Yang, Jinlong, Huang, Deqing, Xia, Jingkang and Li, Yanan (2023) Trajectory deformation with constrained optimization for bilateral rehabilitation robots. IEEE-ASME Transactions on Mechatronics. pp. 1-12. ISSN 1083-4435

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Trajectory Deformation with Constrained Optimization for Bilateral Rehabilitation Robots

Jinlong Yang, Student Member, IEEE, Deqing Huang, Senior Member, IEEE, Jingkang Xia, Student Member, IEEE, Yanan Li, Senior Member, IEEE

Abstract—Robot-aided bilateral treatment has been verified to be an effective training program for hemiplegic rehabilitation. In this article, a reference-free active control framework based on optimal trajectory deformation is proposed to ensure the safety requirements in the leader-follower paradigm of bilateral treatment. A constrained optimization method is developed to handle the motion constraints, which are constructed by quantitative assessments of typical impairment in stroke patients, such as reduced range of motion (ROM), resistance to passive movement (RPM), and disturbed quality of movement (QOM). Then, by optimally deforming the robotic trajectory, abnormal motion patterns that lead to safety issues can be rectified. Furthermore, the physically interactive trajectory deformation is employed to achieve active control without a predefined trajectory. At last, all approaches are verified on a robotic platform with a 2-DoF lower-limb exoskeleton. Experimental results demonstrate the effectiveness of proposed control scheme in rectifying abnormal motion patterns and enhancing human-robot interaction.

Index Terms—bilateral treatment, physical human-robot interaction (pHRI), trajectory deformation, constrained optimization.

I. INTRODUCTION

STROKE is a disease related to cerebral lesion and the majority of its victims suffer from hemiplegia that causes complete or partial loss of movability on one side of their extremities [1]. In hemiparesis rehabilitation, bilateral treatment has been investigated as a rehabilitation intervention with great potential for further activating motor cortex and enhancing motor learning [2]. Also known as bilateral training, it is a generic term for rehabilitation paradigms that require the patient’s both limbs to complete a task. Distinct evidence of efficacy has been presented by a lot of bilateral paradigms, including bilateral isokinematic training, machine-assisted bilateral training, bilateral mirror therapy and bilateral priming [3]. Among those paradigms, the machine-assisted method which employs robotic devices to assist the rehabilitation exhibits poten- tialities, with advantages in less labor-intensive operations and more precise quantitative diagnosis for individuals [4].

The bilateral robot in hemiplegic rehabilitation can be categorized into two types in terms of motion trajectory [5]. One considers the robot with a fixed trajectory, whose control objective is to repeat a predefined task. However, due to the limit of the predetermined trajectory, the robot (e.g., the BULReD in [6]) cannot provide assistance in unstructured tasks. Moreover, seamlessly switching the training tasks according to the patient’s intent is prohibited by this type of robot.

Another type of the bilateral robot delivers the training with a customized trajectory generated under the cooperation of the patient’s both affected and unaffected limbs. For instance, the robot-aided BFIAMT [7], a bilateral force-induced isokinetic arm movement trainer, enables the push and pull forces exerted by the human user’s both hands to drive two cone-shaped handles to move in a symmetrical and smooth manner. Furthermore, the bilateral robot of this kind is compatible with mirror therapy so it can further improve the recovery. In [8], the mirror-image motion enabler (MIME) enables the leader-follower mode to help the patient continuously move the paretic limb to the mirror-image position of the opposite limb, where the rate and range of coordinated movement are under the patient’s own control. The EXO-UL7 from [9] also employs the leader-follower mode, whereas the robot provides only partial assistance, which implies that the paretic limb of the patient is not forced into the precise mirror-image position. With assistance of the EXO, patients are capable of interacting with therapy games in a virtual environment, and they exhibit significant improvements after the treatment. In this point of view, the bilateral robot with leader-follower mode enables the patient to conduct more efficacious training movements. Motivated by these results, the control strategy in this paper is proposed on the basis of leader-follower mode for bilateral robots.

For hemiplegic rehabilitation, on the other hand, active involvement induces neural plasticity, which plays a prominent impact on the recovery particularly for severely to mildly impaired patients [10]. Therefore, the rehabilitation robot is supposed to exhibit compliance in physical human-robot interaction (pHRI), so the participant-active mode is necessary for the robot. In this regard, impedance control is a promising approach for achieving the robotic compliance [11]. Nevertheless, since the healthy limb of patient dominates robotic behaviors in the leader-follower mode, abnormal movements which are generally incurred by either inattention, myotonia or muscle spasm, can lead to safety issues due to the partial loss of mobility on the patient’s paretic limb. [5] in this
sense proposes a trajectory generation method with subject-specific adaptation to compromise between free leader motion and predetermined movement, though the alternative reference trajectories are still preprogrammed and fixed. Furthermore, even within the context that appropriate reference movements are provided during participant-active rehabilitation, the safety issues, which could be excess motion range/speed caused by excessive pHRI between patient’s paretic limb and robotic device, are supposed to be taken into concern. For instance, [12] designs specifications for the actuated DoF of their lower extremity powered exoskeleton (LOPES), where peak torque/force and motion range/speed are chosen based on measured values in the “slow walking cycle”. From the quantified assessment of movement [13], the therapists find some typical impairments in stroke patient, such as reduced range of motion (ROM), resistance to passive movement (RPM), disturbed quality of movement (QOM) and so on. Overall, in order to ensure the patient’s safety during rehabilitation, an active control scheme allowing for the assessment and rectification of abnormal motion patterns is supposed to be employed.

On this issue, analogous scenarios have been discussed in [14], [15], where solutions based on barrier Lyapunov function (BLF) and admittance control are proposed to guarantee the position constraint of robotic manipulators. In their strategies, the reference trajectory is reshaped with the external force based on an impedance model. Since the reference trajectory is supposed to remain within the constrained task space, a soft saturation function is employed to rectify the trajectory. Then, the controller designed with the BLF ensures output-constraints satisfaction during trajectory tracking. By limiting the output within a time-varying and asymmetric barrier around the reference trajectory, the boundary of output will not be transgressed [16]. Beyond the barrier limit to output position, [17] establishes a unified framework to incorporate the robot’s velocity and acceleration constraints. Nonetheless, those strategies generally focus on a structured task with predetermined movement, thereby unable to handle the motion constraints in leader-follower bilateral treatment, where the constraints can be either time-varying or nonlinear.

In this regard, to meet the safety requirements in hemiparesis rehabilitation, we propose a trajectory modification approach based on prediction and optimization. By building the constrained optimization and deforming the robotic movement, typical abnormal motion patterns in stroke patients [13], such as ROM, RPM and QOM, can be quantitatively assessed and rectified. Furthermore, because of the non-causality of optimization programming, and the reference-free movement in leader-follower bilateral training, the physically interactive trajectory deformation [18] is employed to enable the modification on the future segment of trajectory [19], [20]. From above discussions, a novel active control framework based on trajectory deformation with constrained optimization is proposed in this paper. The objective is to enable the stroke patient to actively participate in the bilateral training, and ensure the safety of rehabilitation by assessing and rectifying typical impaired patterns in their movement, such as ROM, RPM and QOM. All abovementioned approaches are examined by experiments on a 2-DoF lower-limb exoskeleton.

Compared with existing control schemes for bilateral treatment, the main contributions of this paper can be summarized as follows:

1) A novel reference-free active control framework, based on trajectory deformation with constrained optimization, is proposed to ensure the safety requirements in hemiparesis rehabilitation.
2) Novel evaluation methods are constructed to quantitatively assess abnormal motion patterns with respect to ROM, RPM and QOM.
3) Compared with traditional output-constrained methods, the proposed control framework enables handling of more complex motion constraints which can be either time-varying or nonlinear.

The remainder of this paper is organized as follows: the architecture of the robotic device is elaborated in Section II; the proposed control framework is formulated in Section III; in Section IV, the experimental results are presented and analyzed; at last, conclusions are drawn and some future research possibilities are suggested in Section V.

II. SYSTEM DEVELOPMENT

A. Description of Robotic Platform

In this paper, a lower-limb robotic control system is developed for leader-follower bilateral training, on which our control scheme is examined. Here, an overview of the structure of our robotic platform is shown in Fig. 1.

Typically, robotic devices for bilateral training are supposed to possess dual robots to fulfill the leader-follower task. This requirement potentially constrains the applicability and flexibility of robot-aided bilateral therapy. In this sense, an attempt is made on employing a lower-limb motion monitoring system to capture the motion of the patient’s healthy limb during bilateral training. While the patient wears the exoskeleton on their paretic limb, the leader-follower strategy can be conveniently achieved by tracking the monitored movement. In this regard, a motion monitoring system based on a wearable IMUs-based device is developed in this paper, and details of the system design are discussed in the next subsection. In terms of the participant-active mode, interaction force between
the patient’s impaired limb and the exoskeleton is measured through an external measurement device with pressure sensors as shown in Fig. 1, by which the disturbance from variable participants with different limb masses is eliminated.

In general, the operating flow drawn in Fig. 1 can be summarized as follows: firstly, the human user continually conducts reference-free bilateral movement, where force and inertial sensors are used to measure the motion of the user’s healthy limb and interaction force of their impaired limb; by the time new measurements arrive, the proposed control system provides rectification on the robotic behavior; at the end of a control period, the modified control input is transmitted to the control cabinet to generate appropriate robotic movement.

B. Development of Lower-Limb Motion Monitoring System

In this part, a measurement system for patient’s healthy limb is designed on a wearable IMU-based device. The measurement is fulfilled by two 9-axis IMUs mounted on the patient’s healthy limb and equipped at the thigh and shank, respectively. In terms of the inertial sensor, measuring the tri-axial orientation through either the gyroscope, accelerometer or magnetometer leads to different types of errors, which are induced by accumulation of drift, external acceleration or magnetic disturbance [21], [22], respectively. Therefore, approaches that incorporate multiple-sensor data fusion and human biomechanical constraint are developed to enable robust joint’s angle estimation [23], [24]. Fig. 2 exhibits the structure of our motion monitoring system, in which, the two-stage real-time monitoring framework [24] is employed.

1) Data Fusion: The first stage is intended to derive the tri-axial orientation of each IMU from raw data of multiple sensors including the gyroscope, accelerometer and magnetometer. In terms of computationally efficient methods, complementary filter (CF)-based techniques are advantageous in convenience to real-time and wearable applications. For example, [22] presents a CF-based robust orientation estimation method that exploits a gradient descent approach to estimate the integrated gyro drift utilizing multiple-sensor data (i.e., angular acceleration, acceleration, and magnetic field). Moreover, as a popular computationally lean alternative to Kalman filter, Madgwick’s algorithm [25] employs an optimized gradient descent approach to compute the orientation error under quaternion representation. Furthermore, an extended CF method is proposed in [26] which offers improved robustness and efficiency over the method in [25].

As shown in Fig. 2, in our development of the motion monitoring system, the complementary filter based on the gradient descent algorithm under Euler angle representation is employed because of its acceptable accuracy and easy-to-implement property [22]. On the other hand, since quaternion representation is utilized in our joint angles estimation approach, the transformation from Euler angle to quaternion could lead to extra noise. Nonetheless, it’s worth mentioning that the two-stage design of the motion monitoring system enables cheap replacement for the data fusion method, i.e., the fusion methods [25], [26] can be chosen according to specific applications.

2) Estimation of Joint Angles: The second stage aims at the estimation of hip and knee joint angles through the incorporation of the sensor orientations and lower-limb biomechanical constraint. [27] utilizes the knowledge of knee joint constraint to estimate the joint angle during dynamic activity. [24] proposes a gradient descent method to iteratively estimate the quaternion-based joint angles while assuming the flexion/extension of the lower-limb joints occurs only around the transverse axis of the body. Motivated by works [24], [27], we develop an optimization method for the estimation of lower-limb joint angles under quaternion representation.

As shown in Fig. 2, the cost function of the optimization is derived from the quaternion rotation between hip and knee joints. While the orientations obtained by data fusion contain external noises, the biomechanical constraint (i.e., the hinged knee joint) is taken into account to refine the coarse estimation. According to the properties of quaternion, the proposed optimization can be formulated as a standard quadratic programming (QP) problem. Details about the estimation method are given in APPENDIX A.

III. CONTROL FRAMEWORK

In this section, a reference-free active control framework is proposed on the basis of trajectory deformation with constrained optimization, where the control behavior is determined by the current human commands only. As shown in Fig. 3, the current motions of the patient’s healthy and parietal limbs are the only external inputs to drive the robotic system, which indicates the control framework is free of a preprogrammed reference behavior. Multiple motion constraints are constructed for assessing and rectifying typical abnormal motion patterns in stroke patients, such as ROM, RPM and QOM, and a motion rectification method is developed to render necessary trajectory modification and to mitigate the conflict with human intention. Furthermore, the optimal trajectory deformation algorithm [18], [20] is employed to provide robot compliance through modification on the future segment of trajectory, and to avoid conflicts with the motion rectification.

This section is organized as follows: in Section III.A, we develop trajectory deformation algorithm (TDA) based on optimal variation, and a deformation scalar different from [18], [20] is designed to achieve output invariances to sampling period and deformation duration simultaneously; in Section III.B, objective assessments of abnormal motion patterns, such as ROM, RPM and QOM, are introduced, for which the
corresponding safety constraints and constrained optimization are proposed.

A. Trajectory Deformation Algorithm

In order to explore the optimal trajectory deformation, we define \( q_d : \mathbb{R} \rightarrow \mathbb{R} \), \( q_h : \mathbb{R} \rightarrow \mathbb{R} \) as the reference and desired trajectories, respectively. \( f_h : \mathbb{R} \rightarrow \mathbb{R} \) is the pHRI force and \( \tau \in \mathbb{R} \) is the sampling period. Initially, the desired trajectory is the same as the reference \( q_d(t) = q_h(t) \). For each step, \( t_i = k\tau \) is defined as current time (i.e., \( k \in \mathbb{N} \) denotes current step), \( t_f = t_i + T_p \) is a future time at which the trajectory deformation of the current step ends and \( T_p \) is a positive constant denoting the duration of the deformation, and \( \gamma_d : [t_i, t_f] \rightarrow \mathbb{R} \) denotes the segment of the desired trajectory \( q_d \) over the time interval \( t \in [t_i, t_f] \). Then we can define \( \hat{\gamma}_d : [t_i, t_f] \rightarrow \mathbb{R} \) as the deformation of \( \gamma_d \). Once the motion planner determines \( \hat{\gamma}_d \), the desired trajectory \( q_d \) over the interval \( t \in [t_i, t_f] \) will be updated as \( q_d(t) = \hat{\gamma}_d(t) \). Analogously to the works in [18], [20], a quadratic optimization is exploited to yield the optimal deformation,

\[
\min_{\hat{\gamma}_d(t)} J(\hat{\gamma}_d(t)) = - \int_{t_i}^{t_f} V(t)f_h(t_i)dt + \frac{1}{2\alpha} \int_{t_i}^{t_f} [\hat{V}(t)]^2dt \\
s.t. \quad V^{(n)}(t_i) = V^{(n)}(t_f) = 0, \quad n \in \mathbb{N} \tag{1}
\]

where \( \alpha \) is a positive scalar; \( V : \mathbb{R} \rightarrow \mathbb{R} \) is defined as a variation field which is a smooth curve satisfying \( \hat{\gamma}_d(t) = \gamma_d(t) + V(t), \forall t \in [t_i, t_f] \).

The solution of (1) satisfies the Euler-Lagrange equation,

\[
\sum_n (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial \dot{\gamma}^{(n)}_d} = 0, \quad \tag{2}
\]

where \( L(\gamma_d, \dot{\gamma}_d, \ddot{\gamma}_d, \dddot{\gamma}_d, \tau) = -V(t)f_h(t_i) + \frac{1}{2\alpha}[\hat{V}(t)]^2 \). By solving (2) we can derive \( \hat{\gamma}^{(n)}_d = -\alpha f_h(t_i) + \gamma^{(n)}_d \), where the superscript \( (n) \) denotes \( n \)-th derivative, \( n \in \mathbb{N} \). Considering the boundary constraints of continuity in (1), the solution of it has the form \( f_h(t_i)V(t) \), where \( V(t) = \alpha \cdot T_p^3(t - t_i)^3(T_f - t)^3, \) \( t \in [t_i, t_f] \).

Utilizing the notion in [18], we define that \( \Gamma : \mathbb{N} \times \mathbb{R} \rightarrow \mathbb{R} \) denotes a family of the variation fields, in which, the formulation of \( \Gamma(k,t) \) can be derived on the basis of \( V(t) \):

\[
\Gamma(k,t) = \begin{cases} 
\alpha \cdot T_p^6(t-k\tau)^3(T_p + k\tau - t)^3, & t \in [k\tau, T_p + k\tau] \\
0, & \text{otherwise}
\end{cases} \tag{3}
\]

Besides, the derivation shows that TDA works as a linear and time-invariant (LTI) system with the interaction force as an input and the deformation curve as an output. Hence, the ultimate deformation curve derives from the linear superimposition of the variation fields at each timestep. Analogously, we define \( h : \mathbb{N} \times \mathbb{R} \rightarrow \mathbb{R} \) as a family of the deformation curves,

\[
h(k,t) = \sum^{k}_{n=0} f_h(n\tau)\Gamma(n,t) \tag{4}
\]

Then in order to discuss our design of the constant \( \alpha > 0 \), we consider the unit step response of TDA, by which we obtain the maximum magnitude of the deformation curve,

\[
\sup_{\tau \in \mathbb{R}^+} \| h(k,t) \| = \frac{1}{\tau} \int_{t_i}^{t_f} 1 \cdot V(t)dt = \alpha \cdot \frac{T_p^7}{\tau c} \tag{5}
\]

where \( c \) is a positive constant. Because \( T_p \) determines the period of one interaction, its adjustment is not supposed to alter the amplitude of deformation. Besides, as the setting of \( \tau \) depends on the computational performance of the processor, the shape of the deformation should be independent of \( \tau \). Therefore we define \( \alpha = \mu \cdot \frac{T_p^7}{\tau c} \) to achieve two types of invariance as shown in Fig. 4 which exhibits the step response of TDA with different control parameters \( \tau \) and \( T_p \). For example, the multiple blue curves in Fig. 4(a) can be regarded as a cluster of the variation fields because of the unitized input \( f_h(t) = 1, t \in [0,1] \). As a result of (4), the figure shows that the pink and red deformation curves, i.e., the output of TDA are the superimposition of variation fields, and the comparisons to Figs. 4(b) and (c) indicate that our design of \( \alpha \) renders the output invariance to \( \tau \) and \( T_p \), respectively. Different from the vector-form TDA proposed in [18] and [20], which achieves the output invariance to either \( \tau \) or \( T_p \), here, the deformation developed in an analytic form is advantageous in ensuring safety.

B. Safety Constraints and Constrained Optimization

Without restrictions on the movements, patients are more prone to be injured during rehabilitation due to the lack of basic motor functions. For this reason, a trajectory rectifying method based on constrained optimization is proposed. Here, safety constraints of typical functional impairments in stroke patients [13], such as ROM, RPM and QOM, that allow for quantitative assessments are developed.

1) Range of Motion: ROM bounds the workspace where the patient could freely flex and extend their joints. The ranges vary among different individuals and are used to differentiate the severity of impairments in general [13]. Defining \( \hat{q}_d : \mathbb{R} \rightarrow \mathbb{R} \) as the desired trajectory after rectified by the
denotes maximum resistance force; \( \kappa \) denotes deformation duration \( \tilde{f} \) area. From (7), we can figure out that the function is in the

The joint's velocity, in which, \( \text{tanh} (\cdot) \) is the joint's angular velocity to restrict robotic movements

where \( \omega_l \) and \( \omega_u \) denote the lower and upper limits of the joint's angular velocity.

3) Quality of Movement: QOM assesses the patient's sensorimotor impairment and motor learning, in which quantitative measures of movement smoothness play an important role [30]. [13] conducts an experiment about the difference between healthy and post-stroke subjects in quality of movement. The results of quantitative analysis show healthy subjects are able to execute smoother movements than the subjects in the post-stroke stage, which implies that the path accuracy and movement smoothness of stroke patients are expected to be enhanced.

Considering the impaired behavior with the motion pattern QOM, the restriction on QOM is formulated as a soft constraint, instead of the form of hard constraint analogous to ROM and RPM. On this premise, based on the idea of shared control that mitigates the conflicts between the patient and the robotic system, we propose an optimization method to enhance QOM while ensuring the robotic movement \( \tilde{q}_d \) to satisfy the safety constraints simultaneously as discussed above. The energy function of the optimization is formulated as follows,

\[
E(u(t)) = \int_{t_i}^{t_f} u^2(t)dt + \frac{1}{2 \beta} \int_{t_i}^{t_f} \omega_d^2(t)dt
\]

where \( \tilde{q}_d(t) = q_d(t) + u(t) \) denotes rectified desired trajectory, \( u(t) \) denotes the modified angle which needs to be solved, and the boundary conditions of \( u(t) \) are formulated as \( u^{(n)}(t_i) \) = 0, \( n = 0, 1, 2 \). Examining (9), the first part denotes the energy of modification, which represents the objective to mitigate the conflicts in pHRI. The second part is the minimum-jerk model accounted for natural human trajectory modifications [31], the same as in TDA. In this sense, the conflicts between pHRI and the motion rectification can be mitigated. The proposed optimization enables the robotic device to correct its behavior once the movement tends to violate the safety constraints. Meanwhile, slight modifications are continually made to enhance the quality of robotic movement.

Nevertheless, improving the QOM by applying only the minimum-jerk model cannot rectify abnormal patterns, e.g., oscillating movements (as shown in Section IV.B). Therefore, we introduce a constrained inequality on the basis of speed arc-length metric [32],

\[
\ln \left( \int_{t_i}^{t_f} \sqrt{\frac{1}{T_p}} + \left( \frac{d\tilde{v}}{dt} \right)^2 \right) \leq \eta_m
\]

where \( \tilde{v}(t) := \frac{\tilde{q}_d(t)}{f_m} \) denotes normalized joint's angular velocity, and \( v_m = \max (|\omega_l|, |\omega_u|) \) is the maximum velocity; \( \eta_m \in \mathbb{R}^+ \) is a predefined smoothness index. Viewing (10), the left side of inequality reflects the arc-length of velocity curve. By setting restriction on the arc-length, the amplitude of oscillation can be significantly reduced (as shown in Section
IV. B). On the other hand, (10) constrains the average of movement acceleration, while retaining the components of lower frequency in the velocity curve.

As discussed above, the energy function and constraints have been given in (6-10). In addition, the state equation corresponding to \( \dot{q}_d(t) \) and \( u(t) \) is supposed to be discussed. Firstly, checking the desired trajectory \( q_d(t) \), we have \( q_d(t) = q_r(t) + h(k, t) \), in which \( k \in \mathbb{N} \) is the current step. Because the reference trajectory \( q_r(t) \) and the interaction force \( f_s(t) \) are not predefined in bilateral training and (9) is non-causal, they need to be predicted. Considering the human motions during rehabilitation as in a low jerk, we assign the constant-acceleration model to the description of predicted reference trajectory \( \dot{q}_r(t) \). Besides, referring to work [18], we assume that future applied forces will be the same as the current input \( \hat{f}_h = f_h(k\tau) \). Then, the state equation of \( \dot{q}_d \) can be formulated as,

\[
\dot{q}_d(t) = \dot{q}_d(t) + u(t) = \dot{q}_r(t) + u(t) + \sum_{n=0}^{[t/\tau]} f_h(n\tau) \Gamma(n, t) + \sum_{n=k+1}^{[t/\tau]} \hat{f}_h \Gamma(n, t)
\]

(11)

From above discussions, an optimization problem related to \( u(t) \) is formulated with (6)-(11) and the boundary conditions. Denote the number of sampling points between the duration \( T_p \) as \( N = \lfloor \frac{T_p}{\tau} \rfloor \). At the current step \( k \), defining \( \hat{q}_{d,k} = [\hat{q}_d(n\tau)] \), \( \hat{q}_{r,k} = [\hat{q}_r(n\tau)] \), \( u_k = [u_k(n\tau)] \in \mathbb{R}^{N+3} \), \( n = [k-3, k + N-1] \in \mathbb{Z} \) as vectors of regulated desired trajectory, predicted reference trajectory and modified angles, respectively. Then, the discretized optimization with fixed sampling time \( \tau \) is formulated as follows:

\[
\min_{u_k} E(u_k) = \|u_k\|_2^2 + \frac{1}{2\beta} \|\hat{q}_{d,k}\|_R^2
\]

\[
\begin{align*}
\dot{q}_{d,k}(t) = & \dot{q}_{r,k}(t) + h(k-1, (k + \ell - 3)\tau) \\
& + u_k(t) + \sum_{n=0}^{\ell} \hat{f}_h \Gamma(k, (k + \ell - 3)\tau) & (a)
\end{align*}
\]

\[
\begin{align*}
\theta_t \cdot \mathbf{I} & \leq \hat{q}_{d,k} \leq \theta_u \cdot \mathbf{I} & (b)
\end{align*}
\]

\[
\begin{align*}
2\omega_n \tau \cdot \mathbf{I} & \leq h_1 \hat{q}_{d,k} \leq 2\omega_n \tau \cdot \mathbf{I} & (c)
\end{align*}
\]

\[
\begin{align*}
-\Lambda_1 \hat{q}_{d,k} \cdot \tanh(\kappa_3 \hat{q}_{d,k}) & \leq 2\omega_n \tau \cdot \mathbf{I} & (d)
\end{align*}
\]

\[
\begin{align*}
\ln \sum_{n=0}^{\ell} \frac{1}{T_p^2} \frac{1}{n^2} + \frac{\|\hat{q}_{d,k}\|_R^2}{n^2 \tau^2 \sigma_m^2} & \leq \eta_m & (e)
\end{align*}
\]

\[
\begin{align*}
u_k(t) = 0, \quad t = 0, 1, 2 & (f)
\end{align*}
\]

in which, \( \hat{q}_{r,k} \) is predicted by

\[
\dot{\hat{X}}_{r,k}(t+1) = \begin{bmatrix}
1 & \tau & \frac{1}{2} \tau^2 \\
0 & 1 & \tau \\
0 & 0 & 1
\end{bmatrix} \cdot \hat{X}_{r,k}(t)
\]

(13)

where \( \hat{X}_{r,k}(t) = [\hat{q}_{r,k}(t), \dot{\hat{q}}_{r,k}(t), \ddot{\hat{q}}_{r,k}(t)]^T \) and it can be calculated from \( q_r(t), t \leq k\tau \). In addition, \( \Lambda_0 \in \mathbb{R}^{N \times (N+3)} \), \( n = 1, 2, 3 \) denote the n-th order finite differencing matrix, and we have \( \hat{q}_{d,k}^{(n)} = \tau^{-n} \cdot \Lambda_0 \hat{q}_{d,k} \), when ignoring boundary conditions [33]. \( R = \Lambda_3^T \Lambda_3, Q_n = \Lambda_2^T e_n \lambda_2^T \Lambda_2 \) are symmetric positive semidefinite matrices, and \( e_n \in \mathbb{R}^N \) are unit vectors so that \( e_n(n) = 1 \).

Besides, we can prove the convexity of this optimization, which indicates the local optimum as equivalent to the global optimal solution [34], therefore the problem can be efficiently solved. The proof of the convexity is given in APPENDIX B. In this paper, the convex optimization problem (12) is resolved by the sequential least squares programming algorithm (SLSQP) [35]. Moreover, as shown in Fig. 3, a position controller is designed for completing trajectory tracking task. For simplicity, a PD-type position controller is employed to track the rectified trajectory \( \dot{q}_d \).

IV. EXPERIMENTS

In this section, experiments are set up and conducted by four healthy subjects with the human-robot interaction test within unconstrained space and the effectiveness test within constrained space. The subjects are required to participate in the two tests in sequence. The results of experiments are presented to demonstrate the effectiveness of the proposed control framework in Section III.

A. Experiment 1: Human-Robot Interaction within Unconstrained Space

As shown in Fig. 5, experiments are carried out on a 2-DoF lower-limb rehabilitation robot. The lengths of aluminum links are adjustable, and velcro straps are used for leg fixing, in order to adapt the subjects with different height. Brushless flat motors (EC 90 flat, Maxon) with rated torque 964mNm and harmonic reducers (LHT-I, LaifuelDrive) with rated ratio 80 : 1 construct two actuator modules, by which the thigh and shank links with respective masses 1.42 kg and 0.87 kg are driven to imitate lower-limb flexion/extension. A

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
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<td>( \tau )</td>
<td>25</td>
<td>ms</td>
<td>Sampling period</td>
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<td>0.5</td>
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</tr>
<tr>
<td>( \mu )</td>
<td>8.73 \times 10^{-4}</td>
<td>rad/(N.s)</td>
<td>Deformation factor</td>
</tr>
<tr>
<td>( \beta )</td>
<td>1</td>
<td>-</td>
<td>Weighted factor</td>
</tr>
<tr>
<td>( M )</td>
<td>diag[18, 18]</td>
<td>N.m/2 rad</td>
<td>Inertia</td>
</tr>
<tr>
<td>( B )</td>
<td>diag[150, 150]</td>
<td>N.m/2 rad</td>
<td>Damping</td>
</tr>
<tr>
<td>( K )</td>
<td>diag[1300, 1300]</td>
<td>N/m rad</td>
<td>Stiffness</td>
</tr>
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Fig. 5. Experimental setup.
motion controller (BAC332E, LeadShine) and a motor driver (Accelnet AE2, Copley) are utilized to control the actuator modules, where the angular position of each joint is sampled by an optical encoder (AFM60A, SICK). An interaction force measurement device is developed by pressure sensors (ZNHM-VII-H6-20, Zhongnuo Sensor) with rated load 20kg and sensitivity 2.0mV/V to measure the interaction force between the exoskeleton and human limb. The motion monitoring is fulfilled by a wearable device consisting of two 9-axis IMUs (N100, WHEELTEC) and an ARM board (RPi4B, Raspberry Pi Foundation). Moreover, a personal computer is served as the master station to process data acquisition with an interface (N100, WHEELTEC) and an ARM board (RPi4B, Raspberry Pi Foundation).

In this part, we compare our proposed strategy with admittance model (AM) and TDA to test their ability to achieve smooth and compliant movement. Analogous to TDA, AM is implemented for trajectory deformation, in which, the deformed trajectory $q_{ad}$ is derived from,

$$M(q_{ad} - \dot{q}_r) + B(q_{ad} - \dot{q}_r) + K(q_{ad} - q_r) = f_h$$  \hspace{1cm} (14)

where $M, B, K$ are the desired inertia, damping and stiffness matrices of AM, respectively. To make comparisons between the three strategies, we select the parameters of $M, B, K$ on the basis that the peak time and peak value of AM are approximate to those of TDA so that the two models render analogous unit-step response. The relevant parameters of control strategies are listed in Table I.

In the experiment, the subjects are asked to wear the measurement device on their left legs, and equip the exoskeleton on the opposite. Each subject excutes symmetrical flexion/extension movement with their lower limbs for respective five trials under each strategy. Unconstrained workspace is assumed in this experiment, i.e., the constraints (12)b-e are not enabled.

Figs. 6(a), (c) and (e) show the robotic behaviors under AM, TDA and our proposed method, respectively. Analogous to AM and TDA, the proposed method exhibits robotic compliance during human-robot interaction. But from Figs. 6(a) and (c), we can find that there are slight ripples on the desired (red) and actual (black) trajectories, when the reference (blue) trajectory or the applied force is oscillating (such as 2-4s, and 10-12s). Here, the desired trajectory denotes the trajectory resolved from the control method, and the actual trajectory represents the actual joint’s position measured by optical encoders. More obvious results can be found in Figs. 6(b) and (d), where the angular accelerations present violent oscillations during active interactions. Instead, the curves with our rectifying strategy show no oscillations, and the amplitude of angular accelerations is much smaller than those of AM and TDA, as shown in Figs. 6(e) and (f).

The quantitative analysis results are shown in Fig. 7. The evaluation measures DSJ, EPUD and NRMSD denote the dimensionless squared jerk, the energy per unit distance, and the normalized root mean square deviation, respectively. A smaller value of DSJ indicates smoother movements, and a smaller EPUD reflects higher robotic compliance. More details of those evaluative methods can be found in [20].

In this experiment, the mean DSJ value of desired trajectory generated by our proposed strategy is 81.10% and 78.35% less than those using AM and TDA, respectively. The mean DSJ value of actual trajectory is reduced by 38.42% and 40.14% with respect to the other two methods. And we show that the reductions on the DSJ value are significant through t-test, which reflects the improvement on smoothness of robotic behaviors by using the proposed method. On the other hand, the quantitative result in Fig. 7 suggests that there is no significant difference on DSJ of the desired trajectory between TDA and AM, different from the conclusion in [20]. This may be addressed by expanding the sample range and introducing Analysis of Variance (ANOVA) for quantitative analysis. In terms of EPUD and NRMSD, the deviations between AM, TDA and the proposed method are not statistically significant. We conclude that the proposed strategy can improve the smoothness of robotic movements, while maintaining the same level of robotic compliance and tracking accuracy, compared with AM and TDA.

B. Experiment 2: Effectiveness Test within Constrained Space

In this part, we set up three groups of tests to demonstrate the validation of the proposed method in assessing and recti-
fying abnormal motion patterns as ROM, RPM and QOM, respectively. The same four subjects are employed in this experiment as those in Experiment 1.

1) ROM: In this test, the restriction on range of motion (12)b is enabled, where the boundary $[\theta_l, \theta_u] = [0.26, 0.69]$. Each subject is required to execute symmetrical flexion/extension movement for ten trials, and other settings are the same as those of Experiment 1.

As shown in Fig. 8(a), the robot (or the actual trajectory) complied with human subject’s interactions within the predefined constrained space. Instead, the unrectified trajectory (green), i.e., the trajectory generated by only TDA, exceeds the predefined ROM in the test, which will cause safety issue in actual rehabilitation. With the constraint (12)b, as shown in Figs. 8(a) and (b), when the robotic behavior is about to exceed the safety threshold, the actual trajectory smoothly converges to the boundary (dotted line), and its velocity converges to zero. Moreover, from Fig. 8(c), no violent change in acceleration curves is found when the robotic device operates at the neighborhood of predefined boundary, which implies that the rectifying process should not cause any discomfort to human subjects.

2) RPM: This experiment is desired to demonstrate the validation of our method for mitigating the RPM. In the test, the restriction on motion speed (12)c-d is enabled, where the scale factor $\kappa = 1.879$, the maximum resistance force $f_m = 100$, and the boundary $[\omega_l, \omega_u] = [-0.70, 0.70]$. For each trial, the subjects need to complete one flexion/extension movement in 10 seconds, in which, five trials require the subject to stretch slowly, and other five trials ask for fast stretch. In addition, right leg of the subject should passively participate in the movement.

To simulate the RPM pattern in stroke patients, a damper-spring model is introduced to describe the dynamics of the patient’s impaired limb,

$$f_h = \begin{cases} -B_H \dot{q} - K_H (q - \theta_m), & q > \theta_m \\ -B_H \dot{q}, & \text{otherwise} \end{cases}$$

(15)

where $B_H = 120, K_H = 850$ are damping and stiffness, respectively; $\theta_m = 0.65$ denotes the range of free motions. Motivated by [13], we incorporate the human limb model [11] with speed thresholds $\omega_l, \omega_u$ and catch angle $\theta_m$ to simulate the mobility impairments in stroke patients. On a related note, the damping and stiffness values $B_H, K_H$ are chosen according to the impaired behaviors assessed in clinical mTS test [28], [29], and in specific, criteria are set to enable the model to 1) model the slight resistance of the muscle before hyperactive stretch reflex at which ‘catch’ motion occurs; 2) allow the passive range of motion beyond ‘catch’ angle; 3) render the hyperactive stretch reflex at the ‘catch’ angle.

As shown in Fig. 9, for ‘Slow Stretch’ test, it is obvious
that both rectified and unrectified trajectories are restricted as the resistance growing, because of the robotic compliance provided by trajectory deformations. From Figs. 9(c) and (e), we can find our method took effect when the resistance was approaching the boundary $f_m$, by limiting the direction and speed of robotic movement (such as $4 - 6\, s$). For ‘Fast Stretch’ test, without the rectification on robotic behavior, the resistance exceeded the preset boundary substantially as the blue curve ‘Unrectified’ shown in Fig. 9(f), which might cause severe safety issues. Instead, after rectifying the robotic behavior, the resistance force was controlled in a certain boundary as the red curve ‘Rectified’ shown in Fig. 9(f). These results indicate that, by dynamically rectifying the movement speed of robotic device, the proposed method can effectively mitigate the RPM. Nevertheless, viewing Figs. 9(d) and (f), we can find major oscillations on the velocity and force curves.

3) QOM: In this test we desire to examine the effect of (10) on mitigating oscillations. Related constraints (12)(c-e) are enabled, in which, $\eta_m = 0.07$ and $v_m = 0.7$. Other parameter settings and requirements on the subjects are the same as those in Experiment 2.2. In addition, the subjects are only required to execute ‘Fast Stretch’ test in this part.

The comparative results are shown in Fig. 10. Viewing plots in Fig. 10(a), we can find that our trajectory regulator made correct response to the increasing resistance, while the amplitude of oscillations in angular velocity and force curves is obviously mitigated in comparison between Figs. 10(b) and (c) with Figs. 9(d) and (f). Furthermore, in order to quantitatively analyze the improvement of our method, the Fourier magnitude spectrum of the force signal is used to describe the noise content of the signal. The noise content at frequency $\omega$ is defined as the ratio between the harmonic content with frequency higher than $\omega$ and the content of fundamental wave. The result of quantitative analysis is exhibited in Fig. 11. It shows significant reduction in the noise content when the frequency is higher than 5Hz after enabling the constraint (10). Quantitatively, the noise content is reduced by 63.40%. Besides, there is a huge drop at 3Hz, which indicates the distribution of the main harmonic frequency.

C. Discussion of Experiment Results

Aforementioned experimental results and comparative analysis show that the proposed control framework enables the robotic device to operate participant-active bilateral training with improved quality of movement. The effectiveness of the developed motion constraints is validated with access to render rectification on the typical impairments, through a simulated impaired mobility model.

Furthermore, with the objective to validate the effectiveness of our method on neurologically impaired subjects, a three-stage plan has been made. 1) Parameter Tuning: the clinical setting for parameters of ROM and RPM depends on individual impairment, whereas the settings for QOM or TDA need not to be subject-specific. The customization for the parameters of ROM and RPM would be conducted through the clinical mTS test [28], [29], where the patient’s muscle reactions to slow and fast stretch manoeuvres would be recorded. 2) Objective Test on ROM and RPM: analogous to Parameter Tuning, the mTS test is utilized to evaluate the performance of our method. For specific patients, the parameters of ROM and RPM would be set as narrower values to their actual reactions in the first stage, i.e., higher level of the patient’s impairment would be presumed. 3) Subjective Test on QOM: as the indicator of QOM, smoothness of recorded movement would be further evaluated through subjective assessment by healthy participants as suggested in [13], instead of quantitative evaluation only, e.g., using DSJ.

V. CONCLUSIONS

In this article, a new robotic system is designed for bilateral treatment in hemiplegic rehabilitation. An IMUs-based motion monitoring scheme is developed which enables unilateral robotic device to extend for conducting bilateral training program. A reference-free control framework on the basis of physically interactive trajectory deformation and constrained optimization is performed to assess and rectify several typical abnormal motion patterns. By constructing pertinent movement constraints for representative impairments in stroke patients, the proposed strategy enables to guarantee the safety in robotic rehabilitation. With multiple experiments that simulate the rehabilitation of the patient, the effectiveness of our strategy is demonstrated. Our future works include further verification of the effectiveness of our method on stroke patients and extension of the proposed strategy to upper-limb rehabilitation with higher DoFs. In addition, efforts will be made in the application of telerehabilitation and teleassessment with our proposed scheme.

APPENDIX A

LOWER-LIMB JOINT ANGLE ESTIMATION

To formally explore the estimation method, we define the frame of upper sensor and lower sensor (i.e., the inertial sensor mounted on the thigh and shank, respectively) as $F_U$ and $F_D$. The Earth frame is defined as $F_E$. On this basis, the definitions of hip and knee joint angles are given as follows: the hip joint’s angle $\alpha_{\text{hip}}$ denotes the complement of included angle between the $z$-axis of $F_E$ and the $y$-axis of $F_U$; the knee joint’s angle $\alpha_{\text{knee}}$ is defined as the included angle between the $y$-axes of $F_U$ and $F_D$. The $y$-axis of each IMU is supposed to be parallel with the corresponding link while the $x$-axis is along the vertical axis of the body. In addition, we represent the rotation from the Earth frame to the upper sensor frame $F_E \rightarrow F_U$, the rotation from the Earth frame to the lower sensor frame $F_E \rightarrow F_D$, and the rotation $F_U \rightarrow F_D$ as quaternions $q^U_E$, $q^D_E$ and $q^D_U$, respectively. According to the definition of the quaternions, $q^U_E$ and $q^D_U$ can be transformed from the IMU orientations.

For the hip joint, the complementary angle of $\alpha_{\text{hip}}$ can be obtained from,

$$\tilde{\alpha}_{\text{hip}} = \cos^{-1}(y_{U/Z}^T E)$$

where $\tilde{\alpha}_{\text{hip}}$ denotes the complementary angle of $\alpha_{\text{hip}}$, and $y_{U} = q^U_E \otimes y_{E} \otimes (q^U_E)^{-1}$. $x,y,z \in \mathbb{R}^4$ are pure quaternions (i.e., the real part is zero) whose vector parts are the directional...
vectors of $x$-, $y$- and $z$- axes in 3-dimensional (3D) space, respectively; subscripts $u$, $p$ and $e$ represent the corresponding reference frames; symbol $\otimes$ denotes the Grassmann product. By substituting $y_U$, the hip joint’s angle $\alpha_{hip}$ can be easily resolved. It should be noted that quaternions are in vector representation, where the first element of a vector is used to depict the real part of a quaternion. Therefore, for two pure quaternions (e.g., $x$ and $y$) their inner product $x^T y$ is equal to the inner product of their vector parts. As a result, (16) is derived because $x, y, z$ are unitized.

For the knee joint, it is reasonably considered that the flexion/extension occurs only around the $x$-axis of the upper sensor frame [24], [27], i.e., the knee is regarded as a hinge joint. As a result, the desired rotation from $F_U$ to $F_D$ can be defined as a quaternion $q$, which satisfies $x_U \otimes q = q \otimes x_U$, where $x_U = q^U_E \otimes x_E \otimes (q^U_E)^{-1}$. However, the quaternion $q^D_U$ representing the actual rotation does not satisfy the above equation, due to the errors from external acceleration and magnetic disturbance. Instead, there is the relation $y_D = q^D_E \otimes y_E \otimes (q^D_E)^{-1}$.

In this regard, with the objective to optimally estimate the desired quaternion $q$, a quadratic programming problem is formulated to minimize $J_0(q) = \|y_D - q \otimes y_U \otimes q^{-1}\|^2$ and subject to $x_U \otimes q = q \otimes x_U$. Transform the Grassmann product with matrix operation, where $p \otimes q = M(p)q = M(q)p$ holds for all non-zero quaternions $p, q \in \mathbb{R}^4$, and $M : \mathbb{R}^4 \rightarrow \mathbb{R}^{4 \times 4}$, $M : \mathbb{R}^4 \rightarrow \mathbb{R}^{4 \times 4}$ are non-singular matrices [36], then it yields,

$$\min_q J_0(q) = \|p - Rq\|^2$$

s.t. $Bq = 0$   \hspace{1cm} (17)

Examining (17), we define the matrices $R = M(q^D_U)$ and $B = M(x_U) - M(x_U)$, and $p$ is used to represent $q^D_E$ for brevity. We know that Rank$(B) = 2$, which indicates that it is not full-ranked. Therefore, the constraint $Bq = 0$ is replaced with $\bar{B}q = 0$ equivalently, in which, $\bar{B}$ is composed by the maximal linearly independent system of the matrix $B$.

Here, we solve this optimization problem by Lagrange multipliers. Define the Lagrangian $\mathcal{L}(q, \lambda) = J_0(q) + \lambda^T Bq$, where $\lambda \in \mathbb{R}^2$ is the Lagrange multipliers. The extremums of $\mathcal{L}$ satisfy $\nabla_q \mathcal{L}(q, \lambda) = 0$ and $\nabla_\lambda \mathcal{L}(q, \lambda) = 0$. Solving the equations, the extremal point is given as follows:

$$q = Q^{-1}(I - \bar{B}^T \bar{B})^{-1} \bar{B}^T p$$

where $Q = 2R^T R$ is positive definite, and $I \in \mathbb{R}^{4 \times 4}$ is the identity matrix. For ensuring that (18) stands for a minimum point, we consider the Hessian matrix $\nabla^2 \mathcal{L}(q, \lambda) = Q$. Because $Q$ is positive definite, we conclude that (18) minimizes the cost. Assuming the estimation $\hat{q} = [\hat{q}_0, \hat{q}_1, \hat{q}_2, \hat{q}_3]^T$, we can obtain the knee joint’s angle from

$$\alpha_{knee} = 2 \arccos \frac{\hat{q}_0}{|\hat{q}|}$$

Finally, the hip and knee joints’ angles are passed through a low-pass filter (e.g. Butterworth filter), to remove the components of high frequency from the estimated angle.

**APPENDIX B**

**PROOF OF THE CONVEX OPTIMIZATION**

For the constrained optimization (12), we consider the Hessian of the energy function $\nabla^2 E(u_k) = \frac{1}{\beta} A^T \Lambda A$. Because $A^T \Lambda A$ is positive semidefinite, we can conclude that the Hessian matrix is also positive semidefinite if defining the weighted constant $\beta > 0$, which implies $E(u_k)$ is a differentiable convex function. Moreover, (12)a-d and (12)f are affine functions so that their feasible sets are convex. As for (12)e, the feasible set is equivalent to the solution of

$$\sum_i \sqrt{r^2 + \|\hat{q}_d,k\|_2^2} \leq \rho$$

where $r = \tau v_m/N, \rho = \tau v_m T_R^2 e^{\eta_m}$. We assume that $w, v \in \mathbb{R}^{N^+3}$ are vectors satisfying the inequality (20), and for arbitrary $\lambda \in [0, 1]$, we have

$$\sum_i \sqrt{r^2 + \|\lambda w + (1 - \lambda)v\|_2^2} \leq \sum_i \left(\lambda^2 (\|w\|_2^2 + r^2) + (1 - \lambda)^2 (\|v\|_2^2 + r^2) + \lambda(1 - \lambda) \sqrt{\|w\|_2^2 + r^2} \sqrt{\|v\|_2^2 + r^2}\right)^{\frac{1}{2}}$$

$$\leq \sum_i \left(\lambda^2 (\|w\|_2^2 + r^2) + (1 - \lambda)^2 (\|v\|_2^2 + r^2) + 2\lambda(1 - \lambda) \sqrt{\|w\|_2^2 + r^2} \sqrt{\|v\|_2^2 + r^2}\right)^{\frac{1}{2}}$$

$$\leq \lambda \sqrt{\|w\|_2^2 + r^2} + (1 - \lambda) \sqrt{\|v\|_2^2 + r^2} \leq \rho$$

From above discussions, the convexity of (12)e is proved, and we can conclude that (12) is a convex optimization problem.

**REFERENCES**


