Understanding Mathematics Anxiety: 
Loss Aversion and Student Engagement

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Abstract
This paper applies the well-known cognitive bias of loss aversion from behavioural economics to student decisions over engagement with mathematically demanding coursework. This bias is shown to predict behaviour that is consistent with mathematics anxiety in a dynamic model of student engagement. In the absence of any countervailing pedagogic interventions, these forces are shown to imply polarization in student outcomes with some students chronically disengaged in a low-attainment equilibrium. However, the model illustrates that chronic disengagement is not equivalent to chronic apathy. Rather, students for whom the short-term cost of failure looms large are shown to be at heightened risk of disengagement. The model is used to understand and evaluate various elements of the mathematics anxiety literature including the role of formative assessment, the fixed and growth mindset models, the formation of functional learner identities, the cognitive interference and motivation enhancement models of test anxiety, the provision of remedial classes, and the prospects for ‘gamification’ of assessment.

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1. Introduction

Mathematics anxiety has rightly received the scholarly attention of pedagogues for almost seventy years (Aiken, 1974; Betz, 1978; Dutton, 1954; Gough, 1954; and Richardson and Suin, 1972 are some notable early contributions). The phenomenon has been documented in students over a variety of subjects and across the spectrum of education levels (Hembree, 1990), and even among some groups of teachers (for example by García González and Sierra, 2020; and Harper and Daane, 1998). In the context of higher education, the issue of mathematics readiness has been a particular concern, especially as it relates to the transition from high-school to university on a range of degrees (such as economics, finance, geography and engineering) that do not specialise in mathematics but are nevertheless mathematically demanding (Feudel and Dietz, 2019; Hodgen, Adkins and Tomei, 2020; and McAliden and Noyes, 2019).

Parallel to these developments, a hitherto unrelated literature has studied biases and inconsistencies in people’s responses to risky prospects following Kahneman and Tversky’s (1979) seminal contribution on prospect theory. This paper uses loss aversion, a specific component of prospect theory, to deepen our understanding of mathematics anxiety. Put simply, loss aversion is the empirical observation that “Losses hurt about twice as much as gains make us feel good” (p.137. Thaler, 2000). This excess sensitivity to losses has been shown induce important and pervasive biases in decision making in areas as diverse as pension contributions, organ donation, tax payment, consumer decisions, and disease prevention behaviour (Behavioural Insights Team, 2012; Cribb and Emmerson, 2016; and O’Keefe and Jensen, 2007). The evidence base for loss aversion has been built largely on surveys of student populations from all over the world: Kahneman and Tversky’s (1979) paper was based on surveys of Israeli, Swedish, and American university students and a single special issue of the Quarterly Journal of Economics in memory of Amos Tversky contained papers based on surveys of students from the Netherlands, the UK and the USA (Bateman et al., 1997; Gneezy and Potters 1997; and Thaler et al. 1997). For the interested reader, Rabin (2003), Rabin and Thaler (2001), and Thaler (2000), celebrate the rich contributions of prospect theory to our understanding of human decision making, while Barberis (2013) provides a more recent assessment of this literature.
The main contribution of the present paper is to serve as a bridge between these literatures and to apply insights from loss aversion to further the pedagogic understanding of mathematics anxiety. To this end, the paper identifies the sharply dichotomous nature of success and failure as a key feature that distinguishes a typical mathematics exercise from other, more discursive, forms of university proficiency. A student typically knows within minutes of starting a mathematically intensive exercise in economics, finance, engineering or chemistry whether or not the attempt has been successful. To the extent that failure to complete an exercise is hedonically costly and success is a hedonic boon, undertaking such an exercise implies a gamble over a student’s short-term perceived well-being. In these circumstances, prospect theory predicts that the decision-making processes of students may be subject to loss aversion.

The paper presents a simplified dynamic model of student attainment which predicts that students who have initially low levels of academic capital in mathematics are at heightened risk of suffering from mathematics anxiety and disengagement. In the absence of any countervailing pedagogic intervention these forces are shown to imply polarization in student outcomes with some students sustaining engagement in a high-attainment equilibrium, while others are left persistently disengaged in a low-attainment equilibrium. These predictions are of particular concern given the substantial evidence base linking gender, ethnicity, and class with heightened risk of disengagement in mathematics education (Hottinger, 2016; Lim, 2008; and Mendick, 2005a). Such concerns are amplified in the current landscape of Higher Education with larger and more heterogeneous student intakes (Commons Library Briefing, 2019 and Department for Education, 2018), where a substantial proportion of students on mathematically demanding programmes hold relatively modest incoming mathematics qualifications (Hodgen, Atkins and Tomei 2020).

The paper then turns to harnessing insights from the loss-aversion based theory of mathematics anxiety developed in Section 2 to critically evaluate several remedial measures that have been proposed in the mathematics anxiety literature. In the view of the theory developed in the present paper, the effectiveness of potential interventions depends on the extent to which they are successful in reframing the perception of failure from a negative hedonic domain to a neutral or positive hedonic domain. Specifically, interventions which mitigate the perceived cost of making a mistake, such as efforts to cast formative assessments as a ‘safe space’ for students to learn how to improve their work (in the spirit of Sadler, 1989), to foster a ‘growth mindset’ among mathematics learners (as proposed in Dweck, 2006 and revisited in Boaler, 2013), and to operationalize functional learner identities (as defined in
Solomon, 2007 and analysed in Mendick, 2005b) are found to be potentially effective responses as each intervention adopts at its heart a positive framing of the threat of failure. The paper goes on to explore how a loss-aversion based theory of mathematics anxiety can improve our understanding of the cognitive interference and motivation enhancement models of test anxiety, and the interrelationships between the two (Cassady and Johnson, 2002; Hembree, 1988; Sarason, 1984; and Seipp 1991 _inter alia_). A traditional response to heterogeneous ability cohorts is to offer remedial support to weaker students, though such support has been less than universally effective (Bahr, 2008; Di Pietro, 2011; and Lagerlöf and Seltzer, 2009). The model developed here serves to illustrate the theoretical rationale for remedial classroom support, but critical reflection on the model parameters reveals why such interventions may be of only limited practical benefit. Finally, the model is used to evaluate the effectiveness of technology enhanced learning tools such as supplementary multimedia resources and adaptive learning technologies which can ‘gamify’ virtual learning environments.

The remainder of the paper is organised as follows. Section 2 develops a simplified, dynamic model of mathematics anxiety and makes predictions about student engagement and attainment under loss aversion. Section 3 discusses the implications of these findings for a range of themes raised within the mathematics anxiety literature, while section 4 concludes.

### 2. Mathematics anxiety under loss aversion

#### 2.1 Ability, engagement and anxiety

The pedagogic function of formative assessment is to provide students with an opportunity to identify and rectify gaps in their learning without the potentially adverse consequences (Sadler, 1989). In theory then, failure to correctly complete a summative assessment should be costless, whereas the hedonic boon associated with ‘getting it right’ might yield a positive payoff, normalised here to equal 1. We may think of a student undertaking a mathematically intensive exercise as experiencing the following utilities:

\[
U (\text{failure}) = 0 \quad (1)
\]

\[
U (\text{success}) = 1
\]

Now suppose that for a given student and a given exercise there is a probability \( p \) that the attempt ends in success, and \((1-p)\) that it ends in failure. Then the expected utility of attempting the exercise is given by:

\[
E(U) = p(U \mid success) + (1 - p)(U \mid failure), \quad (2)
\]
In principle, the decision of whether or not to engage with an exercise should fully internalise the long-term implications of engagement or disengagement on module and degree outcomes. However, a variety of studies have found that students suffer from ‘present bias,’ where they systematically underweight the long-term costs and benefits of their decisions in favour of the present (Gneezy and Potters 1997, Takeuchi 2011, Thaler et al. 1997, among many others). Though there are variety of conceptualisations of present bias – including hyperbolic discounting (Loewenstein and Thaler 1989 provide a useful summary) and models of willpower and personal rules (see for example Ainslie and Haslam 1992, or Benabou and Tirole, 2004) – and each of these is likely to interact with the forces studied here in interesting and nuanced ways, the present paper refrains from considering these in detail so as to maintain a sharp focus on the implications of loss aversion. Therefore, the paper assumes that students are perfectly biased towards the present, or equivalently, that they fully ignore the long-term implications of their actions. Thus, the outside option or the utility of not attempting to engage with an assignment is 0:

\[ U (\text{non-attempt}) = 0 \]  

(3)

The subsequent discussion considers some of the implications that relaxing this admittedly restrictive assumption may have on my results. Comparing equation (1) with equation (2) we observe that in theory costless failure can incentivise students to engage with formative assessments, even when their decision making is fully biased towards the present.

In practice, however, other factors may cause the hedonic effect of a failed attempt to complete an exercise to take on a more negative character. Such failure may induce social, emotional, or psychological penalties on students. For example, students may lose standing among their peers if such exercises are conducted publicly or if results are compared in groups. Failure may perpetuate a negative self-image that is psychologically costly to the student. Students may interpret failure to complete a formative exercise as signal that they are likely to perform poorly in summative assessments. These problems may be exacerbated if students treat some level of proficiency as a reference point and are sensitive to falling short of this reference point. Thus, the implicit hedonic gamble that a student takes on when beginning a mathematical exercise in a formative assessment may have payoffs defined over both the positive and negative domains so that the standard prospect theory payoffs, where “Losses hurt about twice as much as gains make us feel good” (p.137. Thaler, 2000) apply as follows:

\[ U (\text{failure}) = -2. \]  
\[ U (\text{success}) = 1 \]  

(4)
As above, chance chooses with probability $p$ if a given attempt ends in success. I now focus on ability, $a$, and engagement, $e$, as the two key determinants of this probability of success, $p$.

For simplicity, I model engagement as a dichotomous variable taking on a value of 1 if a student decides to attempt a problem, and 0 otherwise. The simplest functional form that allows for the probability of successfully completing an exercise to be increasing in ability and engagement is the unweighted average of these two arguments. Assuming also that student ability, relative to the difficulty of the subject matter, is (initially) uniformly distributed over the open interval between 0 and 1 ensures that the resulting probabilities are well behaved. Thus:

$$p_t = \frac{a_t + e_t}{2} \tag{5}$$

This simple structure is already sufficient to yield some preliminary insights which are summarized in Figure 1. If failure is costless then the expected utility or payoff to engaging with an exercise is increasing in ability but remains non-negative even for the lowest ability students. This is illustrated by the dashed line. This line is computed by substituting equation
(5) and the payoffs in (1) into equation (2). Thus, costless failure and potential benefits from success in theory can incentivise engagement with formative assessments across the distribution of student abilities.

The solid line, by contrast, represents the expected utility of attempting a problem where failure is costly and students are loss averse, that is where the payoffs in (4) and equation (5) are substituted into (2). Here, there is a critical value of ability below which the expected utility of engagement is negative: when students perceive failure in mathematically demanding exercises to be costly formative assessments will only be effective in eliciting engagement from students of relatively high ability, whereas the threat of failure leads the remaining students to disengage in a response that is consistent with experiences of mathematics anxiety. The excess sensitivity to losses predicted by prospect theory amplifies the negative payoff to failure and so biases student decisions toward disengagement.

To the extent that a key function of higher education is to serve as a signalling device for ability (as formalised by Spence, 1973), such patterns of engagement are not necessarily problematic: an educator could set exercises of a level of difficulty so as to allow students above some specified ability threshold to self-select into engaging with assessments. Clearly though, labour market signalling is not the only purpose of higher education – education is a core input to human capital, and universities seek to imbue their students with substantive academic development and personal growth. It is to these more ambitious objectives of higher education that the anxiety and disengagement elicited by loss aversion poses a substantive threat. For students with low levels of incoming academic capital, the fear of failure, intensified by loss aversion, may prevent them from participating effectively in the learning process.

2.2 Dynamics: Polarization in engagement

The paper will now consider some aspects of the dynamics of engagement with loss averse students. A key stylized fact that underpins much of the current delivery of university education is that sustained student engagement is an important input to academic proficiency (Lee, 2014 provides recent empirical evidence). This observation is formalized by assuming that ability in a particular period depends not only on the level of ability in the preceding period, but also on whether or not the student chose to engage with the academic material so that:

$$a_t = \frac{a_{t-1} + e_{t-1}}{2}$$

(6)
where the unweighted average and the single lag are again chosen to favour clarity and simplicity of exposition over complexity. The solid arrow in Figure 2 presents the dynamics that emerge from combining equation (6) with the behaviour predicted for loss averse students by the solid line in Figure 1.

A student who had an ability level greater than 1/3 in the preceding period will have chosen to engage with exercises in that period, as the expected utility from doing so will have been positive and thus greater than that of their outside option, 0. Thus, their ability this period will be given by $\frac{a_{t-1} + 1}{2}$, which is greater than $a_{t-1}$ so that over time the student’s ability evolves towards the north-easterly corner of the diagram. By contrast, a student who had an ability level below 1/3 will not have engaged last period so that ability this period, will be $\frac{a_{t-1}}{2}$ which is less than ability last period. This depreciation in ability should be interpreted as

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1A practical, interesting and valuable avenue for future research would be to use more complex autoregressive structures to empirically estimate the persistence in disengagement and attainment.
occurring relative to the difficulty of academic content, which typically increases as students progress through a course of study. The predicted evolution of ability of students with low endowments of incoming academic capital is represented in Figure 2 by the solid arrow that evolves towards the south-westerly direction. Thus, in the absence of any shocks or countervailing pedagogic intervention, this model predicts that academic capital and engagement can interact with one another to create virtuous and vicious cycles that if left unchecked can lead to polarization in classroom outcomes. Evidence suggests that such polarization interacts with student identities to place women, some ethnic minority groups, and specific socioeconomic class groups at heightened risk (Hottinger, 2016; Lim, 2008; and Mendick, 2005a)

2.3 Engagement transitions

Of course, a number of other forces may also affect whether or not a student chooses to engage in a particular period. Factors such as illness, family emergencies or relationship events can cause a historically engaged student to disengage for a particular period. Of particular concern here is the rapidly increasing number of students declaring a mental health condition (UCAS, 2021). Conversely, a particularly inspirational teaching session, a helpful classmate, or a temporary reprieve from some other form of life stress might cause a chronically disengaged student to temporarily engage.

To understand if short-term ‘shocks’ to engagement can have long-term effects on attainment, recall that in the model, students fall into a vicious cycle of persistent disengagement if ability dips below 1/3. Equation (6) assumes that next period’s ability is the unweighted average of this period’s ability and engagement. From this, a straightforward computation yields the result that that if current ability is below 2/3, disengagement today will imply that ability tomorrow is below 1/3. Therefore, all students with ability between 1/3 and 2/3 are ‘at risk’ of persistent disengagement if exposed to a one-off, adverse shock to engagement. The dotted arrow in Figure 2 highlights these ‘at risk’ students. Of course, these exact numbers are merely illustrative in nature and cannot be used as literal thresholds. Nevertheless, they serve to make the important point that at any given time the set of students who are at-risk of falling into the vicious circle is in principle non-empty.

Conversely, the dashed arrows in Figure 2 indicate the set of students who are ‘remediable’. These students are currently in a vicious cycle of persistent low engagement, but would transition to the high-ability, high-engagement trajectory if subject to a one-off, positive engagement shock. Inspection reveals that for the parameter values assumed here, a student of
any ability level who engages for one period will have a subsequent level of ability greater than 1/3 and will therefore transition from the vicious cycle to the virtuous one. This admittedly optimistic feature of the current parameterization – that all it takes is one period of engagement for any low ability student to transition to the virtuous cycle – is clearly a function of the assumed simplified lag structure and functional form. One can of course devise more complicated structures to generate a lower bound on the ability of students who are remediable with a one-off intervention, though it is not clear that the additional complexity would yield any further pedagogic insight.

3. Discussion

3.1 Positive framing, learner identities and the growth mindset

Formative assessments are designed to elicit student engagement by providing a safe space for essential academic trial and error that necessary for students to identify how to improve their work (Sadler, 1989). The results above have shown that this function can be undermined for students with low levels of incoming academic capital if other factors such as psychological, social or emotional forces intervene to make failure on mathematics intensive exercises costly. In such cases, loss aversion can cause students to become overly sensitive to the costs of failure and so disengage from challenging mathematics content in a way that is consistent with mathematics anxiety. In the behavioural economics literature framing has become a standard response to improving outcomes in areas as important and wide ranging as pension contributions, tax payment, organ donation, and disease prevention (Behavioural Insights Team, 2012; Cribb and Emmerson, 2016; and O’Keefe and Jensen, 2007). To take a concrete example, Cribb and Emerson (2016) show that when employees are asked to opt-in to a pension contributions become framed as losses from salary and relatively few employees choose to contribute. On the other hand, if automatic enrolment is the default and employees are instead offered the opportunity to opt out, enrolment increases by 37 percentage points. The literature has explained this apparently contradictory behaviour by observing that one frame contains an option that is defined over the negative domain (the deduction from salary) and so triggers loss aversion whereas the other is defined over a non-negative domain (the already reduced salary is the status-quo) and so does not. Thus, framing a situation in terms of gains does not remove the adverse prospect (those making pension contributions receive the same take-home pay regardless of what the default is), but rather it changes how the prospect is perceived. This established literature on gain-framed behavioural interventions has clear implications for good teaching practice.
Teachers who desire to ameliorate mathematics anxiety among their students should take steps to gain-frame the risk of failure and so rehabilitate student sensitivity to the threat of failure. Such students would transition from the ‘costly failure’ to the ‘costless failure’ line in Figure 1, implying an increase in engagement and thus subsequent attainment. But what might such frames look like in the classroom? Teachers could leverage the fact that students usually express a desire to be challenged in their coursework (Wilkie and Sullivan 2018 provide recent evidence) by explaining to them that a challenging academic experience necessarily implies some risk of failure. Such an intervention facilitates the formation of mathematics identities that treat failure as part of the learning process rather than personal deficiencies and so fosters more ‘functional’ mathematics learners as espoused, for example by Solomon (2007). This line of reasoning is very much aligned with recent pedagogic innovations in ‘growth mindset’ oriented teaching, as defined by Dweck (2006) and revisited by Boaler (2013), among others. Indeed, the present paper provides an alternative framework to understand why actively promoting a growth mindset in educational settings has had such a profound effect on learning. Under the ‘fixed mindset’ which views student ability as an unchanging endowment, failure to complete an exercise can only be viewed as a loss with a negative payoff. As argued above, such a frame is likely to trigger loss aversion, mathematics anxiety, and student disengagement. However, under the ‘growth mindset’ the experience of failure is framed over a positive domain as an input to learning and a way to track growth and so does not trigger loss aversion or mathematics anxiety, thus eliciting better levels of engagement. An emphasis on inputs to learning such as time and effort spent by students on exercises (Lee 2014 finds evidence that such inputs have a strong relationship with attainment) may also mitigate anxiety as it shifts the focus from risky outcomes such as success and failure, which drive loss averse behaviour and anxiety in the model, to parameters over which students exercise direct control.

Solomon (2007) has also emphasised the role that group work can play in shifting the narrative of failure from that of a personal loss to the creation of ‘a community of learners.’ Given the results above, such a community may be effective in part because groups are less susceptible to being undermined by individual loss aversion. However, group work can also be subject to freeriding and so may itself create disengagement if not properly managed (as outlined for example by Hansen, 2006 and Maiden and Perry, 2011).

3.2 Salience of summative assessments: motivation enhancement or cognitive interference?

An important element of the narrative underpinning the results presented so far is that students are myopic, that is, their decision making does not appropriately account for the
considerable eventual costs that disengagement is likely to entail. This is captured in Figure 3 (and Figure 1 earlier) by assuming that the expected utility of the outside option of disengagement is zero. Persistent disengagement is associated with lower attainment, heightened risk of exam failure and resits, and in the worst case, heightened risk of degree non-completion (Andrew and Everett, 2009; Lee, 2014; Salamonson; and Singh, Granville, and Dika, 2002 among others). To a rational decision-maker the present value of these costs should be substantial. It is plausible, then, that undoing present bias by increasing the salience of contributory, summative assessments may increase engagement.

This possibility is explored theoretically in Figure 3 where the ‘negative outside option’ line illustrates the case where the payoff to disengaging is perceived to be substantial and negative. Comparing this with the ‘costly failure’ line illustrates that even when failure is costly students across the ability distribution have an incentive to engage because the consequences of disengaging are assumed to be even more costly. This is in sharp contrast to the behaviour of relatively low ability students in Figure 1 who were assumed to be not only loss averse, but also fully present biased. In theory then, increasing the salience of exams and the threat of
degree non-completion throughout the learning block can incentivise consistent engagement. This result is consistent with the ‘the motivation enhancement model’ (Seipp 1991, and Struthers, Perry and Menec, 2000) from the literature on test anxiety, where exam stress increases motivation and thus engagement and outcomes.

On the other hand, the test anxiety literature has also discussed the ‘cognitive interference model’ (Cassady and Johnson, 2002; Hembree, 1988; Sarason, 1984; Seipp, 1991; and Zeidner, 2007), where the threat of failure impedes cognitive function more than it enhances motivation. The balance of the empirical evidence in the test anxiety literature appears to validate this the latter view. Furthermore, inducing test anxiety may exacerbate disengagement if the two forms of anxiety are mutually reinforcing. Indeed, research indicates that these two phenomena may be more closely interlinked than previously thought, with many overlapping correlates (Kazelskis et al. 2000). Thus, priming students to interpret failure on formative assessments as a signal of future failure on summative assessments may induce test anxiety and so heighten mathematics anxiety. This is illustrated in Figure 3 by way of the ‘induced anxiety’ line where the expected utility of failure is even more negative than on the original ‘costly failure’ line. If the perceived cost of failure is sufficiently large so that the induced anxiety line is sufficiently steep the model can generate the cognitive interference model where a greater fraction of students chooses to disengage after the threat of exam failure is made more salient than before.

Thus Figure 3 provides an important tool with which to understand both the motivation enhancement and cognitive interference models of attainment and the interplay between the two.

3.3 Polarisation, diverse cohorts and layered curricula

Recent research (Hodgen, Atikins and Tomei, 2020) indicates that in the UK mathematically demanding degrees such as economics, chemistry, and computer science recruit substantial fractions of students both with and without an A-level or AS-level qualification in mathematics. It is also understood that the transition from high school mathematics education to university mathematics education is an especially complex developmental step for students in both the natural and social sciences (McAlindend and Noyes, 2019). The forces analysed above suggest that in cohorts with mixed levels of incoming mathematics experience there is a real danger that students who start their degrees at a relative disadvantage may be at heightened risk of experiencing mathematics anxiety and disengagement if high proficiency is assumed or presented as the norm by their tutors.
In mathematically demanding subjects in the social and physical sciences it is common for specific concepts and the accompanying mathematical tools to be presented to students in layers of increasing complexity over the weeks within a learning block and over the different learning blocks of a degree programme. For example, in an economics degree it is typical for an introductory module to present profit maximization graphically, where an intermediate module might use univariate calculus, and an advanced module might require a bordered Hessian. The dynamics of engagement analysed above suggest that there may be unintended consequences to this type of layering that may disproportionately affect students who enter these degrees with low endowments of incoming mathematics proficiency. In these circumstances, the results of section 3.1 above suggest that the convenors of first-year principles modules need to be especially careful to frame the risk of failure over the positive domain, to build functional learner identities, and to facilitate a growth mindset within in the classroom.

3.5 Remedial support

Understanding mathematics anxiety as rooted in loss aversion provides some justification for the provision of remedial mathematics seminars, but also raises some important questions about this approach. In Figure 1, if students with negative expected utility of engaging are organised into a separate group and offered less complex exercises, the probability of failure would decrease and so the the expected utility of engagement would increase. Thus, in theory supplementary remedial classes can improve engagement among students who are at risk of mathematics anxiety without impeding the learning of others. Empirically however, such remedial mathematics support has proven to be less than universally effective (Bahr, 2008; Di Pietro, 2011; and Lagerlöf and Seltzer, 2009). This may be because unlike some other applications of loss aversion, where losses are measured as some tangible asset such as cash pay-outs or asset returns, the losses in this context arise due to complex interactions between academic outcomes and the psychology, learner identities and social relations between students as discussed above. In this setting obliging students to self-select into a low-capability group in a way that is easily observable to their peers might exacerbate perceived losses, anxiety, and disengagement rather than ameliorating them.

An alternative may be to leverage widely used virtual learning environments to address some of the shortcomings of remedial classes. When students engage with supplementary resources online, they may be less susceptible to the barriers which impede classroom participation (these barriers are carefully documented in Fassinger, 1995 and Fassinger, 2000).
For example, computerized quizzes can provide an important tool for students to identify gaps in their knowledge without having to expose these gaps to their peers or faculty, a concern that may impede participation in interactive classroom sessions. Thus, the cost of failure may be substantially reduced in virtual learning environments in comparison to physical ones. Indeed, stigma may even be reversed as proactive, voluntary engagement with online resources may enhance self-image and learner identity (as in Solomon, 2007) rather than detracting from them.

The developing field of gamification in education (Borges et al. 2014 provides an early review) applies techniques from video game design to stimulate engagement in educational contexts. This field has the potential take the above-mentioned benefits of technology enhanced learning even further. An aspect of gamification that is especially salient to this study is the possibility of programming formative assessments on a virtual learning environment to dynamically match the difficulty level of questions with the ability level of the student taking the assessment (as estimated, for example, by the fraction of correct answers and question difficulty in the student’s recent history). The results of this paper help to understand why such algorithms may be effective: they allow formative assessment to be kept challenging enough to enable engagement and learning, but not so challenging as to induce anxiety and disengagement. However, this positive effect on engagement comes at the cost of circumventing a potentially important learning experience: an overreliance on such algorithms would remove the opportunity for students to learn how to cope with the threat of failure, an important educational outcome in its own right. Nevertheless, in concert with other interventions described above, such algorithms may well become an important component of a multifaceted approach to mitigating the effects of loss aversion and mathematics anxiety in heterogeneous mathematically intensive courses.

4. Conclusion

This paper has leveraged insights from behavioural economics (Kahneman and Tversky, 1979; Rabin and Thaler, 2001; and Thaler, 2000) to develop an understanding of mathematics anxiety. The sharply dichotomous nature of success and failure in mathematically intensive university assessment is shown to imply a gamble in students’ short-term perceived well-being. When sufficient care is not taken on the part of instructors to frame the outcomes of this gamble over a positive domain, loss aversion was found to bias study decisions towards disengagement in behavioural patterns that are consistent with mathematics anxiety. These
forces were shown to imply polarization in student outcomes in a simple, dynamic model of engagement and mathematics ability.

The paper went on to use the loss-aversion based theory of mathematics anxiety as a framework to understand and evaluate interventions that have been proposed in the literature to counteract mathematics anxiety. Interventions that are designed to positively frame the risk of failure in learning contexts were found to be potentially effective. These include efforts to cast formative assessments as a ‘safe space’ for students to learn how to improve their work (Sadler, 1989), to foster a growth mindset in the classroom (Boaler, 2013 and Dweck, 2006), and to operationalize functional learner identities (Solomon, 2007 and Mendick, 2005b) that are resilient to the threat of failure. Prior literature has also asked if the salience of summative assessments can be used to foster engagement with formative assessment in the motivation enhancement and cognitive interference models (Cassady and Johnson, 2002; Hembree, 1988; Sarason, 1984; and Seipp 1991). The dynamics of student engagement proposed here were applied to these models and shown to offer important insights on the interrelationships between them. Critical reflection on the model parameters also helped to understand why the provision of remedial classes while helpful in theory, is not always effective in practice (Bahr, 2008; Di Pietro, 2011; and Lagerlöf and Seltzer, 2009). Technology enhanced learning tools such as supplementary multimedia resources and adaptive learning technologies which can ‘gamify’ virtual learning environments (Borges et al. 2014) were found to be potentially useful in sustaining engagement, but miss an opportunity to teach students how to cope with the threat of failure.

Mathematics struggles with diversity. Women, ethnic minorities and underprivileged groups are at heightened risk of persistent disengagement on mathematically intensive courses of study (Hottinger, 2016; Lim, 2008; and Mendick, 2005a). A theory that helps to understand persistent disengagement and identify measures that might help prevent it has the potential to make mathematics classrooms more diverse and inclusive spaces. Some educators have been tempted to conclude that chronically disengaged students are chronically apathetic to their studies. The theory developed here suggests otherwise. Rather, some students may experience anxiety and disengage because they are overly affected by the threat of failure. The positive teaching practices outlined above can help to equip them with tools to cope with the threat of failure thus alleviating mathematics anxiety and paving the way to persistent improvements in engagement and attainment.
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