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Closed-LSTM Neural Network based Reference Modification for Trajectory Tracking of Piezoelectric Actuator

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Abstract

In this article, we propose a trajectory tracking control method for piezoelectric actuators (PEAs) based on long short-term memory neural network (LSTM-NN). Different from traditional control framework where neural network is used to approximate the open-loop PEA dynamics, LSTM-NN is used to establish the mapping between the actual trajectory and the reference trajectory of the closed-loop PEA, leading to a Closed-LSTM neural network control framework. With this framework, the trained LSTM-NN is used to modify the reference trajectory to compensate for the tracking error without changing the controller. First, we analyze and simplify the modeling of the linear and nonlinear characteristics of the PEA, and select the training input features of the LSTM-NN. Then, we use the actual trajectory and reference trajectory of the closed-loop PEA to train the LSTM-NN. The Closed-LSTM neural network control framework enables independent designs of the baseline feedback controller and feedforward compensator. In particular, the feedback controller is used to guarantee the system stability, and the LSTM-NN reference modification module is used as the feedforward compensator to achieve high-precision trajectory tracking, which does not affect the system stability and can be easily applied to off-the-shelf motion control systems. Its validity is experimentally verified on a PEA platform.

Keywords: Piezoelectric actuators, LSTM-NN, Motion control

1. Introduction

Pactuators (PEAs) are widely used in micro-nano positioning systems and ultra-precision motion control systems. However, hysteresis, creep and vibration characteristics, and unknown external disturbances are inherent factors that affect the performance of PEAs \cite{1}, making the control of the system extremely challenging.

In order to achieve high-performance trajectory tracking control of PEAs for repetitive tasks, iterative learning control (ILC) \cite{2,3} and repetitive control (RC) \cite{4,5} are found to be effective. Combined with ILC, the inversion-based controller can achieve precise output tracking for repetitive tasks \cite{6}. With the nonlinearities compensated for by the inversion model, repetitive controllers are developed for trajectory tracking of both single and dual-stage nanopositioning PEA-based systems \cite{7}.

Although ILC-based methods can be used to achieve high-precision trajectory control, they are not suitable for non-repetitive tasks. Therefore, real-time control of non-repetitive trajectories has been extensively studied. Accurately modeling...
inherent nonlinear characteristics is found useful for designing model-based compensation control strategies [9]. The nonlinear characteristics are treated as bounded interference, and various compensation and suppression strategies are designed [9]. In [10], the Bouc-Wen model of hysteresis is established, and a sliding mode control method based on the hysteresis modeling is proposed to suppress the hysteresis nonlinearity of the piezoelectric driver stage and realize high-precision tracking control. In [11], model predictive control (MPC) based on disturbance observer (DOB) is proposed to suppress hysteresis nonlinearity and model uncertainty. Both friction and the piezoelectric materials have hysteresis, therefore, the friction characteristics can not be ignored. In piezoelectric inertia–friction actuators (PIFAs), the dynamics of the whole system depend on both PEA and frictional contact. There are different models of friction used for modeling the PIFA system, such as the Coulomb model, the LuGre model and the elastoplastic model [12]. Due to the complexity of the nonlinear model, the designed control system can be complicated, and the stability of the system is hard to analyze in rigor.

In recent years, many researchers have applied artificial neural network (ANN) modeling and control methods to trajectory tracking of PEA [13, 14, 15, 16], especially the inverse compensation control based on ANN. In [17], in order to improve the performance of active vibration control for a helicopter driven by piezoelectric stack actuators (PSAs), a hysteresis neural network based on nonlinear autoregressive neural network (NARX) is established, and then another neural network is used to compensate for the hysteresis characteristics of the PSA. In [18], a deep gated recurrent unit (GRU) neural network is proposed to model the nonlinearity and memory of the hysteresis loop, a back propagation (BP) neural network is integrated to improve the generalization ability, and the scanning motion control of piezoelectric driven atomic force microscope is achieved. The inverse compensation control based on neural network has also been applied in other fields. In [19], an algorithm based on deep neural network (DNN) is proposed as an inverse compensation module, which improves the tracking performance of quadrotor. Given a desired trajectory, the DNN compensation module provides a suitable reference trajectory for the controller to realize the identity mapping between the actual trajectory and the reference trajectory.

Figure 1: Comparison of two control frameworks. (a) Traditional control framework. (b) The control framework proposed in this article

Although the control objective of this article is similar to those in the previous studies, there are differences between the control method proposed in this article and the previous control method based on the ANN inverse model, as shown in Fig. 1, where $y_d(k)$, $y_r(k)$ and $y(k)$ denote desired trajectory, reference trajectory and actual trajectory at the sampling instant $k$, respectively. The desired trajectory is the trajectory predefined for the piezoelectric actuator to actually track. The reference trajectory is obtained by using the LSTM module to compensate for and modify the desired trajectory, and is used as the input signal of the closed-loop PEA. While the previous method uses ANN to establish the inverse model of the nonlinear characteristics of the open-loop PEA to perform feedforward compensation [20, 21], the ANN in this article is to establish a mapping between the actual trajectory and the reference trajectory of the stable closed-loop PEA to modify the reference trajectory. The advantages of the proposed method in this article are: (1) The closed-loop PEA can partially compensate for the interference, and its response is more repeatable than the open-loop system, so the generalizability of

\[
y_d(k) \xrightarrow{\text{Feedback Controller}} \text{Piezoelectric Actuator} \xrightarrow{\text{ANN}} y(k)
\]

\[
y_r(k) \xrightarrow{\text{Feedback Controller}} \text{Piezoelectric Actuator} \xrightarrow{\text{ANN}} y(k)
\]
the ANN modeling is relatively high, leading to high compensation control accuracy. In comparison, the traditional method directly establishes the inverse model of the open-loop PEA and the open-loop characteristics are often more complicated and lack repeatability, so the accuracy of the established inverse model is limited by uncertainties, resulting in low compensation control accuracy. (2) ANN, as an additional module to modify the reference trajectory, can predict the tracking error and eliminate it in advance, so the tracking error will not increase due to the increase of velocity. In the compensation control based on the inverse model of open-loop PEA, when the modeling accuracy is low and the effect of feedforward compensation is small, the feedback component will play a major role, bringing a greater tracking error with the increase of velocity. (3) The ANN is connected in series in front of the closed-loop PEA to modify the reference trajectory. It does not need to modify the controller of the closed-loop PEA, so it does not affect the stability of the system and can be easily applied to various motion control systems.

Regarding different neural networks, NARX with external source input connects the output feedback to the input, which enhances the memory ability of historical data, but the memory time is fixed. Recurrent neural network (RNN) adds the horizontal connection between the hidden layer units, which can transfer the value of the last time neural unit to the current neural unit, so that it has the memory function. LSTM-NN is a special RNN that can take into account both long-term memory and short-term memory, and can automatically adjust the influence of historical information in the future, so it can be used to fit time series and the complex dynamics of PEAs. GRU modifies the structure of LSTM-NN, and it also has the function of long and short-term memory. In this article, by comparing the accuracy of LSTM-NN and GRU, LSTM-NN is selected as the neural network model.

Based on the above discussions, this article will propose a trajectory tracking control method based on LSTM-NN modelling of closed-loop PEA, which is named Closed-LSTM. First of all, the physical characteristics of closed-loop PEA are analyzed, and the dynamic model is established. Then, according to the model and dynamic characteristics, the input features of LSTM-NN are selected and training is performed to establish the mapping between the actual trajectory and reference trajectory of closed-loop PEA. The trained LSTM-NN is used to obtain the modified reference trajectory as the input of the closed-loop system.

The main contributions of this article lie in two aspects. On the one hand, we analyze the nonlinear characteristics of closed-loop PEA to select suitable features for LSTM-NN. On the other hand, we propose the Closed-LSTM control framework using LSTM-NN to establish the mapping between the actual trajectory and the reference trajectory of the closed-loop PEA, which can compensate for tracking error in advance and improve the tracking performance.

The rest of this article is organized as follows. Closed-loop PEA model analysis is performed in Section II. Section III introduces the proposed tracking control method based on LSTM-NN. Section IV presents results of experiments. At last, we draw the conclusions of this article.

2. SYSTEM MODELING

PEAs use the reverse voltage effect of piezoelectric materials to realize the conversion from electrical energy to mechanical energy. It is one of the high-precision actuators widely used in flexible micro-nano platforms. PEAs have non-linear characteristics such as friction, hysteresis, creep and vibration. When modeling the PEA, the creep and vibration are simplified into linear models, and the friction and hysteresis nonlinearity are analyzed separately.

2.1. Linear Model of Closed-loop PEA
2.1.1. Modeling of PEA

By connecting each individual sub-model in series, the nonlinear and linear characteristics of the PEA can be decoupled, as shown in Fig. 2, where \( u(t) \) denotes the input voltage at time \( t \).
and $H$, $C$, and $V$ represent the hysteresis, creep and vibration dynamic characteristics of the PEA, respectively. The method of modeling these three characteristics is to decouple them into two main parts. The hysteresis nonlinear unit is connected in series with a linear dynamic unit including creep and vibration characteristics.

$$u(t) \rightarrow \text{Hysteresis Model} \rightarrow \text{Vibration Model} \rightarrow \text{Creep Model} \rightarrow y(t)$$

Figure 2: Physical model of PEA

Creep characteristics can be modeled using transfer function and lumped parameter methods. When creep is modeled as a mass-spring-damping system \(^{(28)}\), the transfer function of creep characteristics is expressed as

$$G_r(s) = \frac{1}{k_0} + \sum_{i=1}^{N} \frac{1}{s c_i + k_i}$$

where $k_i$ and $c_i$ are the spring and damping coefficients respectively. Generally, the creep model of the PEA is approximated as a second-order system, namely

$$G_r(s) = \frac{T_2 s^2 + T_1 s + T_0}{\tau_2 s^2 + \tau_1 s + \tau_0}$$

where $T_2 = c_1 c_2 k_0$, $T_1 = c_1 k_0 k_2 + c_2 k_0 k_1$, $T_0 = k_0 k_1 k_2$, $\tau_2 = c_1 c_2$, $\tau_1 = c_1 k_0 + c_2 k_0 + c_1 k_2 + c_2 k_1$, $\tau_0 = k_0 k_1 + k_0 k_2 + k_1 k_2$.

Similarly, the vibration dynamic characteristics model can be obtained by adding a force balance to a standard mass-spring-damping system \(^{(29)}\), and the transfer function is expressed as

$$G_v(s) = \frac{\alpha}{s^2 + \frac{c_1}{m} s + \frac{k_1}{m}}$$

where $m$, $c$, and $k$ are mass, damping and spring coefficients, respectively, and $\alpha$ is a constant related to the input voltage and the force generated by the PEA.

2.1.2. Servo controller

In motion control systems, a feedback controller is first used to form a closed-loop system, which has anti-interference ability and repeatability. A PI feedback controller is usually used in the closed-loop piezoelectric system, whose transfer function expression is

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s}\right)$$

where $K_p$ is the proportional coefficient, and $T_i$ is the integral time constant. The feedback part is a low-pass filter, and its transfer function is

$$G_f(s) = \frac{G_0 w_c}{s + w_c}$$

where $G_0$ is the passband gain, and $w_c$ is the cutoff frequency of the first-order filter.

According to the above models, when ignoring the hysteresis characteristics of the PEA, a simplified linear model of the closed-loop PEA dynamic model can be obtained as shown in Fig. \(3\), where $Y_r(s)$ and $Y(s)$ are Laplace transformations of $y_r(t)$ and $y(t)$, respectively.

$$y_r(t) + \frac{K_p (1 + \frac{1}{T_i})}{T_s} \rightarrow \frac{\alpha}{s w_c + c_3 + k} \rightarrow \frac{\tau_2 s^2 + \tau_1 s + \tau_0}{T_2 s^2 + T_1 s + T_0} \rightarrow \frac{G_s w_c}{s + w_c} \rightarrow y(s)$$

Figure 3: Simplified model block diagram of closed-loop PEA

According to the simplified linear model block diagram in Fig. \(3\), the transfer function of closed-loop PEA is expressed as

$$Y(s) = \frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

where $b_0$, $b_2$, $\cdots$, $b_5$ and $a_0$, $a_1$, $\cdots$, $a_4$ are the coefficients calculated based on the simplified model in Fig. \(3\).

2.2. Friction and hysteresis nonlinearity analysis

The dynamic model of the piezoelectric drive system is related to the dynamics of the piezoelectric element and the contact friction. Therefore, the friction characteristics must be considered when analyzing the characteristics of the system.
The LuGre model can be used to describe the friction in the PEA system \[12\]. The expression of the model is as follows:

\[
\begin{align*}
F &= \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \\
\frac{dz}{dt} &= v - |v| g(v) \tag{7}
\end{align*}
\]

where $F$, $\sigma_0$, $\sigma_1$, $\sigma_2$ and $z$ are the friction force, average stiffness of the bristles, damping coefficient, the viscous coefficient and the average deflection of the bristles, respectively. $v$ is the relative velocity between the two surfaces, and $g(v)$ is a function corresponding to the Stribeck effect. It can be seen from the above expression that the friction of piezoelectric drive system is mainly related to position and velocity.

Due to the characteristics of piezoelectric materials and the influence of friction and temperature, the piezoelectric drive system will also have hysteresis nonlinearity. The hysteresis characteristic of the closed-loop PEA means that the reference displacement curve and the actual displacement curve do not overlap, but they form a multi-value and memory-related hysteresis loop, as shown in Fig. 4. The existence of this nonlinearity leads to a larger tracking error of the closed-loop PEA.

![Figure 4: Hysteresis characteristics of closed-loop PEA](image)

The characteristics of hysteresis nonlinearity are as follows:

1) Non-local memory: The output value of closed-loop PEA system is not only related to the current value of the input signal, but also related to the historical value of the input signal, especially the historical extreme value.

2) Multi-value mapping: closed-loop PEA system can have a variety of different output values when the input has the same value.

3) Rate correlation: The output of closed-loop PEA system is affected by the frequency of the input signal. The higher the frequency is, the more significant effect the hysteresis has. In Fig. 5, the hysteresis characteristic curves of closed-loop PEA under four sinusoidal signals with different frequencies of 5rad/s, 15rad/s, 30rad/s, and 50rad/s are given.

![Figure 5: Hysteresis curve of closed-loop PEA with different frequencies. (a) 5rad/s. (b) 15rad/s. (c) 30rad/s. (d) 50rad/s.](image)

Considering the various nonlinear characteristics of PEAs, it is difficult to accurately model and identify the parameters of the system using traditional modeling methods. Nevertheless, above modeling and analysis lays the foundation of feature selection for LSTM-NN in the next section.

3. CONTROL STRATEGY AND LSTM-NN CONSTRUCTION

3.1. Control strategy

The traditional control framework based on the ANN inverse model is shown in Fig. 1(a). The ANN inverse model is used as a feedforward controller that can compensate for the tracking error of the system by modifying the control input, and the feedback controller is used to suppress unknown disturbances and parameter uncertainties. Different neural networks can be used to establish the dynamic inverse model of the PEA, and the feedback controller also has different realizations.
In this control framework, an inverse model of the open-loop plant is established, whose accuracy is limited due to inevitable system uncertainties.

The Closed-LSTM trajectory tracking control framework proposed in this article is shown in Fig. 6. In Closed-LSTM, the control system can be divided into two parts. The first part is a stable closed-loop PEA, which includes a feedback controller, where the PID controller is used in this article. The input of the closed-loop PEA is the reference trajectory \( y_r(k) \), and the output is the actual trajectory \( y(k) \). The second part is a trajectory modification module based on LSTM-NN. Its input is the desired trajectory \( y_d(k) \), and the output is the modified reference trajectory \( y_r(k) \). The feedback controller in the closed-loop PEA can ensure the stability of the system and achieve a certain precision of trajectory tracking. For the same reference trajectory, similar tracking error of the closed-loop system can be produced, i.e. the system has better robustness and repeatability compared to the open-loop system. For such a closed-loop system, LSTM-NN with powerful approximation capabilities for time series can be used to learn the mapping between \( y_d(k) \) and \( y_r(k) \). The reference trajectory is modified by using the trained LSTM-NN to improve the tracking performance of the whole closed-loop PEA system.

The proposed Closed-LSTM includes a training phase and a testing phase, and LSTM-NN is used to learn the mapping between \( y_d(k) \) and \( y_r(k) \). In the training phase, we treat \( y(k) \) as \( y_d(k) \), and actual trajectories \( y(k), y(k+1), \ldots, y(k+N) \) are used as training inputs of the LSTM-NN, and reference trajectories \( y_r(k), y_r(k+1), \ldots, y_r(k+N) \) are training outputs. In the testing phase, the trained LSTM-NN is used to modify the desired trajectory to obtain a new reference trajectory of the closed-loop PEA system. The input of LSTM-NN is desired trajectory \( y_d(k) \), and output is the reference trajectory \( y_r(k) \). When the reference trajectory \( y_r(k) \) is input in closed-loop PEA, its actual trajectory \( y(k) \) is equal to desired trajectory \( y_d(k) \) if the LSTM-NN is absolutely accurate.

Here we briefly discuss the stability of the Closed-LSTM system. First, we use \( \hat{y}_r \) to represent the reference displacement calculated by LSTM-NN and it is the input of the piezoelectric driver. The LSTM-NN module has a feedforward structure, and the activation function is global Lipschitz. Because LSTM-NN is a linear combination of Lipschitz functions, the output \( \hat{y}_r \) of the LSTM-NN module is also global Lipschitz, i.e. \[ \| \hat{y}_r \|_\infty \leq L \| y_d \|_\infty \] (8) where \( L \) is a positive scalar. Under the feedback controller, the closed-loop PEA is input-to-state stable. Therefore, with a bounded input, the output of the closed-loop PEA is bounded, and the whole system is stable.

3.2. Feature selection

Feature selection is particularly important to ensure the modeling accuracy of the LSTM-NN. By reasonably selecting input features, the modeling accuracy of LSTM-NN is improved, and then, the tracking performance of the whole motion control system can be improved.

3.2.1. Feature selection based on simplified linear model

Based on the simplified linear model (9) of the closed-loop PEA, the transfer function can be transformed into the discrete-time form

\[
\frac{Y(z)}{Y_r(z)} = \frac{\beta_4 z^4 + \beta_3 z^3 + \beta_2 z^2 + \beta_1 z + \beta_0}{z^5 + \alpha_4 z^4 + \alpha_3 z^3 + \alpha_2 z^2 + \alpha_1 z + \alpha_0}
\]

(9)

where \( Y_r(z) \) and \( Y(z) \) represent the \( z \) transformation expressions of the reference displacement and actual displacement of the closed-loop PEA, respectively, \( \alpha_i, i = 0, \ldots, 4 \) and \( \beta_j, j = 0, 1 \ldots, 4 \).
are the coefficients calculated based on (10). Performing inverse transformation on (10), the relationship between the reference displacement and the actual displacement can be obtained as follows

\[ y_r(k) = \frac{1}{\beta_4} y(k + 1) + \frac{\alpha_4}{\beta_4} y(k) + \cdots + \frac{\alpha_0}{\beta_4} y(k - 4) \]

\[ - \frac{\beta_3}{\beta_4} y_r(k - 1) - \frac{\beta_2}{\beta_4} y_r(k - 2) - \cdots - \frac{\beta_0}{\beta_4} y_r(k - 4) \]

where \( y_r(k) \) and \( y_r(k) \) represent the reference displacement and actual displacement at time \( k \) respectively.

According to the discrete-time expression (10) of the linear part of the closed-loop PEA, the reference displacement \( y_r(k) \) at the sampling time \( k \) is related to the actual displacement \( y(k + j) \), \( j = -4, \cdots, 1 \) and reference displacement \( y_r(k - i) \), \( i = 1, 2, \cdots, 4 \). When using LSTM-NN to approximate the inverse dynamics of closed-loop PEA, \( y(k + j) \), \( j = -4, \cdots, 1 \) are used as the input features and \( y_r(k) \) as the output of LSTM-NN, i.e., the reference displacement of the current time is calculated by using the actual displacement of the past 4 time steps, current time step and the next 1 time step, while reference displacement \( y_r(k - i) \), \( i = 1, 2, \cdots, 4 \) as hidden information is retained and fitted by the long-term and short-term memory characteristics of LSTM-NN. Since the input features are selected based on a simplified linear model that may not be accurate, we use them as a starting point for feature selection. In practice, the specific values of \( n \) and \( m \) can be adjusted according to the accuracy of model fitting, to select appropriate actual displacement \( y(k + j) \), \( j = -n, \cdots, m \).

3.2.2. Feature selection based on friction and hysteresis

The friction of PEAs is mainly related to position and velocity, and hysteresis characteristics of PEAs is rate-dependent as analyzed in Section II. The larger the velocity of the reference trajectory is, the more significant the hysteresis is. Therefore, when using the LSTM-NN to establish the mapping between the actual trajectory and the reference trajectory, we consider adding the velocity information of the past, current and future time steps of the trajectory \( \dot{y}(k-n), \dot{y}(k-n+1), \cdots, \dot{y}(k+m-1), \dot{y}(k+m) \) as the input features of the LSTM-NN. This selection of input features means that the trained LSTM-NN can plan the reference trajectory in advance based on the future state information to improve the trajectory tracking performance.

The method based on LSTM-NN uses the idea of black box to establish the mapping between the desired trajectory and the reference trajectory. Although we only analyze the two nonlinear characteristics of friction and hysteresis to select the features of LSTM-NN, due to its strong nonlinear fitting ability, it can deal with other nonlinear characteristics at the same time.

3.3. Training data

The LSTM-NN is used to establish the mapping between the actual trajectory and the reference trajectory of closed-loop PEA, so the data for training the LSTM-NN should be able to cover most of the feature space and fully stimulate the response characteristics of the system. In this sense, random nonuniform rational B-spline surface (NURBS) curve can be used. Moreover, the position, velocity and acceleration are limited in the range that the system can bear.

The random NURBS trajectory is generated by using random control points [30], which are composed of independent variable vector and dependent variable vector, and the independent variable vector is time

\[ \vec{t} = [t_0, t_0 + \Delta t_1, \cdots, t_0 + \sum_{i=0}^{n} \Delta t_i] \]

We set \( \Delta t_i = 0.04 + \text{rand}(0.04, 0.08) \), where the \( \text{rand}(0.04, 0.08) \) represents a random number between 0.2 ~ 0.4. We set the dependent variable observation value vector \( \vec{x} \) of these control points to satisfy the normal distribution, and the mean value and standard deviation of the normal distribution are 25 and 10, respectively.

According to the independent variable \( \vec{t} \) and the dependent variable \( \vec{x} \), the random control point sequence \( y(\vec{x}, \vec{t}) \) can be obtained, and these random control points are taken as training reference
trajectories. In addition, according to the movement stroke of the closed-loop PEA, these control point sequences are mapped to $0 \sim 60\mu m$, and zero-phase filtering is performed to remove the peak points, and finally the training trajectory is obtained.

3.4. LSTM-NN model

This paper uses the ANN inverse model to modify the reference input to compensate for the tracking error of the system, so the selection of the neural network is very important. The following will compare several common neural networks. There are three main types of ANNs currently used to fit time series: NARX, GRU and LSTM. The NARX with external input connects the output feedback to the input, which enhances the memory ability of historical data. The GRU neural network is a kind of RNN, which is a special LSTM-NN. The advantage of GRU over LSTM-NN is that the structure is simpler and the training efficiency is higher. In practical applications, if training efficiency and hardware computing power are not significant concerns, the one with the better actual performance is often selected.

3.4.1. LSTM-NN structure

LSTM-NN is an RNN with excellent performance for model fitting of sequence-to-sequence response. As a special RNN, LSTM can accommodate long-term memory and short-term memory by adding long-term memory sequences to the standard RNN. In the time domain, the LSTM-NN will pass back the two pieces of information: the cell state and the hidden state. It provides accurate approximation because of the cell state so that the historical information stored therein will not be easily changed, and it has a good memory capability making LSTM-NN suitable for long-term dependence problems.

3.4.2. Training of LSTM-NN

The weight parameters of the LSTM-NN need to be trained offline. In short, the training process is to iterate and update the weight parameters through the back propagation (BP) algorithm to find the optimal weight matrix.

The training input of the LSTM-NN is the actual trajectory information (position, velocity and acceleration), and the network training output is the estimated reference trajectory $\hat{y}_r(t)$. The loss function is defined as root mean square error (RMSE) between the estimated reference trajectory $\hat{y}_r(t)$ and the actual reference trajectory $y_r(t)$, and the expression at time $t$ is

$$J(t) = \frac{1}{2}\|\hat{y}_r(t) - y_r(t)\|^2$$

(12)

where $\|\cdot\|$ denotes the norm. Within $T$ training samples, the total loss function of the LSTM neural network is $J = \sum T \ J(t)$. Finally, we update the weight matrix of the LSTM-NN according to the gradient of $J$.

4. EXPERIMENT RESULTS AND DISCUSSION

4.1. Experimental platform

The experimental platform is mainly composed of PEA, piezoelectric controller, real-time simulation controller, and a computer, as shown in Fig. 7. The PEA used in this system is a cylindrical low-voltage PEA of the Harbin Core Tomorrow Science and Technology Co., Ltd. (model number PST150/10/60VS15), and its physical parameters are shown in Table 1. The piezoelectric controller is the E53.B servo controller of the Harbin Core Tomorrow Science and Technology Co., Ltd., equipped with an SGS displacement sensor with sensitivity of 6um/V and measurement noise of 0.05um. The real-time simulation controller is a product of dSPACE, and its model is DS1103 PPC Controller Board.

The control algorithm is implemented on the computer and the corresponding code and modules are compiled into the target file that can be
Figure 7: Closed-loop PEA experiment platform

Figure 8: Block diagram of closed-loop PEA experimental platform

In order to evaluate the repeatability of the closed-loop PEA, the repetition error at the sampling time $t$ and the actual trajectory variance obtained by running the same reference trajectory multiple times are defined as

$$
\hat{e}(t) = y_{\text{max}}(t) - y_{\text{min}}(t) 
$$

$$
y_{\sigma}(t) = \frac{1}{n} \sum_{i=1}^{n} (y_i(t) - \hat{y}(t))^2 
$$

where $y_i(t)$, $\hat{y}(t)$, $y_{\text{max}}(t)$ and $y_{\text{min}}(t)$ represent the actual displacement of the same reference trajectory for the $i$th repetition, the mean value of $n$ repetitions, the maximum and minimum at the sampling time $t$, respectively.

In order to evaluate the accuracy of LSTM-NN modeling of the closed-loop PEA and the performance of trajectory tracking control, modeling error $\varepsilon$ and tracking error $e$ are defined. For modeling error $\varepsilon$, the average relative error, maximum absolute error and root mean square error are defined as follows:

$$
\varepsilon_{\text{max}} = \max(||\hat{y}_r - y_r||) 
$$

$$
\varepsilon_{\text{rms}} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} ||\hat{y}_r(j) - y_r(j)||^2} 
$$

$$
\varepsilon_{\text{aver}} = \frac{1}{n} \sum_{j=1}^{n} \frac{||\hat{y}_r(j) - y_r(j)||}{y_r(j)} \times 100\% 
$$

For tracking error $e$, the average relative error, maximum absolute error and root mean square error are defined as follows:

$$
e_{\text{max}} = \max(||y - y_d||) 
$$

$$
e_{\text{rms}} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} ||y(j) - y_d(j)||^2} 
$$

$$
e_{\text{aver}} = \frac{1}{n} \sum_{j=1}^{n} \frac{||y(j) - y_d(j)||}{y_d(j)} \times 100\% 
$$

4.2. Feasibility analysis

In Closed-LSTM, we use the trained LSTM-NN to modify the reference trajectory to compensate for the hysteresis characteristics of the system in advance, thereby reducing the tracking error of the system. Since this is an open-loop control method, the closed-loop PEA is required to have good feedback properties, i.e., anti-interference ability and repeatability. On this basis, the LSTM-NN is used to guarantee the trajectory tracking properties of the system.
4.2.1. Anti-interference ability

We apply a power disturbance to the closed-loop PEA, and the ability of the system to suppress the disturbance is analyzed. Starting from \( t = 0 \) s, the output displacement of the closed-loop PEA is in a steady state of 30um, and a power disturbance of 1.5v is applied at \( t = 2s \). Fig. 9(b) shows that the actual displacement quickly returns to the original position after the deviation in about 30ms, so the closed-loop PEA has good anti-interference ability.

![Figure 9: Anti-interference performance of closed-loop PEA. (a) Response curve with disturbance. (b) Partial enlargement.](image)

4.2.2. Repeatability

Since the LSTM-NN is trained offline, in order to guarantee its good generalization performance in the test phase, the closed-loop PEA is required to have good repeatability, i.e., for the same reference trajectory, the actual trajectory obtained by the closed-loop system should be consistent. To test this, we use (11) to generate a random trajectory as a reference trajectory, let the closed-loop PEA repeat the same reference trajectory for 20 times, and then collect the corresponding actual trajectory. The variances of the actual displacement for the same reference trajectory at different times are shown in Fig. 10(a), illustrating a value around \( 3 \times 10^{-3} \)um. As shown in Fig. 10(b), most of the repetition errors are less than 0.3um. It demonstrates that the closed-loop PEA has good repeatability, which guarantees that the offline-trained LSTM-NN can be used as the reference modification module to improve the trajectory tracking performance.

![Figure 10: Repeatability test of closed-loop PEA. (a) Actual trajectory variance. (b) Repetition error.](image)

4.3. Comparison of different input features

As analyzed in Sections II and III, when establishing the trajectory mapping of the closed-loop PEA, the selection of input features is particularly important to the modeling accuracy of the LSTM-NN. According to theoretical modeling, the actual position and velocity should be considered, while the actual acceleration is also considered for comparison. The following three combinations of input features are tested: 1) the actual trajectory points at current and future \( N \) moments; 2) the actual trajectory points and corresponding velocities at current and future \( N \) moments; 3) the actual trajectory points at current and future \( N \) moments and the corresponding velocity and acceleration.

For all the above three input features, the training output is the current reference trajectory point. After the LSTM-NN model is trained, the test reference trajectory is input into the closed-loop PEA, and the corresponding actual trajectory is collected. Then, the actual trajectory is input into the LSTM-NN model. The modelling error of LSTM-NN is obtained by comparing the estimated reference trajectory with the actual reference trajectory, as shown in Fig. 11 and Table 2. From Table 11, it can be seen that the modeling error is the largest when only position information is used. When using position and velocity information as input features, most of the modeling errors are less than 0.1um. It can also be found that adding acceleration information has a very small improvement in modeling accuracy. Therefore, considering the calculation and training efficiency, the feature set 2) is subsequently
Table 2: LSTM-NN Modeling errors under different input features

<table>
<thead>
<tr>
<th></th>
<th>Pos</th>
<th>Pos+Vel</th>
<th>Pos+Vel+Acc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{max}}$ (um)</td>
<td>0.3596</td>
<td>0.2557</td>
<td>0.245</td>
</tr>
<tr>
<td>$\varepsilon_{\text{rms}}$ (um)</td>
<td>0.078</td>
<td>0.0565</td>
<td>0.0568</td>
</tr>
<tr>
<td>$\varepsilon_{\text{aver}}$ (%)</td>
<td>0.25</td>
<td>0.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>

used as the input feature of the LSTM-NN.

Figure 11: Modeling error under different input features

4.4. Comparison of different neural networks

The ANN currently used to predict the mapping between time series mainly includes NARX, GRU and LSTM-NN [17, 18, 25]. Therefore, this experiment compares the accuracy of the three neural networks in establishing the mapping between the actual trajectory and the reference trajectory of the closed-loop PEA. The selected neural network structure is shown in Fig. 12.

Figure 12: Different neural networks structure. (a) NARX. (b) GRU. (c) LSTM.

The NARX, GRU and LSTM-NN are trained respectively under the condition that the training set and the test set are the same and the input and output are consistent. The random NURBS trajectories generated by (11) are used as the test trajectories to compare the accuracy of different neural networks. From Fig. 13, it can be seen that the LSTM-NN has higher accuracy than the GRU and NARX. In particular, the maximum modeling error of NARX is 0.35um, root mean square error 0.071um; the maximum modeling error of GRU is 0.60um, root mean square error 0.093um; the maximum modeling error of LSTM-NN is 0.24um, root mean square error 0.057um. Therefore, LSTM-NN is selected in this article.

Figure 13: Comparison of modeling error of different neural networks. (a) random NURBS trajectory. (b) modeling error.

4.5. Suppression of hysteresis

When training the LSTM-NN model, the rate dependence of hysteresis is considered, and velocity is regarded as one of the training input features. Therefore, after the reference trajectory is modified by LSTM-NN, the delay between the desired trajectory and the actual trajectory is suppressed. The hysteresis curves before and after modification are shown in Fig. 14 to illustrate this result.

Figure 14: Comparison of hysteresis curves before and after modification.

4.6. Tracking control performance

4.6.1. Comparison of different control frameworks

Based on the discussion using Fig. 11, the trajectory tracking control performance of four control frameworks are compared: PID, PI inverse model plus proportional integral sliding mode control (PI-PISMC) [31], control based on LSTM-NN inverse model for open-loop PEA (Open-LSTM) [15] and LSTM-NN trajectory modification for PEA’s closed-loop system (Closed-LSTM). The Open-LSTM uses LSTM-NN to establish the inverse model of the open-loop PEA as the feedforward
compensator, which is combined with the PID feedback controller. The PI-PISMC uses the inverse model of hysteresis characteristics as the feedforward compensation module, and uses proportional integral sliding mode control as feedback controller. The comparison experiments are carried out using different sinusoidal trajectories with frequencies of 5 rad/s, 15 rad/s, 30 rad/s and 50 rad/s. The tracking errors between the actual trajectory and the desired trajectory under the three control frameworks are shown in Fig. 15 and Table 3.

Taking the sinusoidal trajectory of 15 rad/s as an example, after adding the LSTM-NN module to the closed loop system, the maximum tracking error is reduced from 1.5860 μm to 0.1518 μm, which is a reduction of 90.4%. The root mean square error is reduced from 0.2500 μm to 0.0547 μm, a reduction of 95.1%. It can also be seen from Table 3 that the tracking performance of Closed-LSTM has a lower tracking error than Open-LSTM and PI-PISMC.

In addition, since Closed-LSTM modifying the reference trajectory is to perform tracking error compensation before the error occurs, the error can be eliminated in advance. Even if the velocity of the desired trajectory increases, the tracking error does not increase significantly. From Table 3 and Fig. 15, when the frequency of the sinusoidal trajectory is increased by 6 times from 5 rad/s to 30 rad/s, the root mean square errors of PID and Open-LSTM increase by almost 6 times, while that of Closed-LSTM increases by less than 2 times. This is because LSTM-NN has long-term and short-term memory and velocity information is selected as its input feature, and it is thus not sensitive to the frequency change of desired trajectory.

4.6.2. Comparison with ILC

Because iterative learning control (ILC) is proved to be very effective in trajectory tracking control by repeatedly performing the same task, it is used to further compare with the proposed Closed-LSTM
tracking control method. In particular, the online ILC algorithm based on radius basis function neural networks (RBF-NN) in [32] is considered, and again a random NURBS trajectory is used as the desired trajectory. ILC compensates for the tracking error in each iteration by generating a new reference trajectory and update the weight parameters $W$ of RBF-NN, i.e.

$$W = W - \alpha \varepsilon^T S(Z)$$  \hspace{0.5cm} (21)

where $S$ and $Z$ are the basis function and the input of RBF-NN, respectively, and $\alpha = 1.1 \times e^{-4}$. The comparison results are shown in Fig. 17 and Table 4 where five iterations are carried out for ILC and PID is used as the benchmark method.

Table 4: Comparison of tracking error under ILC and Closed-LSTM

<table>
<thead>
<tr>
<th>Method</th>
<th>$\varepsilon_{\text{max}}$ (um)</th>
<th>$\varepsilon_{\text{rms}}$ (um)</th>
<th>$\varepsilon_{\text{aver}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed-LSTM</td>
<td>0.1582</td>
<td>0.0539</td>
<td>0.0220</td>
</tr>
<tr>
<td>ILC</td>
<td>0.0947</td>
<td>0.0303</td>
<td>0.0110</td>
</tr>
<tr>
<td>PID</td>
<td>2.8396</td>
<td>0.8045</td>
<td>2.7900</td>
</tr>
</tbody>
</table>

Experimental results show that the performance of the ILC is slightly better than Closed-LSTM, but the ILC requires multiple iterations. Furthermore, when the desired trajectory changes, the learning process needs to be repeated for the ILC. In contrast, the proposed Closed-LSTM can be used without further training for new trajectories.

5. Conclusion

This article proposes a trajectory tracking control method of closed-loop PEA based on LSTM-NN (Closed-LSTM), which can improve the trajectory tracking performance while preserving the anti-inference capability and repeatability provided by the baseline feedback controller. By establishing a mapping between the actual trajectory and the reference trajectory, the reference trajectory can be modified to compensate for the tracking error in advance. On the one hand, the linear and nonlinear characteristics of the closed-loop PEA are modeled and analyzed to select the input features for LSTM-NN. On the other hand, this method adds a module to modify the reference trajectory on the basis of the closed-loop system without modifying the baseline controller. Compared to the neural network control based on the inverse model of the open-loop plant, this method not only achieves better tracking performance, but also maintains this performance despite the increase of velocity, thus addressing the trade-off between high speed and high accuracy that the current motion control systems generally face. Our future works will focus on the integration of the proposed method with online learning and extension to other motion control systems.

References


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posite proportional-integral sliding mode control with feedforward control for cell puncture mechanism with piezoelectric actuation, ISA transactions (2020).