The supersymmetric standard model from the $\mathbf{Z}_6'$ orientifold?

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Abstract. We construct $\mathcal{N}=1$ supersymmetric fractional branes on the $\mathbf{Z}_6'$ orientifold. Intersecting stacks of such branes are needed to build a supersymmetric standard model. If a,b are the stacks that generate the $SU(3)_c$ and $SU(2)_L$ gauge particles, then, in order to obtain *just* the chiral spectrum of the (supersymmetric) standard model (with non-zero Yukawa couplings to the Higgs mutiplets), it is necessary that the number of intersections $a\cap b$ of the stacks a and b, and the number of intersections $a\cap b'$ of a with the orientifold image b' of b satisfy $(a\cap b, a\cap b')=(2,1)$ or $(1,2)$. It is also necessary that there is no matter in symmetric representations of either gauge group. We have found a number of examples having these properties.

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1. INTRODUCTION

Intersecting D-branes provide an attractive, bottom-up route to standard-like model building. In these models one starts with two stacks, a and b with $N_a=3$ and $N_b=2$, of D6-branes wrapping the three large spatial dimensions plus 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold) on which the theory is compactified. These generate the gauge group $U(3)\times U(2)\supset SU(3)_c\times SU(2)_L$, and the non-abelian component of the standard model gauge group is immediately assured. Further, (four-dimensional) fermions in bifundamental representations $(\mathbf{N}_a,\overline{\mathbf{N}}_b)=(\mathbf{3},\overline{\mathbf{2}})$ of the gauge group can arise at the multiple intersections of the two stacks. These are precisely the representations needed for the quark doublets Q_L of the standard model. In general, intersecting branes yield a non-supersymmetric spectrum, so that, to avoid the hierarchy problem, the string scale associated with such models must be low, no more than a few TeV. Then, the high energy (Planck) scale associated with gravitation does not emerge naturally. Nevertheless, it seems that these problems can be surmounted [1, 2], and indeed an attractive model having just the spectrum of the standard model has been constructed [3]. It uses D6-branes that wrap 3-cycles of an orientifold T^6/Ω, where Ω is the world-sheet parity operator. On an orientifold, for every stack $a,b,...$ there is an orientifold image $a',b',....$ At intersections of a and b there are chiral fermions in the $(\mathbf{3},\overline{\mathbf{2}})$ representation of $U(3)\times U(2)$, where the $\mathbf{3}$ has charge $Q_a=+1$ with respect to the $U(1)_a$ in $U(3)=SU(3)_c\times U(1)_a$, and the $\overline{\mathbf{2}}$ has charge $Q_b=-1$ with respect to the $U(1)_b$ in $U(2)=SU(2)_L\times U(1)_b$. However, at intersections of a and b' there are chiral fermions in the $(\mathbf{3},\mathbf{2})$ representation, where the $\mathbf{2}$ has $U(1)_b$ charge $Q_b=+1$. To get just the standard model spectrum, the number of intersections $a\cap b$ of the stack a with b, and the number of intersections $a\cap b'$ of the stack a with b' must satisfy $(a\cap b, a\cap b')=(1,2)$

or $(2,1)$ (if we demand standard-model Yukawa couplings for all matter).

Despite the attractiveness of the model of reference [3], there remain serious problems in the absence of supersymmetry [4, 5]. Several attempts have been made to construct the MSSM [6, 7, 8, 9, 10] using an orientifold with point group $P = \mathbf{Z}_4, \mathbf{Z}_4 \times \mathbf{Z}_2$ or $\mathbf{Z}_6$. The most successful attempt to date is the last of these. However, none of them yield the required intersection numbers $(a \cap b, a \cap b') = (1,2)$ or $(2,1)$. The question then arises as to whether the use of a different orientifold could circumvent this problem. Here we address this question for the $\mathbf{Z}_6'$ orientifold. Further details of this work may be found in reference [11].

2. $\mathbf{Z}_6'$ ORIENTIFOLD

We assume that the torus T^6 factorises into three 2-tori $T_1^2 \times T_2^2 \times T_3^2$. The 2-tori T_k^2 $(k = 1,2,3)$ are parametrised by complex coordinates z_k. The action of the generator θ of the point group $\mathbf{Z}_6'$ on the coordinates z_k is given by

$$\theta z_k = e^{2\pi i v_k} z_k \tag{1}$$

where

$$(v_1, v_2, v_3) = \frac{1}{6}(1,2,-3) \tag{2}$$

The point group action must be an automorphism of the lattice, so in $T_{1,2}^2$ we may take an $SU(3)$ lattice. Specifically we define the basis 1-cycles by π_1 and $\pi_2 \equiv e^{i\pi/3}\pi_1$ in T_1^2, and π_3 and $\pi_4 \equiv e^{i\pi/3}\pi_3$ in T_2^2. Thus the complex structure of these tori is given by $U_1 = e^{i\pi/3} = U_2$. The orientation of $\pi_{1,3}$ relative to the real and imaginary axes of $z_{1,2}$ is arbitrary. Since θ acts as a reflection in T_3^2, the lattice, with basis 1-cycles π_5 and π_6, is arbitrary.

We consider "bulk" 3-cycles of T^6 which are linear combinations of the 8 3-cycles $\pi_{i,j,k} \equiv \pi_i \otimes \pi_j \otimes \pi_k$ where $i = 1,2,\ j = 3,4,\ k = 5,6$. The basis of 3-cycles that are *invariant* under the action of θ contains 4 elements ρ_p $(p = 1,3,4,6)$ whose intersection numbers are always even. Besides these (untwisted) 3-cycles, there are also exceptional 3-cycles that arise in twisted sectors of the orbifold in which there is a fixed torus. They consist of a collapsed 2-cycle at a fixed point times a 1-cycle in the invariant plane. We shall only be concerned with those that arise in the θ^3 sector, which has T_2^2 as the invariant plane. There are then 8 independent $\mathbf{Z}_6'$-invariant exceptional cycles $\varepsilon_j, \tilde{\varepsilon}_j$ $(j = 1,4,5,6)$. Again, their intersection numbers are always even.

The embedding $\mathscr{R}$ of the world-sheet parity operator Ω may be chosen to act on the three complex coordinates z_k $(k = 1,2,3)$ as complex conjugation $\mathscr{R} z_k = \bar{z}_k$, and we require that this too is an automorphism of the lattice. This fixes the orientation of the basis 1-cycles in each torus relative to the Re z_k axis. It requires them to be in one of two configurations **A** or **B**. In both cases the real part of the complex structure U_3 of T_3^2 is fixed, but the imaginary part is arbitrary. It is then straightforward to determine the action of $\mathscr{R}$ on the bulk 3-cycles ρ_p and on the exceptional cycles ε_j and $\tilde{\varepsilon}_j$. In particular, requiring that a bulk 3-cycle $\Pi_a = \sum_p A_p^a \rho_p$ be invariant under the action of $\mathscr{R}$ gives 2 constraints on the bulk coefficients A^a_p, so that just 2 of the 4 independent bulk 3-cycles are $\mathscr{R}$-invariant. Which 2 depends upon the lattice.

The twist (2) ensures that the closed-string sector is supersymmetric. In order to avoid supersymmetry breaking in the open-string sector, the D6-branes must wrap special Lagrangian cycles. Then a stack a of D6-branes is supersymmetric if $X^a > 0$ and $Y^a = 0$, where X^a and Y^a are linear combinations of the bulk coefficients A^a_p that depend upon the lattice chosen. The (single) requirement that $Y_a = 0$ means that 3 independent combinations of the 4 invariant bulk 3-cycles may be chosen to be supersymmetric. Of these, 2 are the $\mathscr{R}$-invariant combinations. However, unlike in the case of the $\mathbf{Z}_6$ orientifold, there is a third, independent, supersymmetric bulk 3-cycle that is *not* $\mathscr{R}$-invariant.

We noted earlier that the intersection numbers of both the bulk 3-cycles ρ_p and of the exceptional cycles $\varepsilon_j, \tilde{\varepsilon}_j$ are always even. However, in order to get just the (supersymmetric) standard-model spectrum, either $a \cap b$ or $a \cap b'$ must be odd. It is therefore necessary to use fractional branes of the form

$$a = \frac{1}{2}\Pi^{\rm bulk}_a + \frac{1}{2}\Pi^{\rm ex}_a \tag{3}$$

where $\Pi^{\rm bulk}_a = \sum_p A^a_p \rho_p$ is an invariant bulk 3-cycle, and $\Pi^{\rm ex}_a = \sum_j (\alpha^a_j \varepsilon_j + \tilde{\alpha}^a_j \tilde{\varepsilon}_j)$ is an exceptional cycle. Supersymmetry requires that this exceptional cycle is associated with the fixed points in T^2_1 and T^2_3 traversed by $\Pi^{\rm bulk}_a$.

In general, besides the chiral matter in bifundamental representations that occurs at the intersections of brane stacks $a, b, \ldots$, with each other or with their orientifold images $a', b', \ldots$, there is also chiral matter in the symmetric $\mathbf{S}_a$ and antisymmetric representations $\mathbf{A}_a$ of the gauge group $U(N_a)$, and likewise for $U(N_b)$. Orientifolding induces topological defects, O6-planes, which are sources of RR charge. The number of multiplets in the $\mathbf{S}_a$ and $\mathbf{A}_a$ representations is

$$\#(\mathbf{S}_a) = \frac{1}{2}(a \cap a' - a \cap \Pi_{\rm O6}) \tag{4}$$

$$\#(\mathbf{A}_a) = \frac{1}{2}(a \cap a' + a \cap \Pi_{\rm O6}) \tag{5}$$

where $\Pi_{\rm O6}$ is the total O6-brane homology class; it is $\mathscr{R}$-invariant. If $a \cap \Pi_{\rm O6} = \frac{1}{2}\Pi^{\rm bulk}_a \cap \Pi_{\rm O6} \neq 0$, then copies of one or both representations are inevitably present. Since we require supersymmetry, $\Pi^{\rm bulk}_a$ is necessarily supersymmetric. However, we have observed above that this does not require $\Pi^{\rm bulk}_a$ to be $\mathscr{R}$-invariant, as $\Pi_{\rm O6}$ is. Thus, unlike the $\mathbf{Z}_6$ case, in this case $a \cap \Pi_{\rm O6}$ is generally non-zero. Excluding the appearance of the representations $\mathbf{S}_a$ and $\mathbf{S}_b$ requires that $a \cap a' = a \cap \Pi_{\rm O6}$ and likewise for b. The number of multiplets in the antisymmetric representation $\mathbf{A}_a$ is then $a \cap \Pi_{\rm O6}$. Thus to avoid unwanted vector-like quark singlet matter we must also impose the constraint $|a \cap \Pi_{\rm O6}| \leq 3$. A similar constraint on b is required to avoid vector-like lepton singlet matter.

3. RESULTS AND CONCLUSIONS

We have shown [11] that, unlike the $\mathbf{Z}_6$ orientifold, at least on some lattices, the $\mathbf{Z}_6'$ orientifold *can* support supersymmetric stacks a and b of D6-branes with intersection numbers satisfying $(a \circ b, a \circ b') = (2,1)$ or $(1,2)$. Stacks having this property are an indispensable ingredient in any intersecting brane model that has *just* the matter content of the (supersymmetric) standard model. By construction, in all of our solutions there is no matter in symmetric representations of the gauge groups on either stack. Some of our solutions have no antisymmetric (or symmetric) matter on either stack, and we shall attempt in a future work to construct a realistic (supersymmetric) standard model using one of these solutions. Our results also show that different lattices can produce different physics, and this suggests that other lattices are worth investigating in both the $\mathbf{Z}_6$ and $\mathbf{Z}_6'$ orientifolds. In particular, since Z_6 can be realised on a G_2 lattice, as well as on an $SU(3)$ lattice, one or more of all three $SU(3)$ lattices in the $\mathbf{Z}_6$ case, and of the two on $T^2_{1,2}$ in the $\mathbf{Z}_6'$ case, could be replaced by a G_2 lattice. We shall explore this avenue too in future work.

The construction of a realistic model will, of course, entail adding further stacks of D6-branes $c, d, ..$, with just a single brane in each stack, arranging that the matter content is just that of the supersymmetric standard model, the whole set satisfying the condition for RR tadpole cancellation. Even so, some of the moduli will remain unstabilised. Their stabilisation requires the introduction of RR, NSNS and metric fluxes [12, 13] and indeed models similar to the ones we have been discussing can be uplifted [14] into ones with stabilised Kähler moduli using a "rigid corset". In general, fluxes contribute to tadpole cancellation conditions and might make them easier to satisfy. In which case, it may be that one or other of our solutions with antisymmetric matter could be used to obtain just the standard-model spectrum. In contrast, the rigid corset can be added to any RR tadpole-free assembly of D6-branes in order to stabilise all moduli. Thus our results represent an important first step to obtaining a supersymmetric standard model from intersecting branes with all moduli stabilised.

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