This chapter builds on the idea of epistemic quality, as discussed in Hudson (2018, 2019), which arose from the outcomes of the Developing Mathematical Thinking in the Primary Classroom (DMTPC) project (Hudson et al. 2015). It does so by relating epistemic quality to a continuum that reflects a trajectory of epistemic ascent (Winch 2013) in the development of expertise from the novice towards that of an expert in the subject. The significance of epistemic quality stems from the need to maximize the chances that all pupils will have epistemic access (Morrow 2008; Young 2013: 115) to high-quality education in school mathematics. This is regarded as a way of making quality education visible and as a precondition for addressing the challenges of UN Sustainable Development Goal 4 to ensure inclusive and equitable quality education for all (UN 2015). The idea of epistemic quality is in turn used as a theoretical framework for evaluating the quality in primary school mathematics by revisiting the data analysis from an earlier study that considered the development of a topic-based approach to teaching and learning mathematics on the theme of 'The Rainforest' (Hudson 2015, 2019). The analysis addresses the question of how the nature of epistemic quality in school mathematics can be characterized by focussing first on the epistemic quality of the content and second on the epistemic quality of the teacher–student(s) joint action. As such, the chapter addresses KOSS research questions 1 and 2 that look at how the nature of powerful knowledge and epistemic quality in different school subjects can be characterized and how the transformation processes related to powerful knowledge and epistemic quality can be described, respectively. The final discussion focuses on considering the implications for curriculum innovation in schools.
Developing mathematical thinking in the primary classroom project

The DMTPC project was funded by the Scottish Government (2010–12) and involved working with a group of practising teachers ($n = 24$), who were all participants in a newly developed Masters course designed with the aim of promoting the development of mathematical thinking in the primary classroom. The project as a whole was established within a design research framework, which sought to promote curriculum development through the process of classroom-based action research on the part of participants and also research and evaluation of the project as a whole. A Curriculum for Excellence Development partnership group was established to plan the course that included a teacher from each of the Local Education Authorities (LEA) together with LEA advisory staff members and members of the university project team. An important aspect of the case made for support in the grant application was the fact that most mathematics lessons at that time in Scotland still tended to feature some form of teacher-led demonstration followed by children practising skills and procedures from a commercially produced scheme (Scottish Executive Education Department 2005). The more recent Scottish Survey of Literacy and Numeracy highlighted that the highest percentage of pupils reported that the ways in which they participated ‘very often’ were to ‘listen to the teacher talk to the class about a topic’ (62 per cent in P4 and 64 per cent in P7 and S2) and to ‘work on your own’ (between 55 and 61 per cent) (Scottish Government 2012: 13).

The course was designed around three main questions, two core texts and an action research project. The key questions were:

- What is mathematics?
- What is mathematical thinking?
- What is good mathematics teaching?

Participants completed an action research project, with their reports forming the course assignment. The research questions of the associated study focused on the teachers’ confidence, competence, attitudes and beliefs relative to mathematics and their expectations and experiences of the impact on pupil learning arising from this course. Empirical data were drawn from pre- and post-course surveys, interviews, observations of the discussion forums in the online environment and from the teachers’ action research project reports.
Epistemic quality

The term ‘epistemic’ is concerned with the knowledge involved in a didactical or teaching-studying-learning situation (Hudson et al. 2020). In turn, the term ‘epistemic quality’ refers to the quality of what students come to know, make sense of and are able to do in school. The concept of epistemic quality arose from a perspective informed by concepts drawn from the field of subject didactics (Hudson et al. 2015; Hudson 2016) and is seen as a way of thinking that helps articulate aspects of what we mean by ‘powerful knowledge’ (Young 2013, 2015). It is also seen as a way of making quality education visible and as of particular significance for addressing the challenges of UN Sustainable Development Goal 4 to ensure inclusive and equitable quality education for all (UN 2015). It is especially significant in relation to the need to maximize the chances that all pupils will have epistemic access (Morrow 2008; Young 2013: 115) to powerful knowledge through the curriculum which is seen as ‘access to the best knowledge in any field of study they engage in’ (Young 2013: 115). The idea stemmed from the process of applying the theoretical framework of Joint Action Theory in Didactics (JATD), developed within the tradition of French didactics, to the analysis of classroom interaction. JATD is based on the theoretical principle that, in order to understand a didactic activity, which denotes an activity when someone teaches and someone learns, you need to understand a system, the didactic system (Brousseau 1997; Sensevy et al. 2005; Sensevy 2011). This is a system of three subsystems, namely knowledge, the teacher and the student. The didactic system is seen as indivisible, and it is regarded as impossible to grasp the meaning of the teacher’s action without understanding the relations between this action, the students’ action and the structure of the piece of knowledge at stake. A key concept drawn from JATD is that of a ‘game’ based on the concept of ‘language games’ (Wittgenstein 1997) specific to didactic systems, as discussed by Sensevy (2011). ‘Game’ is used to describe what happens in a given didactic situation and allows us to describe the joint dimension of the didactic activity by modelling the interactions between the student and the teacher as participants in the same ‘game’. Particular attention has been given to two games through the ways in which students participate in learning games in connection with epistemic games. With regard to the latter, an epistemic game refers to the game of the mathematician as a subject expert in his or her professional activity and two aspects of this game are distinguished. On the one hand, the source epistemic game refers to the human practices that exist outside the didactic situation while, on the other hand, the actual epistemic game is based on the analysis of class
practices as they occur in situ. Second, the learning game is the reciprocal game of the student in relation to the joint game. In turn, these are seen as games of a particular kind, that is, as games in which some specific pieces of knowledge are involved and as collaborative or joint games within joint action (Gruson, Loquet and Pilet 2012: 65). A second key concept influencing the evolution of the idea of epistemic quality is that of 'didactic transposition', which draws from the wider field of French didactics as outlined by Chevallard (2007). The basic principle underpinning this perspective on learning and teaching is that knowledge is not something that is to be taken as simply given and to be explained. Rather, it is the case that 'knowledge is potentially encapsulated in situations, and it is in going through those situations that the pupil, or whoever, can learn' (Chevallard 2007). This view of learning as 'learning from the situation' is a central principle of French didactics, which sees knowledge as built up and transformed or transposed in didactic situations. Underpinning this theory is an ecological approach to the social dynamic of knowledge (ibid.). The main point in didactic transposition theory is that it considers knowledge as a changing reality, which adapts to its institutional habitat. Accordingly, in relation to the school context, the knowledge in question is not knowledge for acting and solving problems in the social contexts in which it was created and where it is used, but instead is transposed into knowledge to be taught and learned. This is an example of a transformation process, as discussed in Gericke et al. (2018).

A significant situation that highlighted the issue of epistemic quality occurred within the DMTPC course and was a result of the powerful responses arising from the teachers’ readings of the book The Elephant in the Classroom by Boaler (2009). In this book, Boaler highlights the fact that in many mathematics classrooms a very narrow subject is taught to children, one that involves copying methods that teachers demonstrate and reproducing them accurately over and over again. Furthermore, she argues that this narrow subject is not mathematics but that it is a strange mutated version of the subject that is taught in schools. This process of mutation is seen as a transformation process and in particular as an example of didactic transposition. The mutated version of mathematics is characterized by an approach that presents the subject as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain, and also involves an overemphasis on memorization, rule-following of strict procedures and right or wrong answers. In Hudson et al. (2015), we describe this as mathematical fundamentalism (Hudson et al. 2015) and contrast it with mathematical fallibilism based on a heuristic view of mathematics as a human activity (Lakatos 1976). The latter involves an approach that presents mathematics as fallible, refutable and uncertain, and
which promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes. The central role of creative reasoning is considered further in Hudson (2018, 2019) by drawing on the work of Lithner (2006) who offers a conceptual framework that compares and contrasts creative and imitative reasoning in mathematics which fits with the distinctions between mathematical fundamentalism and mathematical fallibilism. In relation to imitative reasoning in mathematics, Lithner (ibid.) highlights two aspects: memorized reasoning and algorithmic reasoning. Memorized reasoning is seen to fulfil two conditions. First, the strategy choice is founded on recalling a complete answer and, second, the strategy implementation consists only of writing it down. In relation to algorithmic reasoning and with reference to Brousseau (1997: 129), an algorithm is defined as 'a finite sequence of executable instructions that allows one to find a definite result for a given class of problems'. Similarly, algorithmic reasoning meets two conditions. First, the strategy choice is to recall a solution algorithm regarding which the predictive argumentation may be of different kinds but does not necessitate the creation of a new solution. Second, the remaining reasoning parts of the strategy implementation are trivial for the reasoner, and only a careless mistake can prevent an answer from being reached. Lithner (ibid.) also stresses how textbooks and teachers can serve to guide such superficial memorized and algorithmic reasoning. In contrast, creative mathematical reasoning involves novelty, plausibility and mathematical foundation, while creativity is seen as an orientation or disposition towards mathematical activity that can be fostered broadly in school. This perspective is reflected in US Standards of the National Council of Teachers of Mathematics (NCTM 2000) that recognize reasoning and proof as fundamental aspects of mathematics. ‘People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove’ (ibid., 56). Such reasoning can have many functions in mathematics, including verification, explanation, systematization, discovery, communication, construction of theory and exploration.

In relation to epistemic quality, the characteristics of mathematical fundamentalism are used to describe school mathematics of low epistemic quality, while those characterizing mathematical fallibilism describe school mathematics of high epistemic quality. It is further argued (Hudson et al. 2015) that high epistemic quality is promoted through an approach based on assessment for learning involving low stakes formative and self-assessment. This is engaging and motivating for individual learners and can create the conditions
leading to a sense of enjoyment and fulfilment of mathematics as a creative human activity. In contrast, the excessive pressure from high-stakes external testing and inspection and the associated heavy emphasis on memorization, drill and practice establish circumstances that can degrade epistemic quality into the mutated form of mathematical fundamentalism and lead to an experience for learners of mathematics that is fearful and anxiety-inducing, boring, demotivating and alienating from the subject itself.

In this chapter, the concept epistemic quality is developed further by considering it in relation to the idea of *epistemic ascent* (Winch 2013) in the development of subject expertise. This is based on a continuum that reflects a trajectory in developing expertise from that of a novice towards that of an expert in the subject. In his discussion of curriculum design and epistemic ascent, Winch (2013: 129) identifies three distinct yet related kinds of knowledge: *knowledge by acquaintance*, propositional knowledge or *knowledge that* and procedural knowledge or *knowledge how*. By doing so, he highlights the importance of each and also the relationships between them for the process of curriculum design. The primary mode of *knowledge by acquaintance* is through the senses so that one may be acquainted with objects, events, processes, states and persons. Examples of how such *knowledge by acquaintance* can arise include hearing a piece of music, smelling a flower and tasting a fruit. Furthermore, it is highlighted that not only is this an important part of any curriculum but also it is often a challenge for the curriculum designer to provide such knowledge. In relation to powerful knowledge, there is a correspondence between *knowledge by acquaintance* and Young’s (2013) use of the term ‘everyday knowledge’. While discussing *knowledge that*, Winch (ibid.) argues that this cannot consist solely in the identification of true but isolated propositions, but that this is embedded within a conceptual structure which is itself embedded within further related propositions. In developing subject expertise, it is stressed that it is not only the quantity of propositions that is important but also the ability to understand and, with reference to Brandom (2000), to make inferences employing the concepts embodied within the subject matter. It is also stressed (Winch 2013: 132) that it is necessary to be able to distinguish within the subject between claims that can be counted as knowledge and those which count as true beliefs. He also highlights that this distinction can be especially hard for a novice because the source of such beliefs is authoritative testimony. Third, as concerns *knowledge how*, it is argued that knowing how to do something is an epistemic capacity that is related to *knowledge by acquaintance* and *knowledge that* given that knowing how to do something usually requires elements of the other two kinds of knowledge. In thinking about the nature of
expertise, it is argued that this applies to both knowledge how and knowledge that and also their inferential relationships with each other. Subject expertise in turn also involves knowledge of how to make and understand inferences to a very significant extent. Beginning to learn a subject involves beginning to use the language associated with the concepts of the subject, and this is primarily a practical ability that is learned. Accordingly, a central dimension of learning about a subject is learning to take part in conversations and discussions that employ those concepts. This process is part of a gradual transition from novice to expert as, with reference to Wittgenstein (1969), 'light dawns gradually over the whole'.

Figure 2.1 Trajectory in the development of subject-specific powerful knowledge in mathematics.
This stress on the crucial role of language in the process of developing subject expertise reflects the emphasis placed on this aspect in JATD through the central focus within this framework on language games. Hence, in the process of epistemic ascent the novice moves from a position in which knowledge by acquaintance and isolated beliefs based on authoritative testimony are predominant towards the more complex and interconnected higher-order knowledge that and knowledge how of an expert in the subject. This trajectory of development in relation to school mathematics is captured in Figure 2.1.

The framework based on epistemic quality outlined above is used in this chapter to evaluate the quality in primary school mathematics by revisiting the data analysis of an earlier study. This focuses on two aspects; first, the epistemic quality of the content knowledge and, second, the epistemic quality of the teacher–student(s) joint action. The idea of joint action is a fundamental starting point for JATD and is based on a view articulated by Loquet (2011) of teaching and learning as co-operative and asymmetric joint action between the teacher and the students.

The study

The study is based on the analysis of one sequence of classroom interaction from a series of mathematics lessons in a primary school. The data collection was part of an action research project on the development of a topic-based approach to teaching and learning mathematics on the theme of ‘The Rainforest’. This was conducted by one teacher who took part in the DMTPC project already referred to. Analysis of data taken from the series of lessons as a whole using the JATD framework is discussed in Hudson (2015, 2019). In revisiting the analysis using the framework based on epistemic quality described above, the following two questions are addressed:

i. How can the epistemic quality of content be characterized?

ii. How can the epistemic quality of teacher–student(s) joint action be characterized?

The action research project addressed the question of the extent to which a topic-based approach to mathematics allows children to demonstrate their mathematical thinking. The teacher (Anna) was also interested in the extent to which topic-based mathematical questions allow children to verbalize their thinking and also the effect of such an approach on children’s levels of engagement. This study took place in a five-class, rural primary school in Angus
in North East Scotland, with a mixed-age class of Year 5 and Year 6 pupils, aged between 9 and 11 during the Spring and Summer terms of 2012. The overall topic for the action research project was ‘The Rainforest’ and, as a result, 23 pupils were given 4 questions to explore, analyse and record during a period of 3 weeks. The mathematical content was ‘measurement’, which chiefly related to measuring length and weight. The four questions the pupils worked on were:

1. How could we measure these life-sized insects accurately?
2. How could we mark out the different layers of the rainforest in our playground?
3. Can you compare the length of the River Tay and the Amazon River?
4. Is there a relationship between the weight of an animal and the layer it lives on in the rainforest?

Accounts of happenings in teaching and learning situations were recorded by collecting data in three ways. First, children’s talk was recorded informally during conversations with peers. Second, notes were made of what children said during class feedback sessions and comments on the lessons in feedback sessions were recorded. Furthermore, observations were made of the levels of engagement within the class and various parts of the activities were filmed to watch and analyse on a later date. In this paper, one extract from Lesson 3 in response to question 3 is used for the analysis.

Data collection

Lesson 3 focused on addressing the question: ‘Can you compare the length of the River Tay and the Amazon River?’ Anna described her expectation for the children to be able to ‘discuss length confidently in kilometres and to demonstrate an understanding of the enormity of the Amazon River in relation to the River Tay’. Prior to the lesson, the children had investigated the length of a kilometre, and most were able to convert between metres and kilometres. To begin the activity, children had 30 minutes to research the lengths of the two rivers by making use of the Internet.

A Year-5 boy (Andrew) explained his thinking as he was beginning to engage in the activity:

‘The Amazon is like a snake, so some people might be measuring it from when it’s just a wee stream and some might just be measuring it from the start of the big river’.
The children were then asked by Anna to present their findings in a sentence, paragraph or diagram. Anna saw this as a successful exercise for observing and recording the children’s understanding of kilometres in a relaxed yet focused atmosphere. Some of the children’s findings are listed below to illustrate the progress of the activity:

A Year-5 boy (Joseph) simply noted that:
   The Tay is shorter than the Amazon.
Another Year-5 boy (Andrew) went further:
   ‘The Amazon is the second biggest river in the world at around an amazing 6,400 kilometres long. So, the Amazon is longer’.
A Year-6 boy (Keith) described more extensive use of the Internet:
   ‘We only found our results in miles but we found an online converter and put them both into kilometres’.
A Year-5 girl (Karen) stated that:
   ‘We took away 193 from 6,360 to find the difference between the two rivers.’
   This comment intrigued the rest of the class and prompted the activity of repeating the calculation to see if Karen had calculated correctly.

Anna describes how, following this activity, a Year-5 boy (John) wanted to explain the diagram that he and his partner had created.

He stated, ‘I drew the Amazon and the River Tay on my piece of paper. I measured the paper and it was about 300 millimetres so we narrowed it down and got that every 5 centimetres was about 1,000 kilometres. The River Tay is only 186 kilometres, so it’s only that size’.

She describes how he then pointed to the part of their diagram labelled ‘River Tay’ at the very bottom of the page shown in Figure 2.2.

Anna noted how this comment and diagram not only confirmed John’s understanding of kilometres, but also how it demonstrated his understanding of centimetres and millimetres. Furthermore, she noted how from the teacher’s perspective it had been an excellent opportunity to not only see the children’s mathematical thinking unfold in their diagram but also listen to their explanation and observe their enthusiasm. The other children in the class were very interested in this and one Year-6 girl (Tracy) continued by commenting that:
   ‘The Tay is tiny compared to that, you could fit like, a hundred of the Tay into the Amazon!’

Anna notes how this comment was explored and extended, leading her to question:
   ‘How many times would the Tay fit into the Amazon River?’ (Anna).
Subsequently, Anna described this as a ‘light bulb moment’ and this aspect is discussed further in the section below.

One Year-6 boy (Michael) suggested that both numbers should be rounded off to make it easier. In response, a Year-6 girl (Charlotte) said:

That would be …. 190 kilometres?
Michael then replied: ‘It'd be good to do it to 200, to the hundred instead.’

As part of the whole class activity, Anna organized a vote on the two suggestions, and it was decided to work to the nearest hundred. Children were then set a task to solve this problem in pairs. Various methods were demonstrated, and two examples of the children's working are shown below in Figures 2.3 and 2.4.

To finish the lesson, the results were discussed and strategies explained. Here are two examples of the children’s writing, which highlight the development of their mathematical thinking.

‘I did 200, 400, 600, 800 and 1,000, so there's 5 River Tays in 1,000. That means there's 30 in 6,000 because 5 times 6 is 30, then I just needed two 200s to get the 400 kilometres left. So, the Amazon is about 32 River Tays.’

‘I did 6,400 divided by 200 as a sum to find out how many River Tays I could get in the Amazon. I said, “How many two hundreds are in 6, none, in 64 none, in 640, 3 with 40 left over”. Then I put that 40 with the zero in the units column and got two hundreds in 400. That made 32.’

Anna describes how the discursive element of this lesson proved to be a very effective tool for assessing the pupils’ understanding and mathematical thinking, and this aspect is discussed more in the section below.

Figure 2.3 First example of a child’s working.
Figure 2.4 Second example of a child's working.
Data analysis

The data is analysed by addressing the question of how the nature of epistemic quality in school mathematics can be characterized, and consideration is paid to the implications for curriculum innovation in the final discussion. With regard to epistemic quality, two aspects are addressed in particular; first, the epistemic quality of the content and, second, the epistemic quality of the teacher–student(s) joint action.

Epistemic quality of the content

In addressing the question ‘Can you compare the length of the River Tay and the Amazon River?’; Anna had designed a classroom activity that drew on the children’s knowledge by acquaintance. One of her aims was that the children would build on their prior knowledge and be able to discuss length confidently in kilometres and demonstrate an understanding of the enormity of the Amazon River in comparison with the River Tay. The children’s knowledge by acquaintance was based on two aspects. The first aspect was their general knowledge of the Amazon River, which may already have been gained through popular culture but was introduced and/or extended through the project on ‘The Rainforest’. The second aspect was their knowledge of the River Tay as a very familiar feature of the local environment in which they lived. Prior to the lesson, the children had developed their knowledge that by investigating the length of a kilometre, and most had developed the knowledge how in terms of converting between metres and kilometres. The classroom activity was developed by the children having some time at the outset to research the lengths of the two rivers by making use of the Internet.

Andrew demonstrated his engagement in the activity by comparing the Amazon to a snake, while Joseph demonstrated a basic knowledge how in terms of comparison by making the inference that the Tay is shorter than the Amazon. Similarly, Andrew concluded that the Amazon is longer by also noting that it is the second-biggest river in the world at around 6,400 kilometres in length. Keith demonstrated how his knowledge how had been extended by the fact that his group could only find distances in miles, leading them to find an online converter to convert them both kilometres. Karen also demonstrated knowledge how of subtraction in finding the difference between the two rivers. This contribution intrigued the rest of the class and was used by Anna to extend the activity by engaging the class in repeating the calculation to see if Karen...
had calculated correctly. She describes how John wanted to explain the diagram he and his partner had created, as illustrated in Figure 2.2. In drawing pictures of both the Amazon and the River Tay, he used his prior knowledge of measuring and also of ratio and scale such that every 5 centimetres represented 1,000 kilometres. This resulted in a very striking overall representation of the size of the two rivers and Tracy’s comment that the River Tay was tiny in comparison. Anna highlighted the extent of both John’s knowledge that and knowledge how in noting that this confirmed his understanding not only of kilometres but also of centimetres and millimetres and the associated inter-relatedness between them. Anna also stressed the importance of language in emphasizing how it had been an excellent opportunity for her to not only see the children’s mathematical thinking unfold in their diagram but to also listen to their explanation and observe their enthusiasm. She goes on to illuminate how she built on Tracy’s comment, leading to her key question of ‘How many times would the Tay fit into the Amazon River?’

In summing up, Anna noted that it was evident the children were able to take their investigation further according to their level of understanding and previous knowledge. A particularly significant aspect of Lesson 3 was the incident Anna described as a ‘light bulb moment’. This marked a situation in which she responded at the moment and built on the children’s contributions to extend the classroom activity in a very spontaneous and creative way. As a result, the epistemic quality of the content was greatly enhanced into one concerned with both knowledge that and knowledge how in relation to estimation, scale and calculations involving addition, subtraction, division and multiplication with up to four-digit numbers.

**Epistemic quality of teacher–student(s) joint action**

In relation to the quality of the teacher–student(s) joint action, Anna describes how the discursive element of this lesson proved to be a very effective tool for assessing the pupils’ understanding and mathematical thinking. Anna’s original learning intention for this lesson was for the children to become confident in measuring with millimetres and be able to convert between centimetres and millimetres. However, the children’s knowledge how and mathematical thinking was extended through a process of creative reasoning to consideration of knowledge that in terms of the concept of scale, which involved estimation and also calculations entailing addition, subtraction, division and multiplication with up to four-digit numbers as outlined above. Since all pupils recognized the
vast difference between the two rivers, their natural inquisitiveness and natural orientation towards creative reasoning drove them to extend their own thinking and the thinking of others. The question in Lesson 3 ‘Can you compare the length of the River Tay and the Amazon River?’ in particular developed tremendously throughout the lesson. It was evident in this study that the children had very different prior knowledge and experiences to bring to the problem-solving elements of the tasks. Moreover, the mathematics became more accessible to all due to their ability to visualize the problems and to engage in collective creative reasoning. This resulted from the high quality of the teacher–student(s) joint action and the high epistemic quality of the content, which in turn can be seen to have produced an evolution in mathematical thinking and epistemic access (Morrow 2008; Young 2013: 115) to powerful knowledge for all.

Discussion

As indicated earlier, an important aspect of the case made for support in the grant application for the DMTPC project was that most mathematics lessons in Scotland at that time still tended to feature some form of teacher-led demonstration followed by children practising skills and procedures from a commercially produced scheme. The more recent Scottish Survey of Literacy and Numeracy of that time also reported the most common aspects of the classroom experience for pupils were ‘listening to the teacher talk to the class about a topic’ and ‘working on your own’. Such approaches serve to reinforce the presentation of mathematics as being predominantly about memorization, rule-following of strict procedures and right or wrong answers that are associated with mathematical fundamentalism and low epistemic quality, as illustrated in Figure 2.1, as an over-emphasis on merely knowing that. In contrast, the case study discussed in this chapter illustrates the central role of children verbalizing their thinking, which was one of Anna’s specific interests at the outset. In particular, this reflects an important aspect in the development of subject expertise from the knowledge by acquaintance of the novice to the knowledge that and knowledge how of an expert in the subject. This development involves knowledge of how to make and understand inferences and the use of language associated with the concepts of the subject, which is largely a practical ability that is learned. As such, a central dimension of learning about a subject is learning to take part in conversations and discussions that employ those concepts. This process can also be seen as part of a gradual transition from novice to expert such that 'light
dawns gradually over the whole’ (Wittgenstein 1969). Such an emphasis serves to develop a fallibilistic view of mathematics as refutable and uncertain and one which promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes that is associated with high epistemic quality.

With regard to the implications for curriculum innovation in schools, Anna’s approach may be seen as exemplary. This involved addressing the central question of the extent to which a topic-based approach to mathematics allows children to demonstrate their mathematical thinking. In addition, she was interested in the extent to which topic-based mathematical questions allow children to verbalize their thinking and the effect of such an approach on children’s levels of engagement. The way in which this was approached as a whole through the DMTPC project is discussed further in Hudson (2022), which focuses on the processes involved in supporting the professional development of teachers as ‘curriculum makers’ (Lambert and Biddulph 2015) for school mathematics of high epistemic quality.

The interplay between curriculum innovation and teacher professional development is captured most effectively by Stenhouse (1980) in his observation below, which is an appropriate point for reflection on which to end this chapter:

What is curriculum as we now understand the word? … It is not a syllabus – a mere list of content to be covered – nor even is it what German speakers would call a Lehrplan … Nor is it in our understanding of a list of objectives. Let me claim that it is a symbolic or meaningful object, like Shakespeare’s first folio, not like a lawnmower; like the pieces and board of chess, not like an apple tree. It has a physical existence but also a meaning incarnate in words or pictures or sound or games or whatever … by virtue of their meaningfulness curricula are not simply means to improve teaching but are expressions of ideas to improve teachers. Of course, they have day-to-day instructional utility: cathedrals must keep the rain out ….

(Stenhouse 1980: 40)

References


