_d \) of Ref. [63], respectively.

**APPENDIX B: MATCHING OF THE MSSM ONTO A 2HDM**

The notation of Sec. III distinguishes between the coefficients \( \hat{\lambda}_i \) and \( \hat{\lambda}_i \). The former quantities contain the results from the supersymmetric loop corrections to the quadrilinear Higgs couplings, whose tree-level values are given in (21). The latter coefficients also include the effect of the wave-function and gauge coupling renormalization constants in Appendix B1.

In the following, we summarize the one-loop matching corrections for the quartic Higgs-coupling constants in the general MSSM. While the calculation of loop corrections to the Higgs sector of the MSSM has a long history, see for example [18–24], and a determined reader could extract part of the matching coefficients below from these works, the results collected in this appendix as a service to the
reader are more complex than those in the literature, capturing the effects of the full set of mass, flavor-violation, and CP-violation parameters of the most general MSSM. The effective potential \( V \) in (12) must be used with \( \lambda_i = \tilde{\lambda}_i \) and the relation between \( \tilde{\lambda}_i \) and \( \lambda_i \) is given in (71); the renormalization constants needed in this relation are given in Appendix B1. In the following subsections, we quote the results for \( \tilde{\lambda}_i \) in the general MSSM and decompose \( \tilde{\lambda}_{1-7} \) as

\[
\tilde{\lambda}_i = \lambda_{i \text{tree}} + \lambda_{i \text{ino}} + \lambda_{i \text{form}} \frac{16\pi^2}{\mu_0^2}.
\]

The tree-level values \( \lambda_{i \text{tree}} \) are given in (21), \( \lambda_{i \text{ino}} \) and \( \lambda_{i \text{form}} \), given in Appendices B2 and B3, contain the contributions from higgsino and gaugino loops and from sfermion loops, respectively. Finally we also list the relevant loop functions in Appendix B4. All these results are given in the super-flavor basis including the most general soft-breaking terms.

1. Renormalization constants

The renormalization of \( g^{(\theta)} \) is related only to the field renormalization of \( W \) and \( B \), \( Z_{W,B} = 1 + \delta Z_{W,B} \), if we decouple the sfermionic, higgsino and gaugino contributions: \( \delta g' = -\delta Z_B/2 \) and \( \delta g = -\delta Z_W/2 \). The finite part of the one-loop wave-function renormalization constants of the gauge bosons are

\[
\delta Z_W = -\frac{g^2}{16\pi^2} \frac{1}{6} \left[ 4\log \left| \frac{\mu_0^2}{\mu_i^2} \right| + \left( \frac{M_0^2}{\mu_0^2} + \frac{\mu_i^2}{\mu_0^2} \right) - 4 \right]
\]

\[
\delta Z_B = \frac{g^2}{16\pi^2} \frac{1}{3} \left[ 2\log \left| \frac{\mu_0^2}{\mu_i^2} \right| + \left( \frac{\mu_i^2}{\mu_0^2} + \frac{\mu_0^2}{\mu_i^2} \right) - 2 \right]
\]

while the respective contributions of the gaugino and higgsino loops read:

\[
\delta Z_{dd} = -\frac{1}{32\pi^2} \sum_{ij} \left[ 3B_0'(\tilde{m}_d, \tilde{m}_Q)T_{dj}T_{dj}^* + 3\left| \mu_i^2 \right| B_0'(\tilde{m}_u, \tilde{m}_Q)Y_{uj}Y_{uj}^* + B_0'(\tilde{m}_e, \tilde{m}_l)T_{ej}T_{ej}^* \right]
\]

\[
\delta Z_{ud} = -\frac{1}{16\pi^2} \sum_{ij} \left[ 3\mu_i^2 B_0'(\tilde{m}_d, \tilde{m}_Q)T_{dj}Y_{uj} + 3\left| \mu_i^2 \right| B_0'(\tilde{m}_u, \tilde{m}_Q)Y_{uj}Y_{uj}^* + \mu_i^2 B_0'(\tilde{m}_e, \tilde{m}_l)T_{ej}Y_{ej}^* \right]
\]

\[
\delta Z_{uu} = -\frac{1}{32\pi^2} \sum_{ij} \left[ 3B_0'(\tilde{m}_u, \tilde{m}_Q)T_{uj}T_{uj}^* + 3\left| \mu_i^2 \right| B_0'(\tilde{m}_d, \tilde{m}_Q)Y_{uj}Y_{uj}^* + \left| \mu_i^2 \right| B_0'(\tilde{m}_e, \tilde{m}_l)Y_{ej}Y_{ej}^* \right]
\]

2. Higgsino-gaugino contributions to \( \lambda_1-\lambda_7 \)

The situation of \( \tilde{\lambda}_5 = \lambda_5 \) is particularly simple: The matching correction only involves the box function and \( \lambda_{5 \text{ino}} \) can be written in a compact form:

\[
\lambda_{5 \text{ino}} = 3g^4 \mu^2 M_2^2 D_0(\langle M_1 \rangle, \langle |\mu| \rangle) + 2g^2 g^2 \mu^2 M_1 M_2 D_0(\langle M_1 \rangle, \langle |\mu| \rangle) + g^6 \mu^2 M_1^2 D_0(\langle M_1 \rangle, \langle |\mu| \rangle),
\]

if we use the loop functions defined in Appendix B4.

We find for \( \lambda_{i \text{ino}} = \lambda_{1 \text{ino}}, \ldots, \lambda_{3 \text{ino}}, \lambda_{4 \text{ino}}, \lambda_{5 \text{ino}} \):

\[
\lambda_{i \text{ino}} = g_i^2(a_+ + a_2 \tilde{D}_2(\langle M_1 \rangle, \langle |\mu| \rangle) + a_4 \tilde{D}_4(\langle M_1 \rangle, \langle |\mu| \rangle)) + g^2 g^2 (a'_+ + a'_2 \tilde{D}_2(\langle M_1 \rangle, \langle |\mu| \rangle) + a'_4 \tilde{D}_4(\langle M_1 \rangle, \langle |\mu| \rangle)),
\]

where the coefficients \( a_+, \ldots, a'_4 \) depend on the index \( i \) labeling \( \lambda_i \) in (B1), which we suppress throughout this appendix. The coefficients \( a^{(\theta)}_{2,4} \) are given in Table IV, while
TABLE IV. Coefficients entering $\lambda^{\text{ino}}_1$, $\lambda^{\text{ino}}_2$, $\lambda^{\text{ino}}_3$, and $\lambda^{\text{ino}}_4$ in Eq. (B6).

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$a_2$</th>
<th>$a_4$</th>
<th>$a'_2$</th>
<th>$a'_4$</th>
<th>$a''_2$</th>
<th>$a''_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>$\frac{1}{2}[M_2^2]$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(M_1^2 + M_2^2)$</td>
<td>$1$</td>
<td>$\frac{1}{2}[M_1^2]$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>$\frac{1}{2}[M_2^2]$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(M_1^2 + M_2^2)$</td>
<td>$1$</td>
<td>$\frac{1}{2}[M_1^2]$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>$3</td>
<td>\mu</td>
<td>^2 + \frac{1}{2}[M_2^2]$</td>
<td>$\frac{1}{2}$</td>
<td>$2</td>
<td>\mu</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>$-3</td>
<td>\mu</td>
<td>^2 - 2[M_2^2]$</td>
<td>$2$</td>
<td>$2</td>
<td>\mu</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>$3\mu M_2$</td>
<td>$0$</td>
<td>$\mu(M_1 + M_2)$</td>
<td>$0$</td>
<td>$\mu M_1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\lambda_6$</td>
<td>$3\mu M_2$</td>
<td>$0$</td>
<td>$\mu(M_1 + M_2)$</td>
<td>$0$</td>
<td>$\mu M_1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

where the slepton contribution is contained in $d^{ijkl}_4$ listed in Table V and $d^{ijkl}_{2-4}$ comprises the squark contribution (Table VI). Only $\lambda_4$ receives a contribution from $d^{ijkl}_4$:

$$d^{ijkl}_4 = -3(T_{u_{ij}}T_{u_{ij}}^* - |\mu|^2 Y_{u_{ij}} Y_{u_{ij}}^*) \times (T_{u_{ij}}T_{d_{ij}} - |\mu|^2 Y_{u_{ij}} Y_{d_{ij}}^*)$$

(9)

while $d^{ijkl}_4 = 0$ for $\lambda_i$ with $i \neq 4$. The contributions to the matching coefficients depend on the Yukawa couplings $Y_{e_{e,d}}$ of the charged leptons, the up-type quarks, and the down-type quarks as well as on the trilinear soft-breaking terms $T_{e_{u,d}}$, defined in the superflavor basis (see Appendix A).

We write $\lambda^{\text{ferm}}_{i-4} = \lambda^{\text{ferm}}_{i-4} + \lambda^{\text{ferm}}_{i-4}^s$, and find for the slepton contribution

$$\lambda^{\text{ferm}}_{i-4} = (b_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_2 Y_{e_{e}}, T_{ij}, b_3 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_e, \tilde{m}_i)$$

+(b_4 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_5 Y_{e_{e}}, T_{ei}, b_6 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_i, \tilde{m}_e)$$

+(c_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_2 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i)$$

+(c_3 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_4 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_i, \tilde{m}_e, \tilde{m}_i)$$

+(c_5 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_6 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$$

+(c_7 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_8 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_i, \tilde{m}_i, \tilde{m}_i)$$

+d^{ijkl}_1 B_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$

(10)

while the squark contribution reads:

3. Sfermion contributions to $\lambda_{1-7}$

The sfermion contribution to $\lambda_{1-7}$ are products of loop functions and flavor dependent coefficients if we sum over the generation index of the internal sfermions. For $\lambda_5$, our results then take the simple form

$$\lambda_5^\text{sferm} = d^{ijkl}_1 D_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$$

$$+ d^{ijkl}_2 D_0(\tilde{m}_d, \tilde{m}_d, \tilde{m}_Q, \tilde{m}_Q)$$

$$+ d^{ijkl}_3 D_0(\tilde{m}_Q, \tilde{m}_Q, \tilde{m}_u, \tilde{m}_u)$$

(11)

where the slepton contribution is contained in $d^{ijkl}_4$ listed in Table V and $d^{ijkl}_{2-4}$ comprises the squark contribution (Table VI). Only $\lambda_4$ receives a contribution from $d^{ijkl}_4$:

$$d^{ijkl}_4 = -3(T_{u_{ij}}T_{u_{ij}}^* - |\mu|^2 Y_{u_{ij}} Y_{u_{ij}}^*) \times (T_{u_{ij}}T_{d_{ij}} - |\mu|^2 Y_{u_{ij}} Y_{d_{ij}}^*)$$

(9)

while $d^{ijkl}_4 = 0$ for $\lambda_i$ with $i \neq 4$. The contributions to the matching coefficients depend on the Yukawa couplings $Y_{e_{e,d}}$ of the charged leptons, the up-type quarks, and the down-type quarks as well as on the trilinear soft-breaking terms $T_{e_{u,d}}$, defined in the superflavor basis (see Appendix A).

We write $\lambda^{\text{ferm}}_{i-4} = \lambda^{\text{ferm}}_{i-4} + \lambda^{\text{ferm}}_{i-4}$, and find for the slepton contribution

$$\lambda^{\text{ferm}}_{i-4} = (b_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_2 Y_{e_{e}}, T_{ij}, b_3 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_e, \tilde{m}_i)$$

+(b_4 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_5 Y_{e_{e}}, T_{ei}, b_6 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_i, \tilde{m}_e)$$

+(c_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_2 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i)$$

+(c_3 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_4 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_i, \tilde{m}_e, \tilde{m}_i)$$

+(c_5 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_6 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$$

+(c_7 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_8 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_i, \tilde{m}_i, \tilde{m}_i)$$

+d^{ijkl}_1 B_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$

(10)

while the squark contribution reads:

$$\lambda^{\text{ferm}}_{i-4} = (b_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_2 Y_{e_{e}}, T_{ij}, b_3 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_e, \tilde{m}_i)$$

+(b_4 T_{ij}, T_{ei}, T_{ei}, T_{ei}, b_5 Y_{e_{e}}, T_{ei}, b_6 Y_{e_{e}}, Y_{e_{e}}) B_0(\tilde{m}_i, \tilde{m}_e)$$

+(c_1 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_2 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i)$$

+(c_3 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_4 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_i, \tilde{m}_e, \tilde{m}_i)$$

+(c_5 T_{ij}, T_{ei}, T_{ei}, T_{ei}, c_6 T_{ei}, T_{ei}, T_{ei}, T_{ei}) C_0(\tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$$

+d^{ijkl}_1 B_0(\tilde{m}_e, \tilde{m}_e, \tilde{m}_i, \tilde{m}_i)$$

(10)
Here, we introduced shorthand notations for the products of two Yukawa coupling matrices:

\[ Y_{xyij} = Y_{xj}^\dagger Y_{yi}, \quad \tilde{Y}_{xyij} = Y_{xi}^\dagger Y_{yi}^\dagger \]  

(B12)

where \( x, y = e, u, d \) and we sum over the internal index \( l \). The coefficients \( b_n, c_n \), and \( d_n^{ijkl} \) for each \( \lambda_1, \ldots, \lambda_4 \) are given in Tables V, VI, VIII, and IX.

We finally give the slepton and squark contributions to \( \lambda_{6,7} \):

\[ \lambda^{\text{Sferm}}_{6,7} = (c_1' \mu T_{e_i} Y_{e_i}^* \delta_{ij} + c_2' \mu T_{e_i} Y_{e_i'} Y_{e_i'} C_0(\bar{m}_{e_i}, \bar{m}_{e_i'}, \mu_{11}, \mu_{11}) + (c_3' \mu T_{e_i} Y_{e_i}^* \delta_{ij} + c_4' \mu T_{e_i} Y_{e_i'} Y_{e_i'} C_0(\bar{m}_{e_i}, \bar{m}_{e_i'}, \mu_{11}, \mu_{11})) \]

\[ + (c'_1 \mu T_{d_i} Y_{d_i}^* \delta_{ij} + c'_2 \mu T_{d_i} Y_{d_i'} Y_{d_i'} C_0(\bar{m}_{d_i}, \bar{m}_{d_i'}, \mu_{11}, \mu_{11}) + (c'_3 \mu T_{d_i} Y_{d_i}^* \delta_{ij} + c'_4 \mu T_{d_i} Y_{d_i'} Y_{d_i'} C_0(\bar{m}_{d_i}, \bar{m}_{d_i'}, \mu_{11}, \mu_{11})) \]

\[ + (c'_1 \mu T_{u_i} Y_{u_i}^* \delta_{ij} + c'_2 \mu T_{u_i} Y_{u_i'} Y_{u_i'} C_0(\bar{m}_{u_i}, \bar{m}_{u_i'}, \mu_{11}, \mu_{11}) + (c'_3 \mu T_{u_i} Y_{u_i}^* \delta_{ij} + c'_4 \mu T_{u_i} Y_{u_i'} Y_{u_i'} C_0(\bar{m}_{u_i}, \bar{m}_{u_i'}, \mu_{11}, \mu_{11})) \]

\[ + d_1^{ijkl} D_0(\bar{m}_{e_i}, \bar{m}_{e_i'}, \bar{m}_{u_i}, \bar{m}_{u_i}) + d_2^{ijkl} D_0(\bar{m}_{d_i}, \bar{m}_{d_i'}, \bar{m}_{d_i}, \bar{m}_{d_i'}) + d_3^{ijkl} D_0(\bar{m}_{Q_i}, \bar{m}_{Q_i'}, \bar{m}_{u_i}, \bar{m}_{u_i}) \]  

(B13)

where the coefficients \( c_n' \) and \( d_n^{ijkl} \) are given in Tables V, VI, and VII.

4. Loop functions

In the UV-divergent loop functions we set \( \epsilon = (4 - D)/2 \). The loop functions are defined as

\[ \frac{i}{(4\pi)^2} A_0(m_1) \left( \frac{4\pi}{\mu_0} e^{-\gamma_0} \right)^\epsilon = \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m_1^2}, \]

\[ \frac{i}{(4\pi)^2} B_0(k^2; m_1, m_2) \left( \frac{4\pi}{\mu_0} e^{-\gamma_0} \right)^\epsilon = \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q + k)^2 - m_1^2} \frac{1}{q^2 - m_2^2}, \]

\[ \frac{i}{(4\pi)^2} C_0(m_1, m_2, m_3) \left( \frac{4\pi}{\mu_0} e^{-\gamma_0} \right)^\epsilon = \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m_1^2} \frac{1}{q^2 - m_2^2} \frac{1}{q^2 - m_3^2}, \]

\[ \frac{i}{(4\pi)^2} D_0(m_1, m_2, m_3, m_4) \left( \frac{4\pi}{\mu_0} e^{-\gamma_0} \right)^\epsilon = \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m_1^2} \frac{1}{q^2 - m_2^2} \frac{1}{q^2 - m_3^2} \frac{1}{q^2 - m_4^2}, \]

\[ \frac{i}{(4\pi)^2} W(m_1, m_2) \left( \frac{4\pi}{\mu_0} e^{-\gamma_0} \right)^\epsilon = -\frac{d}{dk^2} \left| \int_{k^2=0}^{d} \frac{d^D q}{(2\pi)^D} \text{Tr}((q - k)(q^2 - m_1^2)(q^2 - m_2^2)), \right. \]

(B14)

and we also write

\[ B_0(m_1^2, m_2^2) = B_0(0; m_1^2, m_2^2), \quad B'_0(m_1^2, m_2^2) = -\frac{d}{dk^2} \left| \int_{k^2=0}^{d} B_0(k^2; m_1^2, m_2^2). \right. \]

(B15)

These functions read:
\[ A_0(m_1) = \frac{m_1^2}{\epsilon} + m_1^3 + m_1^4 \log \left( \frac{\mu_0^2}{m_1^2} \right) \]
\[ B_0(m_1, m_2) = \frac{m_2^2}{\epsilon} + m_2^3 + m_2^4 \log \left( \frac{\mu_0^2}{m_2^2} \right) \]
\[ B'_0(m_1, m_2) = \frac{m_1^2 - m_2^2 + 2m_2^2 \log \left( \frac{m_2^2}{m_1^2} \right)}{(2m_1^2 - m_2^2)^2} \]
\[ C_0(m_1, m_2, m_3) = m_2^2 m_3^2 \log \left( \frac{m_2^2}{m_3^2} \right) + m_2^3 m_3^2 \log \left( \frac{m_2^2}{m_3^2} \right) + m_2^3 m_3^2 \log \left( \frac{m_2^2}{m_3^2} \right) \]
\[ D_0(m_1, m_2, m_3, m_4) = \sum_{\{a, b, c, d\}} a^2 b c \log \left( \frac{\mu^2}{\mu_0^2} \right) + a b^2 c \log \left( \frac{\mu^2}{\mu_0^2} \right) + a b c^2 \log \left( \frac{\mu^2}{\mu_0^2} \right) \]
\[ W(m_1, m_2) = \frac{2}{\epsilon} + 2 \log \left( \frac{\mu_0^2}{m_1^2} \right) \log \left( \frac{m_2^2}{m_1^2} \right) \left[ 2m_2^2 - 6m_1^2 \right] + \frac{m_1^4 - 6m_1^2 m_2^2 + m_2^4}{(m_1^2 - m_2^2)^2} \]
\[ \tilde{D}_2(m_1, m_2, m_3, m_4) = C_0(m_1, m_2, m_3, m_4) + m_2^3 D_0(m_1, m_2, m_3, m_4) \]
\[ \tilde{D}_4(m_1, m_2, m_3, m_4) = B_0(m_1, m_4) + (m_1^2 + m_2^2) C_0(m_2, m_3, m_4) + m_1^2 D_0(m_1, m_2, m_3, m_4) \]

A further loop function, \( H_2 \), is defined in (34).

**APPENDIX C: RENORMALIZATION-GROUP AND BAG PARAMETERS**

The standard-model contribution to \( B - \bar{B} \) mixing involves the operator \( \bar{Q}^{\text{YLL}}_1 = (\bar{b}_L \gamma_\mu q_L) (\bar{b}_L \gamma_\mu q_L) \) of (6). The main new supersymmetric contribution to \( B - \bar{B} \) mixing presented in this paper comes with the four-quark operator \( Q^{\text{SLL}}_1 = (\bar{b}_R q_L) (\bar{b}_R q_L) \) with \( q = d \) or \( q = s \), see (2). \( \bar{Q}^{\text{SLL}}_1 \) mixes under renormalization with

\[ \bar{Q}^{\text{SLL}}_1 = (\bar{b}_R q_L) (\bar{b}_R q_L) \]

where \( i, j \) are color indices. The operators \( Q^{\text{SLL}}_1 \) and \( \bar{Q}^{\text{SLL}}_1 \) are widely studied in the context of the width difference \( \Delta \Gamma \) among the two mass eigenstates in the \( B - \bar{B} \) mixing system and the CP asymmetry \( a_{fs} \) in flavor-specific decays [64,65].

Yet the next-to-leading-order (NLO) anomalous dimensions have been calculated for an equivalent operator basis in Ref. [28]. These operators,

**TABLE VII.** Coefficients of \( \lambda_{\text{SLL}} \) and \( \lambda_{\text{SLL}} \) in Eq. (B13).

<table>
<thead>
<tr>
<th>( \lambda_{\text{SLL}} )</th>
<th>( \lambda_{\text{SLL}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>( -\frac{g^2}{2} )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>( \frac{1}{3} (g^2 - 3g^2) )</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>( \frac{1}{3} (g^2 + 3g^2) )</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>( -\frac{g^2}{2} )</td>
</tr>
<tr>
<td>( c_6 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( c_7 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( c_8 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( c_9 )</td>
<td>( \frac{1}{3} (3g^2 - g^2) )</td>
</tr>
<tr>
<td>( c_{10} )</td>
<td>( \frac{1}{3} (3g^2 - g^2) )</td>
</tr>
<tr>
<td>( c_{11} )</td>
<td>( g^2 )</td>
</tr>
<tr>
<td>( c_{12} )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

**TABLE VIII.** Slepтон loop contributions to \( \lambda_{\text{SLL}} \) and \( \lambda_{\text{SLL}} \) in Eq. (B10).

<table>
<thead>
<tr>
<th>( \lambda_{1} )</th>
<th>( \lambda_{2} )</th>
<th>( \lambda_{3} )</th>
<th>( \lambda_{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>( -\frac{g^2}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_2 )</td>
<td>( g^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_3 )</td>
<td>( -1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_4 )</td>
<td>( \frac{1}{2} (g^4 - g^6) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_5 )</td>
<td>( \frac{1}{8} (g^4 - g^6) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_6 )</td>
<td>( 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_1 )</td>
<td>( 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_2 )</td>
<td>( g^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_3 )</td>
<td>( 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_4 )</td>
<td>( -\frac{g^2}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_5 )</td>
<td>( 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_6 )</td>
<td>( \frac{1}{2} (g^2 - g^4) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_7 )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( c_8 )</td>
<td>( -2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_{10} )</td>
<td>( \frac{1}{2} (g^2 - g^4) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_{11} )</td>
<td>( g^2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_{12} )</td>
<td>( 0 )</td>
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</table>
TABLE IX. Squark loop contributions to $\lambda^i_{\text{form}} \ldots \lambda^4_{\text{form}}$ in Eq. (B11).

<table>
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<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
</tr>
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<td>$b_7$</td>
<td>$-\frac{e^a}{\pi^2}$</td>
<td>$-\frac{e^a}{\pi^2}$</td>
<td>0</td>
</tr>
<tr>
<td>$b_8$</td>
<td>$g^a$</td>
<td>0</td>
<td>$-\frac{e^a}{\pi^2}$</td>
</tr>
<tr>
<td>$b_9$</td>
<td>$-3$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b_{10}$</td>
<td>0</td>
<td>0</td>
<td>$-3$</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>$-\frac{e^a}{\pi^2}$</td>
<td>$-\frac{e^a}{\pi^2}$</td>
<td>$\frac{e^a}{\pi^2}$</td>
</tr>
<tr>
<td>$b_{12}$</td>
<td>0</td>
<td>$2g^a$</td>
<td>$-g^a$</td>
</tr>
<tr>
<td>$b_{13}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b_{14}$</td>
<td>$\frac{1}{ \pi^2} (-9g^a - g^b)$</td>
<td>$\frac{1}{ \pi^2} (-9g^a - g^b)$</td>
<td>$\frac{1}{ \pi^2} (9g^a + g^b)$</td>
</tr>
<tr>
<td>$b_{15}$</td>
<td>$\frac{1}{2} (3g^2 + g^b)$</td>
<td>0</td>
<td>$\frac{1}{2} (-3g^2 - g^b)$</td>
</tr>
<tr>
<td>$b_{16}$</td>
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<td>$\frac{1}{2} (g^2 - 3g^b)$</td>
</tr>
<tr>
<td>$b_{17}$</td>
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<td>0</td>
</tr>
<tr>
<td>$b_{18}$</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$c_9$</td>
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<td>$-g^a$</td>
<td>$\frac{e^a}{\pi^2}$</td>
</tr>
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<tr>
<td>$c_{11}$</td>
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<td>0</td>
<td>$-3$</td>
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<td>$c_{12}$</td>
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<td>0</td>
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<tr>
<td>$c_{13}$</td>
<td>0</td>
<td>$\frac{1}{2} (3g^2 + g^b)$</td>
<td>$\frac{1}{2} (3g^2 + g^b)$</td>
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<tr>
<td>$c_{14}$</td>
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<td>0</td>
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<tr>
<td>$c_{15}$</td>
<td>$\frac{1}{2} (3g^2 + g^b)$</td>
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<td>$c_{16}$</td>
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<tr>
<td>$c_{19}$</td>
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<td>$\frac{1}{2} (3g^2 - g^b)$</td>
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<td>$\frac{1}{2} (g^2 - 3g^b)$</td>
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<td>$g^a$</td>
</tr>
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</tr>
<tr>
<td>$c_{26}$</td>
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<td>$2g^a$</td>
<td>$-g^a$</td>
</tr>
<tr>
<td>$c_{27}$</td>
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<td>$-6$</td>
<td>0</td>
</tr>
</tbody>
</table>

enter the NLO results as counterterms. In particular for the SLL sector, the evanescent operators of Ref. [28] read:

\[
E^{\text{SLL}}_1 = (\bar{b}_R q_L^i) (\bar{b}_R q_L^i) + \frac{1}{2} Q^{\text{SLL}}_1 - \frac{1}{8} Q^{\text{SLL}}_2,
\]

\[
E^{\text{SLL}}_2 = -(\bar{b}_R^c \sigma_{\mu\nu} q_L^i) (\bar{b}_R^c \sigma_{\mu\nu} q_L^i) + 6 Q^{\text{SLL}}_1 - \frac{1}{2} Q^{\text{SLL}}_2,
\]

\[
E^{\text{SLL}}_3 = (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) + (-64 + 96 \epsilon) Q^{\text{SLL}}_1 + (-16 + 8 \epsilon) Q^{\text{SLL}}_2,
\]

\[
E^{\text{SLL}}_4 = (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) - 64 Q^{\text{SLL}}_1 + (-16 + 16 \epsilon) Q^{\text{SLL}}_2,
\]

The operator basis

\[
Q^{\text{SLL}}_1 = \bar{Q}^{\text{SLL}}_1, \quad Q^{\text{LR}}_1 = \bar{Q}^{\text{LR}}_1, \quad Q^{\text{LR}}_2 = \bar{Q}^{\text{LR}}_2,
\]

\[
Q^{\text{SLL}}_2 = \bar{Q}^{\text{SLL}}_2, \quad Q^{\text{SLL}}_4 = (\bar{b}_R q_L^i) (\bar{b}_R q_L^i), (C4)
\]

which we adopt in this work agrees with the one of (C2) except for the SLL sector and the SRR sector. The evanescent operators are defined as in Refs. [59,64]:

\[
E^{\text{SLL}}_1 = (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) + 8(1-\epsilon) Q^{\text{SLL}}_1,
\]

\[
E^{\text{SLL}}_2 = (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) (\bar{b}_R^c \gamma_\mu \gamma_\rho \gamma_\sigma q_L^i) + 8(1-\epsilon) Q^{\text{SLL}}_1.
\]

The hadronic matrix elements in this basis are parametrized in terms of “bag” parameters $B^{\text{SLL}}_1$, $B^{\text{SLL}}_L$, and $B^{\text{SLL}}_R$ defined as

\[
\langle B_q | Q^{\text{SLL}}_1 (\mu) | B_q \rangle = \frac{2}{3} M^{\text{B}}_{R} f_{R} B^{\text{SLL}}_1 (\mu),
\]

\[
\langle B_q | Q^{\text{SLL}}_2 (\mu) | B_q \rangle = -\frac{5}{12} M^{\text{B}}_{R} f_{R}^2 B^{\text{SLL}}_2 (\mu),
\]

\[
\langle B_q | \bar{Q}^{\text{SLL}}_1 (\mu) | B_q \rangle = \frac{1}{12} M^{\text{B}}_{R} f_{R}^2 B^{\text{SLL}}_R (\mu).
\]

Here, $\mu$ is the renormalization scale at which the matrix element is computed and $f_{B_q}$ is the $B_q$ meson decay constant. While $f_{B_q}$ exceeds $f_{B_R}$ by 10–30%, no nonperturbative calculation finds any dependence of a bag parameter on the flavor of the light valence quark. In the vacuum insertion approximation the bag parameters equal $B^{\text{SLL}}_1(\mu) = 1$ and $B^{\text{SLL}}_L(\mu) = B^{\text{SLL}}_R(\mu) = M^{\text{B}}_{R}/[m_b(\mu) + m_q(\mu)]^2$. Lattice computations determine the matrix elements at a low scale around 1 GeV and results are quoted for $\mu = m_{b}(m_{b})$. In order to use the lattice results in our calculation, we need the renormalization-group (RG) evolution of the bag parameters to the high-scale $\mu_R$ which is set by the masses of the Higgs bosons exchanged in our $B - \bar{B}$ mixing diagrams. The matrix elements computed on a finite lattice are converted to continuum QCD by a matching calculation. This lattice-continuum matching is only meaningful beyond the leading-order of perturbative QCD. Thus, the dependence of the bag parameters on the chosen (continuum) renormalization scheme must be addressed: The NLO anomalous dimension matrices entering the RG evolution must be defined in the same renormalization scheme as the bag parameters, so that the scheme dependence properly cancels from physical observables. The NLO anomalous dimensions have been calculated for $Q^{\text{SLL}}_1$ in Ref. [66]. As said previously, in the case of $(Q^{\text{SLL}}_1, Q^{\text{SLL}}_2)$ the NLO anomalous dimensions have been calculated for the equivalent operator basis $(\bar{Q}^{\text{SLL}}_1, \bar{Q}^{\text{SLL}}_2)$ with the evanescent operators of (C3) [28].

The purpose of this section is twofold: First, we present the transformation of the results of Ref. [28] to the
\[ 
\gamma = \frac{\alpha_s(\mu)}{4\pi} \gamma^{(0)} + \left( \frac{\alpha_s(\mu)}{4\pi} \right)^2 \gamma^{(1)} + O(\alpha_s^3). \]  
(C7)

The NLO correction \( \gamma^{(1)} \) has been computed for the basis \((Q_1^{SLL}, Q_2^{SLL})\) in Ref. [28]. In four dimensions it is related to the basis (C4) by a simple Fierz identity:

\[ 
\tilde{Q} = \left( \frac{Q_1^{SLL}}{Q_2^{SLL}} \right)^D \hat{R} \left( \frac{Q_1^{SLL}}{Q_2^{SLL}} \right) = \hat{R} \tilde{Q}. \]  
(C8)

where \( \hat{R} \) is given in Eq. (C10) below.

Yet in \( D \) dimensions our change of basis involves a rotation of the operator basis—including the evanescent operators \( \tilde{E} = (E_1^{SLL}, E_2^{SLL})^T \)—and a change of the renormalization scheme. We follow Ref. [67] and write the rotation as\(^{14}\)

\[ 
\tilde{Q} = \hat{R} (\tilde{Q} + \tilde{W} \tilde{E}), \quad \tilde{E} = \hat{M} (\epsilon \tilde{U} \tilde{Q} + [1 + \epsilon \tilde{U} \tilde{W}] \tilde{E}), \]  
(C9)

with

\[ 
\hat{R} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{1}{8} \end{pmatrix}, \quad \tilde{W} = \begin{pmatrix} 0 & 0 \\ 8 & 0 \end{pmatrix},
\]

\[ 
\tilde{U} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{8} \\ \frac{8}{8} & -\frac{1}{4} \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} -8 & 0 \\ -4 & 1 \end{pmatrix}. \]  
(C10)

The information on the definition of the evanescent operators in Eqs. (C3) and (C4) is contained in the matrices \( \tilde{U} \) and \( \hat{M} \). Now Eq. (C9) corresponds to a finite renormalization with renormalization constants [67]

\[ 
\gamma^{(1)} = \hat{R} \chi^{(0)} \hat{R}^{-1}, \]  
(C12)

with the one-loop anomalous dimension matrix just rotated, the two-loop anomalous dimension matrix undergoes an additional scheme transformation:

\[ 
\chi^{(1)} = \hat{R} \chi^{(1)} \hat{R}^{-1} - [\hat{R} \chi^{(0)} \hat{R}^{-1} - 2\beta^{(0)}] \hat{R} \chi^{(0)} \hat{R}^{-1}, \]  
(C13)

We can now calculate the new two-loop anomalous dimension matrix \( \gamma \) from the NLO anomalous dimension matrix \( \chi \) of Ref. [28],

\[ 
\chi^{(1)} = \begin{pmatrix} -10 & 0.5 \\ -40 & 2.5 \end{pmatrix}, \]  
(C14)

\[ 
\chi^{(2)} = \begin{pmatrix} -1459 & 27 \\ -6332 & 12 \end{pmatrix}, \]  
(C15)

We obtain

\[ 
\gamma^{(0)} = \begin{pmatrix} -28 & 3 \\ 16 & 1 \end{pmatrix}, \quad \gamma^{(1)} = \begin{pmatrix} -260 & 88 \\ 242 & -76 \end{pmatrix}, \]  
(C16)

Here, \( f \) denotes the number of active flavors and \( \gamma^{(0)} \) coincides with the result in [64]. As a check, we have calculated the result of Eq. (C15) also in a different way: It is possible to define evanescent operators such that the Fierz identity holds for the one-loop matrix elements. This choice fixes the definitions of both \( E_1^{SLL} \) and \( E_2^{SLL} \) in Eq. (C5) and of the evanescent operators on the \((Q_1^{SLL}, Q_2^{SLL})\) basis. (One of the latter operators equals \( \epsilon \) times a physical operator. Its impact is equivalent to a finite multiplicative renormalization of \( Q_1^{SLL} \).) In this approach, one can simply rotate \( \gamma^{(1)} \) in the same way as \( \gamma^{(0)} \) in Eq. (C12). Finally, the result is transformed to the scheme of Ref. [28] using the scheme transformation formula of Ref. [68].

Next, we calculate the matrices governing the RG evolution in the \((Q_1^{SLL}, Q_2^{SLL})\) basis. The bag factors at the scale \( \mu_b \) are obtained from those at the low scale \( \mu_b = \tilde{O}(m_b) \) via

\[ 
\left( \begin{array}{c} -5B_1^{SLL}(\mu_b) \\ B_1^{SLL}(\mu_b) \end{array} \right) = U(\mu_b, \mu_b)^T \left( \begin{array}{c} -5B_1^{SLL}(\mu_b) \\ B_1^{SLL}(\mu_b) \end{array} \right) \]  
(C16)
In the spirit of [29], we write the evolution matrix as
\[ U(\mu_b, \mu_b) = U^{(0)}(\alpha_s(\mu_b)) \frac{\alpha_s(\mu_b)}{4\pi} \Delta U(\alpha_s(\mu_b)) \]
where \( U^{(0)} \) is the LO evolution matrix and the NLO correction reads
\[ \Delta U(\eta) = J_f U^{(0)}(\eta) - \eta U^{(0)}(\eta) J_f. \]  
(C18)

The 2 \times 2 matrix \( J_f \) is calculated from the anomalous dimension matrix \( \gamma \) [69]. We only need \( J_f \), since we run with 5 active flavors to the scale \( \mu_b \). For applications in kaon physics, one also involves \( J_4 \) and \( J_3 \). We quote all three matrices here, so that the formulas of Ref. [29] can be easily extended to the \( \langle Q_1^{SLL}, \bar{Q}_1^{SLL} \rangle \) basis:
\[ \Delta U_{J-f}(\eta) = \left( \begin{array}{cc} 1.4040 - 1.3707 \eta & -0.3680 - 2.0731 \eta \\ 0.6454 + 0.0898 \eta & -0.1692 + 0.1358 \eta \end{array} \right) \eta^{0.6315} + \left( \begin{array}{cc} 0.0704 - 0.1037 \eta & 1.0746 + 1.3665 \eta \\ -0.3395 - 0.3958 \eta & -5.1807 + 5.2141 \eta \end{array} \right) \eta^{0.7184}. \]  
(C21)

In our numerical analysis we drop the terms which are linear in \( \eta \) in the two matrices in Eq. (C21), because they are scheme-dependent. The scheme dependence of these terms cancels with that of the NLO QCD corrections to the \( B \rightarrow \bar{B} \) mixing diagrams with SUSY Higgs exchange. Yet these QCD corrections are unknown.

2. Hadronic matrix elements and heavy-quark relations

The three bag factors \( B_{VLL}^{SLL}(\mu_b) \), \( B_{VLL}^{SLL}(\mu_b) \), and \( \bar{B}_{VLL}^{SLL}(\mu_b) \) obey a heavy-quark relation [65]:
\[ B_{VLL}^{SLL}(\mu_b) = \frac{4}{5} \alpha_2(\mu_b) B_{VLL}^{SLL} + \frac{1}{5} \alpha_1(\mu_b) \bar{B}_{VLL}^{SLL} + \mathcal{O}(\Lambda_{QCD}/m_b). \]  
(C22)

Here \( \alpha_1(\mu) \) and \( \alpha_2(\mu) \) comprise NLO QCD corrections [59,64]:
\[ \alpha_1(\mu_b) = 1 + \frac{\alpha_s(\mu_b)}{4\pi} \left( 16 \log \frac{\mu_b}{m_b} + 8 \right). \]  
(C23)
\[ \alpha_2(\mu_b) = 1 + \frac{\alpha_s(\mu_b)}{4\pi} \left( 8 \log \frac{\mu_b}{m_b} + 26 \right). \]

These values are specific to the definition of the evanescent operators as in Eq. (C5). As mentioned in Appendix C1, this definition allows to maintain the validity of Fierz identities at the loop level. Such a definition is preferred, if the bag factors are meant to parametrize the deviation of matrix elements from the vacuum insertion approximation (VIA), because the calculation of matrix elements in VIA approximation involves a Fierz transformation. In particular the choice in Eq. (C5) is crucial for Eq. (C22) to hold in the limit of a large number \( N_C \) of colors [64].

\[ J_5 = \left( \begin{array}{cc} 1.474 & 0.707 \\ 0.306 & -5.350 \end{array} \right), \quad J_4 = \left( \begin{array}{cc} 0.964 & 1.452 \\ 0.375 & -4.982 \end{array} \right). \]  
(C19)

We quote handy formulas for the five-flavor evolution matrix, similarly to Ref. [29]:
\[ U_{f-s}(\eta) = \left( \begin{array}{cc} 0.9831 & -0.2577 \\ -0.0644 & 0.0169 \end{array} \right) \eta^{-0.6315} + \left( \begin{array}{cc} 0.0169 & 0.2577 \\ 0.0644 & 0.9831 \end{array} \right) \eta^{0.7184}. \]  
(C20)

The NLO correction reads:
\[ B_{VLL}^{SLL}(m_b) = 0.93 + 0.23 \frac{\bar{B}_{VLL}^{SLL}(m_b)}{B_{VLL}^{SLL}(m_b)} + (0.23 \pm 0.05) \frac{1}{B_{VLL}^{SLL}(m_b)}. \]  
(C24)

quite precisely, even if \( \bar{B}_{VLL}^{SLL} \) is only poorly known, because its coefficient in Eq. (C24) is small. The last term in Eq. (C24) quantifies the \( \Lambda_{QCD}/m_b \) corrections, see [59] for details. The lattice results of [70] have been combined in Ref. [59] to
\[ B_{VLL}^{SLL}(m_b) = 0.85 \pm 0.06 \quad \text{and} \quad \bar{B}_{VLL}^{SLL}(m_b) = 1.41 \pm 0.12. \]  
(C25)

Inserting these values into Eq. (C24) yields
\[ \frac{B_{VLL}^{SLL}(m_b)}{B_{VLL}^{SLL}(m_b)} = 1.57 \pm 0.08, \]  
(C26)

which is consistent with the direct determination
\[ \frac{B_{VLL}^{SLL}(m_b)}{B_{VLL}^{SLL}(m_b)} = 1.34 \pm 0.12 \]  
(C27)

from the lattice [70].

We are now in the position to accurately predict the bag factors at the high-scale \( \mu_b \). Choosing \( \mu_b = \tilde{m}_b(m_\tau) = 164 \text{ GeV} \), \( \alpha_s(M_Z) = 0.1189 \) and \( \tilde{m}_b(m_\tau) = 4.2 \text{ GeV} \) and using Eqs. (C26) and (C27) we find
SUPERSYMMETRIC HIGGS SECTOR AND $B - \bar{B}$ …

$$B_1^{SLL}(m_b) = 1.62 P_1^{SLL}(m_b) + 0.01 \tilde{B}_1^{SLL}(m_b)$$

$$= (2.54 \pm 0.13) B_1^{VLL}(m_b) + 0.01$$

$$\tilde{B}_1^{SLL}(m_b) = 1.29 P_1^{SLL}(m_b) + 0.54 \tilde{B}_1^{SLL}(m_b)$$

$$= (2.03 \pm 0.10) B_1^{VLL}(m_b) + 0.77 \pm 0.07$$

(C28)

Here, we have omitted the scheme-dependent terms proportional to $\eta$ in Eq. (C21). The small (2, 1) element of $U^{(0)}_{f-5}$ in Eq. (C20) ensures that $B_1^{SLL}(m_b)$ is inessential for $B_1^{SLL}(m_b)$. One realises from Eq. (C28) that the uncertainty of the high-scale bag factors stems almost completely from the error of the lattice result for $B_1^{VLL}(m_b)$.

Switching finally to the $P_i$’s defined in Eq. (8) we get

$$P_1^{SLL} = - \frac{5}{8} B_1^{SLL}(m_b)$$

$$= -(1.59 \pm 0.08) B_1^{VLL}(m_b) - 0.01$$

$$= -1.36 \pm 0.12$$

$$P_1^{VLL} = B_1^{VLL}(m_b) = 0.83 B_1^{VLL}(m_b) = 0.71 \pm 0.05.$$  

(C29)

In the last line, the full NLO result of [66] has been used. We do not need $\tilde{B}_1^{SLL} = \tilde{B}_1^{SLL}(m_b)/8$ for our analysis. Parity ensures that $\tilde{Q}_1^{SLL}$ and the chirality-flipped operator $O_1^{SRR}$ defined in Eq. (7) have the same matrix element, i.e., $P_1^{SRR} = P_1^{SLL}$.

Finally, we compute $P_2^{LR}$ using the formulas of Ref. [29] with the bag factors of Bečirević et al. [70]. This time the conversion between the bases of Ref. [29,70] is straightforward, since the renormalization scheme used in Refs. [28,29] respects the Fierz symmetry and lattice results are already quoted for this scheme. The result is

$$P_2^{LR} = 3.2 \pm 0.2.$$  

(C30)

The number in Eq. (C30) is significantly larger than $P_2^{LR} = 2.46$ quoted in Ref. [29], because our value for $m_b$ is smaller and the lattice bag factors are larger than 1. The error in Eq. (C30) does not include the systematic error from the quenching approximation.

APPENDIX D: TRILINEAR HIGGS COUPLINGS

The trilinear terms of the effective Lagrangian at $\tan \beta = \infty$ introduced in Sec. IIC read

$$V_{hb}^{(3)} = \frac{v}{\sqrt{2}} \left[ \lambda_5 (r_u^0 + (G^0)^2 + 2|h_u^0|^2) + \sqrt{2} \lambda_7 r_u H_d^1 H_d \right.$$  

$$\lambda_5 (2r_u (r_u + iG^0) - h_u^0 h_u^0)$$  

$$+ \left[ \lambda_5 h_u^0 \left( \frac{r_u + iG^0}{\sqrt{2}} - r_u h_u^0 + H.c. \right) + \lambda_6 (H_d^1 H_d) h_u^0 \right.$$  

$$\lambda_5 \left( r_u^0 + \frac{2}{\sqrt{2}} \left( \frac{r_u^0}{2} + i r_u G^0 + \frac{1}{2} (G^0)^2 + |h_u^0|^2 \right) \right.$$  

$$\lambda_7 \left( h_u^0 \right) \right] + H.c.$$  

(D1)

Again, the first two lines respect the $U(1)$ symmetry introduced in Sec. IIC, while the last two lines break it, and the breaking is proportional to loop-induced couplings. Finally, the quartic Lagrangian is obtained from the quartic terms in Eq. (12) by substituting $H_u \rightarrow (h_u^0, \frac{1}{\sqrt{2}} \phi_u^0)$ and $H_d \rightarrow (h_d^0, -h_d^0)$. Also there, only $\lambda_5$, $\lambda_6$ and $\lambda_7$ break the symmetry.

[30] see e.g., H. E. Haber, in Perspectives on Higgs physics II, edited by Gordon L. Kane (World Scientific, Singapore, 1997), and references therein.