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Article (Published Version)

Skinner, J C, Prance, H, Stiffell, P B and Prance, R J (2010) Sisyphus effects in a microwave-excited flux-qubit resonator system. Physical Review Letters, 105 (25). pp. 257002-1. ISSN 0031-9007

Sisyphus Effects in a Microwave-Excited Flux-Qubit Resonator System

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(Received 21 April 2010; published 14 December 2010)

Sisyphus amplification, familiar from quantum optics, has recently been reported as a mechanism to explain the enhanced quality factor of a classical resonant (tank) circuit coupled to a superconducting flux qubit. Here we present data from a coupled system, comprising a quantum mechanical rf SQUID (flux qubit) reactively monitored by an ultrahigh quality factor noise driven rf resonator and excited by microwaves. The system exhibits enhancement of the tank-circuit resonance, bringing it significantly closer (within 1%) to the lasing limit, than previously reported results.

DOI: 10.1103/PhysRevLett.105.257002 PACS numbers: 85.25.Dq, 03.67.Lx, 42.50.Hz, 74.50.+r

Superconducting qubits can be designed in a range of configurations, whereby various degrees of freedom are chosen to create a two level quantum system. The flux qubit is currently under investigation for its suitability as a quantum device [1,2]. One implementation of a flux qubit is based on the familiar, single Josephson weak link, rf SQUID (radio frequency superconducting quantum inteference device). In this case, external control of the weak link parameters allows the damping of the rf SQUID to be tuned so as to access the regime where the external flux dependent potential comprises a double well with just a few energy levels. Typically the lowest two levels are flux localized eigenstates of the system, characterized by macroscopic screening currents which are well spaced from the next highest, unlocalised levels [3] The current flips direction as the flux contained within the ring (Φ_x) passes the degeneracy point for $\Phi_x = (\frac{1}{2} + n)\Phi_0$, where Φ_0 ($= h/2e$) is the flux quantum. It has been shown that a powerful mechanism for reading out the state of an rf SQUID qubit is through reactive monitoring, by inductively coupling the ring to a lumped component rf resonator (tank-circuit) [2–5]

It has recently been proposed that for a coupled SQUID-tank-circuit system, excited by microwaves, so called Sisyphus damping and amplification of the tank-circuit energy can result [6]. The Sisyphus mechanism was first discussed in relation to atom optics [7], adding further support for the analogies between superconducting circuits and established atomic, molecular and optical phenomena [8,9]. The mechanism involves the tank-circuit oscillation driving an adiabatic modulation of the qubit energy around a given dc flux bias point, which causes the level separation of the ground and first excited states of the qubit to modulate accordingly. For such an adiabatic modulation of the SQUID, the tank-circuit does either positive or negative work, depending on the slope of the energy band and the evolution caused by the rf excursions [6]. If the qubit remains in the same energy state, then the net work per rf cycle will average to zero. However, at specific points in total external flux bias, the gap between the lowest two energy levels will allow applied microwave photons to excite the qubit from the ground state. At some time later, depending on the coupling of the qubit to its environment, the system will relax back to its ground state. It can be shown that for particular values of external flux bias and microwave frequency (energy), these transitions between energy levels will lead to an asymmetrical work function for the tank circuit over the period of oscillation.

When the qubit is blue detuned in dc flux bias away from the microwave transition point, the tank circuit experiences a net energy gain (warming) over the rf period. The additional energy relates to the difference between the energy of the microwaves used to transition the SQUID to an excited state and that associated with the spontaneous relaxation back to the ground state. This is a direct consequence of modulation of the level separation by the rf bias and means that, for the case of a red detuned qubit, the difference results in a net loss to the tank circuit and hence a cooling effect.

Advanced cooling of atoms and optical systems is well developed, but the results presented here and previously [6] show that heating and cooling has been achieved with a macroscopic ensemble of condensed particles, with an associated classically measured effect. The implications of the work are significant as a specialised technique for temperature control of macroscopic quantum systems using classically defined voltages and currents. For example, a similar charge based qubit has been used to create a superconducting micro-maser [10].

The Sisyphus effect in a coupled rf SQUID-tank-circuit system can be modeled as an effective bath coupled to the tank circuit with a quality factor Q_{Sis}, which satisfies the equation [6]:

$$\frac{1}{Q_{\mathrm{tank}}} = \frac{1}{Q_0} + \frac{1}{Q_{\mathrm{Sis}}} \tag{1}$$

Where Q_0 is the intrinsic (coupled) tank-circuit resonance quality factor and Q_{tank} is the quality factor of the tank-circuit resonance under Sisyphus damping or amplification conditions. $Q_{\rm Sis}$ can take positive or negative values and is a modeled parameter, not a physical quantity. Positive values of $|Q_{\rm Sis}|$ give rise to tank-circuit Q damping and negative values to amplification. In the latter case, as $|Q_{\rm Sis}|$ approaches Q_0 from above the observed tank-circuit quality factor, $Q_{\rm tank}$, increases asymptotically towards infinity at $|Q_{\rm Sis}| = Q_0$, the lasing instability. In practice, this regime is more likely to be observed in resonant systems with the highest intrinsic quality factors, such as nanomechanical resonators [6]. We report here observation of highly efficient lasing using an ultrahigh-Q lumped component tank-circuit.

By selecting the SQUID ring loop inductance (Λ), the weak link capacitance(C_q) and the environmental temperature of the system (T), it is possible to configure the flux-qubit ground state to be well separated from the higher excited states of the qubit, such that excitations from the ground state have a very low probability. This is formalised by $k_B T \ll \hbar\omega_s$, where k_B is Boltzmann's constant, ω_s is the level separation in radian frequency units and $\hbar$ is the modified Plank constant. Figure 1 shows the computed, uncoupled SQUID energy levels for the point contact device considered in this paper.

It has previously been shown that it is possible to excite the qubit from the ground state by means of a multiphoton process [11,12], as well as with single photons. The Sisyphus mechanism considered here is applicable in both cases and it is therefore to be expected that complex patterns of warming and cooling will exist in the response of such systems with $\omega_m \ll \omega_s$ under microwave irradiation at a frequency ω_m. Key factors in determining whether the effect is observed include the underlying flux dependent energy band structure of the SQUID, the possible

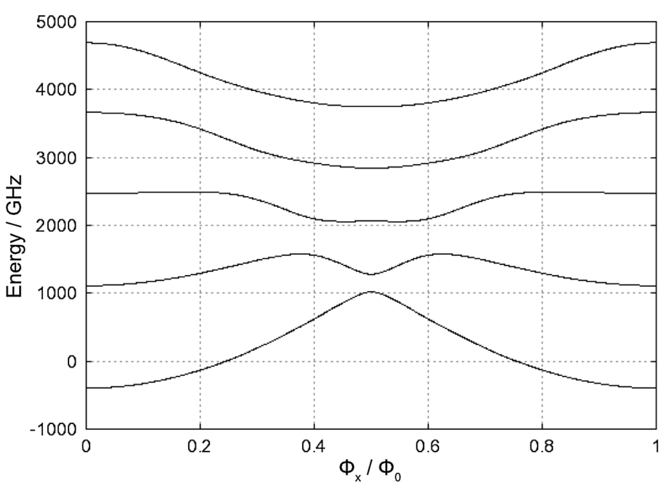

FIG. 1. The first five energy levels of the SQUID under investigation, computed by fitting the coupled tank-circuit frequency shift, measured with the SQUID in its ground state and using the measured SQUID inductance [5,19]. At the half integer bias ($0.5\Phi_0$) the gap between the ground and first excited state is around 240 GHz.

excitation trajectories across that flux space for a given microwave energy (frequency) and the available dissipation modes and associated dissipation rates. What is important is that both a pumping from the ground state and a subsequent relaxation occur with a net gain or loss of energy to the tank circuit.

Experimentally the truly macroscopic quantum device can be controlled and probed by a number of different techniques. Inductively coupled dc SQUID magnetometers [1,13], SQUID bifurcation amplifiers [14], directly connected Josephson junctions [15] and inductively coupled resonant circuits [16,17] all find application in the monitoring of SQUID based qubits. Quantum-non-demolition measurements, are also becoming a subject of interest [18].

Inductive coupling of a (linear) tank circuit to the qubit was used to obtain the results in this paper and has been used previously to reveal features of the dynamical behavior of rf SQUID systems, including the underlying ground state energy of the system [19]. Recently the same method has seen renewed interest in the field of qubit measurement, being termed the "impedance measurement technique" (IMT) [2]. Our system has also been used to study very different behavior for SQUIDs configured in the hysteretic regimen, where highly nonlinear dynamics [20] have been observed.

A two-hole Zimmermann type SQUID ring topology was used [21], with an open loop ring inductance, $\Lambda \approx 30$ nH [16]. The weak link capacitance is in the region of 10^{-16} F [22]. By careful design of the tank circuit, we can perform very high fidelity measurements on the rf SQUID qubit without introducing excessive decoherence. Extensive shielding was employed, including primary and secondary superconducting screening around the 4.2 K SQUID, mu-metal shielding around the cryostat and enclosure of the entire experiment within a Faraday cage.

Figure 2 shows a simplified schematic of the experimental setup. The LC tank circuit has a capacitance of $C_T = 330$ pF and an inductance $L_T \approx 146$ nH, giving an oscillation frequency of ≈ 20 MHz. The losses in the bare

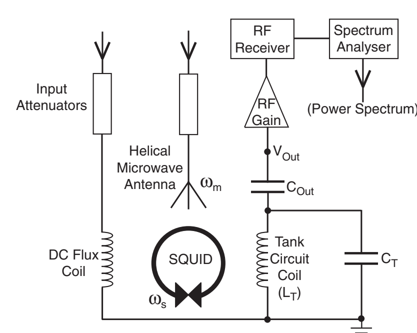

FIG. 2. Circuit diagram showing the rf SQUID coupled to a dc bias coil, a helical microwave antenna and a 20 MHz tank circuit. The incoming signals are attenuated for improved noise performance and the tank-circuit output is amplified at 4.2 K. The tank circuit is undriven (see text).

tank circuit have been designed to be minimum, such that a quality factor at resonance (Q_0), has been measured exceeding 9000 when uncoupled to the SQUID ring. The ultrahigh-Q of the system allows us to observe the undriven tank-circuit resonance above the noise present in the 4.2 K environment. A mutual inductance (M) between the (resonant) tank circuit (L_T) and the SQUID loop (Λ) defines their interaction energy, which is usually considered in terms of a dimensionless coupling parameter, $k^2 = M^2/L_T\Lambda$ ($= 0.92 \times 10^{-3}$ for the experiments described here). Two further coils were inductively coupled to the SQUID, a helical coil to deliver the microwave excitation signal (ω_m) and a solenoid for dc flux biasing of the ring.

In the experiments described here we consider the effect of exciting a macroscopic SQUID-tank system with microwaves, where the driving frequency, $\omega_m/2\pi$, is around 6.7 GHz. Our experimental evidence shows that this frequency allows good coupling between the microwave source and the SQUID, but as a subsequent paper will show, many frequencies around this value and also across a much wider range show similar effects, indeed we have observed instances of tank-circuit amplification and cooling at frequencies up to 40 GHz.

Figure 3 shows the spectral response of the system under three conditions. Trace (a) shows the noise driven resonance without microwaves applied and flux biased furthest away from degeneracy, i.e., at $n\Phi_0$ (n integer). The resonance is clearly visible 8 dB over the background and with a quality factor of $Q_0 = 4240$. As the dc bias signal is changed to position the ring at the degeneracy point, where the flux threading the ring is $(\frac{1}{2} + n)\Phi_0$, trace (b) shows the resonance above the noise floor. The amplitude and quality factor has reduced and there has been a shift in the frequency by around -60 kHz. This is typical behavior around the degenerate region and should not be confused with the Sisyphus effect.

Trace (c) in Fig. 3 shows the effect of applying 6.17 GHz and -61 dBm (at room temperature) microwave excitation to the system and selecting dc flux biasing to blue detune the qubit, configuring conditions conducive to give Sisyphus amplification of the tank-circuit resonance. We see that the peak level of the resonance is now amplified 24 dB above the integer bias level without microwaves and that the quality factor of the resonance has increased markedly. It is of note that there is now an asymmetry in the resonance, which is here strongly driven by the SQUID ring. This is consistent with the results of previous experiments, where nonlinearity and bifurcations of the resonance under strong driving conditions have been documented [23].

In Fig. 4 we show data for a similar experimental configuration. The y axis encompasses a full period in static dc bias and the range of applied microwave voltages are plotted on the x axis. The microwave input frequency is fixed at 6.72 GHz in this data set, but many similar plots have been generated for different frequencies. The color scale represents the tank-circuit output, as measured by the spectrum analyzer. The white regions correspond to an unmodified tank-circuit amplitude. This is the level seen with no applied microwaves and for a dc bias of $n\Phi_0$. At around 450 μV the power detected deviates markedly from the unmodified regions shown as white. The red regions show an increase in the detected power at room temperature, representing amplification of the tank-circuit quality factor and the blue regions show damping. The modest level of cooling is attributed to the lack of available dynamic range in the damping regime, where the underlying noise floor of the system presents a lower threshold.

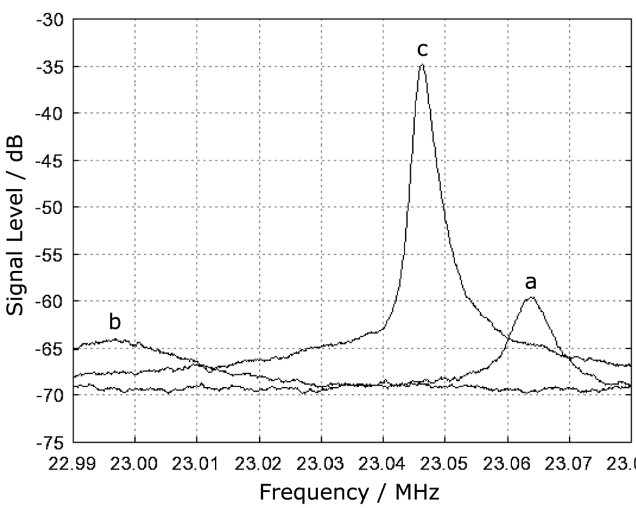

FIG. 3. The spectral responses of the noise driven tank circuit shown for $\Phi_x = n\Phi_0$ (a) and $\Phi_x = (\frac{1}{2} + n)\Phi_0$ (b) between these extremes of operation the frequency shift is 60 KHz and the amplitude drops by about 4 dB. Trace (c) shows the effect of applying microwaves at a frequency of 6.173 GHz with an intermediate bias in dc flux causing the Sisyphus cycle to become manifest.

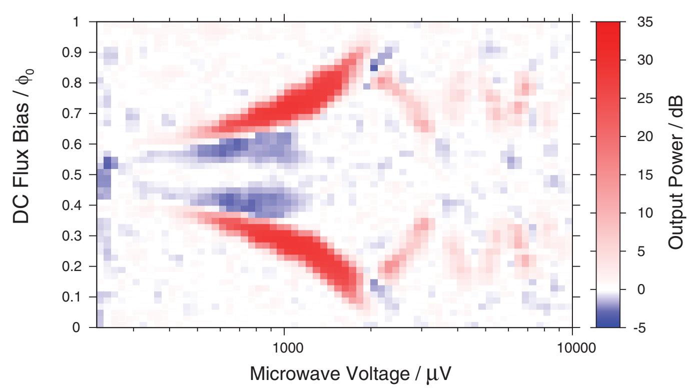

FIG. 4 (color). Color scale plot of the tank-circuit amplification over a range of dc bias settings on the vertical scale and applied microwave voltage on the horizontal axis. It can be seen that amplification and damping of the tank-circuit resonance occurs at distinct regions in the plot. A microwave frequency of 6.72 GHz was used.

For low values of microwave voltage ($\approx 200\ \mu$V) and $0.5n\Phi_0$ bias it is observed that there is a dip of around 5 dB in the power detected at room temperature. This is caused by the switching of the ring screening current at the half integer bias point and is unrelated to the Sisyphus effect.

Considering the two data sets presented in Figs. 3 and 4, the proximity to the theoretical lasing condition (when $|Q_{\mathrm{Sis}}| = Q_0$) for each case can be evaluated. The quality factor of the amplified resonance, Q_{tank}, is measured from the data and used in Eq. (1) to calculate Q_{Sis}, which is then compared to the unenhanced quality factor Q_0. The measurement of very high quality factors from the shape of the power spectrum resonance is difficult. However, since there is a linear relationship between the quality factor of a resonant circuit and its output level, an accurate measure of Q_{tank} may be obtained from the increase in the output, measured at room temperature as a power by the spectrum analyzer. For the data in Fig. 3, we note an amplification of 24 db, compared to the unenhanced resonance at $\Phi_x = n\Phi_0$. This equates to an increase of 316 in the quality factor. From Eq. (1), we therefore calculate a value of Q_{Sis} of -4253. For the data of Fig. 4, the maximum amplification is 31 dB, giving a calculated Q_{Sis} of -4243. For a given system the Sisyphus mechanism (configured to operate under optimum conditions [6]), is able to provide a finite energy to the tank circuit. If this energy equals or exceeds that of the intrinsic losses of the tank circuit, the system will be acting as a laser. Minimizing these losses is thus key to achieving lasing in such circuits. For example, following the analysis in Grajcar (Ref. [6], Eq. (5) in Appendix), for our system we would require a tank circuit with a Q_0 of 4250 to reach lasing, where our current experimental value is 4240.

In conclusion, we have shown, using an ultrahigh quality factor linear resonator coupled to a macroscopic SQUID ring and under the Sisyphus mechanism discussed in previous work [6], that our system is only 0.1% away from the theoretical lasing limit. This proves experimentally the importance of using intrinsically high quality factor resonators to exploit this mechanism.

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