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ANALYSIS OF NONLINEAR MODAL DAMPING DUE TO FRICTION AT BLADE ROOTS IN MISTUNED BLADED DISCS

Junjie Chen^a, Chaoping Zang^a, Biao Zhou^a, E.P. Petrov^b

^a Nanjing University of Aeronautics and Astronautics , Jiangsu Province Key Laboratory of Aerospace
Power System, Nanjing 210016, China

e-mail: chen_junjie@nuaa.edu.cn; c.zang@nuaa.edu.cn; biao.zhou@nuaa.edu.cn;

^b University of Sussex, Brighton BN1 9QT, United Kingdom, e-mail: y.petrov@sussex.ac.uk

ABSTRACT

A method is proposed to analyse the modal damping in mistuned bladed-disc with root joints using large finite element models and the detailed description of frictional interactions at contact interfaces. The influence of mistuning on the dissipated energy for different blades on a bladed-disc and the modal damping factors for different vibration levels for any family of modes can be investigated.

The dissipated energy and damping factors due to micro-slip are simulated by multitude of surface-to-surface elements modelling the friction contact interactions at root joints. The analysis is performed in the time domain and an original reduction method is developed to obtain the results with acceptable computational times. The model reduction method allows the calculation of the modal damping of the mistuned assembly by evaluation of the energy dissipated at root joint of each individual blade using small parts of bladed disc sectors.

The dependency of modal damping factor on blade mode shapes, engine-order excitation numbers, nodal diameter numbers and vibration amplitudes are studied and the distributions of amplitude and dissipated energy on the mistuned bladed-disc are investigated using a realistic blade disc model.

INTRODUCTION

The bladed discs of gas-turbine engines operate under high temperatures and high aerodynamic loading in the field of centrifugal forces. The spectrum of the dynamic excitation loads is dense and the spectrum of natural frequencies of the bladed discs is also very dense. The use of friction devices such as under-platform damper (see Refs.[1] and [2]) and interlock shrouds is one of most common measures to suppress, limit dangerous resonance vibrations or shift the resonance frequencies away from the operating rotation speed range. The root damping is always present in bladed disc assemblies and the micro-slip motion at the root joints transforms the vibration

energy into heat and the dissipation energy produces additional damping and helps to control the vibration level. The root damping is impossible to ignore in the analysis of assembled bladed disks and its modelling and optimization represent current significant interest for the industry.

There are so many studies considering the blade root damping modelling. An assessment of the friction modelling in different friction devices is made in Ref.[3]. With a comparison between some experimental results and simulations, it is concluded that the macro-slip model is more suitable for under-platform damper while it is not appropriate to be used in blade root joints simulation. Studies aimed at the development of methods for numerical analysis of the interaction at the contact interfaces and the influence of root damping on vibration reduction of the nonlinear forced response amplitudes are reported in Refs.[4]-[9].

A method allowing using available finite element codes and surface-to-surface friction contact elements to simulate the micro-slip occurring at the contact surface of blade root joints is proposed in Ref.[10], where the effects of different model parameters on micro-slip friction damping and hysteresis loops are explored. A special model reduction technique has been developed in Ref.[11], where the root damping is investigated for a single blade and for cyclically symmetric bladed disks.

The blades of real bladed disks have inevitably small scatter which although being small and within acceptable manufacture tolerances can cause significant influence on mode shapes and forced response levels (see Refs[12] - [14]). Blade mistuning destroys the cyclic symmetry of the bladed disc. Therefore, the sector model which is commonly used for analysis of tuned, cannot be used for the analysis of mistuned bladed discs. For a mistuned bladed disk calculations, finite element models of a whole bladed disc have to be used. The size of a whole bladed disk model is large for finite element models used by the industry and the direct use of such model is not practical and usually one or another reduction technique is applied in the mistuned bladed disk analysis.

Most studies of friction in turbine blades used cyclic symmetry assumption and one sector, although there are recent attempts to analyse nonlinear forced response in the mistuned bladed disks (see Ref.[15]). Many researchers have begun to consider the performance of damping devices on whole bladed discs using rotating test rigs (Refs [16] and [17]).

This paper studies the influence of mistuning on the dissipated energy and modal damping produced by micro-slip at blade root joints. A method for analysis of modal damping produced by micro-slip friction at contact surfaces in bladed disc vibration has been developed to investigate the influence of mistuning on dissipated energy and damping factors. The method uses high-fidelity models of bladed disc with nonlinear contact surfaces

The paper starts from the method developed for the analysis of modal damping in mistuned bladed disc. Then, the methods is applied for thorough analysis of dissipated energy and modal damping factors and effects of vibration amplitudes, contact interfaces parameters, the rotation speed and different mistuning factors are explored. The influence of mistuning on modal damping of different families of modes is also analysed.

METHOD FOR THE NONLINEAR MODAL DAMPING ANALYSIS

The method developed performs the analysis of modal damping in time domain by the integrating the equation of motion for a structure with contact interfaces. The use of available commercial and free-domain finite element (FE) software codes is intended. In order to reduce the computational expense, an approach is suggested which allows performing the time domain calculations only for a part of the structure. The reduction is based on the assumption that the effect of the root damping on bladed disc mode shapes is small, especially for that amplitude levels where the microslip friction is prevalent, and can be neglected for the modal damping calculation. This assumption is expected to be satisfied in many realistic operating conditions where there is no significant contact-separation transitions at root contact surfaces of bladed disks. The validity of this assumption is supported by the numerical studies performed by multiharmonic balance method in Ref.[18], which provided very close results with one blade mode included in the model and with 128 blade modes for a realistic bladed disk model. The part of the whole structure, selected for the reduced model, includes the nonlinear contact interfaces. For any mode shape analysed, the interaction of the selected part with the rest of the considered structure is accounted for by the application of the displacements corresponding to the analysed mode shape at all boundary nodes. The energy dissipated at blade-root joints is calculated by considering the work over the vibration period of the friction stresses under the microslip relative motion at contact interfaces. This reduction approach has been proposed and implemented in Ref.[11] for tuned bladed discs and here we developed it further for the case of mistuned bladed discs.

Two models (shown in Fig 1) are used for the analysis of realistic bladed discs. The mistuned bladed disc model (see Fig 1a) is used for the calculation of the mode shapes of a mistuned bladed disc. The model of a blade root joint with adjacent parts of the blade and disc are used to calculate the dissipated energy at root joints, which is done individually for each blade of the mistuned bladed disc (see Fig.1b). The plane marked in Fig.1b by letter 'B' separates the blade root joint model from blade and planes 'D1', 'D2' and 'D3' separate it from the disc.

Fig 1 Bladed disc models: a) a whole bladed disc; b) a sector model and selected root joint part

Mistuned Bladed Discs: Modelling and Modal Analysis

The detailed finite element models are used for the bladed disc modelling. The mistuning is introduced by specifying different material density for each blade. Random mistuning patterns are generated. The modal analysis is carried out for the mistuned bladed disc to obtain the natural frequencies and the corresponding mode shapes. The mistuning destroys the cyclic symmetry and the mode shapes become asymmetric, which requires the use of the finite element model for a whole bladed disc. The mode shapes are calculated for the rotating bladed discs and the static analysis is performed for the mistuned bladed disc under action of centrifugal forces. The stress field obtained from the static problem solution is taken into account in the modal analysis problem and the coupled equations for the static and modal problems for the mistuned bladed disc can be written in the following form:

$$\mathbf{K}\mathbf{X}_0 = \Omega^2 \mathbf{F}_{cf} \quad (1)$$

$$[\mathbf{K} + \mathbf{K}_0(\mathbf{X}_0) - \mathbf{M}_0(\Omega) - \omega_j^2 \mathbf{M}] \boldsymbol{\phi} = 0 \quad (2)$$

where $\mathbf{K}$ and $\mathbf{M}$ are stiffness and mass matrices of the whole mistuned bladed disc; Ω is the rotation speed and $\mathbf{F}_{cf}$ is the centrifugal force vector calculated at unit value of the rotation speed; $\mathbf{X}_0$ is the nodal displacements obtained from solution of the static; $\mathbf{K}_0(\mathbf{X}_0)$ is the geometric stiffening matrix

depending on the static deformation and $M_0(\Omega)$ is mass-softening term describing the dynamic forces occurring due to the variation of the centrifugal force directions during vibrations.

Blade Root Joints: Reduced Model And Nonlinear Forced Response Analysis

Based on the assumption that mode shape is not affected significantly, the nonlinear transient dynamic analysis is performed for blade root joint models for any mode of interested calculated for a mistuned bladed disc.

The transient analysis is chosen since it is available for analysis of strongly nonlinear vibrations performed in most FE codes. The developed reduction models allow the use of the time-domain nonlinear solvers for the analysis of the steady-state vibrations, which is necessary to calculate the modal damping in the structure. The numerical experiments (see Refs.[10] and [11]) show that the vibration process convergence to steady-state vibrations after relatively small numbers of loading cycles.

For modelling the friction contacts at root joints, the surface-to-surface 4 node contact elements from ANSYS are applied between two contact surfaces to model the interaction of the blade and disc. This type of friction elements available in ANSYS uses Coulomb friction model. The friction coefficient describes the relationship between friction force and normal load in the slipping state; the normal contact stiffness, k_n , establishes the relationship between normal pressure and relative displacement along the direction normal to the contact surface and the tangential contact stiffness, k_t , provides the relationship between tangential stress and relative displacement in the plane of contact when the contact surfaces are stuck.

The time integration of the vibrations is performed separately for each of the blade root joint models, i.e. for each of the blades included in the mistuned bladed disc. The equation of motion with the prescribed motion at the boundaries of each root joint is integrated:

$$M_i \ddot{x}_i(t) + C_i \dot{x}_i(t) + K_i x_i(t) + f(x_i(t)) = 0; \quad (i = 1..N) \quad (3)$$

where i is the blade number; N is the total number of blades; M_i , C_i and K_i are mass, viscous damping and stiffness matrices of the blade root joint and $f(x_i(t))$ is the nonlinear contact forces. For each of the blade root joint the displacement controlled boundary conditions are applied at all planes where the joint interact with the disc and the considered blade. Displacement controlled excitation of the joint model is provided for all nodes on boundaries of the considered root joints. These displacements are selected from solutions of the static problem and modal problem:

$$x_i^\gamma = x_{0i}^\gamma + a u_i^\gamma \cos \omega_j t; \quad i = 1..N \quad (4)$$

where $\gamma = B, D_1, D_2, D_3$ indicate the boundary plane, x_i^γ is the displacement applied at the chosen boundary of the blade

root joint; x_{0i}^γ is the vector of displacements at the boundary obtained from the static problem; u_i^γ is the mode shape displacements; ω_j is the j -th frequency of mistuned bladed disc and a is the multiplier used to determine the level of dynamic excitation. For the analysis of mistuned bladed discs, the level of vibrations is determined in the way to provide the desired displacement value for a mean value of blade tip amplitudes calculated for all blades:

$$a^{mean} = \frac{1}{N} \sum_{i=1}^N a_i \quad (5)$$

where a_i is the tip amplitude for i -th sector.

It should be noted that the results of static and modal problems for mistuned bladed disc are obtained in the global Cartesian system of coordinates and the used blade joint model is the same for all blades in the bladed disc. Because of this, the modal displacement vectors x_i^γ used in Eq.(5) are not only extracted from mode shape ϕ_j but also is transformed in the system of coordinate of first blade in the mistuned bladed disc, i.e.:

$$u_i^\gamma = \text{diag}\{r(\alpha(i-1)), \dots, r(\alpha(i-1))\} \phi_j^\gamma; \quad i = 1..N \quad (6)$$

$$x_{0i}^\gamma = \text{diag}\{r(\alpha(i-1)), \dots, r(\alpha(i-1))\} x_0^\gamma; \quad i = 1..N \quad (7)$$

where $\alpha = 2\pi/N$ is the bladed disc sector angle and $r(\alpha(i-1))(3 \times 3)$ is the rotation matrix describing the rotation of the coordinate system to i -th blade. A special APDL code has been created to extract the nodal displacements on four boundaries in ANSYS static and modal analysis.

The important feature of the method is its natural parallelization capability: the transient analysis can be performed for each of the blade root joints of a mistuned assembly independently: we can perform parallel computations on N computers for N blades.

The analysis of the vibration allows the calculation of distributions of contact stresses: frictional and normal at any time instant. The friction stress distribution at each contact surface varies over time. An example of the calculated variation of the friction stresses at contact interface over half a loading cycle is shown in Fig 2 for one root joint of the mistuned bladed disc. The time instants corresponding to the shown 4 friction stress plots are distributed uniformly over the half of the vibration period and the intensity of blue colour indicates the friction stress level.

Fig 2 Friction stress distribution over the loading process

Calculation Of The Dissipated Energy

For the calculation of the energy dissipated at blade roots by friction under a prescribed blade tip vibration amplitude level, the friction stresses and nodal displacements at contact nodes are extracted for all contact surfaces from the results of time integrations using APDL code. In addition the areas represented by friction contact elements are also extracted. The dissipated energy is calculated using a specially created MATLAB code.

The values of friction stresses and the relative displacements determined as a result of integration of Eq.(3) are transformed to the local coordinate system of the contact patch. The traction stresses due to friction are expressed by a two-dimensional vector $\tau = \{\tau_1, \tau_2\}^T$. The relative displacements between a pairing contact surface are also determined in the local coordinated system. The relative displacement vector is obtained in the form:

$$u^{rel} = \{u_1^{contact} - u_1^{target}, u_2^{contact} - u_2^{target}\}^T \quad (8)$$

where, $u_1^{contact}$, u_1^{target} and $u_2^{contact}$, u_2^{target} are nodal displacements of the contact pair in two directions of the local coordinate system.

The dissipated energy ΔW_i^S is calculated for each i -th sector of a bladed disc by integration of the work of the friction stresses over all contact interfaces and over the period of the excitation. The spatial integration is performed using Gauss quadrature formulas with N_G points (see Ref.[19]) and for the time integration the rectangular integration scheme is applied:

$$\begin{aligned} \Delta W_i^S &= \sum_{\gamma=B_i D_i^1 D_i^2 D_i^3} \int_0^{2\pi/\omega_j} \int_{A_e} \tau^T(t) \dot{u}^{rel}(t) dA dt \\ &= \sum_{\gamma=B_i D_i^1 D_i^2 D_i^3} \sum_{k=1}^{N_{step}} \sum_{e=1}^{N_{elem}} \iint_{A_e} \tau_{ie}^k(x, y) u_{ie}^{k,rel}(x, y) dx dy \quad (9) \\ &\approx \sum_{\gamma=B_i D_i^1 D_i^2 D_i^3} \sum_{k=1}^{N_{step}} \sum_{e=1}^{N_{elem}} \sum_{g=1}^{N_g} w_g \tau_{ie}^k(P_g) u_{ie}^{k,rel}(P_g) \end{aligned}$$

where τ_{il}^k and $u_{il}^{k,rel}$ are friction stress and relative displacement at e -th element and k -th time step, A_e is the area of e -th element, N_{step} and N_{elem} are the number of time steps and the number of elements, P_g and w_g are the Gaussian quadrature points and weights with g - the number of the Gaussian point.

The energy dissipated in all root joints of the mistuned assembly, ΔW is the sum of energy dissipated in all sectors, and it is calculated in form:

$$\Delta W = \sum_{i=1}^N \Delta W_i^S \quad (10)$$

Calculation Of Damping Factors

The damping factor is the ratio of the dissipated energy and the maximum strain energy. For a considered mode shape, the damping factor can be calculated in form:

$$\eta_j(a_j^{mean}) = \Delta W_j(a_j^{mean}) / 2\pi \left(\frac{a_j^{mean}}{a_j^0} \right)^2 W_j \quad (11)$$

where ΔW_j is the dissipated energy calculated for all root contact interfaces of a mistuned bladed disk for the j -th mode shape, $W_j = W(\phi_j)$ is the strain energy calculated for a mistuned bladed disc for the considered mode shape – this is done by using ANSYS APDL commands, a_j^0 is the mean amplitude of the tip node for which the strain energy W_j is calculated for the j -th mode shape; a_j^{mean} is the level of mean bladed disk amplitude for which the modal damping generated by the micro-slip friction is calculated through forced response analysis.

NUMERICAL STUDIES OF THE MODAL DAMPING FOR MISTUNED BLADED DISCS

The Analyzed Bladed Disc Model

A model of a bladed disc with 24 blades is considered. The three-dimensional finite element model used in analysis for mistuned case is shown in Fig 3a. It contains 10.6 millions DOFs and is used to calculate the mistuned mode shapes. The blade joint model is shown in Fig 3b. This model contains only about 43.5 thousands DOFs. It is used in the transient analysis for the calculation of the dissipated energy, and the model reduction saves the computational efforts drastically. The periodic vibration regimes were achieved after 20 cycles of the excitation load variation. The computation time for one blade-disk reduced model was about 1 hour even though different considered parameters can change the computation time a little. The total simulation performed for all 24 blades of the mistuned bladed disk was performed for this bladed disk during can still take a 1 hour if 24 CPUs or PCs are used for the calculation concurrently using the natural parallelization of the method. In our case we used 4 PCs and the total computation time was about 6 hours.

Fig 3 Finite element models: a) a bladed disc b) a root joint

The natural frequencies and mode shapes were also calculated for a tuned bladed disc for all possible nodal diameters (ND) from 0 to 12. The relationship between natural frequencies and NDs are plotted in Fig 4 for first 5 families of natural frequencies.

In order to facilitate the determination of steady-state vibration regimes, two types of the damping are applied: (i) structural, physical damping and (ii) numerical damping. The influence of beta damping and numerical damping on calculation has been assessed in Ref.[11]. It has been found that the values for beta and numerical damping 0.01 can be a good choice for predictive analysis of the modal damping.

Fig 4 Natural frequencies of the tuned bladed disc

In all calculations, the friction coefficient is 0.4, and the normal and tangential contact stiffness coefficients are: $k_n = k_t = 10^5 \text{ N/mm}^3$, if they are not provided in the text specially. The rotation speed is assumed in most calculations 1000rad/s, and the mean amplitude value of the bladed disc at blade tips is 0.5mm – if other values are not specified. The mean value of the maximum magnitude of displacement vector at a blade tip is used for characterization of the vibration level (see Eq.(5)).

For some cases the modal damping of the mistuned bladed disc is compared with the damping in its tuned counterpart. For mode shapes where the disc is dominant the correspondence between mode shapes of tuned and mistuned bladed discs is easy to establish. For modes where blades are dominant (e.g. modes 6ND-1st to 11ND-1st considered below in our examples) the mode shapes of a mistuned bladed disc become so distorted that such correspondence is very difficult to find and a special procedure based on sorting natural frequencies and harmonic analysis of mode shapes is applied.

The mistuning patterns are generated randomly within mistuning range $\pm 5\%$ by specifying different material density for each blade. It should be noted that the method proposed does not impose any restriction on the way how the mistuning is introduced, including the mistuning by blade geometry scatter.

Verification Of The Method

The new method developed for the analysis of mistuned bladed discs was verified by the comparison with the results obtained using a sector model using the method and code developed in Ref.[11]. To be able to perform such comparison a tuned bladed disc is considered. The mode shapes considered here are 1st family modes of 0 and 6 NDs. The dependency of the modal damping factor on the blade tip amplitude is shown in Figs. 5 and 6.

Fig 5 Modal damping factor for mode: 0ND, 1st family

Fig 6 Modal damping factor for mode: 6ND, 1st family

We can see that the obtained results are very close. For 0ND mode the damping factor increases with amplitude growth for low amplitude levels, reaches the critical level and for higher amplitude it starts to decrease. Such behaviour is observed for this mode shape because for higher amplitude, the dissipation energy increases with the vibration amplitude but it increases slower than the strain energy of the bladed disc.

Dissipation Energy In Blade Roots Of Mistuned Bladed Discs

The mistuning distorts the mode shapes of a bladed disc and, in contrast to a tuned assembly, the dissipation of energy at blade root becomes very different for different blades of the bladed disc. An example of the analysed mode shapes for two different mistuning patterns of a mistuned bladed disc is shown in Fig 7, where we can see strong localization of the amplitudes.

Fig 7 Mode shapes of 6ND 1st family for: a) mistuned pattern 1
b) mistuned pattern 2

The choice of the number of cycles needed for achieving the steady-state response in the transient analysis and for obtaining the convergent values of energy dissipated for a vibration cycle have been thoroughly analysed. An example illustrating this convergence is shown in Fig 8, where the dissipated energy is shown for all 24 blade root joints as a function of the vibration cycle number for the case when the mean amplitude level is 0.5mm. It is obvious that the energy dissipation has reached convergence after 6 cycles in this calculation.

Fig 8 Energy dissipation changes with vibration cycle number

The dissipated energy distributions over blades of a mistuned bladed disc for different vibration levels are plotted in Fig 9. Here the mode shapes shown in Fig.7 is considered and the tuned bladed disc dissipation energies are shown also. We can see that the energy dissipated at different blades can differ by several orders of magnitudes and that the distributions of the dissipation energy is dependent on the amplitudes level, although general trend is preserved for all considered vibration levels.

Fig 9 Energy dissipated at root joints for different levels of vibration

The rotation speed variation produces different static normal pressure, therefore the micro-slip friction dissipation. The distributions of dissipated energy over blade joints was calculated for static loading by centrifugal forces at different rotation speeds. Examples of such distributions are shown in Fig 10. One can see that with the increase of rotation speed (and therefore the static normal pressures at contact interfaces) the energy dissipated at blades with low amplitudes and dissipation levels decreases further. This results from the fact that when the tip amplitude is small, the increase of the rotation speed suppresses slip at the contact interface and the dissipated energy decreases. For blades experienced large amplitudes, the increase of the normal pressure is not sufficient to reduce the micro-slip areas sufficiently in order to reduce the dissipation energy.

Fig 10 Energy dissipated at root joints for different rotation speeds

The most commonly considered contact parameters in friction problems are friction coefficient and tangential and normal contact stiffness, and their influence on the dissipated energy distribution was explored here.

In Fig 11, the distribution of energy dissipated in one vibration period for friction coefficients from 0.3 to 0.5 is displayed. We can see that for the blades with smaller tip amplitudes the increase of friction coefficient decreases the dissipation amplitudes, and for the larger blade amplitudes the dissipation energy stays almost constant. The causes of this behaviour are similar to what we observe for rotation speed influence.

Fig 11 Energy dissipated at root joints for different friction coefficients

The distribution of dissipated energy for different normal and tangential contact stiffness values are shown in Fig 12 and Fig 13. We can see that when the magnitudes of contact stiffness coefficients are large enough, the dissipated energy changes very little with the variation: for both types of the contact stiffness.

Fig 12 Energy dissipated at root joints for different normal contact stiffness values

The increase of normal contact stiffness makes the dissipated energy smaller for blades with small amplitude. This is due to the increase of the normal pressure defined from the relative motion at contact patches. When the tangential contact stiffness is too small, a quite large nodal relative displacement is needed to ensure a slip, so dissipated energy is small in Fig.13.

Fig 13 Energy dissipated at root joints for different tangential contact stiffness values

Modal Damping Analysis for Blade-Dominated Modes

One of the randomly generated mistuning patterns was chosen to investigate the difference between damping factors in tuned and mistuned cases. The mistuning splits the double frequencies of a tuned assembly and the mode shape of the lower frequency is chosen from these two frequencies to compare with the tuned bladed disc mode. The mode shapes considered here are blade dominated modes of 1st family with nodal diameters varying from 6ND to 11ND, all these modes have very close natural frequencies.

The effect of vibration amplitudes on modal damping of tuned and mistuned bladed discs has been compared. The dependency of the damping factor with the amplitude variation is shown in Fig 14. The amplitude levels are varied within the range from 0.01mm to 1mm. One can see that for the case of tuned bladed disc the modal damping increases monotonically for all considered mode shapes. For a mistuned bladed disc, they grow till certain amplitude level and then they start slow decreasing. The reason for such behaviour is the large scatter in amplitudes and dissipation energies over blades of a mistuned bladed disc. For example, for the considered here mode shapes, when the mean amplitude of mistuned bladed disc is 0.5mm, the most of the blades have amplitudes well below 0.2mm and they produce much less dissipated energy than several blades with high amplitudes. After some vibration level is achieved the blades with high amplitudes does not produce the dissipation energy increase with the same rate as the strain energy. So the damping factor have the trend to decrease after certain level. For the considered modes in the tuned bladed disc, vibration with this range of amplitude has not reached the critical level, so the damping factor continues increasing in the considered amplitude range.

Fig 14 Dependency of the damping factor on the amplitude

Fig 16 Dependency of the damping factor on the friction coefficient

Fig 15 Dependency of the damping factor on the rotation speed

The damping factors variation with the rotation speed in the range from 500 to 1250rad/s for tuned and mistuned bladed discs is shown in Fig 15. We can see small dependency of the damping factor on the rotation speed for the mistuned bladed disc and significant dependency for its tuned counterpart.

Fig 16 shows how the damping factor changes with friction coefficient variation from 0.3 to 0.5. Similar to the influence of rotation speed, the modal damping factor variation is rather small for the mistuned bladed disc. When amplitude of tuned bladed disc is small enough, most of the contact surface is stuck, and the increase of friction coefficient contribute to the damping factor reduction due to decrease of slipping areas. While for mistuned case, some sectors may have macro-slip and others have micro-slip or are stuck, so that the damping energy produced at different blades can decrease and increase, and the damping factor of the mistuning bladed disc changes little with the variation of friction coefficient for the considered case. When the friction coefficient is large, the damping factor for a tuned disc is small due to large stuck areas, while some blades in the mistuned disc still experience slipping, so the damping factor become larger than for the tuned disc.

Fig 17 Dependency of the damping factor on the normal contact stiffness

Fig 18 Dependency of the damping factor on the tangential contact stiffness

The variations of modal damping factors changes with normal and tangential contact stiffness variation from 10^4 to 10^6 N/mm^3 are shown in Fig 17 and Fig 18. The dependencies

for all considered modes are similar for tuned and mistuned bladed discs.

When normal and tangential contact stiffness is large enough, the change of their value has little influence on the modal damping factor. The amplitudes of some sectors in mistuned bladed disc are large compared with tuned one, so the dissipated energy generated makes the damping factor to be larger.

Effect Of Mistuning On Disc-Dominated Modes

The disc dominated vibrations differ significantly from the blade dominated vibration modes since the disc experience large vibration levels and the deformation modes of blade root joints differ. The classification of such modes is easier for mistuned bladed discs since it is quite easy to find a mode shape in the frequency-NDs diagram of the corresponding tuned bladed disc. For the analysis of modal damping the following disc-dominated modes are chose: 2ND 1st, 6ND 2nd and 8ND 2nd (see Fig 4). The mode shapes of these mods for a mistuned bladed disc are shown in Fig 19. We can see that 2ND 1st is a mode shape where the disc vibrations are evident, mode shapes 6ND 2nd and 8ND 2nd are still regular although the disc contribution is much smaller. The influence of mistuning on all mode shapes is small and visually difficult to distinguish these mode shapes from mode shapes of a tuned structure.

Fig 19 Mode shapes for disc dominated vibration: a) 2ND 1st b) 6ND 2nd C) 8ND 2nd

The effect of vibration amplitudes on modal damping factors for the mistuned bladed disc for these modes shapes is shown in Fig 20. The mean amplitude is the mean value of tip amplitude of all sectors, which varies from 0.05 to 1mm. Here the modal damping factors of a tuned bladed disc are also plotted. Mistuning splits the double frequencies existing in tuned bladed discs and for the mistuned bladed disc the modal damping factors are calculated for both split modes. These two split mode shapes are marked by words 'lower' and 'higher' – accordingly to the natural frequency value in the split pair. We can see that the modal damping factors calculated for split modes shapes are very close. The difference between modal factors of tuned and mistuned is small for 2ND 1st mode shape – the mode shape where the vibrations are fully defined by the disc. The values of the modal damping factors are smaller than for the other two considered modes by factor of 6 in some cases. The modal factors calculated for the mistuned bladed disc for modes 6ND 2nd and 8ND differ significantly from the values obtained for these modes for the tuned bladed disc. This

is due to the fact that in the mistuned bladed disc under traveling wave excitation all blades have identical vibration amplitudes and dissipation energy, but in the mistuned bladed disc the mean amplitude (see Eq.(5)) is used to characterize the bladed disc vibration level. Therefore, large number of blades in the mistuned bladed disc have higher amplitudes than the mean bladed disc amplitude and further the increase of amplitude of vibration does not increase the dissipation energy significantly in these blades. The decrease of the modal factors with increase of the amplitude (as we already observed in Fig.14) is achieved at lower levels of amplitudes in mistuned than in the tuned bladed disc.

Fig 20 Dependency of the damping factor on the amplitude

The dependency of damping factor in the mistuned bladed disc dominated vibration on rotation speed is shown in Fig 21. The rotation speed varies from 500 to 1250rad/s. We can see that the modal damping factors decreases with rotation speed increase for modes 6ND 2nd and 8ND 2nd, while for the mistuned bladed disc they increase. The modal damping factor for mode 2ND 1st for the tuned bladed disc increases at the beginning and then starts decreasing; for the mistuned disc this modal factor grows and reaches the saturation at the end of the considered rotation frequency range.

Fig 21 Dependency of the damping factor on the rotation speed

The dependency of modal damping factors on contact parameters such as friction coefficient, normal and tangential

contact stiffness coefficient are plotted in Figs.22-24. The damping factor for mistuned disc dominated vibration is smaller than that in tuned one. The explanation of this effect is similar to the discussion provided for the rotation speed effects.

Fig 22 Dependency of the damping factor on the friction coefficient

Fig 23 Dependency of the damping factor on the normal contact stiffness

Fig 24 Dependency of the damping factor on the tangential contact stiffness

It should be noted that for all considered here, the mode

shapes for disc dominated vibration which corresponding to the split frequencies do not differ significantly and the modal damping factors are nearly the same for 'lower' and 'higher' frequencies.

Effect Of Mistuning Patterns On Modal Damping

The effects of mistuning patterns on the modal damping is performed by generation four different mistuning patterns and performing the analysis of modal damping factor. 6ND 1st mode is chosen for the analysis.

The dependency of the modal damping factor on the mean amplitude for four mistuning patterns and the tuned bladed disc is plotted in Fig 25. It should be noted that although the distributions of mistuning factors over sectors for four mistuning patterns differ significantly, but the dependencies of the modal factor on the vibration amplitude are close to each other.

Fig 25 Dependency of the damping factor on the amplitude

Fig 26 Dependency of the damping factor on the rotation speed

The mean amplitude used to evaluate the influence of mistuning on damping factor is 0.5mm and the rotation speed range is from 500 to 1250 rad/s. The dependency of the modal damping on the rotation speed is shown in Fig.26. One can see that the mistuning makes the dependency of the modal damping on the rotation speed for mistuned bladed disc different from

that obtained for the tuned bladed disc. The curves corresponding to four patterns are changed a little with the rotation speed variation.

The friction coefficient considered in the analysis of its effects varies from 0.3 to 0.5. The results obtained for 4 mistuning patterns are shown in Fig 27. The modal damping factors for mistuned cases change within relatively small range.

Fig 27 Dependency of the damping factor on the friction coefficient

The effect of different mistuning patterns on modal damping factor can be seen in Fig 28 and Fig 29 for normal and tangential contact stiffness separately. We can see that the stiffness coefficient affects the modal damping factors little if the contact stiffness value is chosen higher than 5×10^5 .

Fig 28 Dependency of the damping factor on the normal contact stiffness

Fig 29 Dependency of the damping factor on the tangential contact stiffness

CONCLUSIONS

A method for analysis of modal damping produced by micro-slip friction at contact surfaces in bladed disc vibration has been developed to investigate the influence of mistuning on dissipated energy and damping factors. The method uses high-fidelity models of bladed disc with nonlinear contact surfaces.

An effective model reduction technique is proposed to enable the simulation of vibration of large scaled mistuned bladed disc with a small part containing the friction joints in transient calculation. The interaction at the contact surface of the selected part can accurately imitate the actions of corresponding sector in the whole disc. The reduction technique is based on repeating calculation with the part model, and the assumption that the influence of friction interactions at contact surfaces of blade root on mode shapes at the boundaries of the part is not dramatic. The dissipated energy of the whole bladed disc with mistuning factors is calculated by computing the sum of that of all sectors.

The method can be used for calculation of modal damping for any mistuning factors and any nodal diameter and mode shape numbers of the whole bladed disc.

Comprehensive numerical studies of the modal damping factors and dissipated energy distribution affected by mistuning have been performed on large-scale three-dimensional finite element model of a bladed disc with root joints.

The influence of mistuning on dissipated energy and modal damping factors on the blade and disc dominated vibrations are studied. The effects of the rotation speed value, vibration amplitude level, contact interfaces parameters, and mistuning patterns on the modal damping factors are investigated. The comparison between modal damping in tuned and mistuned bladed discs is performed.

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