Performance analysis of clustering methods for balanced multi-robot task allocations

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Performance Analysis of Clustering Methods for Balanced Multi-Robot Task Allocations

Abstract

This paper models the Multi-Robot Task Allocation (MRTA) problem with a balance constraint to improve the utilization (completion time) of the robots. Our balancing constraint attempts to minimize the travel distance difference among the robots as well as allocates an equal set of tasks to these robots. The clustering-based approach is employed to solve the Balanced Multi-Robot Task Allocation (BMRTA) problem for two principal reasons. That is, this approach clusters given tasks into groups using various clustering techniques for each robot, and sequences the route for each robot using the travelling salesman problem (TSP) conhull algorithm. This work analyses the suitability and performance of the clustering techniques with respect to the balancing criteria using a benchmark dataset. Our findings suggest that K-means clustering is the most suitable for the solving BMRTA problem with complex topologies and it is scalable to deal with any number of tasks and robots compared with Gaussian Mixtures Models (GMM) and hierarchical clustering methods.

Keywords: Balanced multi-robot task allocation problem, Clustering, Multiple travelling salesperson problem, Heuristics approach

1 Introduction

The multi-robot task allocation (MRTA) problem is a generalization of the well-known multiple travelling salesperson problem (MTSP), where more than one salesperson is involved to achieve the given tasks (Sariel and Balch, 2006). Few MTSP challenges reported in the literature are (i) how to allocate/assign group of cities to a salesperson? (ii) how to formulate MTSP using different objectives (minisum, minimax, miniave) to take care of other aspects (e.g. time) other than cost/distance? (iii) What would be the best solution approach that takes care of assignment of cities and routing of persons for a well-known non deterministic polynomial time (NP-hard) problem (Sariel et al., 2009)? To date, very few studies have proposed potential solution approaches to MTSP problems that deal with assignment using minisum objective (Bektas 2006). To the best of our knowledge, efficient solution approach for assignment and routing is very hard to find. Most of the studies modelled MTSP with minisum objective and mandate researchers to try with other objectives (Lagoudakis et al., 2005; Wei et al., 2020).

It is interesting to note from the detailed MRTA taxonomy presented by Gerkey and Mataric (2004) that due attention by researchers is on assignment of tasks rather than dealing with routing or path planning. However, grouping of tasks to each robot and the routing for tasks is essential to design, build and coordinate multi-robot systems (Trigui et al., 2017; Hashemi-Petrood et al., 2021). Therefore, it is obvious that combined allocation of set of tasks to a robot along with routing is essential for multi-robot systems (He and stecke, 2021).

One of the ways to model MRTA application to minimize the total travel distance of the team of robots is through MTSP formulation. Petersen and Madsen (2009) proposed heuristic solution approaches to solve the MTSP problem. MRTA depends on parameters such as distance and utilization (defined in terms of time required to complete various operations in multiple tasks). The difficulty of task allocation grows with the nature of task (number of operations required in single task), size (number of task) and capability of the system under study (Lee, 2018). Furthermore, direct use of MTSP formulation to MRTA application would only consider minimisation of total travel distance and not the total time taken to complete the tasks. It would lead to unbalanced allocation and it needs suitable modification in formulation to accommodate both travel distance and task completion time. Jose and Pratihar (2016) used heuristic methods for multi-robot task allocation and collision free path planning to centralized multi-robot system. Recent study by Girit. and Azizoğlu. (2020) rebalanced the assembly line considering the trade-off between workload balancing (fairness measure) and total replacement distance for the tasks assigned to the different workstations (stability measure).

Unbalanced allocation can be explained through a sample multi-robot task allocation problem which involves scheduling and sequencing of 13 tasks with 4 robots that starts from a reference point (0, 0). In this case it is evident that the MTSP solution would allot 10 tasks to one robot and one task each to the other three robots. This unequal allocation would make one robot to travel more distance than the other three. Ultimately the allocation would minimise the total travel distance of all robots whereas it will certainly increase the time for completing all tasks (Leiber and reinhart, 2020). This further overload a single robot and may be prone to fail in due course of its operation. Thus the multi-robot system becomes less robust.

The objective function should be appropriate to achieve balancing in MRTA. Tovey et al. (2005) have reported three team objective functions such as minisum, minimax, miniave for different applications. The minisum objective function tries to allocate tasks to robots based on the minimum sum of the travel distance and this objective addresses only the space coordinate (Figure 1a) and is widely used in planetary surface exploration (Berhault et al 2003, Dias and Stentz 2000, Dias and Stentz 2002; Sullivan et al., 2019). The minimax objective attempts to allocate tasks based on the minimum total completion time which minimizes the maximum path length of a robot and it is extensively used in facility surveillance, mine clearing etc.(Lagoudakis et al 2005; Sullivan et al., 2019). The minimax objective to certain extent considers both time and space coordinates (Figure 1b). The final objective measure is the miniave objective and it would minimize the average service and arrival time of all robots. It is mostly used in search and rescue applications (Tovey et al., 2005).

Klodt and Willert (2015) used the minimum spanning tree algorithm and optimize pairwise for balanced partitioning of workload. Trigui et al (2017) proposed fuzzy logic approach to solve the MTSP problem for multi-robot system by converting multi-objective into single objective problem using fuzzy logic system. Lee (2018) presented a resource-based task allocation algorithm for multirobot coordination which minimize both the task completion time as well as the resource consumption. Sullivan et al (2019) used single-item auction for heterogenous multi-robot routing with minimax objective to reduce energy usage and task completion time for both indoor and outdoor robots. Pareto artificial bee colony algorithm was proposed by Zhang et al (2019) to reduce energy consumption and enhance productivity improvement in the U shaped robotic assembly lines. Wei et al (2020) applied particle swarm optimization technique for cooperative multi-robot task allocation problem which minimize the highest travel distance of each robot.

All the recent minimax objective used for multi robot task allocation is minimize the maximum travel distance of the robot. The balancing parameter for number of task and total distance travel by each robot is not addressed. Thus, to the best of our knowledge till date all work address only the multirobot task allocation problem with minimax objective but an efficient solution approach for separate parameter for balancing of number of task and individual path balancing for each robot is not addressed. Thus, this paper formulates the problem as balanced multi robot task allocation with separate balancing parameter for balancing of number of task and individual path balancing for each robot

Most of the multi-robot exploration task problem uses the minisum objective. Lagoudakis et al (2005) suggested that the future research should validate results obtained through minisum objective with other potential objectives such as minimax objective. Thus, validation of the minimax objective solution with its minisum objective is a potential research gap in MRTA. Application of minimax objective function to MRTA application is referred in this paper as a balanced multi robot task allocation model (BMRTA).

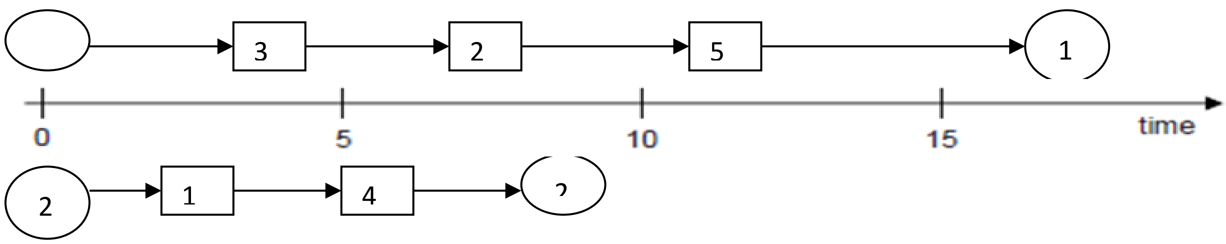

Figure 1a: Robot(○) Task(□) Distance (⟶) minisum objective (Distance unbalanced , Time maximum)

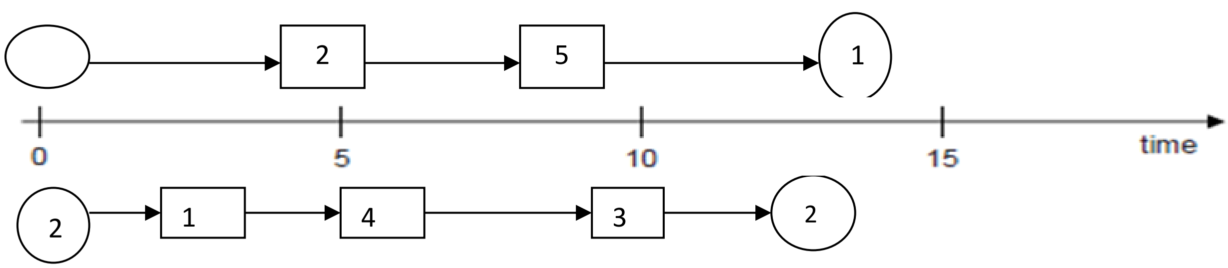

Figure 1b: Robot(○) Task(□) Distance (⟶) minimax objective (Distance balanced , Time minimum)

Figure 1: Minisum and Minimax Objective Functions (Source: Authors)

The global optimal solution for minimax objective models with Integer Programming approach is considered to be more complex (Dias et al., 2006).The literature suggests heuristics approach to solve problems of realistic sizes due to the combinatorial complexity of the MRTA problem (Dias and Stentz 2002). To date, some researchers (Chandran et al., 2006, Lagoudakis et al., 2005, Wu et al., 2010, Solanas and Garcia 2005, Chan et al., 2006; Trigui et al., 2017) meticulously used clustering-based approach to solve the MTSP and multi-robot exploration problem. MRTA problem is often solved by combinatorial optimization method, which generally requires a clustering algorithm where the tasks are grouped with some similarities (Berhault et al., 2003). Recently Elango et al (2016) used an agent-based approach that includes K-means clustering technique to reduce the search space to solve a distributed task allocation model for a dual nozzle head 3D printing machine application which is based on MRTA formulation.Thus, selecting a best cluster-based heuristic methodology either parametric or non-parametric is the emergent need for MRTA. K-means clustering and Gaussian Mixtures Models (GMM) clustering methods are chosen from the parametric type. Hierarchical clustering is chosen from the non-parametric type clustering approach to solve MRTA with minmax objective function (BMRTA). The routing for each robot is determined by a conhull TSP algorithm.

The major contributions of this paper are to solve BMRTA with (i) modified MTSP formulation to balance the workload evenly to all robots by including additional balancing constraint (ii) minimax objective and validate its closeness with the results of minsum objective, and(iii) various clustering-based methods and to select the best clustering-based method.

The rest of the paper is organized as follows. Section 2 highlights the clustering techniques used in this study. Section 3 defines the BMRTA problem and Section 4 presents the model for the BMRTA problem. Section 5 provides the solution methodologies. Section 6 applies a sample benchmark data set on the techniques. Section 7 compares and discusses the solution quality of the different clustering techniques. The paper is concluded in Section 8.

2 Clustering techniques

Clusters are formed in such a way that the objects in the same cluster are similar and objects in different clusters are distinct. The measures of similarity depend on the dataset. Clustering techniques are broadly classified as parametric and non-parametric (see Figure 2).

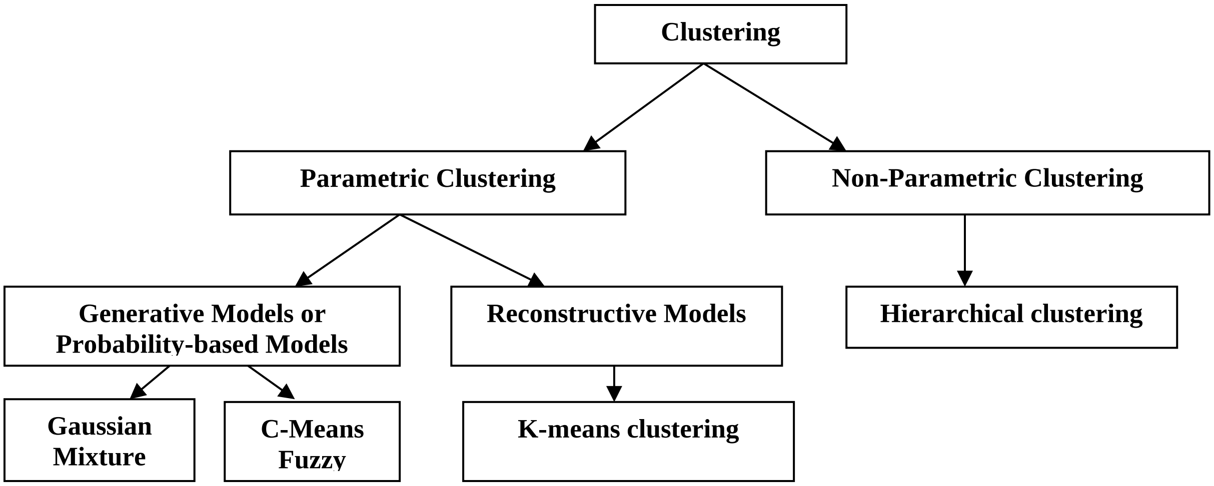

Figure 2: Taxonomy of Clustering Approaches (Source: Authors)

Parametric clustering further divides into two groups: generative or probability-based models and reconstructive models. The Gaussian mixture and C-means fuzzy are the most popular methods under the generative models category. The K-means method falls under the reconstructive models category while hierarchical clustering falls under the non-parametric clustering category. Of the two methods in generative models, the Gaussian mixture clustering method outperforms the fuzzy C-means method on computational time in most cases (Fung et al 2010). For comparison, one method under each classification, i.e. K-means, Gaussian mixture, and the hierarchical clustering is taken toanalyse the BMRTA problem.

K-means clustering, or the partitioning method, partitions the data into K mutually exclusive clusters, and returns the index of the cluster to which it has assigned under each observation. Unlike hierarchical clustering, K-means clustering operates on actual observations (rather than the larger set of the dissimilarity measures), and creates a single level of clusters. This distinction makes K-means clustering more suitable than hierarchical clustering for large datasets (Oyelade et al., 2010). Al-sultan (1997) successfully applied K-means for a large part family problem in a group technology scenario to find the best grouping of parts into families such that the parts within each family are as similar to each other as possible. Further, K-means algorithms are capable of manipulating very large data sets efficiently (Papamichail and Papamichail, 2007, Ünler et al.,2009).Clustering method coupled with an auction mechanism is applied in the context of multi-robot task allocation (Elango et al., 2011).Recently Das et al., (2016) uses PSO-DV algorithm for multi-robot path planning. (Francoisa et al., 2016) proposed ALNS heuristics method for solving multi-trip vehicle routing problem.

The hierarchical clustering method groups data over a variety of scales by creating a cluster tree or dendrogram. The tree is not a single set of clusters, but rather a multilevel hierarchy, where the clusters at one level are joined as clusters at the next level. This allows one to decide on the level or scale of clustering that is most appropriate for an application (Suib and Deris, 2008). Research has attempted to improve the performance of the K-means clustering and hierarchical clustering by combining it with meta-heuristics such as ant colony optimisation, particle swarm optimization (Azzag et al., 2007). By combing with meta-heuristics makes it amenable for managing the tasks clustering for a complex, irregular, and high dimensional data set. Hierarchical and non-hierarchical clustering techniques (with several proximity functions) are also integrated with a sequential heuristic algorithm to solve location routing problems (LRP) and the performance of these technique are analyzed to guide the choice of a suitable clustering technique for solving the LRP (Barreto et al., 2007).

Gaussian Mixture Models (GMM) form clusters by representing the probability density function of observed variables as a mixture of multivariate normal densities. Mixture models of the GMM distribution class are fitted to data using the expectation maximization algorithm (Kavaliauskas and Rudzkis, 2005), which assigns posterior probabilities to each component density with respect to each observation. Clusters are then assigned by selecting the component that maximizes the posterior probability. GMM is preferred to K-means clustering when the clusters have different sizes and correlations exist within them (Fraley and Raftery, 2002).

In general, for overlapping clusters, the increase in the number of clusters and variables (dimensions) decreases the performance of the clustering algorithm. Also K-means and hierarchical clustering methods are good performers and are simpler to implement when compared with other methods (Mingoti and Lima, 2007). Hence, this paper seeks to compare the solution quality obtained by different clustering techniques such as K-means, GMM, and Hierarchical clustering through solving the BMRTA problem. The aim of this study is to identify the best of three techniques for a given problem size. This paper also analyzes the suitability of a particular technique based on the balancing value.

3 Problem definition

The MRTA problem requires a team of robots to perform several tasks. Our work is concerned with the minimization of total distance travel and minimization of total completion time for all robots, This paper consider the problem as a BMRTA problem. Balancing means that (i) the total travel made by all the robots should be about equal, and (ii) the number of tasks allocated to the robots should be the same. Balancing the total travel of the robots is captured as the ratio α_p of the minimum travel made by all the robots to the maximum travel made by all the robots. In the case of the number of task allocation, balancing is represented as the ratio α_n of the minimum number of tasks allocated to a robot to the maximum number of tasks allocated to a robot. Balancing is introduced as a constraint while solving the MRTA problem. This paper refers the current problem as the Balanced Multi-Robot Task Allocation (BMRTA) problem. If the balancing value is “0”, then the problem reduces to a standard MRTA and if the balancing value is “1”, then it is said to be fully balanced. The initial analysis for the various benchmark problems (http://vrp.atd-lab.inf.puc-rio.br/index.php/en/) has been carried out for single fixed balancing value α_p/α_n to analyze the suitability of a particular clustering technique. Balancing concept has been used by Elango et al (2016) to reduce uneven allocation and idleness of nozzles between nozzle heads in a 3D printer. However, this paper focuses on BMRTA problem that is applicable to space exploration application where the distance travelled by multiple robots is directly proportional to the consumption of battery energy. The predominant requirement for this application is to reduce energy consumption or minimize total distance travelled by each robot.

The location of ‘m’ robots and ‘n’ tasks, as well as a distance ‘d’ that specifies the distance to be moved from one location to another. The objective is to find an allocation of tasks to each robot (see Figure 3a) and a path for each robot to visit the tasks allocated to it (Figure 3b), so that the total travel distance is minimized and the balancing value is within the given range for better robot utilization. Each robot starts from its depot (multi depot i.e. the start task location of each robot) and visits a set of tasks.

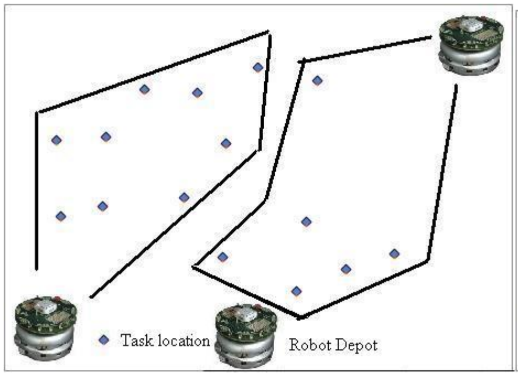

Figure 3a: Task allocation for a robot

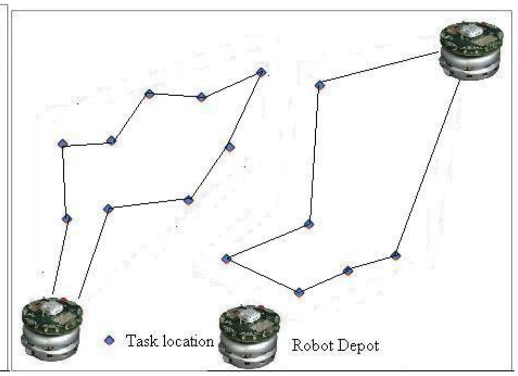

Figure 3b: Path planning for a robot

Figure 3: Task Allocation and Path Planning for a robot (Source: Authors)

4 Model formulation

The model minimizes the total travel distance travelled by the robots. While minimizing the total travel distance the model ensures that all the robots travel approximately the same distance as well as assigns the number of tasks approximately equally between the robots. The model is formulated to enhance the balanced multi-robot task allocation by considering both path balancing and the number of tasks balancing as constraints. This formulation is different from Elango et al. (2016)'s where the model is solved as MTSP problem for each task location. Thus, takes more time to solve the model. Here we cluster the task location in exact number of robot and allocate one cluster for each robot make the model simpler and solve quickly.

<u>Indices and parameters</u>

$k = \{1, 2, \ldots, K\}$ a set of robot
$n = \{1, 2, \ldots, N\}$ be a set of clusters
v = {1, 2, . . . , V} be a set of depot vertices
$n_k = \text{T}otal\ number\ of\ tasks\ assigned\ to\ robot\ k$
$d_k =$ Total distance travelled by the robot k

$$x_{k,n} = \begin{Bmatrix} 1 & if\ \ robot\ k\ \ assigned\ to\ \ cluster\ n \\ 0 & otherwise \end{Bmatrix}$$

$$minimize \sum_{k \in K} \sum_{n \in N} d_k x_{k,n} \tag{1}$$

Subject to

$$x_{k,n} \in \{0,1\} \;\; for\; n = 1,..,N\; and, k = 1,...,K \tag{2}$$

$$\sum_{k \in K} x_{k,n} = 1 \;\; for \quad n = 1,....,N \tag{3}$$

$$\frac{\min_{k \in K}(d_k)}{\max_{k \in K}(d_k)} \geq \alpha_p, \;\; 0 \leq \alpha_p \geq 1 \tag{4}$$

(OR)

$$\frac{\min_{k \in K}(n_k)}{\max_{k \in K}(n_k)} \geq \alpha_n, \;\; 0 \leq \alpha_n \geq 1 \tag{5}$$

$$\sum_{i \in V} x_{i0} = K \qquad \text{(Only for Skewed data)} \tag{6}$$

$$\sum_{j \in V} x_{0j} = K \qquad \text{(Only for Skewed data)} \tag{7}$$

The objective function (1) is to minimize the total distance travelled by robots. Constraint (2) state the condition of the binary decision variable $x_{k,n}$.The constraint (3) enforce that each cluster is assigned to exactly to one robot. Constraint (4) applies when the objective is to balance the travel made by the robots and to determine the extent to which the K robots are utilized (i.e. when the 'α_p' value is 0, the model is suitable for the minimization of total distance without balancing, when the alpha 'α_p' value is 1 the model is suitable for balancing the path (minimize the travel distance difference between robots) without considering the number of tasks allocated to each robot. Constraint (5) is applicable when the objective is to balance the number of tasks allocated to a particular robot and to determine the extent to which the n tasks are shared between the robots (i.e. when 'α_n' is '0', the model is suitable for minimizing the total distance without considering the number of tasks allocated to each robot whereas the 'α_n' value of 1 represents the model is suitable for an equal sharing of the number of tasks to each robot and not for minimization travel distance difference). In a single setting, either constraint (4) or constraint (5) would be used one at a time. Constraints (6) and (7) say that the number of vehicles leaving the depot is the same as the number entering. This constraint (6) and (7) is added to solve when task location is skewed towards any one depot. The model formulated incorporates the capability of both minisum and minimax objective functions, if the balancing constraint (4) and (5) is taking a value very near to zero then the formulation is for minisum. This minisum formulation is solved mathematically and the results are compared. But since optimal solution of minimax formulation is NP hard, the proposed clustering method is used for solving the minimax formulation.

5. Method of clustering

The proposed K-means, GMM, and hierarchical clustering methods for solving BMRTA consider two parameters namely travel distance and robot utilization (task completion time), and thereby strikes a good balance between minimizing the travel distance difference and efficient sharing of the workload. The flow chart for the proposed methods is shown in Figure 4 and explained below.

a. The K-means clustering method is composed of the following steps:

   i. Place cluster centroid (m1 and m2) points into the space represented by the objects that are being clustered. These points represent the initial group centroids.
   ii. Assign each object to the group that has the closest centroid.
   iii. When all objects have been assigned, recalculate the positions of the centroid (m1 and m2) to get the required α_p, α_n parameters, used to calculate the balance value.
   iv. Repeat Steps 2 and 3 until the centroids no longer move. The iterative process will try to separate objects into groups from which the parameter to be minimized is calculated.

b. GMM uses an iterative algorithm and is composed of the following steps:

   i. Assign each data point to a cluster based on probability distribution function score that includes the closeness of data point to mean for a mixture of Gaussian distributions.
   ii. Examine the task allocation to a cluster as per the balancing (α_p, α_n) parameters and finalise the task allocation to the clusters.

c. The hierarchical clustering consists of the following steps:

   i. Consider each data set as a cluster
   ii. By Single linkage, two clusters with the *closest minimum distance* are merged. This process repeats until there is only a single cluster left. (Root cluster) (Figure 5a).
   iii. The Hierarchy of the cluster linkage is stored as dendrogram (Figure 5b). Each vertical line represents a cluster

iv. The horizontal line drawn at different y values cut the vertical linkage line which gives required number of clusters Cut the hierarchical linkage tree for the suitable y values to get the required α_p, α_n value.

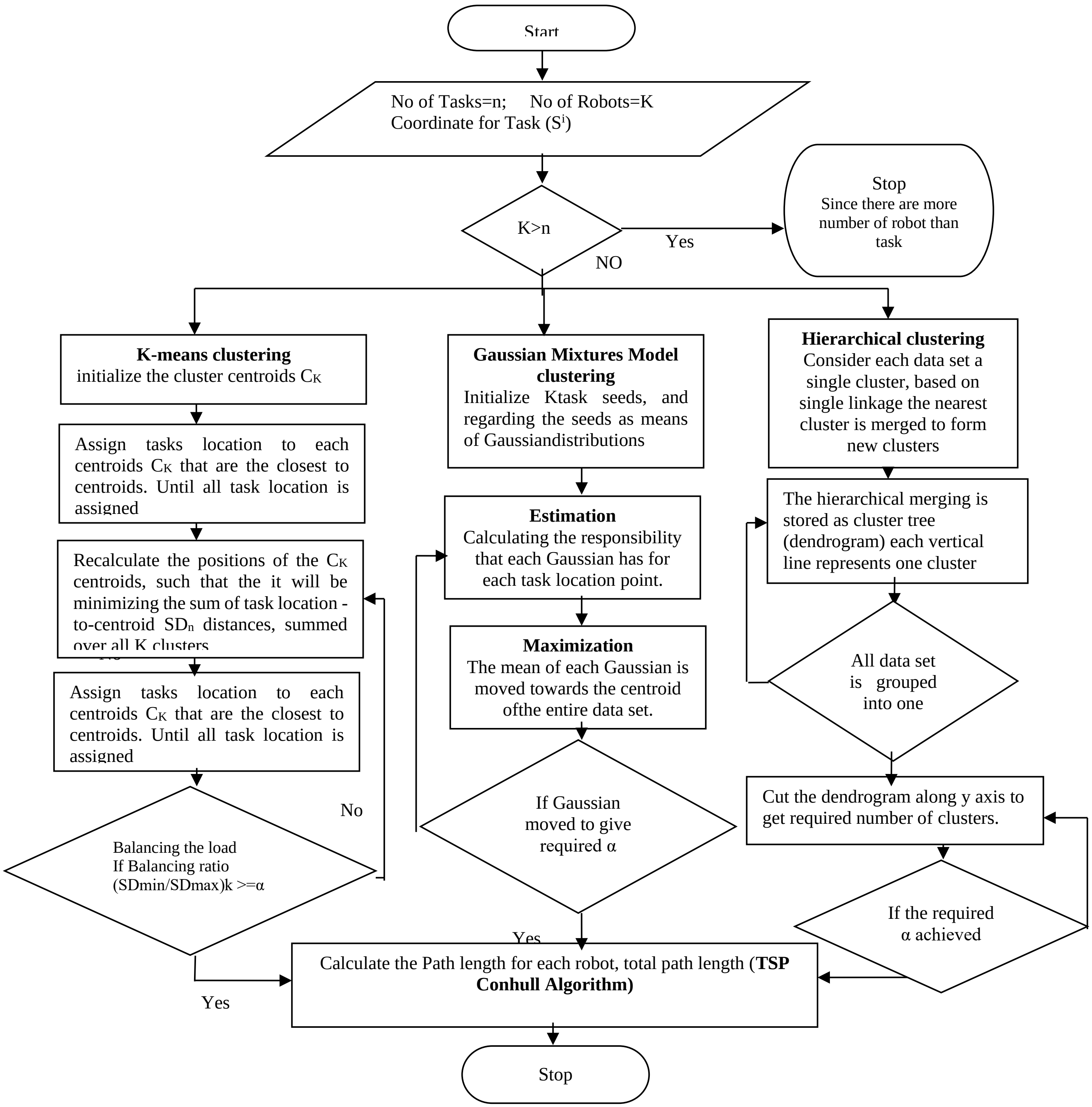

Figure 4: Proposed clustering methodologies for balanced task allocation and path minimization for multi robot systems (Source: Authors)

After the clustering phase the clusters are allocated to the robot iteratively. The routing within each robot is determined by a conhull TSP algorithm (Deineko et al., 1994). Despite the difficulty of TSP problems, most studies have reported that human performance on visual versions compares well with computational procedures (Vickers et al., 2001). The convex hull is a boundary, so that no line joining any two nodes in the array can fall outside it. First, these arrays were randomly generated, the number of interior points would be confounded with the total number of points in each array. Finally, the convex hull boundary is drawn to give the route for each robot.

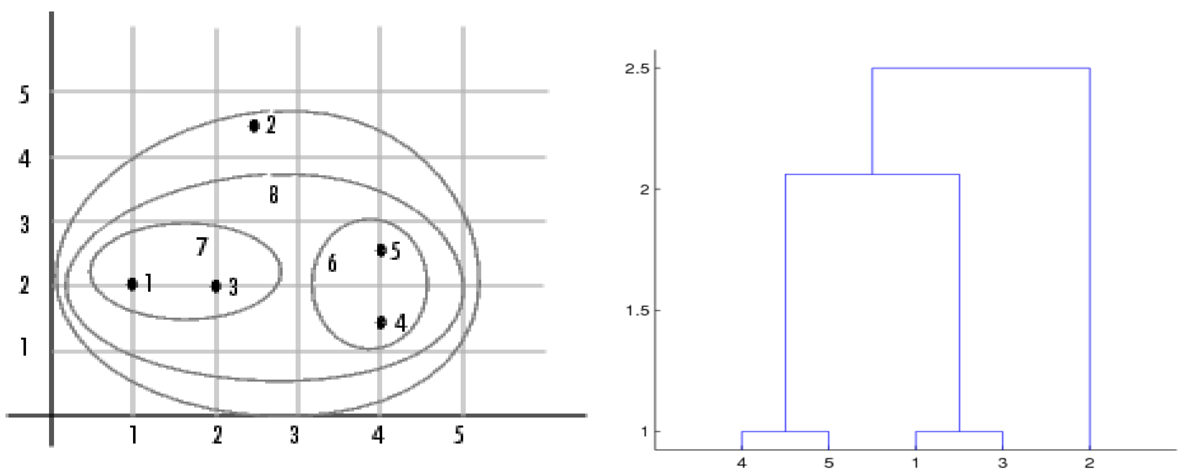

Figure 5a:Merging of atomic clusters into Figure 5b:Hierarchical cluster tree orlarger and larger clusters (MATLAB, 2008)dendrogram (MATLAB, 2008)

Figure 5: Clustering Methods (Source: Authors)

6. Numerical example

The dataset for the clustering techniques used for solving the BMRTA problem is obtained from Chandran et al. (2006). Chandran et al. (2006) proposed clustering heuristics to solve mTSP problems with dataset shown in Table 1. Since it is an mTSP data set, the same dataset for our BMRTA formulation to verify the applicability of the clustering techniques.The dataset comprises 15 task locations (x and y coordinates are shown in Table 1). The program is coded in Matlab Version 7.0 and runs on a PC Pentium-D 2.80 GHz processor.

Table 1: Data sets (Si) for task coordinate (Source: Chandran et al., 2006)

Tasks	x	y
1	4	61
2	40	43
3	61	84
4	29	77
5	86	20
6	5	37
7	78	80
8	44	76
9	18	62
10	72	13
11	17	40
12	52	60
13	75	35
14	100	25
15	51	24

The proposed method consists of two phases. In the first phase, the tasks are assigned to each robot by dividing the tasks into groups which are well separated to give a good distance buffer between the robots and to avoid inter robot collisions. In the second phase, each robot is moved to the first task location of each cluster. Thus, the first node of each cluster acts as a depot for that robot or each robot will move to the nearest task location of a cluster from the current robot location. Finally, the TSP conhull algorithm finds a non-intersecting path for each robot. Figures 8a and 8b show the allocation of tasks and the path to 2 robots and 3 robots respectively with balancing value $\alpha_p = 0.7$, using the K-means clustering combined with the TSP conhull algorithm. Similarly, Figure 6c shows the allocation and path for 2 robots using GMM combined with the TSP conhull algorithm and hierarchical clustering combined with the TSP conhull algorithm. Whereas, Figure8d demonstrates the allocation and path for 3 robots using GMM combined with the TSP conhull algorithm. The results by applying the balancing value $\alpha_p/\alpha_n = 0.7$ to all the clustering-methods with for minimizing both the travel distance difference between the robots as well as assigning the number of tasks equally to a robot in a team are shown in Table 2a. It is interesting to note that for the given dataset, the K-means algorithm performs well on achieving a balancing value $\alpha_p/\alpha_n > 0.7$ but the other two techniques cannot satisfy the balancing constraint.

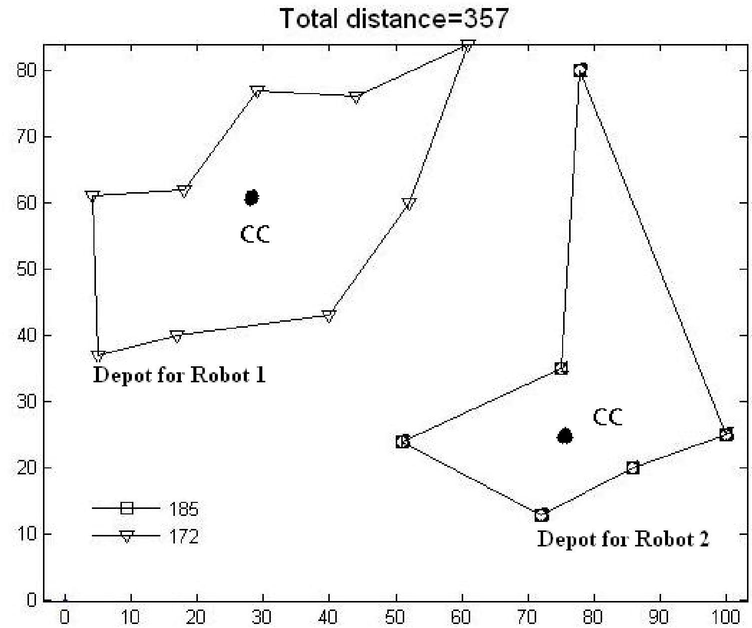

Figure 6a: Path for 2 Robots using K-means clustering: Input K=2 (Δ,□), n=15, task location as per S^i, the cluster centre (CC) is the point through which the tasks are brought to a positive Synergies. This reduces the computational complexity of combinatorial optimization in the MRTA problem (Source: Authors)

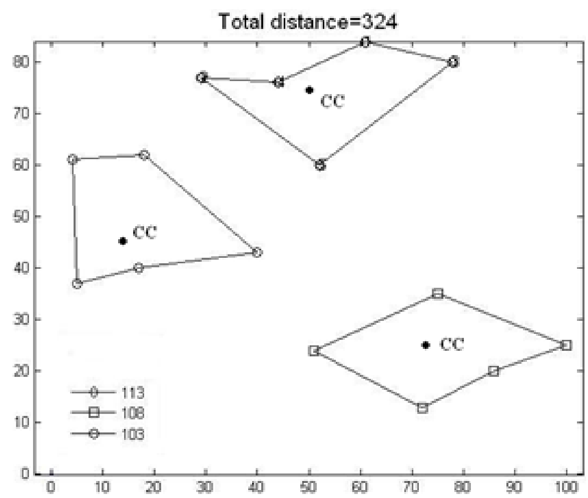

Figure 6b: Path for 3 Robots (refer Table 2a), K-means clustering: Input K=3 (◊,□,o), n=15, task location as per S^i (Source: Authors)

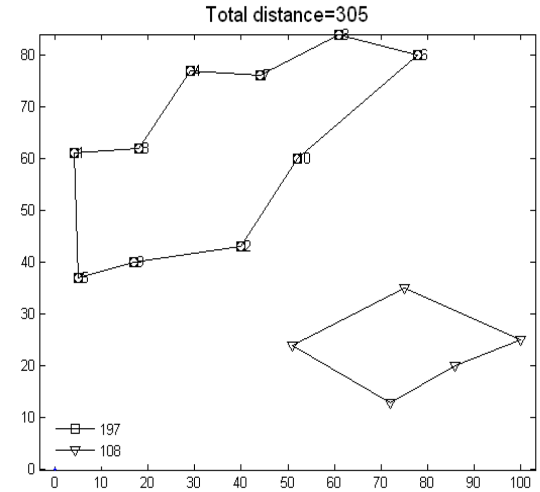

Figure 6c: Path for 2 Robots using GMM clustering and Hierarchical clustering, input K=2 (Δ,□), n=15, task location as per S^i (Source: Authors)

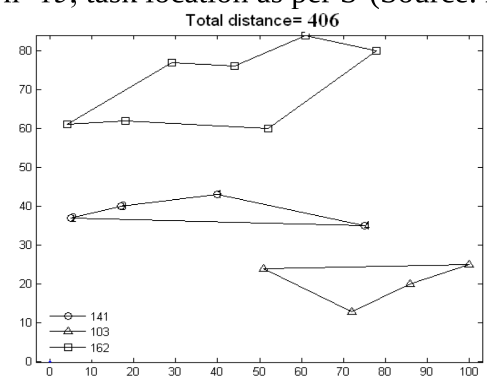

Figure 6d: Path for 3 Robots using GMM clustering, input K=3 (Δ,□,o), n=15, task location as per S^i (Source: Authors)

Figure 6: Different Paths for Different Settings

7. Results and discussion

Based on the results obtained, it is decided to check the results obtained by different clustering methods for it suitability with respect to balancing ratio for larger datasets. Since the BMRTA problem is our new formulation and it requires task locations (i.e. x and y coordinates of each task location) to validate the results with other types of formulations. But in order to verify the validity of the proposed methods different kinds of standard data from vehicle routing problem library (CVRPLIB) is consider as the mission map (http://vrp.atd-lab.inf.puc-rio.br/index.php/en/,A-n33-k5.vrp; A-n53-k7.vrp; A-n80-k10.vrp,P-n101-k4.vrp). The proposed methods are further evaluated by solving larger BMRTA problems and to check the stability of the results when the number of robot increases. All the datasets are solved with a balancing value $\alpha_p/\alpha_n >= 0.7$ for minimizing both the travel distance difference between the robots as well as equally assigning the number of tasks to a single robot in a team. The values (in bold) in Table 2b indicate that their values are greater than the set balancing value of 0.7. The capacity constraint has not been considered in the benchmark problem. The results obtained by the three clustering methods are shown in Tables 2b. The findings from the analysis is presented below:

- When the number of robots is 2 or 3, it is found that the K-means method is superior in terms of achieving both path balancing and the number of task balancing when compared to the other two methods.
- When the number of robots increases more than three (i.e., more resources), the K-means method yields better results in terms of path balancing rather than the number of tasks balancing.
- For only the A-n80-k10.vrp dataset, GMM performs well in terms of the balancing ratio when the number of robots is increased to 5. For task locations skewed towards depot GMM clustering method satisfies well both the balancing constraints for the sample dataset (B -n57-k7.vrp) considered in the study.
- On hierarchical clustering, the results are mixed and in only one case (i.e., A-n80-k10.vrp) the method is capable of yielding good results in terms of the balancing value.

It is evident from Table 2b that the K-means clustering method has consistent path balancing values when the number of robots is increased from 2 to 5. It is obvious that K-means clustering performs well for larger datasets, consisting of 101 task locations. Thus, the K-means clustering is most suitable and scalable in terms of the number of tasks and robots. These finding reveals that the balancing value is critical and it has a significant role in the performance of the method considered. When try to minimize the travel distance difference between the robots, it increases total path length and vice versa. A similar outcome applies for balancing the number of tasks allocated to a single robot. Hence, a trade-off has to be made when fixing the balancing value. A recent study by Nícolas et al (2021) considered multi-skilled workforce scenario for allocation in an assembly line balancing problem. The decision maker has to balance efficiency versus utilization and can choose a method for analysis. In this paper, the balancing value vary from 0 to 1 to see how the various clustering-methods perform. This analysis is useful to determine the required level of balancing and the preferred technique for a given dataset. Table 2c shows the results for the various balancing values. Figure9a and Figure 7b display the performance of a particular clustering technique with respect to path balancing and the number of tasks. For a given dataset, K-means clustering is able to optimize the difference in travel distance between the robots whereas, for balancing the number of tasks assigned to the robots, GMM and hierarchal clustering perform better.

To check the performance of proposed method for task location skewed towards depot. The VRP benchmark data set B -n57-k7.vrp is considered, which consists of 57 data set shown in Figure 8. The result for the data set where task location skewed towards depot is listed in table 2d. It has been shown for the table 2d that the model also solve the task location skewed towards depot for the required balancing ratio greater than 70 %.. Thus the model can be used for both unifomely distributed and also for task locations skewed towards depot cases also.

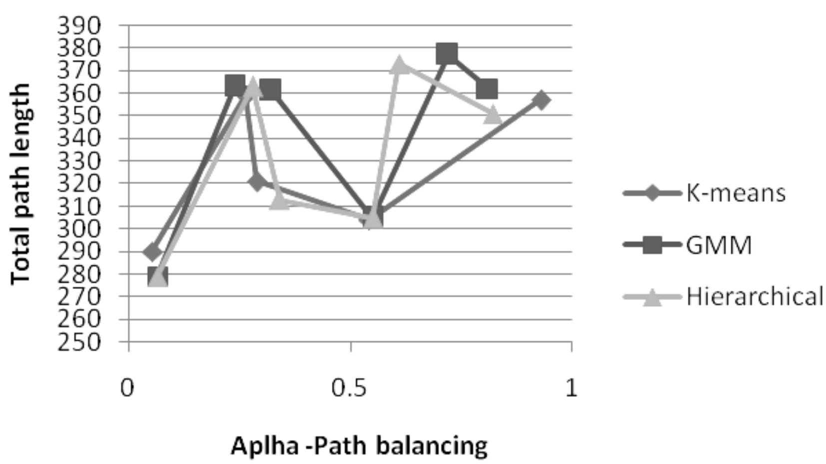

Figure 7a: Based on alpha path balancing(Source: Authors)

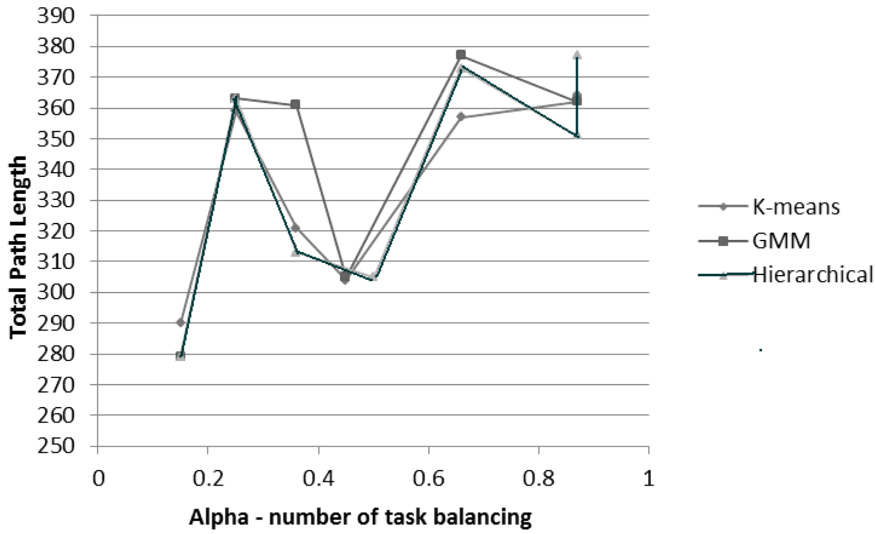

Figure 7b: Based on alpha-number of task balancing

Figure 7: Comparison of methods based on balancing value (α_n)(Source: Authors)

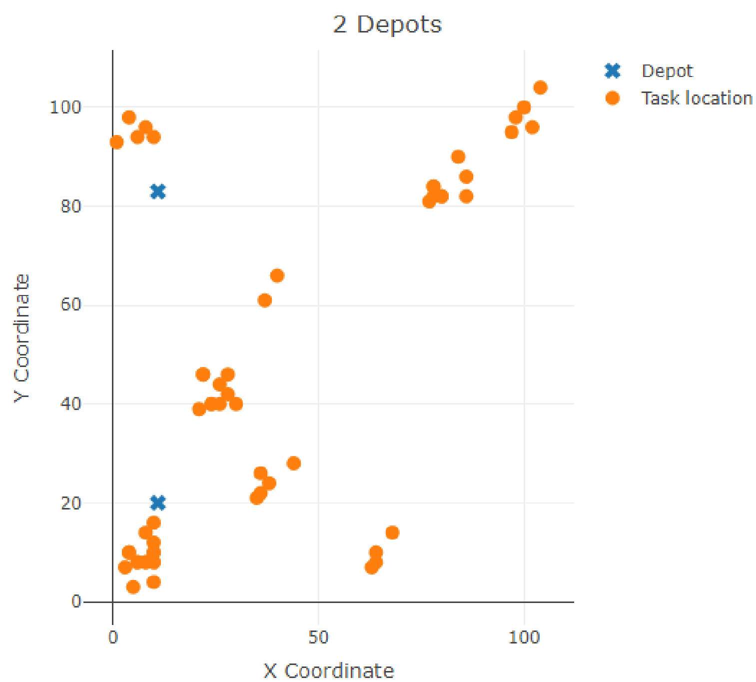

Figure 8: Task location skewed towards depot for the data set B -n57-k7.vrp

Table 2a: Clustering results for data set (S^i 15 task Problem) (Bold value indicates α > = 0.7)

No of robots	K-means					GMM					Hierarchical				
	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing
2	**185-172**	**9-6**	**357**	**0.93**	0.67	108-197	5-10	305	0.55	0.50	108-197	5-10	305	0.55	0.50
3	**103-113-108**	**5-5-5**	**324**	**0.91**	**1**	162-103-141	7-4-4	406	0.64	0.57	72-141-108	4-6-5	321	0.51	0.67

Table 2b: Clustering results for different data set (Bold value indicates α > = 0.7)

Data set	No of robots	K-means					GMM					Hierarchical				
		Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing
A-n33-k5.vrp	**2**	**236-216**	**18-15**	**452**	**0.92**	**0.83**	**238-213**	**17-16**	**451**	**0.90**	**0.94**	116-332	11-22	448	0.35	0.50
	3	**175-134-145**	**15-5-13**	**454**	**0.77**	0.33	**175-134-145**	**15-5-13**	**454**	**0.77**	0.33	217-134-116	17-5-11	467	0.53	0.29
	4	183-105-81-160	12-5-6-10	529	0.44	0.42	134-175-85-108	5-15-7-6	502	0.49	0.33	--	--	--	--	--
	5	122-105-138-55-104	10-4-9-5-5	524	0.40	0.40	68-175-85-105-160	3-15-7-4-4	593	0.38	0.27	--	--	--	--	--
A-n53-k7.vrp	**2**	**322-305**	**27-26**	**627**	**0.95**	**0.96**	377-219	35-18	596	0.58	0.51	144-444	13-40	588	0.32	0.32
	3	188-147-256	15-15-23	591	0.57	0.65	219-144-226	18-13-22	589	0.63	0.59	--	1-39-13	Tour not possible	--	--
	4	73-144-170-219	7-13-15-18	606	0.33	0.39	144-116-109-240	13-11-9-20	609	0.45	0.45	--	17-22-1-13	-do-	--	--
	5	108-170-73-128-166	9-15-7-10-12	645	0.42	0.47	118-169-166-147-105	8-15-10-10-10	705	0.62	0.53	--	--	--	--	--
A-n80-	**2**	**443-362**	**49-31**	**805**	**0.82**	**0.96**	575-246	59-21	821	0.43	0.36	279-506	32-48	785	0.55	0.66
	3	**307-274-237**	**32-25-23**	**818**	**0.77**	0.65	84-240-509	8-19-53	833	0.16	0.15	**225-310-279**	**23-25-32**	**814**	**0.72**	**0.72**

	4	**194-211-184-251**	**18-21-19-22**	**840**	**0.73**	0.39	308-193-180-162	31-18-21-10	843	0.52	0.48	105-181-225-279	9-16-23-32	790	0.38	0.28
	5	**194-137-183-158-184**	**18-13-19-11-19**	**856**	**0.71**	0.47	**150-202-179-164-169**	**15-16-18-15-16**	**864**	**0.74**	**0.83**	134-145-105-181-225	16-16-9-16-23	790	0.46	0.39
P-n101-k4.vrp	2	**406-284**	**60-41**	**690**	**0.70**	0.68	**304-388**	**50-51**	**692**	**0.78**	**0.98**	-	1-100	Tour not possible	-	--
	3	274-210-188	40-38-23	672	0.69	0.57	187-381-133	26-50-25	701	0.35	0.50	--	--	--	--	--
	4	**171-192-181-136**	**21-34-26-20**	**680**	**0.70**	0.59	239-94-169-181	30-14-24-33	683	0.39	0.42	--	--	--	--	--
	5	115-168-139-139-168	14-20-25-23-19	729	0.68	0.56	155-194-187-58-146	18-24-34-10-15	740	0.30	0.29	--	--	--	--	--

Table 2c: Trade-off for various ‘α’ values for 2 robots and 15 tasks problem using three clustering methods

Number of tasks in clusters	K-means				GMM				Hierarchical				Optimal
	Cluster path length	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Total Path length	Path Balancing	Number of task balancing	Lingo minimax
13-02	278-15	293	0.054	0.15	262-17	279	0.065	0.15	262-17	279	0.065	0.15	286.09-14.87
12-03	284-75	359	0.264	0.25	292-71	363	0.24	0.25	284-79	363	0.28	0.25	
11-04	249-72	321	0.289	0.36	273-88	361	0.322	0.36	233-80	313	0.34	0.36	
10-05	197-107	304	0.543	0.50	108-197	305	0.55	0.50	108-197	305	0.55	0.50	
09-06	172-185	357	0.929	0.66	219-158	377	0.72	0.66	141-232	373	0.61	0.66	
08-07	200-162	362	0.81	0.87	200-162	362	0.81	0.87	193-158	351	0.82	0.87	
07-08	164-200	364	0.82	0.87	140-205	345	0.68	0.87	199-178	377	0.89	0.87	
06-09	137-231	368	0.60	0.66	137-231	368	0.60	0.66	212-129	341	0.61	0.66	

Table 2: Results on Different Datasets

Table 2d: Clustering results for data set where task location skewed towards depot (B -n57-k7.vrp)
(Bold value indicates α > = 0.7)

No of robots	K-means					GMM					Hierarchical				
	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing	Cluster path length	Number of tasks in clusters	Total Path length	Path Balancing	Number of task balancing
Robots starts from depot location (11,83)															
2	**311.50 - 287.44**	**35-21**	**589.94**	**0.92**	0.6	**279.98-355.70**	**30-26**	**635.68**	**0.79**	**0.87**	370-220	42-14	590.6	0.59	0.33
3	311-213-111	35-14-7	636	0.36	0.2	283-172-207	26-14-16	662.5	0.61	0.54	346-220-41	37-14-5	608	0.11	0.13
Robots starts from depot locations (11,83) and (11,20) respectively															
2	**238.24-287.44**	**35-21**	**525.69**	**0.83**	0.6	**240.93-346.30**	**30-26**	**587.24**	**0.70**	**0.86**	370-12-220.53	42-14	590.65	0.60	0.33
3	238.25-213.03-111.36	35-14-7	562.64	0.47	0.2	283.22-172.39-48.98	26-14-16	504.59	0.17	0.54	326.48-220.53-41.47	37-14-5	588.48	0.12	0.13

The aim is to validate the proposed method with respect to a specific exploration application.To verify the closeness ofour results with the conventional minisum objective function a sample data set with 2robots starting from the reference (0,0) that visits 15 task locations. The LINGO solver(LINGO,a computer-aided optimization software that solves linear, non-linear, and mixed integer linear and non-linear programs, http://www.lindo.com)is used to test for the conventional objective function. The total travel distance of 453.78 meters is obtained for 2 robots with LINGO solver for conventional minisum objective function. Our proposed K-means clustering method with minimax objective function yielded 531.70 meters as the total travel distance which is 17.16 % higher than minisum objective as shown in Table 3a. The total time to complete all tasks is 23.29% less than the conventional objective function with 0.05m/sec as average velocity of robot (NASA Mars mission website). The similar pattern of results when the number of robots is increased from two to three as shown in Table 3b. Therefore, it can be concluded that the solution obtained through minimax objective function using the proposed clustering-based approach method is close to its optimal minisum solution. It is interesting to notice the computational complexity for realistic problem sizes (2 robots and 80 tasks) it is hard to get minisum optimal solution with conventional LINGO solver after running the program for 3.16 hours.In conclusion, it is viable to get optimal solution for MRTA problems with existing solvers for limited number of task locations and robots. On the other hand, when the number of task locations and robots are greatly increased the clustering-based methods would be more appropriate to solve MRTA and BMRTA formulations in order to achieve solutions closer to minisum objective.

Table 3: Validation of proposed method for different settings

Table 3a: Validation of proposed method for 2 robots and 15 tasks problem (depot location for two robots is (0,0))

No of robots	K-means path length in meters			Lingo path length in meters (minisum)			% deviation from minisum $\frac{\lvert Dmin-Dkm\rvert}{Dmin}$	Total Time for robot average velocity of 0.05 m/sec		% Saving in time from minisum $\frac{\lvert Tmin-Tkm\rvert}{Tmin}$
	Robot 1	Robot 2	Total Dkm	Robot 1	Robot 2	Total Dmin		K-means sec Tkm	Minisum Sec Tmin	
2	240.91	290.79	531.70	379.11	74.67	453.78	17.16	5815.8	7582.2	23.29

Table 3b: Validation of proposed method for 3 robots and 15 tasks problem (depot location for two robot is (0,0)

No of robots	K-means path length in meters				Lingo path Length in meters (minisum)				% deviation from minisum |Dmin-Dkm| / Dmin	Total Time for robot average velocity of 0.05 m/sec		% Saving in time from minisum |Tmin-Tkm| / Tmin
	Robot 1	Robot 2	Robot 3	Total Dkm	Robot 1	Robot 2	Robot 3	Total Dmin		K-means sec Tkm	Minisum Sec Tmin	
3	245.77	171.12	213.37	630.26	74.67	86.92	372.08	533.68	18.1	4915.4	7441.6	33.94

The Balanced multirobot task allocation solution approach would be highly helpful to the mission managers to balance the mobile-robot route planning when they are more concerned with utilisation of robots. It is evident from Table2a-2b that the total path length for the same mission map depends on both number of task location and number of robots. It is interesting to note direct positive relationship between total path length and number of robots as well as the direct positive relationship between total path length and number of task locations. Total path increases when there is a change in one variable irrespective of other as shown in Figure 9. Few possible future extensions are (i) usage of suitable other decomposition algorithms to convert the non-linear model into linear, and(ii) potential to use meta-heuristics to group tasks and assign robots to each task group.

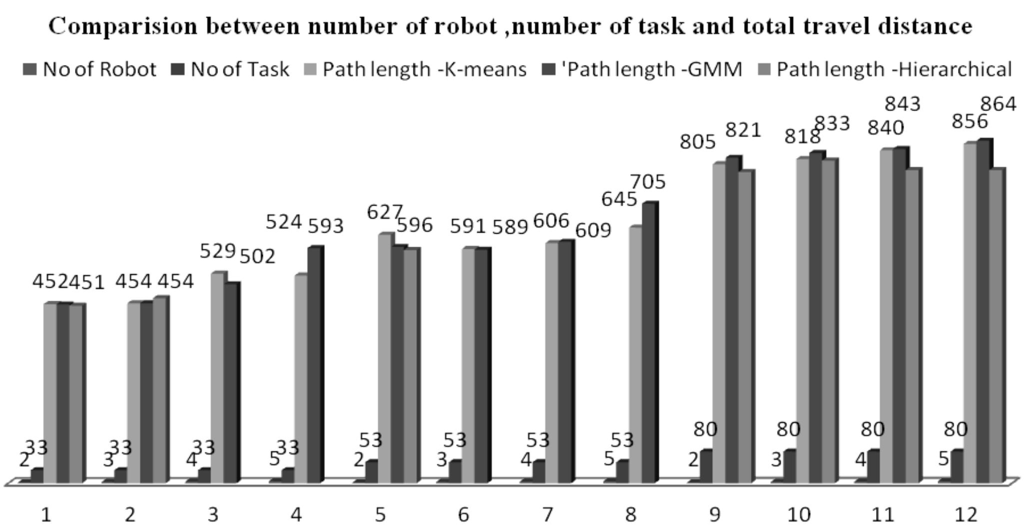

Figure 9: Total path length perfomace comparision (Source: Authors)

8 Conclusion

This paper applies the concept of utilization (task completion time) and incorporates it as an important criterion in the MRTA problem and develops a BMRTA model to balance the travel distance among a team of robots and simultaneously minimizes the total tasks completion time. Clustering based techniques are used to solve the BMRTA problem. Three clustering methods are chosen to assign tasks to robots and they simultaneously try to balance the distance and minimize the total travel length to achieve the minimax objective. It is found that the minimization of total travel distance is also close to minisum objective valve. The K-means clustering method is superior in most cases in achieving the balancing values between the robots for the uniform data set. The K-means is best suitable for large data set and complex topologies than other clustering method and also give wide spread solution for the entire $\alpha_{p,}\alpha_{n}$ range (0 to 1)value. However, for task location skewed toward depot case, GMM is able to satisfy the balancing constraints for sample cases considered in the study. It is found that application of minimax objective is most suitable for space exploration problem since it minimize the travel distance and task completion time. Future work can consider the dynamic cases instead of static case for instance in a team if one of the robots fails during the mission then other robots can complete the tasks assigned to the failed robot. The current assumption in the study could be relaxed by including heterogeneous tasks and robots. In terms of objective function of heterogeneous robot allocation future study could include both distance along with assignment of robot as per the requirements of the task.

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