Mean-maximum drawdown optimization of buy-and-hold portfolios using a multi-objective evolutionary algorithm


This version is available from Sussex Research Online: http://sro.sussex.ac.uk/id/eprint/100714/

This document is made available in accordance with publisher policies and may differ from the published version or from the version of record. If you wish to cite this item you are advised to consult the publisher’s version. Please see the URL above for details on accessing the published version.

Copyright and reuse:
Sussex Research Online is a digital repository of the research output of the University.

Copyright and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable, the material made available in SRO has been checked for eligibility before being made available.

Copies of full text items generally can be reproduced, displayed or performed and given to third parties in any format or medium for personal research or study, educational, or not-for-profit purposes without prior permission or charge, provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.
Mean-Maximum Drawdown Optimization of Buy-and-Hold Portfolios Using a Multi-objective Evolutionary Algorithm

Mikica Drenovak\textsuperscript{a}, Vladimir Ranković\textsuperscript{b}, Branko Urošević\textsuperscript{c}, and Ranko Jelic\textsuperscript{d}

\textsuperscript{a} Faculty of Economics, University of Kragujevac, Licej Kneževine Srbije 3, 34000 Kragujevac, Serbia; +381638186060; mikicadrenovak@uni.kg.ac.rs.
\textsuperscript{b} Faculty of Economics, University of Kragujevac, Licej Kneževine Srbije 3, 34000 Kragujevac, Serbia; +381641336396; vladimir.rankovic@uni.kg.ac.rs.
\textsuperscript{c} School of Computing - Union University, Kneza Mihaila 6/VI, Belgrade, 11000, Serbia; CESifo Group Munich, Munich, Bayern, DE 81679, Germany; +381695355810; bureovic@raf.rs.
\textsuperscript{d} Corresponding author: University of Sussex Business School, Brighton, BN1 9SL, UK; phone: +44(0)1273872597; fax: +44(0)1273876677; email: r.jelic@sussex.ac.uk.

Forthcoming in Finance Research Letters

Abstract

We develop a novel Mean-Max Drawdown portfolio optimization approach using buy-and-hold portfolios. The optimization is performed utilizing a multi-objective evolutionary algorithm on a sample of S&P 100 constituents. Our optimization procedure provides portfolios with better Mean-Max Drawdown trade-offs compared to relevant benchmarks, regardless of the selected subsamples and market conditions. The superior performance of our approach is particularly pronounced in periods with reversing market trends (i.e. a market rally and a fall in the same subsample).

Key words: Maximum drawdown; Genetic algorithm; Portfolio optimization; Risk management

JEL: C61; G11; G18; G23
1. Introduction

Contemporary risk management practices and the regulation of financial institutions rely on quantile measures of risk, and require back-testing of internal risk models to be performed on currently held portfolios.\footnote{For banking (Basel III) regulation see BIS (2019). For insurance (Solvency II) regulation see EU (2015/35).} While the recent financial crisis highlighted the need to properly measure and manage the risk of extreme events, it also exposed some limitations of the quantile risk measures in that regard. Specifically, neither \textit{VaR} nor \textit{CVaR} provide insights into the trends and sequencing of portfolio returns. This is a serious drawback since small but persistent losses may cumulate into large overall falls in portfolio value.

Maximum drawdown (\textit{MDD}) measures the loss that an investor would suffer if she enters into a portfolio position at its peak value and exits at its trough. For institutional investors facing stochastic withdrawal of funds (e.g. pension and mutual funds) large losses from a peak portfolio value can trigger significant fund redemptions. Consideration of \textit{MDD} could, therefore, provide an important complementary picture of downside risk for both investors and regulators. We conjecture that if \textit{MDD} were to be introduced as an additional regulatory measure of downside risk, it would be applied in line with the regulation requiring that back-testing of risk models is performed on currently held portfolios (thus fixing asset holdings rather than portfolio weights). Fixed asset holdings are also preferred by large institutional investors who buy and hold portfolios for a considerable length of time.

Some of previous optimization studies use variance (or a quantile risk measure) and then impose constraints on drawdowns in order to prevent excessive losses (see Grossman and Zhou 1993; Alexander and Baptista 2006). Mendes and Lavrado (2017) propose a simulation based methodology for estimating \(\alpha\)-quantile of the \textit{MDD} distribution and validate the procedure using portfolios with stock indices. Almahdi and Yang (2017) propose the variable weight recurrent reinforcement learning (RRL) long-short approach for optimization of portfolio weights. They rebalance portfolios over a predefined time horizon and examine the expected maximum drawdown \(E(\text{MDD})\) effect on portfolio performance with joint interaction of transaction costs. In this paper, we adopt more direct approach by using \textit{MDD} as the risk measure and examining optimal trade-offs between portfolio mean and \textit{MDD}. Portfolio \textit{MDD}s are estimated using fixed asset holdings, based on the Actual Portfolio Framework (\textit{APF}) developed by Rankovic et al. (2016). To the best of our knowledge, this is the first study of \textit{Mean-MDD} portfolio optimization with fixed asset holdings (hereafter \textit{APF Mean-MDD}).

We test our approach using an opportunity set of the S&P 100 constituents, during 2006-2017. In contrast to the \textit{Mean-MDD} problem with fixed portfolio weights, \textit{APF Mean-MDD} optimization cannot be transformed into a standard optimization problem. In order to solve the problem, we develop software in which we implement the Non-dominated Sorting Genetic Algorithm (\textit{NSGA-II}) from Deb et al. (2002). Results suggest that our portfolio optimization procedure provides buy-and-hold portfolios with better \textit{Mean-MDD} trade-offs, compared to relevant benchmarks (e.g. Chekhlov et al., 2005), regardless of the selected subsamples and market conditions. The superior performance of our approach is particularly pronounced in periods where reversal market trends are observed (i.e. a fall and a rally in the same subsample). Our analysis of the (ex-post) optimal \textit{MDD} portfolios, therefore, helps investors and portfolio managers to detect characteristics of buy-and-hold \textit{MDD} efficient portfolios in different market regimes. In this respect our contribution resembles that of Chekhlov et al. (2005), except that the authors provide algorithm for delivering optimal...
fixed-weight MDD portfolios. In practice, this approach induces a dynamic portfolio strategy with frequent portfolio rebalancing. Our results, thus, provide comparison between (ex-post) portfolios with fixed weights and portfolios with fixed-holdings.

The paper is organized as follows. Section 2 presents relevant literature and hypotheses. The optimization problem and our proposed solution is introduced in Section 3. Section 4 presents data together with criteria for sample selection. The empirical results are presented and discussed in Section 5. Results of our further analysis with robustness checks is presented in Section 6. Section 7 concludes.

2. Literature review

The closest to our work is Chekhlov et al. (2005) who introduce Conditional Drawdown at Risk (CDaR) as the mean of the worst (1-α) *100 drawdowns. Instead of imposing additional constraints to limit excessive losses, Chekhlov et al. (2000; 2005) use CDaR as the risk measure and perform Mean-CDaR optimization directly. The authors solve for the Mean-CDaR efficient frontier by mapping the problem into an equivalent linear programming (LP) model. The approach can be employed for different levels of the parameter α, including α=0 in which case CDaR becomes MDD and algorithm solves Mean-MDD problem. However, their algorithm is constructed under the assumption that investors keep portfolio weights (rather than asset holdings) fixed over time. It could be argued that one could use the methodology of Chekhlov et al. (2005) as an approximate solution to the APF Mean-MDD problem. Namely, for each LP Mean-MDD trade-off one could derive initial holdings, fix these holdings henceforth and calculate corresponding MDD. However, obtained Mean-MDD trade-offs will differ from corresponding LP trade-offs. We, therefore, conjecture that the LP approach would not appropriately approximate efficient buy-and-hold portfolios over a longer time period. To test the above conjecture, we obtained the results of the LP approach and compared these with our results from the APF Mean-MDD approach. We are, therefore, able to show how good an approximation of the actual portfolio Mean-MDD efficient frontier is obtained when the LP approach is used.

In contrast to the Mean-MDD problem with fixed portfolio weights, APF Mean-MDD optimization cannot be simplified to a standard optimization problem. Instead, we use the NSGA-II evolutionary algorithm, an alternative optimization method from the class of multi-objective evolutionary algorithms (MOEAs). MOEAs are efficient search techniques for solving complex multi-objective optimization problems through an application of natural selection. NSGA-II is one of the most frequently employed MOEAs for portfolio optimization problems (see Anagnostopoulos and Mamanis 2010; 2011, Branke et al., 2009, Deb et al., 2011, Ranković et al., 2016, and Drenovak et al., 2017).

3. APF Mean-MDD optimization

Drawdown (DD) is a risk measure that determines the relative change in a portfolio value at a given point in time t, with respect to the maximum portfolio value achieved up to

\footnote{The approach of Chekhlov et al. (2005) is particularly relevant for active investment strategies adopted, for example, by hedge funds. Our approach, on the other hand, is better suited for buy-and-hold investment strategies adopted by, for example, sovereign wealth funds.}
time \( t \). Formally, \( DD \) measured at time \( t \) and over the observed period \([0, t]\), is given by the expression:

\[
DD(t) = \frac{V_{p,t}}{\max_{s} V_{p,s}} - 1, \quad s = 0, t
\]

Here, \( V_{p,0} \) and \( V_{p,t} \) are portfolio values at time 0 (the starting point of the period under consideration) and time \( t \) (the ending point), respectively. Contrary to simple returns that measure the relative change in value with respect to equidistant points in time (say, a day, a week, or a month), \( DD \) is relative change in a portfolio’s value with respect to the highest value achieved up to that point. By definition \( DD(t) \leq 0 \). Investors have strong aversion to large drawdowns. In fact, such drawdowns often trigger fund withdrawals and mutual and pension funds try to avoid them as much as possible.

\( MDD \) is the relative change in a portfolio’s value under the most unfavourable market timing decisions. \( MDD \) is the lowest (i.e. the most negative \( DD \)) of all realized drawdowns over an observed period \([0, T]\).

\[
MDD_T = \min_{i} DD(i), \quad t = 0, T
\]

Our optimization problem takes the following general form:

\[
\min_{w_T} \quad -MDD_T \quad \text{eq. 3}
\]

\[
\max_{w_T} \quad E(r) \quad \text{eq. 4}
\]

subject to \( \sum_{i=1}^{N} w_{i,T} = 1 \) \quad \text{eq. 5}

\[
0 \leq w_{i,T} \leq 1, \quad i = 1, N
\]

Here, \( w_T \) denotes the vector of portfolio weights \( w_{i,T} \) at optimization day \( T \); \( MDD_T \) denotes portfolio Maximum Drawdown calculated at day \( T \); and \( E(r) \) is expected (i.e. mean) portfolio return at day \( T \). \( N \) is the number of assets in opportunity set. Decision variables are weights \( w_{i,T} \) at day \( T \). Equation 5 describes the standard budget constraint which asserts that portfolio weights must sum up to 1. Equation 6 states that no short sales are allowed, a common constraint imposed on large regulated institutional investors such as mutual or pension funds.

Rankovic et al. (2016) and Drenovak et al. (2017) applied the APF framework for portfolio optimization in the context of the VaR, and Basel II and Basel 2.5 regulations, respectively. Here, we adopt their approach when estimating portfolio \( MDD \).

Let \( n_i \) denote the holdings (the number of shares) of an asset \( i \) at day \( T \). In order to calculate time series of portfolio values \( V_{p,t} \) we use time series of constituent prices \( P_{i,t} \) and number of holdings of each asset \( n_i \). Since we examine portfolios with fixed holdings, \( n_i \) is fixed over period \([0, T]\). The dollar portfolio value \( V_{p,t} \) at day \( t \) is given by the expression:
where \( P_{i,t} \) is price of asset \( i \) at day \( t \).

Since our optimization algorithm uses weights at day \( T, w_{i,T} \), as decision variables, in order to derive corresponding holdings \( n_i \) from given set of portfolio weights we use following expression:

\[
n_i = \frac{w_{i,T} V_{p,T}}{P_{i,T}} \quad eq. 8
\]

Equations 7 and 8 show that portfolio value at day \( t \) is function of decision variables \( w_{i,T} \):

\[
V_{p,t} = \sum_{i=1}^{N} n_i P_{i,t} = \sum_{i=1}^{N} \frac{w_{i,T} V_{p,T}}{P_{i,T}} P_{i,t}, \quad t = 0, T \quad eq. 9
\]

Here \( V_{p,T} \) and \( P_{i,T} \) are the dollar portfolio value and the price of asset \( i \) at day \( T \), respectively. Without loss of generality, we assume that at optimization day \( T \), the dollar portfolio value is normalized to one, i.e. that \( V_{p,T}=1 \).

Simple returns on actual portfolio at day \( t \) read:

\[
r_t = \frac{V_{p,t}}{V_{p,t-1}} - 1, \quad t = 1, T \quad eq. 10
\]

Expected return (mean) \( E(r) \) (Eq. 4) over the observed period \([0, T]\) is calculated using formula:

\[
E(r) = \frac{1}{T} \sum_{t=1}^{T} r_t \quad eq. 11
\]

We calculate actual portfolio \( MDD \) by applying equation 1 and 2 on time series \( V_{p,t} \).

Like all other evolutionary algorithms, \( NSGA-II \) starts with a set of randomly generated candidate solutions, referred to as a population. In each of the iterations (generations), a set of new candidate solutions (offspring solutions) is generated by applying the evolutionary processes consisting of selection, crossover and mutation. By performing these processes over a number of generations, the solutions evolve and improve in terms of the chosen objectives (in our case, the expected portfolio return and \( MDD \)).

The implementation of \( NSGA-II \) implies adoption of algorithm settings such as solution representation, the population size, the crossover and mutation operator types and the termination condition. Solution represents portfolio and is defined as a non-negative real-valued vector of portfolio weights. The population size is set to 100. In each generation the offspring population is generated by employing a uniform crossover operator. The uniform crossover operator randomly selects two solutions from the current population and recombines them with a predefined crossover probability, here set to 1, yielding two

---

\(^3\) See Deb et al. (2002) for more on \( NSGA-II \) evolutionary algorithm.
offspring solutions. The recombination implies that every asset weight is exchanged between the selected pair of solutions with a so-called swapping probability (see Sastry et al., 2005). We set the swapping probability to 0.5, as suggested in the previous literature. The crossover operator is executed 50 times yielding offspring population of size 100. Mutation process is performed on the offspring population by using a uniform mutation operator. The operator implies that each weight is selected with a pre-defined mutation probability and replaced with a realization of a random variable, uniformly distributed in the range [0,1]. Mutation probability is set to 0.05. The maximum number of generations is set to 200. When applying uniform crossover and mutation operators the constraint defined by Eq. (6) is satisfied for each offspring. However, the budget constraint (Eq. (5)) is likely violated. Thus, each of the offspring solutions is normalized by dividing each weight by sum of all weights.

4. Data and sample selection

For the purposes of this study we start with 102 constituent stocks of the S&P 100 Total Return Index, as of March 28th 2016. We collect end-of-day data for time series of constituent stocks for the period Jan 1st, 2007 - March 28th, 2016. Each of these series consists of 2,411 trading dates. We excluded eight out of the 102 stocks for which we could not find daily prices for the entire period.

We then create four subsamples which include time series of end-of-day prices for the 94 stocks. The subsamples correspond to four different time periods (see Table 1). The fourth subsample is, in fact, the total sample. The subsamples capture different market trends and have different lengths. This is needed in order to control for MDD’s characteristics such as path dependency and sensitivity to the length of observed periods. Table 1 defines the four subsamples and presents the number of daily observations (K), K=T+1, in each of the subsamples.

<table>
<thead>
<tr>
<th>Subsamples</th>
<th>Period</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Falling market trend</td>
<td>January 1st, 2007- March 6th 2009</td>
<td>570</td>
</tr>
<tr>
<td>2. Falling and rising market</td>
<td>January 1st, 2007- November 1st, 2010</td>
<td>1,001</td>
</tr>
<tr>
<td>3. Rising market trend</td>
<td>March 9th 2009- March 28th, 2016</td>
<td>1,841</td>
</tr>
</tbody>
</table>

Figure 1 presents the evolution of the S&P 100 Total Return Index, together with its DD, during the total sample period. The first three subsample periods are marked with double-headed arrows. Subsample 1 covers the period of a sharp market decline during the recent financial crisis including the day when Lehman Brothers filed for bankruptcy (falling market trend). The minimum value of the market index marks the end of the first subsample period. Subsample two covers both the period of a sharp decline followed by a recovery (falling and rising market trend). Subsample three covers rising market trend and begins on the date the index exhibits its minimum value. Subsample four corresponds to the total sample as indicated on the x-axis.
Figure 1: Evolution of S&P 100 Total return Index and its DD during total sample period (January 1st, 2007-March 28th, 2016). Initial value of index is normalized to 1 (100%).

5. Results of APF Mean-MDD optimization

5.1. Efficient portfolio frontiers in different subsamples

Figures 2-5 depict efficient frontiers, in the Mean-MDD coordinate system, under different market conditions (i.e. for different subsamples). Each frontier consists of 100 portfolios. The (white) circles on each figure represent our APF Mean-MDD solutions. In order to compare our solutions with the corresponding LP solutions based on fixed weights, we perform the following procedure: i) Input MDD values of our APF Mean-MDD solutions into the LP model and obtain solutions corresponding to these MDD values; ii) Calculate mean and MDD values of the obtained LP solutions by applying equations 9, 10 and 2, respectively. In other words, we use weights of every benchmark portfolio to recalculate its mean and MDD values as if the holdings (rather than weights) were fixed over the observed period;⁴ iii) Finally, we compare the two efficient frontiers expressed in the Mean and MDD coordinates. The LP solutions are presented by (black) triangles.

Our APF Mean-MDD solutions outperform the LP solutions in all subsamples. For example, in Total sample (Figure 2) we identified two portfolios (A and L), laying on different frontiers, exhibiting almost identical means with markedly different MDDs. MDD of portfolio L is more than two times larger than MDD of portfolio A. The respective losses, expressed as MDD, are around 45% and 20%, respectively. The difference between the two approaches is particularly pronounced when the market exhibits a reversal of trends (Figures 2 and 3). In periods of one directional market movement (i.e. falling or rising markets) the outperformance of the APF Mean-MDD efficient portfolios is less pronounced (see Figures 4 and 5, respectively).

⁴ Note that the recalculated LP frontier is not necessarily monotonic.
Figure 2: Efficient frontiers for Subsample 4 (Jan 1th, 2007 – March 28th, 2016), covering total sample period. The (white) circles represent our APF Mean-MDD solutions. The LP solutions are presented by (black) triangles. Portfolios A and L exhibit solutions with similar means but markedly different MDD.

Figure 3: Efficient frontiers for Subsample 2 (Jan 1st, 2007 – Nov 1st, 2010), depicting both falling and rising market trends. The (white) circles represent our APF Mean-MDD solutions. The LP solutions are presented by (black) triangles.
Figure 4: Efficient Mean-MDD frontiers for Subsample 1 (Jan 1st, 2007 – March 6th, 2009), depicting falling market trends. The (white) circles represent our APF Mean-MDD solutions. The LP solutions are presented by (black) triangles.

Figure 5: Efficient frontiers for Subsample 3 (March 9th, 2009 – March 28th, 2016), depicting rising market trend. The (white) circles represent our APF Mean-MDD solutions. The LP solutions are presented by (black) triangles.
5.2. Cardinality and composition of efficient portfolios

Table 2 documents the cardinality of the efficient portfolios presented in Figures 2-5. It compares the maximum number of key constituents (i.e. with weights >1%) for the APF Mean-MDD and LP solutions in different subsamples. Maximum cardinality is higher in the subsample depicting raising market trends than in the subsample depicting falling market trends (i.e. financial crisis). This is in line with findings that correlations between all risky assets tend to strengthen during periods of financial crisis, consequently, resulting in less diversified portfolios (see Pedersen 2009).

<table>
<thead>
<tr>
<th>Subsamples</th>
<th>Period</th>
<th>Max Cardinality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Falling market trend</td>
<td>January 1st, 2007- March 6th 2009</td>
<td>5</td>
</tr>
<tr>
<td>2. Falling and rising market</td>
<td>January 1st, 2007- November 1st, 2010</td>
<td>6</td>
</tr>
<tr>
<td>3. Rising market trend</td>
<td>March 9th 2009- March 28th, 2016</td>
<td>11</td>
</tr>
<tr>
<td>4. Total sample</td>
<td>January 1st, 2007-March 28th, 2016</td>
<td>8</td>
</tr>
</tbody>
</table>

In order to highlight differences between APF Mean-MDD and LP solutions, we examine composition of the efficient portfolios, A and L, identified earlier in Figure 2. First, we present the evolution of constituent weights of the optimal portfolio A (Figure 6). Here, the optimal holdings, kept unchanged over time, are determined directly by the APF Mean-MDD approach (at date T). There are six key portfolio constituents (those with weights above 1%) in the efficient portfolio A.\(^5\) Figure 7 depicts the evolution of weights for the key constituents of the portfolio L.\(^6\) Here, we fixed the holdings based on the weights of the portfolio L obtained by LP algorithm (using Eq. 8). For the fixed holdings we then determine the resulting constituents’ weights during the sample period. The results, again, depict six key constituents with weights above 1%.

It can be observed that the weights of the portfolio constituents fluctuate significantly in both figures. Since APF Mean-MDD approach does not promise to deliver fixed weights the fluctuations in Figure 6 are expected. However, fixing initial holdings, corresponding to the weights of the portfolio L, does not maintain the same weights during the sample period (see Figure 7). For example, the holding of Priceline Group increased approximately 5-fold, from January 2007 to April 2012. In essence, the LP method requires frequent trading in order to keep the constituents’ weights fixed to their optimal values.

Cardinality of efficient portfolios in Table 2 suggests comparable number of assets in both APF and LP model solutions. Comparing significant constituents of the optimal portfolios A and L we see that only two stocks, Allergan and Priceline Group, appear both in Figure 6 and 7. The other four stocks in the respective optimal portfolios (A and L) are different suggesting that APF’s outperformance stems from selection of different constituents rather than from differences in levels of diversification.

\(^5\) The key constituents are: Wall Mart Stores, McDonalds, Allergan, Priceline Group, Abbot Laboratories, and Southern.

\(^6\) The key constituents are: Wells Fargo & Co., Gilead Sciences, Allergan, Bristol Myers Squibb, Priceline Group, and Biogen.
Figure 6: Evolution of portfolio weights (y-axis) for the key constituents (those with weights above 1%) of the efficient portfolio A during the entire sample period. Optimal holdings are kept unchanged and determined directly by the APF approach.

Figure 7: Evolution of portfolio weights (y-axis) for the key constituents (those with weights above 1%) of the efficient portfolio L during the entire sample period. The initial holdings were fixed based on the weights of the portfolio L. For these fixed holdings we then determine the resulting constituents’ weights during the sample period.

6. Further analysis and robustness checks

6.1. Constituents of the entire efficient frontiers

We provide graphical representation of composition of each portfolio from the efficient frontiers for the Total sample in Figures 8 (APF frontier) and 9 (LP frontier). Each frontier consists of 100 efficient portfolios (represented on x-axis). Constituents (with weight above 1%) of each of the 100 portfolios are presented by one vertical bar (y-axis). There are,
therefore, 100 bars representing constituents of the entire efficient frontier. Each of the constituents is presented in a different colour. Number of colours of the single bar, therefore, determines the portfolio cardinality. An area represented by a single colour, within a bar, is proportionate to a constituent’s weight, on the optimization day $T$. Legend connects colours with company names representing companies which appear in least one of 100 efficient portfolios with weight above 1%. For example, 15 (out of 94) sample companies appear in different APF efficient portfolios (Figure 8).

**Figure 8:** Constituents of the APF efficient portfolios for Subsample 4, covering total sample period are presented on x-axis. Portfolio weights (y-axis) for the key constituents (those with weights above 1%) are presented on y-axis.

**Figure 9:** Constituents of the LP efficient portfolios for Subsample, 4 covering total sample period are presented on x-axis. Portfolio weights (y-axis) for the key constituents (those with weights above 1%) are presented on y-axis.
Figures 8 and 9 illustrate significant differences between constituents of APF and LP efficient portfolios. They provide further support for our conclusions that APF’s outperformance stems from selection of constituents. Above analysis also shows that efficient buy-and-hold portfolios in general cannot be appropriately approximated by the optimal fixed weight portfolios over longer time periods.

6.2. Analysis of an additional investment opportunity set

We check for the robustness of our results using constituents of the MSCI Developed Markets Total Return Index as an alternative investment opportunity set. The MSCI index is one of the most respected and widely used benchmarks in the financial industry. It is a broad global equity index that represents large and mid-cap equity performance across 23 developed countries. The index covers approximately 85% of the free float-adjusted market capitalization in each country. Use of the global equity investment opportunity set is, therefore, relevant for institutional investors (e.g. sovereign wealth and pension funds) who adopt buy-and-hold strategies. Overall, MSCI Developed Markets Total Return the index is dominated by US (weight around 66% in 2020) therefore exhibiting similar trends to S&P 100 index. We, therefore, use identical subperiods as in our main analysis (see Table 2) and replicate our results using time series of the 23 MSCI sub-indices. Results for Total sample (Subsample 4) are presented in Figure 10.

![Efficient frontiers for Subsample 4 (Jan 1<sup>th</sup>, 2007 – March 28<sup>th</sup>, 2016), covering total sample period. The (white) circles represent our APF Mean-MDD solutions. The LP solutions are presented by (black) triangles.](image)

Overall, the results are in line with the results presented in the manuscript. For example, APF solutions clearly dominate LP solutions. Unreported results suggest low

---

7 See https://www.msci.com/developed-markets.
8 LP solutions were obtained using Portfolio Safeguard (PSG) optimization package for solving nonlinear and mixed-integer nonlinear optimization problems in Windows. See http://www.aorda.com/index.php/portfolio-safeguard/.
cardinality (for both APF and LP solutions). Constituents of APF and LP optimal portfolios are different, in line with our results reported in Section 5.9

6. Conclusions

In this paper we propose a novel Mean-Max Drawdown portfolio optimization approach in line with regulatory standards for the back-testing of financial institutions’ risk models. Efficient portfolios obtained by our optimization approach outperform relevant benchmarks. Importantly, we find that fixed-weight efficient portfolios tend to be a relatively poor approximation of buy-and-hold efficient portfolios especially in periods with reversals in market trends. Results of our study provide valuable insights regarding measurement and management of the risk of extreme events and inform debates regarding the use of artificial intelligence in financial regulation.

References


Evidence for other subsamples is also in line with or results presented in Section 5. Unreported results are available upon request from authors.


