Introduction

This chapter focuses on the change in teachers’ practice and their experiences of the impact on student learning that arose from a research and development project on developing mathematical thinking in the primary school classroom. In particular, it draws on outcomes from the project supported by the Scottish Government (2010–12) called Developing Mathematical Thinking in the Primary Classroom (DMTPC) conducted in the context of the Scottish Curriculum for Excellence reform and reported on in Hudson et al. (2015). It does so by revisiting empirical data from the original study, mainly drawn from post-trial interviews with and action research reports by the participants. The analysis of the data is informed by a theoretical framework that combines the ideas of epistemic quality as discussed in Hudson (2022), of ‘teachers as curriculum makers’ (Lambert and Biddulph 2015) and of teachers’ ‘powerful professional knowledge’ (Furlong and Whitty 2017). The DMTPC project was based at the University of Dundee and carried out in collaboration with a group of primary school teachers (n=24) and Local Education Authority (LEA) advisory staff members across several LEAs in North East Scotland. The project’s overall aim was to develop and implement a postgraduate course for serving teachers on developing mathematical thinking in the primary school classroom. The project as a whole was established within a design research framework, which aimed to promote curriculum development through the process of classroom-based action research on the part of participants and also research and evaluation with respect to the project as a whole. A ‘Curriculum for Excellence’ development partnership group was established at the outset of the project, including teachers.
from each of the Local Education Authorities (LEA) together with LEA advisory staff and members of the university project team. The design of the course of study was based on developing an ‘open collective cycle’ model of a professional learning community (Huberman 1995) enhanced by blended learning through use of a technology-supported online learning environment. In conclusion, this chapter especially focuses on KOSS research question 3 by addressing the question of how the nature of teachers’ powerful professional knowledge in school mathematics can be characterized with reference to the field of subject didactics. Finally, consideration is given to the implications for innovation in teacher education policy and practice.

Theoretical Framework

Epistemic Quality

This chapter draws on the discussion in Hudson (2022) in relation to evaluating epistemic quality in primary school mathematics. The term ‘epistemic’ is concerned with the knowledge involved in a didactical or teaching-studying-learning situation (Hudson et al. 2022). In turn, the term ‘epistemic quality’ refers to the quality of what students come to know, make sense of and are able to do in school. The concept of epistemic quality arose from a perspective informed by concepts drawn from the field of subject didactics (Hudson et al. 2015; Hudson 2016) and is seen as a way of thinking that helps articulate aspects of what we mean by ‘powerful knowledge’ (Young 2013, 2015). It is also seen as a way of making quality education visible and as of particular significance while addressing the challenges of UN Sustainable Development Goal 4 to ensure inclusive and equitable quality education for all (UN 2015). It is especially significant as it concerns the need to maximize the chances that all pupils will have epistemic access (Young 2013: 115; Morrow 2008) to powerful knowledge through the curriculum which is regarded as ‘access to the best knowledge in any field of study they engage in’ (Young 2013: 115). In Hudson et al. (2015), low epistemic quality is characterized by an approach that presents school mathematics as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain, while also entailing an overemphasis on memorization, rule-following of strict procedures, and right or wrong answers. This is described as mathematical fundamentalism (2015) and contrasted with mathematical fallibilism, which is based on a heuristic view of mathematics as a human activity
(Lakatos 1976). The latter involves an approach that presents mathematics as fallible, refutable and uncertain, and which promotes critical thinking, creative reasoning, the generation of multiple solutions and learning from errors and mistakes. The concept arose from a perspective informed by concepts drawn from the field of subject didactics, particularly from a consideration of the transformation process of didactic transposition (Chevallard 2007), which recognizes that knowledge is not something that is to be taken as simply given and to be explained but rather that it is potentially encapsulated in situations, and it is in going through those situations that the pupil, or whoever, can learn. This view of learning as ‘learning from the situation’ sees knowledge as built up and transformed or transposed in didactic situations.

The concept of epistemic quality is discussed further in Hudson (2022) by considering it in relation to that of epistemic ascent (Winch 2013) as concerns subject expertise. This is based on a continuum that reflects a trajectory in the development of subject expertise from that of the novice towards that of an expert in the subject. By drawing on Winch (2013: 129), attention is paid to three distinct, yet related, kinds of knowledge: knowledge by acquaintance, propositional knowledge or knowledge that and procedural knowledge or knowledge how. The primary mode of knowledge by acquaintance is seen to be through the senses so that one may be acquainted with objects, events, processes, states and persons. In turn, it is argued that knowledge that cannot consist solely in the identification of true, though isolated, propositions, but is embedded in a conceptual structure, which is itself embedded within further related propositions. Finally, as concerns knowledge how, it is argued that knowing how to do something is an epistemic capacity related to knowledge by acquaintance and knowledge that, given that knowing how to do something typically requires elements of the other two kinds of knowledge. In developing subject expertise, it is viewed as crucial for enhancing the ability to understand and to make inferences (Brandom 2000) by employing the concepts embodied within the subject matter. It is also stressed that it is necessary to be able to distinguish within the subject between claims that can be counted as knowledge and those that count simply as beliefs. This distinction can be especially hard for a novice because the source of such beliefs is authoritative testimony. Beginning to learn a subject involves starting to use the language associated with the concepts of the subject, and this is primarily a practical ability that is learned. Accordingly, a central dimension of learning about a subject is learning to take part in conversations and discussions that employ those concepts. This aspect is developed further in the following section with reference to the discussion by Lambert and Biddulph (2015) about the
dialogic space offered by curriculum-making in the process of learning to teach and the creation of a progressive knowledge-led curriculum.

Teachers as Curriculum Makers

In their discussion on the process of learning to teach, Lambert and Biddulph (2015) argue that the curriculum-making process is a ‘signature part’ of a teacher’s identity. Referring to Hart (2001), they emphasize the complexity of the teacher’s role and argue that teachers need to hold three interrelated priorities in balance. The first priority relates to the needs, prior knowledge and experiences of students; the second to the nature and purposes of the discipline, and the third to the understanding and performative craft of pedagogic technique. From this perspective, curriculum-making is seen as curriculum thinking in practical action, taking on a ‘trinity of educational practice’ (2001) that involves subject, child and teacher. This way of thinking resonates strongly with the Continental European tradition of didactics and corresponds closely with the idea of ‘holding complexity’ (Hudson 2002: 53; Hudson 2016: 112) as being a central part of a teacher’s reflective practice and highlights the way in which curriculum and pedagogy effectively merge at the classroom level. A key tool for the analysis of the complex relations between teacher, student and the content in the teaching-studying-learning process is the didactic triad. The didactic triad should be treated as a whole, although this is almost impossible to do in practice. The most common approach is to take the pedagogical relation between the teacher and the student(s) as a starting point. The pedagogical relation between the teacher and the student is taken as the significant starting point in Geisteswissenschaftliche, that is, human science pedagogy. In their discussion of the didactic relation in the teaching-studying-learning process, Kansanen and Meri (1999) draw attention to the influence of the thinking of Herman Nohl and also to the writing of Klafki (1970: 55–65) who summarized this relation by stating that it is important to consider from the point of view of a young person and it aims to draw out his or her best. They highlight that the content of this relation must be thought through in each situation, and that it must be interactive in nature such that a student cannot be compelled or forced into it. Moreover, they stress that it is not a permanent relation, but one from which the young person gradually grows out of as that individual develops their own independence. They also highlight the way in which this relation also gradually takes shape since the development of the young person brings different perspectives along with it. Further they argue that this characteristic has often been referred in pedagogical discussions
to as ‘the pedagogical suicide of the teacher’ or the ‘pedagogical paradox’, with reference to Immanuel Kant. The relation is illustrated in Figure 8.1 based on the figure in Hudson (2016: 112).

However, the relationship between the teacher and the content must also be considered, thereby bringing the teacher’s competence into focus. It should also be noted that teaching in itself does not necessarily imply learning and that the term ‘studying’ therefore provides a more accurate description of the activities of students. It is through studying that the instructional process can be observed, while the invisible part of this relation may be learning. Accordingly, a key aspect of the teacher’s role is guiding this didactic relation, as shown in Figure 8.2 similar to the earlier one based on the figure in Hudson (2016: 112).

Further, it is important to stress the way in which this analysis takes place within a school and societal context, as emphasized in Hudson and Meyer (2011: 8). In expanding the perspective, the first step is to bring the classroom situation into focus, while the second step in widening the vision is to consider the school as

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**Figure 8.1** The pedagogical relation in the didactic triad.

**Figure 8.2** The didactic relation in the didactic triad.
a subsystem of wider society which in turn reflects broader societal changes as illustrated in Figure 8.3 and adapted from the figure in Hudson (2016: 112). This framework reflects that of Gericke et al. (2018), who describe processes of knowledge transformation that occur on societal, institutional and classroom levels.

In their discussion, Lambert and Biddulph (2015) draw attention to the dialogic space offered by curriculum-making in the process of learning to teach and the creation of a progressive knowledge-led curriculum. This space is seen as the basis for dialogic teaching that involves ongoing talk between teachers and students such that teachers can build on the everyday knowledge that students bring into the classroom. As indicated earlier, there is a strong correspondence with the emphasis placed by Winch (2013) within the framework of epistemic ascent on the importance of inferential relationships. These inferential relationships are important for the development of both knowledge how and knowledge that and for the interrelationships between them. Accordingly, use of the language associated with the concepts of the subject is a crucial aspect of the process of beginning to develop expertise in that subject, and this is primarily a practical ability that is learned by taking part in conversations and discussions which employ those concepts. Lambert and Biddulph (2015) also highlight a set of principles as a means to guide students’ collaboration with each other, with their teachers and with the discipline in their reflections on the Young People’s Geography project. Central to these is the principle of ‘valuing conversation,

Figure 8.3 The didactic triad within the wider school and societal context.
Teachers as Curriculum Makers for School Mathematics

involving the use of conversation between teacher–student, student–student, and teacher–teacher as a basis for curricula possibilities and knowledge building. In relation to this aspect, they define conversation in a broader sense as 'dialogue between student and teacher, teacher and subject, experience and disciplined thought, curriculum and pedagogy, school and everyday' (2015: 220). They also draw attention to the potential for creating dialogic spaces for young people in the school curriculum and also to how such spaces illustrated the way in which students can be part of the curriculum-making process as a result. In turn, this example is seen as an illustration of where curriculum meets pedagogy.

Powerful Professional Knowledge

The aim of supporting the development of teachers as curriculum makers holds implications for the knowledge base of teachers, in turn raising the question of what may constitute the powerful professional knowledge required for teacher education (Furlong and Whitty 2017: 47–51). In their discussion (Furlong and Whitty 2017: 47–51), they refer to the work of Young and Muller (2010) in reasserting the importance of disciplinary knowledge in professional contexts. Yet, they argue that they do not necessarily exclude other knowledges and emphasize there is a critical pedagogical element to this. They also highlight a key challenge for professional disciplines as being the need to establish precisely how disciplinary knowledge, that is epistemologically strong, articulates with other forms of knowledge and how it can thereby impact practice. In turn, they ask how can disciplinary knowledge and other external knowledges be brought together with professionals’ reflective practice and practical theorizing in professional arenas to produce really powerful professional knowledge and learning. This leads to one of the key KOSS research questions addressed by this book in terms of how the nature of teachers’ powerful professional knowledge can be characterized and what the implications are for teacher education policy and practice.

Following the work of Furlong and Whitty (2017) in the same volume, Tattoro and Hordern (2017) write about the configuration of teacher education as a professional field of practice through a comparative study of mathematics education. In their study, they analyse the role of educational studies in the secondary teacher education curriculum in Germany, Poland, Singapore and the United States. The data come from an analysis of syllabi from cross-national representative samples of pre-service programmes. The analysis focuses on knowledge of the discipline, knowledge of the school curriculum for the discipline, the pedagogy of the discipline, the general pedagogy and the
practicum. The authors use Bernstein’s sociology of knowledge as a theoretical framework to discuss the differential emphasis given to these five domains within and across countries ‘as an expression of the re-contextualisation of knowledge from singulars to regions’ (2017: 255) in relation to how the foundation disciplines and elements of mathematical knowledge are decontextualized to address the preparation of knowledgeable mathematics teachers.

Their approach to the analysis reflects the curriculum tradition’s influence through the very constructive way in which they draw on Schulman’s (1987) typology in teacher education programmes related to mathematics. This enables them to differentiate between mathematical content knowledge (MCK) and general pedagogical knowledge (GPK). This then leads to a focus on mathematical pedagogical content knowledge (MPCK) that ‘involves multiple forms of knowledge contextualisation, including knowledge of the discipline, the curriculum and instruction, and of students thinking’ (1987: 259). In their study, the MPCK assessment framework includes three sub-themes in order to address the complexity of learning to teach mathematics and the role of teacher education in the process. These three themes are mathematics curricular knowledge, knowledge for planning mathematics teaching and learning, and enacting mathematics for teaching and learning.

The analysis leads to a number of significant points regarding the configuration of mathematics teacher education in different national contexts. Notably, it highlights the contrast between a narrowly conceived technical model of teacher education and broader more disciplinary-based forms of education studies. They conclude that where broader pedagogic norms persist and there are strong traditions of educational theory there may be greater openness to extending the teacher’s role in order to develop research-capable practitioners with pastoral and counselling skills. Still, the system-level approach in their analysis means no comparison is made of the differences in approaches to classroom practice across the four countries. In particular, the very significant role of subject didactics in Germany is overlooked in this analysis. This chapter accordingly aims to address this gap by considering the question of how the nature of teachers’ powerful professional knowledge can be characterized from the perspective of subject didactics.

**Background to the Study**

The research study associated with the DMTPC project (Hudson et al. 2015) was carried out with serving teachers and built on an earlier study involving students
in initial teacher education that addressed the question of the nature of subject content knowledge in mathematics (Henderson and Hudson 2011). The DMTPC study mainly focused on the teachers’ confidence, competence, attitudes and beliefs in relation to teaching mathematics. As outlined in the introduction, the project’s overall aim was to develop and implement a postgraduate course for teachers on the theme of developing mathematical thinking in the primary classroom. The project was based at the University of Dundee and carried out in collaboration with a group of teachers (n=24) and Local Education Authority (LEA) advisory staff members across several LEAs in North East Scotland. The design of the course of study was based on developing an ‘open collective cycle’ (OCC) model of a professional learning community (Huberman 1995) enhanced by blended learning through use of a technology-supported online learning environment. The framing of the project as a whole was established within a design research framework (Bannan-Ritland 2003), which aimed to promote curriculum development through the process of classroom-based action research on the part of participants and also research and evaluation of the project overall. The Curriculum for Excellence Development partnership group that planned the course included teachers with one from each of the Local Education Authorities (LEA) together with LEA advisory staff members and members of the university project team. An important aspect of the case made for support in the grant application was the fact that most mathematics lessons at that time in Scotland still tended to feature some form of teacher-led demonstration followed by children practising skills and procedures from a commercially produced scheme (Scottish Executive Education Department 2005). The more recent Scottish Survey of Literacy and Numeracy reported the activities in which the highest percentage of pupils reported that the ways in which they had participated ‘very often’ were to ‘listen to the teacher talk to the class about a topic’ (62 per cent in P4 and 64 per cent in P7 and S2) and to ‘work on your own’ (between 55 and 61 per cent) (Scottish Government 2012: 13).

The course of study was structured around three main questions, two core texts and an action research project. The key questions were:

1. What is mathematics?
2. What is mathematical thinking?
3. What is good mathematics teaching?

As part of the module assessment, participants designed an action research project that would consider the implications of current literature on the
development of mathematical thinking; identify the strategies, skills and attitudes to be enhanced; identify methods of data collection and analysis that provide evidence of impact; be feasible and realistic in terms of the resources available in school; pay consideration to progression and assessment and identify the issues which emerged from the study.

The Study

This chapter focuses on the change in the teachers’ practice and their experiences of the impact on student learning by revisiting empirical data from the original study. In particular, it addresses the KOSS research question of how the nature of teachers’ powerful professional knowledge in school mathematics can be characterized. The data are mainly drawn from post-trial interviews with four of the participants and action research reports completed by another four group members. The initial phase of the design research framework took place in the first year of the project and involved the informed exploration and design of instruments and tools to gather the data. This was carried out alongside the processes of curriculum development and recruiting the participants. Further phases that ran parallel to this involved the enactment of the module and subsequently the action research projects carried out by the participants on the classroom level.

Data Analysis

The data from the post-trial interviews with the course participants (n=24) were revisited by reading them through the lens of the theoretical framework outlined in this chapter with particular attention to concepts of epistemic quality, teachers as curriculum makers, and powerful professional knowledge. This entailed rereading the interview transcripts in the first stage and coding the data for incidences that related to ‘epistemic quality’, ‘teachers as curriculum makers’ and ‘powerful professional knowledge’. The coding process was based on a deductive approach involving these three categories and led to the emergence of several subcategories or themes discussed here. The second stage involved rereading the action research reports written by four participants according to the same approach. The results of these processes are discussed in the following sections.
Stage 1 Data Analysis: Post-trial Interviews

The most frequently recurring theme while analysing the interview data related to the importance of language and classroom talk, both of which are central to the theoretical framework in relation to developing the dialogic space in the classroom and the epistemic quality of classroom interaction. The second-most frequently recurring theme concerned the importance of developing the dialogic space with colleagues. The third theme related to developing the practice of teacher as curriculum maker and involved the role of time and the development of a variety of methods against the background of the Curriculum for Excellence in particular. Several more issues that emerged are clustered around the fourth theme concerned with developing the epistemic quality of the content. It became clear during this process that all of these four themes related in some way to the development of powerful professional knowledge, with this aspect being returned to in the final discussion. With this in mind, each theme is discussed further here and illustrated with examples of what the participants stated in the interviews.

Developing the Dialogic Space in the Classroom and the Epistemic Quality of Classroom Interaction

The most frequently recurring theme in the analysis of the interview data related to the importance of starting to use the language associated with the concepts of the subject and to take part in conversations and discussions that employ those concepts, namely, a central dimension of the previous discussion on the epistemic quality of the classroom interaction. This aspect corresponds with the emphasis placed by Lambert and Biddulph (2015) concerning the dialogic space at the classroom level. All interviewees referred to the importance of developing their practice of teacher questioning in relation to this aspect.

For example, in commenting on the change in her practice and the impact on student learning, Alice highlighted the change in her practice and the resulting impact on her students in terms of motivation:

I’ve changed the way I teach maths, again, it’s away from . . . we were very much of the textbook and, you know, worksheets etc. and I am very much into more questioning, I have more group sessions, we’ll do a lot more oral work, practical work than we ever did in the past . . . they’re actually almost excited to be working together in maths.

While reflecting on a recent lesson observation by her deputy head teacher, Bridget said:
I was using my questioning skills . . . to actually develop their thinking even further and I got a really good feedback from that lesson so yeah, I think I had the skill of questioning but I think it’s kind of confirmed it and probably pushed on my confidence more because I know I’m doing it right.

Bridget also highlighted the impact on her students of greater emphasis on conversation in terms of motivation:

Oh, I think that they have probably seen a kind of revival within me . . . when we’re doing the thinking skills, they really enjoy it and they like the fact that they’re working together or they can work on their own and they can actually come up with any kind of method that they think that they’ve used and then they can explain it and they like going up to the board and explaining it to everybody how they got this, that and the other.

In her responses, Caroline emphasized the change in her practice, her students’ increased levels of engagement as a result as well as the students’ use of language:

the main difference in my teaching is now I do more asking than telling, do you know . . . and to give the children something to start with and then to let them almost tell you what maths is it they need to know . . .

. . . I did ask them the question last week; I said what would someone say if they came into our classroom when we’re doing maths, what do you think they would say? and that straight away the kids were saying ‘oh, they would say good group discussions, they would say that we’re listening to one another, they would say that we’re sharing our ideas and that we’re talking about the strategies’.

Caroline also reflected on the development of her practice in terms of the balance between providing knowledge and experience for her students in relation to this aspect:

I think I’ve been getting a good mix between, you know, giving them the, sort of, knowledge and letting them have experience, if that makes sense?

With regard to Denise’s responses, she emphasized that her questioning was the most significant aspect of her practice to change and also noted the importance of building her confidence to make this change.

I’ve found is that my questioning is the biggest thing that has changed and it’s changing and it’s . . . confidence yes and questioning without a doubt has been a big thing.

In addition, she highlighted how this had an impact on her students’ attitudes and levels of motivation:
I would certainly say that there seems to be quite high motivation and little anxiety which I suppose is what we're aiming for.

**Developing the Dialogic Space with Colleagues**

Three of the four interviewees also referred to the importance of a dialogic space with peers and more widely. This corresponds with the broader sense of the term 'conversation' as used by Lambert and Biddulph (2015) as involving dialogue between student and teacher, teacher and subject, experience and disciplined thought, curriculum and pedagogy, school and everyday.

For example, Alice referred to the importance of colleagues sharing their experiences, resources and practices for her own professional learning:

> drawing upon other people's experiences in discussion groups and seeing, finding out about resources that are out there, finding out about other peoples' methodology. I think that aspect I've benefitted from.

In her responses, Caroline pointed to the role of the online environment for sharing resources and developing a dialogic space with her peers:

> I like how everything is done, you know access to everything online . . . I love it, it's so accessible, having the discussion forums . . . and I think people are great people who really are willing to share everything and people are posting their things up and they're putting on their documents. Yeah, I think there's very much a sense of we're all in this together so, yeah. Very positive, very, very positive.

In relation to Denise's response, she stressed the importance of the key texts and the dialogue between experience and disciplined thought through reading and reflection on practice:

> I think by reading these quite academic texts there have been quite a few, sort of, light bulb moments, it's like 'oh yeah, ok, I understand that, I can see that happening' and I think through talking with S (University tutor) as well I think what's really come over is how taking very small steps, very slowly is actually ok because I think we get there in the end and we get there with more understanding in the end as well.

**Developing the Practice of Teacher as Curriculum Maker**

With regard to the practice of becoming teacher as curriculum maker, the role of time and the development of a variety of methods was emphasized by interviewees against the background of the Curriculum for Excellence.
For example, Alice stressed how the interaction between the course and the Curriculum for Excellence had led to a change in her practice, and she also highlighted the importance of time and the development of a variety of methods:

I think it is a combination of the being part of the group doing the study and also the introduction of Curriculum for Excellence, they both impacted . . .

I think being allowed to actually take time over a piece of mathematics . . . I’ll maybe find some games that they can work on just to better their understanding . . . I am using different methods, a variety of methods.

**Developing the Epistemic Quality of the Content**

Several more issues that emerged are clustered around the aspect of developing the epistemic quality of the content. These included placing an emphasis on the purpose of mathematics, real-life applications of the subject, and breaking down concepts in the subject.

Alice’s responses show the importance of both the need to break down the mathematical concepts and to think about the purpose, as illustrated here:

I think in breaking things down a bit more, it’s almost like getting to the nitty gritty of it, I think, but the purposes behind maths, the how it came about, I suppose the history of the maths and the necessity of it, why we actually do it . . . It’s breaking down the mathematical concepts . . . it’s helping children to enjoy mathematics and to see the purpose behind it and to see how enjoyable it can be as well.

On this aspect, Bridget stressed real-life applications, the changes in her practice and the impact on her students:

I’ve also looked at literature for real life maths and things like that and actually being able to, you know, include that into my teaching as well . . . I think it’s changed them . . ., I think doing this course has made me look at it in a different way to real life so then they actually see more of a purpose to what they are doing.

Bridget also commented on the impact on her own thinking about and attitude to the subject:

I think it’s kind of reignited my interest in maths and . . . backed up my feeling that actually yeah that was my stronger subject and I do enjoy it.

In relation to Caroline’s response, she emphasized the way in which she was paying greater attention to building on her students’ prior knowledge:
had really begun to think more about the cycle of talking to the kids, actually finding out what it is they know before you then, sort of, move on rather than just, sort of, ploughing on relentlessly, do you know, we're in this term we have to be doing fractions or now we have to be doing multiplication or whatever.

With regard to Denise's responses, she stressed the way in which she had placed greater emphasis on sharing the learning intentions of the lessons with her students in a way that was also resonant with the idea of students as curriculum makers:

I'm far more aware of sharing the learning intentions with the children . . . I completely understand learning intentions, I know how to set learning intentions, I know about success criteria but was I actually using them? Probably not, no. Sharing that with the children and then, having high expectations of what they will then achieve but in a low stress way.

Stage 2 Data Analysis: Action Research Reports

The second stage involved rereading action research reports written by four participants based on the same approach outlined above. The four themes that emerged in the analysis of the interviews were also evident while analysing the action reports. However, in overall terms more attention was given to the detail of the mathematical content in the action research reports. For example, in her conclusion Anna underscored the value of a topic-based approach for teaching new concepts and the importance of vocabulary in this process. Elsa's project focused on how her questioning could impact the mathematical thinking of her students in relation to the topic of time and in particular the distinction between analogue and digital representations. In her report, she emphasizes how a shift towards group work and discussion led to increased levels of confidence and motivation as well as improved attitudes to mathematics. The project conducted by Florence involved the teaching of odd and even numbers in which she commented on her greater awareness of the need for children to have the time to expand their thinking – whether alone or with others. She also observed that her students were beginning to use the mathematical language modelled by the teachers, for example 'match', 'pair' and 'even'. Gabrielle's project also concentrated on children's understanding of odd and even numbers. In her reflections, she also stressed how the Curriculum for Excellence was seen to allow more time to be spent teaching mathematics and in empowering teachers to engage in activities involving spatial representations that aim to develop deep
understanding. Finally, she expressed her strong commitment to principles of inclusion arguing that:

as pupils are involved in all activities; they have a better share of power within lessons – the environment for learning is one of trust where pupils are confident about articulating their thinking without fear of being wrong . . . As making conjectures is a key principle of mathematical thinking, it is of paramount importance that pupils feel secure about guessing/estimating or offering responses to be further explored.

Discussion

As discussed, this chapter aims to address the question of how the nature of teachers’ powerful professional knowledge can be characterized from the perspective of subject didactics. This approach recognizes the complexity of the teacher’s role as highlighted by Lambert and Biddulph (2015), the complexity of learning to teach as emphasized by Tatto and Hordern (2017), and builds on the idea of ‘holding complexity’ as being a central part of a teacher’s role as already referred to elsewhere (Hudson 2002: 53; Hudson 2016: 112). Considering the role of the teacher from the perspective of subject didactics brings a focus onto the didactic triad, particularly the didactic relation. As mentioned, a key aspect of the teacher’s role relates to guiding this didactic relation that is a relation to another relation, that is, the relation between the student and the content. Thus, one focus in this set of relationships is on the core of a teacher’s professionalism, and, in view of the complexity of this set of relations as manifested in any situation, it is difficult to think that the didactic relation could be organized universally, or according to a set of recipes or technical rules. Teachers’ own practical theories and pedagogical thinking are consequently seen to be vital. The tradition of didactics hence provides a framework that places the teacher at the heart of the teaching-studying-learning process. Moreover, it provides a framework for teachers’ thinking about the most basic how, what and why questions concerning their work, and strongly resonates with the work of Shulman (1987).

In considering the four themes emerging from this study, we can see how these map onto various aspects of Shulman’s model overlaid onto the didactic triad in Figure 8.4 which is adapted from that illustrated in Hudson (2016: 113). In particular, there is a close correspondence between the didactic triad
and Shulman's (2016: 113) concept of pedagogical content knowledge (PCK), which can be seen as the professional knowledge required by teachers in guiding the didactic relation. This also helps to distinguish between the associated pedagogical knowledge (PK) which relates to the pedagogical relation that is distinct from the content knowledge (CK) required and which maps onto the teacher’s relation with their subject-specific content knowledge.

With regard to the theme of developing a dialogic space in the classroom and the epistemic quality of classroom interaction, we can see how this maps onto the pedagogical relation associated with pedagogical knowledge (PK). The focus of this aspect emerging from this study is the development of teachers’ knowledge how as exemplified in the emphasis given to developing teacher questioning skills and student group working, promoting thinking skills and the students’ use of language and building their confidence.

In relation to developing the epistemic quality of the content, this is clearly associated with the content knowledge (CK) of the teacher. The study participants stressed the purposes of mathematics, the value of drawing on real-life applications of the subject and breaking down the concepts in the subject. The latter aspect of breaking down concepts reflects a process evident in the work of most of the teachers. It is the reverse of knowledge building and is referred to as ‘decompression’ by Ball and Bass (2000: 99) in their work on interweaving content and pedagogy in relation to teaching and learning to teach mathematics. This kind of knowledge how is not necessarily the kind of understanding a mathematician would possess. Rather, it involves the ability to deconstruct one’s own mathematical knowledge into a less polished and final form, in which elemental components are accessible and visible. In contrast, while most personal knowledge of subject matter in mathematics is highly

Figure 8.4 Mapping Shulman's categories onto the didactic triad.
compressed, in this form it might be completely inadequate for teaching. The polished and compressed form of mathematics as a discipline can actually obscure the ability to discern how learners are thinking at the roots of that knowledge. Teachers of mathematics must therefore be able to work backwards from a mature and compressed understanding of the content in order to unpack its constituent elements.

The remaining two themes of developing the practice of teacher as curriculum maker and developing the dialogic space with colleagues can be seen as related to the didactic relation and teachers’ pedagogical content knowledge (PCK). In considering the former, we can see how the introduction of the Curriculum for Excellence enabled these teachers to take the time to develop a variety of methods as they interweaved content and pedagogy in relation to teaching and learning to mathematics. This involved the development of activities that reflected the importance of developing a mathematical vocabulary in this process and specific examples like addressing the distinction between analogue and digital representations of time, beginning to use the mathematical language modelled by the teachers such as ‘match’, ‘pair’ and ‘even’, and understanding the concept of odd and even through pedagogical activities based on spatial representations. This dialogical process lies at the heart of the didactic relation and the development of teachers’ pedagogical content knowledge (PCK). Further, a vital aspect in developing this knowledge how was the process of developing the dialogic space with colleagues that involved sharing experiences, resources and practices in addition to reading and reflection on practice.

In this chapter, it has been argued that the concept of epistemic quality helps articulate aspects of what we mean by powerful knowledge. It is especially significant in relation to the need to maximize the chances that all pupils will have epistemic access (Young 2013: 115; Morrow 2008) to powerful knowledge through the curriculum, which is seen as ‘access to the best knowledge in any field of study they engage in’ (Young 2013: 115). It follows that teachers who are able to develop powerful professional knowledge will demonstrate the abilities to support their students in developing knowledge of high epistemic quality. This is central to ensuring inclusive and equitable quality education for all (UN 2015) and in maximizing the chances that all pupils will have epistemic access to powerful knowledge through the curriculum. In relation to the question of how the nature of teachers’ powerful professional knowledge can be characterized, this study has illustrated some aspects of such powerful professional knowledge being enacted by teachers as curriculum makers in teaching primary school mathematics. The study has highlighted the associated knowledge how in particular through the
four themes of developing the dialogic space in the classroom and the epistemic quality of classroom interaction, developing the epistemic quality of the content, developing the practice of teacher as curriculum maker and developing the dialogic space with colleagues (Figure 8.5). With regard to the question of epistemic access, Gabrielle’s comments in the conclusion of her action research report are especially powerful in which she argues for her pupils to have a better share of power within lessons and stresses the environment for learning as one of trust in which pupils are confident about articulating their thinking without fear of being wrong.

Finally, in relation to the implications for innovation in teacher education policy and practice this study reveals several significant issues. A major implication relates to recognizing teacher education as a lifelong process that entails initial teacher education, induction and continuing professional development. The design of the course of study had the aim to develop an ‘open collective cycle’ model of a professional learning community, enhanced by blended learning through use of a technology-supported online learning environment. This aspect was central to the success of the course in terms of enhancing the practice of teacher as curriculum maker and in supporting that process by developing a dialogic space with colleagues. In a post Covid-19 world, the potential of technology to support professional learning in this way has become self-evident. A second major implication relates to the importance of addressing the development of mathematics as a subject at primary school.
level, which is a highly formative stage in terms of developing attitudes and beliefs in relation to the subject. The importance of Subject-Specific Educational Content Knowledge (SSECK) across school subjects in general is highlighted in the final chapter of this book (Stolare et al., this volume). The challenge for teacher education policy making in relation to school mathematics is to ensure that the development of SSECK is an essential dimension of teacher education in order to support the development of such teachers as curriculum makers in relation to subject-specific education in mathematics at all stages of the lifelong process of teacher education.

References


