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Path Planning and Control of a Quadrotor UAV Based on an Improved APF Using Parallel Search

Tianpeng Huang, Deqing Huang*, Na Qin, and Yanan Li

Abstract—Control and path planning are two essential and challenging issues in quadrotor unmanned aerial vehicle (UAV). In this paper, an approach for moving around nearest obstacle is integrated into artificial potential field (APF) to avoid the trap of local minimum of APF. The advantage of this approach is that it can help UAV successfully escape from the local minimum without collision with any obstacles. Moreover, UV may encounter the problem of unreachable target when there are too many obstacles near its target. To address the problem, a parallel search algorithm is proposed, which requires UAV to simultaneously detect obstacles between current point and target point when it moves around the nearest obstacle to approach the target. Then, to achieve tracking of the planned path, the desired attitude states are calculated. Considering the external disturbance acting on the quadrotor, a nonlinear disturbance observer (NDO) is developed to guarantee observation error to exponentially converge to zero. Furthermore, a backstepping controller synthesized with the NDO is designed to eliminate tracking errors of attitude. Finally, comparative simulations are carried out to illustrate the effectiveness of the proposed path planning algorithm and controller.

Index Terms—Quadrotor unmanned aerial vehicle, nonlinear disturbance observer, Backstepping control, Path planning, Artificial potential field, Parallel search

I. INTRODUCTION

In recent years, UAV has been used in various applications, such as infrastructure management [1], logistics delivery [2] and estimation of above-ground biomass of mangrove ecosystems [3]. The implementation of UAV in all these applications requires it to follow a predefined path. In addition, to achieve path tracking, a good control system should be provided. Therefore, path planning with automatic obstacle avoidance and control are two essential tasks in UAV.

Research on UAV control has been extensively reported in the literature. In [4], a proportional-integral-derivative (PID) controller is designed to accomplish altitude and attitude tracking for a quadrotor helicopter. A fuzzy PID control method is proposed in [5], where the fuzzy rules are employed to automatically adjust the three parameters of PID controller. In [6] and [7], an active disturbance rejection control (ADRC) scheme is developed to achieve trajectory tracking of a quadrotor UAV. PID and ADRC are model-free control strategies, which have an advantage of simple control structure. However, it is difficult to tune the parameters of these two controllers.

In [8], an adaptive sliding mode control (SMC) is investigated to stabilize a quadrotor system subject to unknown external disturbance. [9] presents a continuous SMC approach to follow the predefined trajectories in position and attitude channels for a four-rotor UAV. An adaptive finite-time attitude tracking algorithm is applied to a quadrotor in the presence of uncertainty and disturbance in [10]. In [11], a disturbance observer is integrated to $H_\infty$ technique to realize hovering control of a quadrotor. The robustness of such a method has been verified by experiments. In [12], the differential flatness is used in tracking control of translational and rotational movements of an UAV system considering modeling uncertainty and disturbance. An adaptive linear quadratic control strategy is proposed in [13] to stabilize three attitude angles of a quadrotor. To achieve trajectory following of position and attitude subsystems, a nonlinear disturbance observer (NDO) based backstepping controller is proposed in [14], where the NDO is utilized to estimate external disturbance.

The bio-inspired algorithms have been applied to path planning of UAV. In [15], a chaotic artificial bee colony (ABC) method is developed to design a path in complex environments. In [16], an ant colony optimization (ACO) algorithm is proposed to achieve trajectory planning for a UAV. [17] presents a particle swarm optimization (PSO) algorithm to address path planning of UAV. Genetic algorithm (GA), as a popular optimization algorithm, has been employed to plan a path in UAV [18]. Furthermore, a comparison of GA and PSO for real-time path planning of UAV is carried out in [19], where the results indicate that, under the consideration of statistical significance, the trajectories produced by GA are superior compared to that produced by PSO when using the same encoding. In [20], [21], a grey wolf optimizer is used to search a feasible and effective path for a UAV. An improved fruit fly optimization algorithm is introduced in [22] to address the problem of path planning of multiple UAVs in 3D complicated environments with online changing tasks. In [23], a flower pollination algorithm based on neighborhood global learning is employed to complete route planning of a UAV. [24] offers an evolutionary algorithm based on a novel separate evolution strategy to plan an optimized path for a single UAV. Furthermore, a constrained differential evolution is put forward to achieve path planning of a UAV in [25].

Besides the aforementioned bio-inspired intelligent algorithms, there are many effective strategies to solve problem of path planning of UAV. [26] studies a distributed multi-agent path planning algorithm for quadrotors in dynamic environments. An energy-based path planning framework is used to improve flight endurance for a solar-powered UAV.
in [27]. A multiobjective path planning is presented in [28] to design a feasible path for a UAV, where safety is considered in the proposed algorithm. [29] introduces a path planning system based on elliptical tangent model to reduce computational burden for a quadrotor UAV in an unknown unstructured outdoor environment. In [30], a ground feature oriented approach is investigated to generate a suitable path for UAV mapping. Two path planning algorithms are designed in [31], one of which is based on the exact penalty method and successive convex approximation, and the other adopts a heuristic approach. In addition, [32] presents an improved A-star algorithm to generate a path for target tracking of a UAV. In [33], a Voronoi diagram based multiple UAVs path planning method is proposed to cooperatively attack multiple targets in a static environment. An improved rapidly-exploring random tree (RRT) algorithm is introduced in [34] to realize 3D path planning of a UAV.

As an efficient path planning algorithm, APF has been applied to some scenarios, such as mobile robots [35], [36] and automated vehicles [37]. The traditional APF (TAPF) has two shortcomings, i.e., local minimum and unreachable goal. To address these problems, the repulsive potential function of TAPF is replaced by Gaussian function in [38]. However, UAV still might fall into a local minimum when using the improved APF in [38] when obstacle is on the line connecting current position and target position. Moreover, when multiple obstacles are around target and repulsive gain is large, the resultant repulsive forces might be equal to attractive force, in which case UAV could not approach target. Motivated by the above analysis, a novel APF based on parallel search is proposed for path planning of UAV in this paper.

The main contributions of the paper are summarized as follows:

1) A parallel search algorithm is proposed to address local minimum and unreachable target with obstacles nearby in TAPF.

2) Compared with existing results of path planning algorithms [34], [39], [40], a shorter path and less time consumption are obtained using the proposed algorithm.

3) Compared with ADRC [6], [7], better tracking performance is obtained by the proposed controller based on NDO with exponential convergence when following the planned path.

The remainder of this paper is organized as follows. In Section II, TAPF is introduced and the problems of local minimum and unreachable goal of TAPF are analyzed. In Section III, a novel APF based on parallel search is presented. IV introduces the design of observer and controller of quadrotor. In Section V, comparative simulations are conducted to verify the effectiveness of the proposed algorithm and controller. Section VI concludes the work.

II. TAPF APPLIED TO PATH PLANNING OF UAV

A. TAPF

APF is a virtual potential field in space, which consists of attractive potential field generated by target position and repulsive potential field generated by obstacle. The UAV automatically plans a path to destination under the influence of attractive potential field and repulsive potential field.

Let $P_{\text{cur}}(x_{\text{cur}}, y_{\text{cur}})$ and $P_{\text{tar}}(x_{\text{tar}}, y_{\text{tar}})$ represent current position and target position of UAV, respectively. Then, the attractive potential field is given by

$$U_a(P_{\text{cur}}) = \frac{1}{2} k_a d_a(P_{\text{cur}}, P_{\text{tar}})^2,$$

(1)

where $k_a$ is a positive gain and $d_a(P_{\text{cur}}, P_{\text{tar}})$ is minimum distance between current position and the target position, in a 2D case defined as

$$d_a = \sqrt{(x_{\text{cur}} - x_{\text{tar}})^2 + (y_{\text{cur}} - y_{\text{tar}})^2}.$$

(2)

Let $P_{\text{ob}_i}(x_{\text{ob}_i}, y_{\text{ob}_i})$ denote position of the $i$th obstacle, where $i \in N^+$. Then, the repulsive potential field of the $i$th obstacle is defined as

$$U_{ri}(P_{\text{cur}}) = \begin{cases} \frac{1}{2} k_{ri} \frac{1}{d_{ri}(P_{\text{cur}}, P_{\text{ob}_i})^2} - \frac{1}{d_0} \leq d_0, \\ 0 \quad d_0 > d_0, \end{cases}$$

(3)

where $k_{ri}$ and $d_0$ are positive constants and $d_{ri}(P_{\text{cur}}, P_{\text{ob}_i})$ is minimum distance between current position and the $i$th obstacle, in a 2D case defined as

$$d_{ri} = \sqrt{(x_{\text{cur}} - x_{\text{ob}_i})^2 + (y_{\text{cur}} - y_{\text{ob}_i})^2}.$$

(4)

It is worth noting that $d_0$ in (3) shows the influence range of repulsive potential field of obstacle. Obviously, attractive potential field is global, while repulsive potential field is local. Furthermore, the attractive force from target is obtained by (1)

$$F_a(P_{\text{cur}}) = k_a d_a(P_{\text{cur}}, P_{\text{tar}}).$$

(5)

Meanwhile, the repulsive force of the $i$th obstacle is obtained from (3)

$$F_{ri}(P_{\text{cur}}) = \begin{cases} k_{ri} \left( 1 - \frac{1}{d_0} \right) \left( \frac{1}{d_{ri}} \right)^2 \leq d_0, \\ 0 \quad d_0 > d_0. \end{cases}$$

(6)

Therefore, the path planning for UAV based on TAPF algorithm is shown in Algorithm 1.

B. Local minimum

The local minimum is an inherent disadvantage of TAPF. When attractive force and repulsive force reach a balance, the UAV would encounter a trap of a local minimum, which means that the UAV stops moving towards target, as shown in Figs. 1 and 2, where $F_a$, $F_r$ represent attractive force and resultant repulsive force at current position respectively and $F_{ri}, i = 1, 2$, represents repulsive force of the $i$th obstacle. It is evident that single obstacle or multiple obstacles may cause UAV to fall into a local minimum when a balance of attractive force and repulsive force is reached.
Algorithm 1 path planning algorithm based on TAPF

**Input:** The current position $\mathbf{P}_{\text{cur}}(x_{\text{cur}}, y_{\text{cur}})$ of UAV, target position $\mathbf{P}_{\text{tar}}(x_{\text{tar}}, y_{\text{tar}})$, obstacles position;

**Output:** The path to the target;

1. Update path matrix $M \leftarrow \mathbf{P}_{\text{cur}}$;
2. **while** $d_a > c$ (*c* is a small positive constant) **do**
   1. Calculate attractive force $F_a$, using (2) and (5);
   2. Calculate repulsive force $F_{ri}$, using (4) and (6);
   3. Calculate the components of attractive force along the $x$ and $y$ directions respectively;
   4. Calculate the components of repulsive force along the $x$ and $y$ directions respectively;
   5. Calculate the next point on the path $\mathbf{P}_{\text{next}}$ according to the resultant of attractive force and repulsive force;
   6. Update $M \leftarrow \mathbf{P}_{\text{next}}$;
3. **end while**

---

**C. Unreachable target**

Another shortcoming of TAPF is that the goal might be inaccessible for UAV, when obstacles are near the target, as shown in Figs. 3 and 4, where $\text{Obs}_i, i = 1, \cdots, 8$, denotes the $i$th obstacle. In fact, the attraction of goal to UAV is gradually decreasing, as the UAV approaches destination from (5), while the repulsive force of obstacles to UAV is gradually increasing. Therefore, the UAV fails to plan a path to destination using TAPF.

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**III. NOVEL APF BASED ON PARALLEL SEARCH**

To address the problems of local minimum and unreachable goal of TAPF, a novel parallel search based APF algorithm is proposed to achieve path planning of a UAV. Such an algorithm consists of two key ideas. The first idea is that UAV moves around nearest obstacle when it encounters a local minimum, as shown in Fig. 5, from which it is observed that $d_{r3}$ is smallest. To avoid collision with other obstacles, the UAV makes a circular motion with radius $d_{r3}$ around the third obstacle. Then, the balance of attractive force and repulsive force will be broken once the UAV moves a step around the third obstacle, which would guide the UAV to escape from the local minimum under the APF framework.

The second idea of the proposed algorithm is that when the goal is unreachable for the UAV, the movement around the
nearest obstacle and the detection of obstacle between current position and target position will be performed in parallel. Two examples in Figs. 6 and 7 are used to further illustrate this idea.

In Fig. 6, \( L_1 \) represents a line that goes through current position of UAV and target position. When the target is unreachable for UAV, the points on the line \( L_1 \) will be scanned continuously to find the intersection points of the line \( L_1 \) and the obstacle. If the number of intersection points is equal to zero, it means the UAV can move directly to the target.

In the following, we will explain the scenario in Fig. 7, where \( L_2 \) represents a line that goes through current position of UAV and target position and \( \overline{L} \) denotes a line connecting the current position of UAV and the obstacle. It is obvious that the number of intersection points of \( L_2 \) and the obstacle is greater than zero. In these circumstances, the UAV moves firstly along the line \( L \) to the point \( A(x_A, y_A) \). Then, the UAV moves to the point \( D \) around the first obstacle with radius \( R_A \), where

\[
R_A = \sqrt{(y_{ob1} - y_A)^2 + (x_{ob1} - x_A)^2}. \tag{7}
\]

Finally, the UAV successfully reaches the destination along the feasible path \( L_3 \). It is worth noting that the detection of obstacle between current position and target position will also be performed simultaneously when UAV is at point \( B \) or \( C \).

In summary, the flow of the proposed algorithm is shown in Algorithm 2.

**Algorithm 2** a novel APF algorithm based on parallel search for path planning of UAV

**Input:** The current position \( P_{cur}(x_{cur}, y_{cur}) \) of UAV, target position \( P_{tar}(x_{tar}, y_{tar}) \), obstacles position;  

**Output:** The path to the target;  

1. Calculate \( d_a \);  
2. Update path matrix \( M \leftarrow P_{cur} \);  
3. while \( d_a > c \) do  
   4. Calculate attractive force \( F_a \), using (2) and (5);  
   5. Calculate repulsive force \( F_r \), using (4) and (6);  
   6. Calculate the components of attractive force along the \( x \) and \( y \) directions respectively;  
   7. Calculate the components of repulsive force along the \( x \) and \( y \) directions respectively;  
   8. Calculate the next point on the path \( P_{next} \) according to the resultant of attractive force and repulsive force;  
   9. Update matrix \( M \leftarrow P_{next} \);  
10. Calculate \( d_1 = |M_j - M_{j-3}| \);  
11. if local minimum then  
   12. Find the obstacle closest to the current position, then calculate the distance \( R \) between the obstacle and current position, and moves around the obstacle with radius \( R \);  
13. end if  
14. if goal unreachable then  
   15. Search for obstacles between the current position and the target position;  
   16. if Fig. 6 then  
   17. The UAV moves directly to target along \( L_1 \);  
   18. else  
   19. The UAV moves to the point \( A \) along \( \overline{L} \), then calculates \( R_A \) and moves around the obstacle in Fig. 7 with radius \( R_A \) to the point \( D \). Finally, the UAV reaches the target from \( D \);  
   19. end if  
16. end if  
5. end while

**Remark 1:** In simulation, it is easy to judge the local minimum and the unreachable goal, since the obstacle position and the target position are known. However, the obstacle position may be unknown in real experiments. Therefore, the UAV firstly moves one step around nearest obstacle within scanning range of sensor of UAV when it cannot continue moving towards the target. If the current position is a local minimum,
the balance of forces will be broken and the UAV will escape this trap. Otherwise, the UAV moves to unreachable target.

IV. CONTROLLER DESIGN BASED ON NDO

A. Mathematical model of quadrotor UAV

To track the planned path in 2D space, only the attitude angles need to be regulated. Therefore, attitude dynamics of the quadrotor subject to external disturbances are introduced here.

\[
\begin{align*}
\dot{\phi} &= (J_y - J_z)\dot{\theta} \dot{\phi} - J_r \dot{\theta} \Omega + LF_x + \bar{d}_\phi, \\
\dot{\theta} &= (J_z - J_x)\dot{\phi} \dot{\phi} + J_r \dot{\phi} \Omega + LF_y + \bar{d}_\theta, \\
\dot{\phi} &= (J_x - J_y)\dot{\theta} \dot{\phi} + f F_x + \bar{d}_\phi,
\end{align*}
\]

where \([\phi, \theta, \varphi]\) are altitude, roll angle, pitch angle and yaw angle of quadrotor, respectively; \(J_1, \chi = \phi, \theta, \varphi\) are the control inputs of the system; \(L, f, J_r, J_n, n = x, y, z\) denote the distance from rotor center to mass center, force to moment factor, inertia of each propeller, inertia of the quadrotor around \(x-, y-\) and \(z-\) axes, respectively; \(\Omega\) is the difference in angular speed of the rotors on the diagonal of the quadrotor. The terms \(\bar{d}_\phi, \bar{d}_\theta\) and \(\bar{d}_\varphi\) denote the effect of wind on the translational and rotational subsystems of the quadrotor in the form of external disturbances. Compared with brushless motor, propeller of quadrotor is very light, therefore, the terms \(\frac{L \Omega \chi}{J_x}\) and \(\frac{J_x \Omega \chi}{J_y}\) are ignored here.

B. Assumptions

To make the subsequent analysis rigorous, the following assumption is given.

Assumption 1: It is assumed that the disturbances change slowly, i.e., \(\bar{d}_\phi = d_\phi = \dot{d}_\phi = 0\).

Assumption 2: In the design of controller for the quadrotor, to avoid any singularity, we set \(-\frac{\pi}{2} < \phi < \frac{\pi}{2}\) and \(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\).

C. Observer design

Some of the involved disturbance components in (8) are redefined as \(d_\phi = J_x d_\phi, d_\theta = J_y d_\theta\) and \(d_\varphi = J_z d_\varphi\). To compensate for external disturbance, a NDO with exponential convergence is proposed. Define observation error as

\[
\begin{align*}
\bar{d}_\chi &= d_\chi - \hat{d}_\chi, \\
\end{align*}
\]

where \(\hat{d}_\chi\) is the estimate of \(d_\chi\) with \(\chi = \phi, \theta, \varphi\). Considering Assumption 1, the derivative of observation error \(\dot{\hat{d}}_\chi\) in (9) in attitude channel is obtained by

\[
\dot{\hat{d}}_\chi = -\hat{d}_\chi.
\]

Then the NDO in attitude channel is designed as

\[
\begin{align*}
\dot{\hat{x}}_\chi &= P_\chi (W_j - Y_j) - P_\chi \hat{d}_\chi, \\
\hat{d}_\chi &= x_\chi + P_\chi J_n \hat{x}_\chi,
\end{align*}
\]

where \(j = 1, 2, 3\), \(n = x, y, z\), \(W_1 = (J_z - J_y) \dot{\phi} \dot{\phi}, Y_1 = L U_2, W_2 = (J_x - J_z) \dot{\phi} \dot{\phi}, Y_2 = L U_3, W_3 = (J_y - J_x) \dot{\phi} \dot{\phi}, Y_3 = f U_4, x_\chi\) is an auxiliary variable and \(P_\chi\) is a positive gain.

Theorem 1: If Assumption 1 holds and the observer is designed as (11), then the observation error \(\bar{d}_\chi\) in (9) will exponentially converge to zero.

Proof 1: See Appendix.

Remark 1: Note that \(\hat{d}_\chi\) is not the direct estimates of \(\bar{d}_\chi\) in (8). If we define \(\bar{d}_\chi = \hat{d}_\chi / \varepsilon, \varepsilon = J_x, J_y, J_z\), then \(\bar{d}_\chi\) estimates the means of \(\bar{d}_\chi\) in (8).

D. Controller design

To address the problem of tracking control in attitude channel, a backstepping scheme is proposed. Define the tracking error of attitude as

\[
e_\chi = \chi_d - \chi,
\]

where \(\chi_d\) is desired altitude signal. Then attitude controller is designed as

\[
F_\chi = R_j + G_j (c_{x1} \hat{e}_\chi + e_\chi + c_{x2} e_\chi) + Q_j
\]

where \(R_1 = \frac{J_x - J_y}{L} \dot{\phi} \dot{\phi}, G_1 = \frac{\dot{\theta}}{L}, Q_1 = \frac{\dot{\phi}}{L}, R_2 = \frac{J_x - J_y}{L} \dot{\phi} \dot{\phi}, G_2 = \frac{J_y}{L}, Q_3 = \frac{\dot{\phi}}{L}, R_3 = \frac{J_x - J_y}{L} \dot{\phi} \dot{\phi}, G_3 = \frac{\dot{\phi}}{L}, Q_3 = \frac{\dot{\phi}}{L}, c_{x1}, c_{x2} > \frac{\lambda_\chi}{2}\) with \(\lambda_\chi\) being a positive constant and

\[
e_{x1} = -\dot{\chi} + c_{x1} e_\chi + \dot{\chi}_d
\]

Theorem 2: Under Assumption 1 and Assumption 2, for the altitude dynamics in (8), if the control input \(F_\chi\) is designed as (13), then the tracking error for desired attitude is guaranteed to converge to zero exponentially, i.e., \(e_\chi \rightarrow 0\) as \(t \rightarrow \infty\).

Proof 2: See Appendix.

V. SIMULATION

A. Comparison with TAPF

The simulation parameters including start position, target position and obstacle positions in Figs. 8-10 are listed in Tab. I. To demonstrate the effectiveness of the proposed path planning framework to deal with the traps of local minimum and unreachable goal of TAPF in Figs. 1 - 4, simulations are carried out, as shown in Figs. 8 and 9, where we can observe that the UAV can plan a feasible path to the destination with obstacle avoidance. Furthermore, the proposed algorithm is verified in complex environments with multiple obstacles in Fig. 10.

B. Comparison with BUG1, BUG2 and RRT

Considering the characteristics of the proposed algorithm, BUG1, BUG2 and RRT, and to fairly compare their abilities of path planning, the obstacles of different shapes are placed around the line connecting the start point and the target point.

The simulation parameters including the start position, target position and obstacle positions in Figs. 11-14 are shown in Tab. II. Furthermore, the comparisons with the three path planning algorithms, namely BUG1, BUG2 and RRT, are presented in Figs. 11-14. Figs. 11 and 12 show the results of path planning of UAV with a single obstacle, while the
TABLE I
THE SIMULATION PARAMETERS IN FIGS. 8-10.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fig. 8</th>
<th>Fig. 9</th>
<th>Fig. 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start position</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Target position</td>
<td>(24,24)</td>
<td>(24,24)</td>
<td>(32,32)</td>
</tr>
<tr>
<td>Obstacle1 position</td>
<td>(16,16)</td>
<td>(13,16)</td>
<td>(10,9)</td>
</tr>
<tr>
<td>Obstacle2 position</td>
<td>(22,24)</td>
<td>(16,13)</td>
<td>(11,16)</td>
</tr>
<tr>
<td>Obstacle3 position</td>
<td>(23,25)</td>
<td>(20.5,23.5)</td>
<td>(18,14)</td>
</tr>
<tr>
<td>Obstacle4 position</td>
<td>(24.7,25.2)</td>
<td>(23,25)</td>
<td>(17,24)</td>
</tr>
<tr>
<td>Obstacle5 position</td>
<td>(25.5,24)</td>
<td>(24,26.5)</td>
<td>(24,22)</td>
</tr>
<tr>
<td>Obstacle6 position</td>
<td>(25.2,23.5)</td>
<td>(25,22.5)</td>
<td>(28,30)</td>
</tr>
<tr>
<td>Obstacle7 position</td>
<td>×</td>
<td>(26.5,24)</td>
<td>(29,28)</td>
</tr>
<tr>
<td>Obstacle8 position</td>
<td>×</td>
<td>(25.3,22.5)</td>
<td>(31,28)</td>
</tr>
<tr>
<td>Obstacle9 position</td>
<td>×</td>
<td>(24.21)</td>
<td>×</td>
</tr>
<tr>
<td>Obstacle10 position</td>
<td>×</td>
<td>(23,23)</td>
<td>×</td>
</tr>
</tbody>
</table>

Fig. 8. Scenario 1: the result of path planning of the proposed algorithm.

results of path planning of UAV with multiple obstacles are shown in Figs. 13 and 14. From the results, the feasible path to the target with obstacle avoidance can be obtained when the proposed algorithm, BUG₁, BUG₂ and RRT are applied to path planning of the UAV, respectively. Tab. III shows the time consumption of the four algorithms. It is obvious that in either the environment with a single obstacle or with multiple obstacles, the time consumption of the proposed algorithm is the least. The path length in Figs. 11-14 are listed in Tab. IV, from which we find that compared with paths generated by BUG₁, BUG₂ and RRT, a shorter path is obtained using the proposed algorithm. For the environment with a single obstacle, the shape of obstacle has a greater effect on length of path provided by BUG₁ and BUG₂. Meanwhile, RRT has the worst performance in terms of time consumption and path length. Also, the UAV cannot reach the target accurately when RRT is applied to the UAV.

Overall, compared with BUG₁, BUG₂ and RRT, the proposed algorithm has advantages in time consumption and path length, which means that less time and energy are required to reach the target for the UAV.

TABLE II
THE SIMULATION PARAMETERS IN FIGS. 11-14.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fig. 11</th>
<th>Fig. 12</th>
<th>Fig. 13</th>
<th>Fig. 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start position</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
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<tr>
<td>Target position</td>
<td>(24,24)</td>
<td>(24,24)</td>
<td>(36,36)</td>
<td>(36,36)</td>
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<tr>
<td>Obstacle1 position</td>
<td>(16,15)</td>
<td>(12,13)</td>
<td>(9,9)</td>
<td>(9,10)</td>
</tr>
<tr>
<td>Obstacle2 position</td>
<td>×</td>
<td>×</td>
<td>(14,14)</td>
<td>(14,16)</td>
</tr>
<tr>
<td>Obstacle3 position</td>
<td>×</td>
<td>×</td>
<td>(19,18)</td>
<td>(20,10)</td>
</tr>
<tr>
<td>Obstacle4 position</td>
<td>×</td>
<td>×</td>
<td>(24,23)</td>
<td>(20,20)</td>
</tr>
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<td>Obstacle5 position</td>
<td>×</td>
<td>×</td>
<td>(31,30)</td>
<td>(29,17)</td>
</tr>
<tr>
<td>Obstacle6 position</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>(28,28)</td>
</tr>
<tr>
<td>Obstacle7 position</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>(35,34)</td>
</tr>
</tbody>
</table>

Fig. 10. Scenario 3: the result of path planning of the proposed algorithm in a complex environment.

Fig. 11. Scenario 1: the result of path planning of the four algorithms with an obstacle.

Fig. 9. Scenario 2: the result of path planning of the proposed algorithm.
Fig. 12. Scenario 2: the result of path planning of the four algorithms with an obstacle.

Fig. 13. Scenario 1: the result of path planning of the four algorithms with multiple obstacles.

Fig. 14. Scenario 2: the result of path planning of the four algorithms with multiple obstacles.

C. Trajectory tracking

The physical parameters of quadrotor are set as follows: $L = 0.4$ m, $J_x = 0.16$ kgm$^2$, $J_y = 0.16$ kgm$^2$, $J_z = 0.32$ kgm$^2$, $f = 0.05$ m. The initial attitude of the quadrotor is set as $0$ rad. Furthermore, to follow the planned path, the desired attitude angles need to be addressed. Fig. 11 is used as an example here, whose desired attitude states are calculated as follows: $d_\phi = 0$, $d_\theta = 1$, $d_\psi = 0$, $d_{\phi_1} = 0.785$, $d_{\phi_2} = 1.415$, $d_{\phi_3} = 2.597$, $d_{\phi_4} = 1.537$, $d_{\phi_5} = 1.865$, $d_{\phi_6} = 1.617$, $d_{\phi_7} = 1.514$, $d_{\phi_8} = 1.399$, $d_{\theta_9} = 1.288$, $d_{\theta_{10}} = 1.181$, $d_{\phi_{11}} = 1.011$, $d_{\phi_{12}} = 0.985$, $d_{\phi_{13}} = 0.813$, $d_{\phi_{14}} = 0.78$, $d_{\phi_{15}} = 0.737$, $d_{\phi_{16}} = 0.669$, $d_{\phi_{17}} = 0.607$, $d_{\phi_{18}} = 0.552$, $d_{\phi_{19}} = 0.506$, $d_{\phi_{20}} = 0.47$. It should be noted that: 1) the yaw angle is used to adjust the forward direction of the quadrotor, and the pitch angle is used to control the forward speed of the quadrotor, while the roll angle is required to maintain at $0$ rad; 2) when the yaw angle is trying to maintain one of the above states, a desired pitch angle of $0.1$ rad will be tracked and when the yaw angle is switched between the two states, the desired pitch angle is set as $0$ rad. In addition, the disturbances in attitude channels are given as

$$d_\phi = \begin{cases} 
0.4t - 0.2, & 0 \leq t < 6, \\
2.2, & 6 \leq t < 14, \\
-0.1t + 3.6, & 14 \leq t < 20, \\
-0.1t + 5.6, & 20 \leq t < 33,
\end{cases} \quad (15)$$

$$d_\theta = \cos\left(\frac{\pi}{6}t\right) + 0.4, \quad (16)$$

$$d_\psi = \begin{cases} 
1, & 0 \leq t < 5 \text{ or } 10 \leq t < 15 \cdots, \\
0, & 5 \leq t < 10 \text{ or } 15 \leq t < 20 \cdots.
\end{cases} \quad (17)$$

To verify the effectiveness of the proposed NDO, the time-varying disturbance with different frequencies are injected into the quadrotor system. The estimations of external disturbances are shown in Fig. 15, where we can see that the disturbances (15), (16), (17) can be estimated, even if the derivatives of the disturbances are assumed to be 0 in the design of the disturbance observer. However, the disturbance estimations of the pitch and yaw channels have small fluctuations.

The corresponding tracking results for the desired signals are presented in Fig. 16. Obviously, tracking and frequent switching of so many states raises a challenging problem for the quadrotor controller, especially in the pitch and yaw channels. However, compared to ADRC, the proposed controller has better tracking performance. In addition, in the roll channel, oscillation is generated in the initial stage, and a large spike is produced at the 20th second when ADRC is applied.
to the quadrotor, while a smoother tracking performance is provided by the proposed controller

Fig. 15. Estimations of external disturbances.

VI. CONCLUSION

In this paper, a novel APF algorithm based on parallel search is proposed for path planning of a UAV. An algorithm for moving around nearest obstacle is synthesized into APF to avoid the trap of local minimum. A parallel search algorithm is presented to address the problem of unreachable target. Furthermore, to achieve tracking of planned trajectory of quadrotor UAV subject to external disturbance, a backstepping control strategy with a NDO is designed. The comparative simulations are performed to verify the effectiveness of the proposed path planning algorithm and the proposed controller.

VII. DATA AVAILABILITY

The data used to support this study are included within the article.

VIII. CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

IX. ACKNOWLEDGEMENT

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X. APPENDIX

Proof of Theorem 1

Proof: From the second equation in (11), one has

\[ \dot{\hat{d}}_\chi = \dot{x}_\chi + P_\chi J_n \ddot{x}_\chi \]  

(18)

Substituting the first equation to (18) yields

\[ \dot{\hat{d}}_\chi = P_\chi (W_j - Y_j + J_n \ddot{x}_\chi) - P_\chi \dot{d}_\chi \]  

(19)

Considering the relationship \( d_\chi = J_n \bar{d}_\chi \) and applying (8) to (19), we have

\[ \dot{\hat{d}}_\chi = P_\chi (d_\chi - \hat{d}_\chi) = P_\chi \tilde{d}_\chi. \]  

(20)

Combining (10), the dynamic equation of observation error is obtained

\[ \dot{\tilde{d}}_\chi + P_\chi \tilde{d}_\chi = 0. \]  

(21)

Solving (21), we get

\[ \tilde{d}_\chi(t) = \tilde{d}_\chi(t_0) e^{-P_\chi t}, \]  

(22)

where \( \tilde{d}_\chi(t_0) \) is the initial value of the observation error. (22) indicates that the observation error \( \tilde{d}_\chi \) will exponentially converge to zero as \( t \to \infty \), i.e., \( \tilde{d}_\chi \) will exponentially converge to \( d_\chi \) as \( t \to \infty \) under Assumption 1.

This completes the proof.

Proof of Theorem 2

Proof: The whole proof is divided into two steps. Step 1: we define a Lyapunov function candidate

\[ V_{\chi 1} = \frac{1}{2} J_n e_\chi^2. \]  

(23)

The time derivative of \( V_{\chi 1} \) is

\[ \dot{V}_{\chi 1} = J_n e_\chi \dot{e}_\chi. \]  

(24)
Substituting (12) and (14) to (24), it is obtained that
\[
\dot{V}_x = J_n e_x (\dot{x}_d - \chi)
\]
\[
= J_n e_x (\dot{x}_1 + \chi - c_{x_1} e_x - \hat{\chi})
\]
\[
= - J_n c_{x_1} e_x^2 + J_n e_x e_x_1.
\]
(25)

Step 2: We define the following Lyapunov function candidate
\[
V_x = V_x + \frac{1}{2} J_n e_x^2.
\]
(26)
The time derivative of \(V_x\) is
\[
\dot{V}_x = \dot{V}_x + J_n e_x e_x_1.
\]
(27)

Combining (14) and (25), (27) can be rewritten as
\[
\dot{V}_x = - J_n c_{x_1} e_x^2 + J_n e_x e_x_1 + J_n e_x_1 (\frac{R_j}{G_j} - \frac{F_j}{G_j} + \hat{d}_x
\]
\[
+ c_{x_1} e_x^1 + \chi_d).
\]
(29)

Substituting the attitude controller (13) into (30) yields
\[
\dot{V}_x = - J_n c_{x_1} e_x^2 - J_n c_{x_2} e_x^2 + J_n e_x (d_x - \hat{d}_x).
\]
(30)
The term \(d_x - \hat{d}_x\) will vanish from Theorem 1 as \(t \to \infty\). Hence, according to (23), (26) and the relationship \(c_{x_1}, c_{x_2} \geq \frac{\lambda_2}{2}\), (30) is further derived as
\[
\dot{V}_x = - J_n c_{x_1} e_x^2 - J_n c_{x_2} e_x^2
\]
\[
= - \lambda_2 V_x - (c_{x_1} - \frac{\lambda_2}{2}) J_n e_x^2 - (c_{x_2} - \frac{\lambda_2}{2}) J_n e_x^2
\]
\[
\leq - \lambda_2 V_x.
\]
\[
(31)
\]
Solving (31), it is obtained
\[
V_x(t) \leq V_x(0) e^{-\lambda_2 t},
\]
(32)
where \(V_x(0)\) is the initial value of \(V_x(t)\). From (32), it is concluded that \(e_x\) will exponentially converge to zero as \(t \to \infty\).

This completes the proof.

REFERENCES


