Abstract—In this paper, adaptive impedance control is proposed for a robot collaborating with a human partner, in the presence of unknown motion intention of the human partner and unknown robot dynamics. Human motion intention is defined as the desired trajectory in the limb model of the human partner, which is extremely difficult to obtain considering the nonlinear and time-varying property of the limb model. Neural networks are employed to cope with this problem, based on which an online estimation method is developed. The estimated motion intention is integrated into the developed adaptive impedance control, which makes the robot follow a given target impedance model. Under the proposed method, the robot is able to actively collaborate with its human partner, which is verified through experiment studies.

Index Terms—Motion intention estimation, human-robot collaboration, neural networks.

I. INTRODUCTION

The society has already recognized the needs for human-robot collaboration to reduce human workload, costs and fatigue risk, and to increase the productivity and efficiency [1]. With the advancement of industrial production, most emerging manufacturing tasks that are either too complex to automate or too heavy to manipulate manually are impractical and even impossible to be solely taken by either fully automated robots or human beings, which earnestly requests robots to work alongside human beings collaboratively. The thrusts of human-robot collaboration rely on the observation that robots and human beings share the same workspace and have complementary advantages. The robots’ strength lies in their superior efficiencies in carrying out regular tasks at high speed with guaranteed performance, while human beings with their cognitive skills excel in understanding the circumstances, reasoning, and problem solving.

In human-robot collaboration, one of the most critical problems is to make the robot understand the motion intention of its human partner so that the robot is able to “actively” collaborate with its human partner. In this regard, to make the robot track a prescribed trajectory is not applicable. Force control can be an option for interaction control, but it is limited by its poor robustness [2]. Proposed in [3] and further developed in many other works [4], [5], [6], [7], [8], [9], impedance control is acknowledged to be a promising approach for interaction control. By employing impedance control, the robot is controlled to be compliant to the force exerted by the human partner. In this way, the robot passively follows the motion of its human partner, and human-robot collaboration becomes possible. Nevertheless, as the robot refines its motion according to the force exerted by the human partner, it will act as a load when the human partner intends to change the motion [10]. To solve this problem, the motion intention of the human partner is expected to be estimated and integrated into robot control.

As a matter of fact, understanding the motion intention of the other party is essential in human-human collaboration. Both collaboration parties usually keep communicating with each other through kinds of medias. In this paper, we consider that the force and position sensors are available and they represent the communication medias between a robot arm and a human limb. In the first part, we investigate the problem of how to estimate the motion intention of the human partner from available sensory information. There has been much effort made in this direction in the literature. In [11], the motion characteristics of the human limb is investigated, which is used to generate a point-to-point cooperative movement in [12]. In [13], under the assumption that the momentum is preserved during an interaction task, the motion intention of the human partner is represented by the change of the interaction force, which is estimated by the change of the control effort. In [14], the motion intention state is deemed as a stochastic process and it is estimated by employing the Hidden Markov Model (HMM). In this method, parameters of the human limb model are estimated online, and two intention states (active and passive) are defined to indicate that the human partner leads and follows, respectively. In [15], a crane robot is designed to aid the walking of the elderly and handicapped, and the user’s intentional walking direction is estimated using the Kalman filter. However, human motion intention is typically a time-varying trajectory, which cannot be represented by only several states as in [14] or motion directions as in [15]. In this regard, we employ the human limb model as in [16], and define the desired trajectory in this model as the motion intention of the human partner. A related work can be found in [17], in which the desired trajectory in the human limb model is calculated with unknown parameters of the human limb as design parameters. Considering nonlinear and time-varying properties of the human limb model [18], [19], we estimate the desired trajectory in this model based on neural networks (NN), which are acknowledged to possess excellent universal
approximation ability [20]. In the preliminary study [21], NN
have been employed to develop an off-line estimation method.
It has two obvious disadvantages: (i) the human partner may
change his intention during the collaboration and then the
training process has to be re-conducted; and (ii) the real human
motion intention is needed in the training phase which is
difficult to obtain in practice. Therefore, in this paper, an
updating law is developed to online adjust the NN weights
such that the estimation accuracy is guaranteed even when hu-
man motion intention changes. Besides, the real human motion
intention is not required in the proposed method. Thereafter,
the estimated motion intention is integrated into impedance
control as the rest position of a given target impedance model.
Adaptive control is designed to make the robot follow the
target impedance model, subject to unknown robot dynamics.
As a result, the robot “actively” moves towards its human
partner’s intended position rather than “passively” comply
into interaction force, and the collaboration efficiency i
developing a NN method; and the estimated motion intention is
integrated into impedance control to make the robot “acti-
the rest of the paper is organized as follows. In Section II,
a specific human-robot collaboration system under study is
described and the problem of unknown motion intention of
the human partner is formulated. In Section III, the proposed
motion intention estimation method is introduced in details.
In Section IV, adaptive impedance control is developed and
it is rigorously proven that the robot dynamics are governed
by a given target impedance model. In Section V, an intensive
experiment study is used to verify the effectiveness of the
proposed method. Concluding remarks are given in Section VI.

II. PROBLEM FORMULATION

A. System Description

In this paper, we investigate a typical human-robot collabora-
tion system, which includes a human limb and a robot arm
with a configurable end-effector and a force sensing handle,
as shown in Fig. 1. The robot arm provides \( n \) degrees-of-the-
freedom (DOF) at the force sensing handle, which is mounted
at the end-effector and measures the force exerted by the
human partner to the robot arm. The end-effector is selected
in order to flexibly pick and place objects with different sizes
and shapes. According to the force exerted by the human
partner and detected by the sensor mounted on the handle,
the control system generates control input for each joint of
the robot arm and drives the end-effector to the destination.
In the whole system, human partner leads the task by simply
applying forces to the handle, and the robot arm carries the
object load. The critical problem to be discussed in this paper
is how to estimate the motion intention of the human partner
and make the robot achieve “active” following.

Assumption 1: The object is tightly grasped by the robot
arm and there is no relative motion between the object and the
end-effector. Furthermore, the object is deemed as “a part” of
the robot arm.

Consider the robot kinematics given by

\[
x(t) = \psi(q) \tag{1}
\]

where \( x(t) \in \mathbb{R}^n \) and \( q \in \mathbb{R}^n \) are positions/oritations in the
Cartesian space and coordinates in the joint space, respectively.
Differentiating (1) with respect to time results in

\[
\dot{x}(t) = \dot{J}(q)\dot{q} \tag{2}
\]

where \( J(q) \in \mathbb{R}^{n \times n} \) is the Jacobian matrix. Further differentiating (2) with respect to time results in

\[
\ddot{x}(t) = \ddot{J}(q)\dot{q} + J(q)\ddot{q} \tag{3}
\]

Assumption 2: The Jacobian matrix \( J(q) \) is assumed to be
known and nonsingular in a finite workspace.

The robot arm dynamics in the joint space are described as

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + J^T(q)f(t) \tag{4}
\]

where \( M(q) \in \mathbb{R}^{n \times n} \) is the symmetric bounded positive
definite inertia matrix; \( C(q, \dot{q})\dot{q} \in \mathbb{R}^n \) denotes the Coriolis
and Centrifugal force; \( G(q) \in \mathbb{R}^n \) is the gravitational force;
\( \tau \in \mathbb{R}^n \) is the vector of control input; and \( f(t) \in \mathbb{R}^n \) denotes
the force exerted by the human limb, which is 0 when there
is no contact between the robot arm and human limb.

Property 1: [22] There exist unknown finite scalars \( \theta_j > 0, j = 1, \ldots, 4 \), such that for \( \forall q, \dot{q} \in \mathbb{R}^n \), \( \|M\| \leq \theta_1, \|C\| \leq \theta_2 + \theta_3\|\dot{q}\| \), and \( \|G\| \leq \theta_4 \).

Since the interaction is at the handle near the end-effector,
we consider the robot dynamics in the Cartesian space by
substituting the kinematic constraints (1)-(3) into the dynamic
model (4), as follows

\[
M_R(q)\ddot{x} + C_R(q, \dot{q})\dot{x} + G_R(q) = u + f(t) \tag{5}
\]

where

\[
M_R(q) = J^T(q)M(q)J^{-1}(q),
C_R(q, \dot{q}) = J^T(q)(C(q, \dot{q}) - M(q)J^{-1}(q)\dot{J}(q))J^{-1}(q),
G_R(q) = J^T(q)G(q), \quad u = J^{-T}(q)\tau \tag{6}
\]

Property 2: [23] Matrix \( M_R(q) \) is symmetric and positive
definite.

Property 3: [23] Matrix \( 2C_R(q, \dot{q}) - M_R(q) \) is a skew-
symmetric matrix if \( C_R(q, \dot{q}) \) is in the Christoffel form, i.e.,
\( \xi^T(2C_R(q, \dot{q}) - M_R(q))\xi = 0, \forall \xi \in \mathbb{R}^n \).
B. Problem Statement

In a predefined task, the desired trajectory of the robot arm is prescribed and available for the control design. In the human-robot collaboration task under study in this paper, the desired trajectory is determined by the human partner, which is unknown to the control design. In the literature, impedance control is employed such that the robot arm is controlled to be compliant to the force exerted by the human partner. Equivalently, the robot arm dynamics are governed by a target impedance model as below

\[ M_d(\ddot{x} - \ddot{x}_d) + C_d(\dot{x} - \dot{x}_d) + G_d(x - x_d) = f \]  

(7)

where \( x_d \) is the rest position, and \( M_d, C_d, \) and \( G_d \) are the desired inertia, damping, and stiffness matrices, respectively.

From the above impedance model, we find that the actual position of the robot arm \( x \) will be refined according to the interaction force \( f \). Seen from the perspective of the human partner, he will feel like moving an object with inertial/mass \( M_d \), damping \( C_d \), and stiffness \( G_d \) from the rest position \( x_d \) to \( x \). In this regard, if \( x_d \) is designed to be far away from \( x \), the human partner need consume lots of energy to move the robot arm. Conversely, if the robot “knows” the motion intention of the human partner and changes \( x_d \) accordingly, the human partner will consume less energy to move the robot arm.

In many cases, \( x_d \) can be designed based on the designer’s prediction of the motion intention of the human partner. For example, in the application of human-robot handshaking, although it is impossible to exactly predict human’s actual movement, it is possible to design \( x_d \) based on the basic understanding of the handshaking motion of the human partner. Nevertheless, this empirical method is obviously lack of flexibility and cannot guarantee a good performance. Therefore, in the first part of this paper, we will propose a method to design \( x_d \) based on the estimation of the motion intention of the human partner. After that, we will develop an adaptive control to guarantee the robot dynamics (5) to be governed by the above impedance model (7), subject to unknown robot dynamics.

III. Motion Intention Estimation

A. Human Limb Model

This section is dedicated to define the motion intention of the human partner by employing a human limb model. A general model to describe the dynamics of a human limb is supposed to include its mass-damper-spring property, as in [16]

\[ -M_H\ddot{x} - C_H\dot{x} + G_H(x_{Hd} - x) = f \]  

(8)

where \( M_H, C_H, \) and \( G_H \) are the mass, damper, and spring matrices of the human limb, respectively and they are diagonal, and \( x_{Hd} \) is the trajectory planned in the human partner’s CNS which is referred as the motion intention of the human partner in this paper.

As discussed and verified in [16], the damper and spring components usually dominate human limb model. Thus, we have the following simplified model

\[ -C_H\dot{x} + G_H(x_{Hd} - x) = f \]  

(9)

Suppose that \( C_H \) and \( G_H \) are unknown functions of \( x \) and \( \dot{x} \), i.e., \( C_H(x, \dot{x}) \) and \( G_H(x) \), similarly as in the robot dynamics (4). Then, we may assume that the motion intention \( x_{Hd} \) can be estimated by the interaction force \( f \), actual position \( x \) and velocity \( \dot{x} \), i.e.,

\[ x_{Hd} = F(f, \dot{x}, x) \]  

(10)

where \( F(\cdot) \) is an unknown function. Equivalently, we have the following assumption:

Assumption 3: In a typical collaborative task, the motion intention of the human partner (in each direction), i.e., \( x_{Hd} \) in (9), is determined by the interaction force \( f \), actual position \( x \) and velocity \( \dot{x} \) at the interaction point (in the corresponding direction) of the human limb and robot arm.

Remark 1: The above assumption is the fundamental of the estimation method to be developed in this paper. Its validity will be verified by experiments at the end of this paper.

In (10), function \( F(\cdot) \) is typically unknown and nonlinear considering the time-varying property and uncertainty of \( C_H \) and \( G_H \). Indeed, human partner may change his limb impedance \( (C_H \) and \( G_H) \) during the collaboration. This makes the estimation of \( x_{Hd} \) based on (10) become difficult. In this regard, we employ machine learning to cope with this problem, which can discover intrinsic information, map unknown relationship and approximate functions. The basic idea is to approximate \( x_{Hd} \) in (10) by a linearly parameterized function of \( f, \dot{x} \) and \( x \), and an adaptive method is developed to estimate the ideal weights of the parameterized function.

B. Neural Networks Based Motion Intention Estimation

As one of the popular machine learning methods, radial basis function neural networks (RBFNN) are employed in this paper. The structure of RBFNN is expressed as follows [23]

\[ \varphi(W; r) = W^T S(r), \quad S(r) \in R^p, \]

\[ S(r) = [s_1(r), s_2(r), \ldots, s_p(r)]^T, \]

\[ s_k(r) = \exp\left(-\frac{(r - \mu_k)^T(r - \mu_k)}{\eta_k}\right), \quad k = 1, 2, \ldots, p \]

(11)

where \( \varphi(W; r) \) is a continuous function of \( r, r \in \Omega, \subset R^m \) is the input to RBFNN, \( p \) is the NN nodes number, \( \mu_k = [\mu_{k,1}, \mu_{k,2}, \ldots, \mu_{k,m}]^T \) is the center of the receptive field and \( \eta_k \) is the width of the Gaussian function, and \( W \) is an adjustable synaptic weight vector.

By employing RBFNN, the motion intention of the human partner and its estimation are respectively given by

\[ x_{Hd,i} = \hat{W}_i^T S_i(r_i) + \epsilon_i \]

\[ \hat{x}_{Hd,i} = \hat{W}_i^T S_i(r_i), \quad i = 1, 2, \ldots, n \]

(12)

where \((\cdot)_i\) is the \(i\)th component of \((\cdot), r_i = [f_i^T, x_i^T, \dot{x}_i^T]^T \) is the input of RBFNN, \( \epsilon_i \) is the estimation error, \( \hat{W}_i \) is the estimate of the ideal weight \( W_i \), and \( S_i \) has the same meaning as that in (11). It is known that \( \epsilon_i \) can be made arbitrarily small, if \( p \) is sufficiently large.

Remark 2: One underlying assumption of the above NN estimation is that \( F(\cdot) \) in (10) is a continuous function. This
Impedance may change his motion intention at any time. With the above equation, we obtain the estimated motion dynamics and unknown, it is absorbed by the back propagation algorithm [24] to obtain 

\[ \dot{\alpha}_i = \alpha_i \frac{\partial f_i}{\partial \vec{x}_{H,i}} \]  

According to the discussion in the Introduction and Section II, the control objective is to make the robot “actively” move towards its human partner’s intended position and thus the interaction force \( f_i \) as small as possible. Therefore, \( \dot{\vec{W}}_i \) is adjusted online in the direction of the steepest descent with respect to the following cost function

\[ E_i = \frac{1}{2} \dot{f}_i^2 \]  

Equivalently, we have

\[ \dot{W}_i(t) = -\alpha_i \frac{\partial E_i}{\partial \vec{W}_i} = -\alpha_i \frac{\partial f_i}{\partial \vec{x}_{H,i}} \frac{\partial \vec{x}_{H,i}}{\partial \vec{W}_i} = -\alpha_i f_i \frac{\partial f_i}{\partial \vec{x}_{H,i}} \frac{\partial \vec{x}_{H,i}}{\partial \vec{W}_i} \] (14)

where \( \alpha_i \) is a positive scalar.

In the above equation, \( \frac{\partial f_i}{\partial \vec{x}_{H,i}} \) can be obtained according to (9) as follows

\[ \frac{\partial f_i}{\partial \vec{x}_{H,i}} = G_{H,i} \] (15)

and \( \frac{\partial \vec{x}_{H,i}}{\partial \vec{W}_i} \) can be obtained according to (12) as follows

\[ \frac{\partial \vec{x}_{H,i}}{\partial \vec{W}_i} = S_i(r_i) \] (16)

Substituting (15) and (16) into (14) leads to

\[ \dot{W}_i(t) = -\alpha_i f_i S_i(r_i) \] (17)

where \( \alpha_i = \alpha_i G_{H,i} \). As \( G_{H,i} \) is the parameter of human limb dynamics and unknown, it is absorbed by \( \alpha_i \).

Remark 3: Note that \( G_H \) may be time-varying but it can be still absorbed by \( \alpha_i \), which is set by the designer and does not necessarily have the real value of \( \alpha_i G_H \). The same approach has been used in [25].

Then, we obtain the updating law of \( \dot{\vec{W}}_i \) as below

\[ \dot{\vec{W}}_i(t) = \dot{\vec{W}}_i(0) - \alpha_i \int_0^t \left[ f_i(\omega) S_i(r_i(\omega)) \right] d\omega \] (18)

With the above equation, we obtain the estimated motion intention \( \dot{x}_{H,i} \) according to (12).

Remark 4: Note that \( \dot{\vec{W}}_i \) can be obtained online as in (18). This is a favorable property in the sense that the human partner may change his motion intention at any time.

Remark 5: In the practical implementation, the adaptation of \( \dot{\vec{W}}_i \) can be switched off to simplify the computation and improve the system robustness. The condition to switch the adaptation can be designed as: the adaptation is switched off if \( f_i < f_c \), where \( f_c \) is a design parameter. This condition indicates that the adaptation is switched off when \( x \) is close to \( x_{H,i} \).

As the estimation error with NN is unavoidable and NN estimation usually falls into local minimum, \( x_{H,i} \) cannot be exactly the same as \( x_{H,i} \). Therefore, it is improper to use position control to make the actual position \( x \) track the estimated motion intention \( \dot{x}_{H,i} \). Instead of that, \( x_{H,i} \) can be used as the rest position in the target impedance model (7), such that the error between the actual position \( x \) and the estimated motion intention \( \dot{x}_{H,i} \) can be accommodated partly by impedance control. This will be discussed in the following section. Nevertheless, it is important to note that this is different from the pure impedance control with a fixed rest position, where the error between the actual position and the motion intention is much larger and thus the human partner consumes much more energy to move the robot arm.

IV. ADAPTIVE IMPEDANCE CONTROL

As \( x_{H,i} \) is obtained in the above section, we let \( x_d = x_{H,i} \) and design adaptive impedance control to make the robot arm dynamics (5) track the given impedance model (7). The control diagram is shown in Fig. 2.

Fig. 2: Adaptive impedance control with estimated motion intention

Construct the error signal \( w = M_d \dot{e} + C_d \ddot{e} + G_d e - f \) with \( e = x - x_d \) as in [26], the control objective is to make \( \lim_{t \to \infty} w(t) = 0 \). In [27], an auxiliary variable \( z \) is defined for the analysis convenience, which is briefly introduced in the following.

First, we define an augmented impedance error

\[ \ddot{w} = K_f \dot{w} = \dot{e} + K_d \dot{e} + K_p e - K_f f \] (19)

where \( K_d = M_d^{-1} C_d \), \( K_p = M_d^{-1} G_d \), and \( K_f = M_d^{-1} \).

Choose two positive definite matrices \( \Lambda \) and \( \Gamma \) such that

\[ \Lambda + \Gamma = K_d \]  
\[ \dot{\Lambda} + \Gamma \Lambda = K_p \] (20)

and define

\[ \dot{f}_i + \Gamma f_i = K_f f \] (21)

Thus, we can rewrite (19) as

\[ \ddot{w} = \dot{e} + (\Lambda + \Gamma) e - \dot{f}_i - \Gamma f_i \] (22)

By defining

\[ z = \dot{e} + \Lambda e - f_i \] (23)

we obtain

\[ \ddot{w} = \dot{z} + \Gamma z \] (24)
Suppose that $\lim_{t \to \infty} \dot{z}(t)$ exists, $\lim_{t \to \infty} z(t) = 0$ will lead to $\lim_{t \to \infty} \dot{z}(t) = 0$. Therefore, considering (24) and (19), we have $\lim_{t \to \infty} w(t) = 0$ if $\lim_{t \to \infty} z(t) = 0$. Based on this fact, the control objective finally becomes

$$\lim_{t \to \infty} z(t) = 0$$ (25)

Define an augmented state variable

$$\dot{x}_r = x_d - \Lambda e + f_t$$ (26)

(26) and (23) immediately result in

$$\dot{z} = \dot{x}_r - \dot{x}_r$$ (27)

which will be used in the following control performance analysis.

We propose the adaptive impedance control as below

$$u = -Kz - \sum_{j=1}^{4} \frac{\hat{\theta}_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j z - f$$ (28)

$$\dot{\hat{\theta}}_j = -a_j \hat{\theta}_j - \frac{b_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j$$ (29)

where $j = 1, \ldots, 4$, $\hat{\theta}_j$ is the estimation of $\theta_j$ in Property 1, $K$ is a positive definite matrix, $b_j > 0$, $a_j$ and $\sigma_j$ are time varying positive functions satisfying $\lim_{t \to \infty} a_j = 0$, $\int_0^t a_j(\omega) d\omega = c_j < \infty$, $\lim_{t \to \infty} \sigma_j = 0$ and $\int_0^t \sigma_j(\omega) d\omega = d_j < \infty$, and $\phi_1 = \|J^{-T}||J^{-1}||\dot{x}_r|| + \|J^{-1}||\ddot{x}_r||$, $\phi_2 = \|J^{-T}||J^{-1}||\dot{x}_r||$, $\phi_3 = \|J^{-T}||J^{-1}||\dot{\phi}||\dot{x}_r||$ and $\phi_4 = \|J^{-T}||J^{-1}||\dot{\phi}||\dot{x}_r||$.

Considering (23), we rewrite (5) as

$$M_R \dot{z} + C_R z = u + f - (M_R \ddot{x}_r + C_R \dot{x}_r + G_R)$$ (30)

Substituting the control input (28) into the above equation, we have

$$M_R \dot{z} + C_R z = -Kz - \sum_{j=1}^{4} \frac{\hat{\theta}_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j z - (M_R \ddot{x}_r + C_R \dot{x}_r + G_R)$$ (31)

**Theorem 1:** Considering the robot dynamics described by (4), control (28) with the updating law (29) guarantees the following results:

(i) the defined impedance error asymptotically converges to 0 as $t \to \infty$, i.e., $\lim_{t \to \infty} \dot{z}(t) = 0$; and

(ii) all the signals in the closed-loop are bounded.

**Proof:** Consider the following Lyapunov function candidate

$$V = \frac{1}{2} z^T M_R z + \sum_{j=1}^{4} \frac{1}{2b_j} \dot{\theta}_j^2$$ (32)

where $\hat{\theta}_j = \theta_j - \hat{\theta}_j$.

The derivative of $V$ with respect to time is

$$\dot{V} = \frac{1}{2} z^T M_R \dot{z} + z^T M_R \dot{z} + \sum_{j=1}^{4} \frac{1}{b_j} \dot{\theta}_j \dot{\theta}_j$$ (33)

Considering Property 3, we have

$$\dot{V} = z^T C_R z + z^T M_R \dot{z} + \sum_{j=1}^{4} \frac{1}{b_j} \dot{\theta}_j \dot{\theta}_j$$ (34)

According to the dynamics (31), we obtain

$$\dot{V} = z^T (-Kz - \sum_{j=1}^{4} \frac{\hat{\theta}_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j z - (M_R \ddot{x}_r + C_R \dot{x}_r + G_R)) + \sum_{j=1}^{4} \frac{1}{b_j} \dot{\theta}_j \dot{\theta}_j$$ (35)

According to (29), we have

$$\dot{\hat{\theta}}_j = -\dot{\theta}_j - a_j \dot{\theta}_j - \frac{b_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j$$ (36)

Substituting the above equation to (35) leads to

$$\dot{V} = z^T (-Kz - \sum_{j=1}^{4} \frac{\theta_j}{\phi_j} \dot{\phi}_j^2 ||z|| + \sigma_j z - (M_R \ddot{x}_r + C_R \dot{x}_r + G_R)) + \sum_{j=1}^{4} \frac{a_j}{b_j} \dot{\theta}_j \dot{\theta}_j$$ (37)

Considering the definitions of $\phi_j$, we have

$$-z^T (M_R \ddot{x}_r + C_R \dot{x}_r + G_R) \leq \|z\| (||M_R \ddot{x}_r + C_R \dot{x}_r + G_R||)$$

$$\leq \|z\| (||M_R \ddot{x}_r|| + ||C_R \ddot{x}_r|| + ||G_R||)$$

$$= \|z\| (||J^{-T} M \ddot{\theta}_j|| \ddot{x}_r|| + ||J^{-T} C \ddot{\phi}_j|| \ddot{x}_r|| + ||J^{-T} G \ddot{\phi}_j|| \ddot{x}_r||)$$

$$\leq \|z\| \|J^{-T} \||M_R||J^{-1}||\ddot{x}_r|| + \|C_R||J^{-1}||\ddot{x}_r|| + ||G_R||$$

$$\leq \|z\| (\{||J^{-T} \||M_R||J^{-1}||\ddot{x}_r|| + \{||C_R||J^{-1}||\ddot{x}_r|| + \{||G_R||$$

$$= \|z\| \sum_{j=1}^{4} \sigma_j \dot{\theta}_j$$ (38)

Substituting the above inequality to (37), we obtain

$$\dot{V} \leq -z^T Kz + \sum_{j=1}^{4} \sigma_j \dot{\theta}_j + \sum_{j=1}^{4} \frac{a_j}{b_j} \dot{\theta}_j \dot{\theta}_j$$

$$\leq -z^T Kz + \sum_{j=1}^{4} \sigma_j \dot{\theta}_j + \frac{1}{4} \sum_{j=1}^{4} \frac{a_j}{b_j} \dot{\theta}_j^2$$

$$= -z^T Kz + \delta$$ (39)

where $\delta = \sum_{j=1}^{4} \sigma_j \dot{\theta}_j + \frac{1}{4} \sum_{j=1}^{4} \frac{a_j}{b_j} \dot{\theta}_j^2$, and the last inequality comes from

$$\dot{\theta}_j \dot{\theta}_j = (\dot{\theta}_j - \dot{\theta}_j) \dot{\theta}_j = \frac{1}{4} \dot{\theta}_j^2 - \frac{1}{4} \dot{\theta}_j^2 \leq \frac{1}{4} \dot{\theta}_j^2$$ (40)

Because $\lim_{t \to \infty} a_j = 0$ and $\lim_{t \to \infty} \sigma_j = 0$, we have $\lim_{t \to \infty} \delta = 0$. It indicates that there exists $t_1$ such that when
t > t₁, δ ≤ ε, where ε is a small finite constant. Then we obtain z ∈ Lₙ∞. According to the definition of z in (23), x ∈ Lₙ∞, ˙x ∈ Lₙ∞, and thus ˙z ∈ Lₙ∞, ˙π ∈ Lₙ∞. Considering (31), we have ˙z ∈ Lₙ∞.

Integrating both sides of (39) leads to

\[ V(t) - V(0) \leq - \int_0^t z^T(\omega)Kz(\omega)d\omega + \int_0^t \delta(\omega)d\omega \]  

(41)

which leads to

\[ \int_0^t z^T(\omega)Kz(\omega)d\omega \leq V(0) - V(t) + \int_0^t \delta(\omega)d\omega \leq V(0) + \int_0^t \delta(\omega)d\omega \]  

(42)

because V(t) ≥ 0.

According to the definition of δ, we have

\[ \int_0^t \delta(\omega)d\omega = \frac{4}{3} \sum_{j=1}^4 \theta_j \int_0^t \sigma_j(\omega)d\omega + \frac{1}{3} \sum_{j=1}^4 \theta_j^2 \int_0^t a_j(\omega)d\omega \]

\[ = \frac{4}{3} \sum_{j=1}^4 \theta_j d_j + \frac{4}{3} \sum_{j=1}^4 \theta_j^2 c_j \]  

(43)

The above equation indicates that \( \int_0^t \delta(\omega)d\omega \) is bounded. According to (42), \( \int_0^t z^T(\omega)Kz(\omega)d\omega \) is bounded because V(0) is bounded, which results in z ∈ Lₙ². According to Barbelaet’s Lemma, z ∈ Lₙ² and ˙z ∈ Lₙ∞ lead to z → 0 as t → ∞, which completes the proof.

Remark 6: While the control input u is developed in the Cartesian space, we need transform it to the joint space for the control of each joint. In the non-redundancy case, the transformation is uniquely determined as \( \tau = J^Tu \), as discussed above and shown in Fig. 2. In the redundancy case, the transformation is not uniquely determined and there exists freedom to improve some measures of the system performance, such as singularity avoidance, obstacle avoidance, kinetic energy minimization and posture control. More details can be found in [28], [29].

V. EXPERIMENT

In this section, the proposed method is examined through experiments. The experiments are carried out on Nancy which is a humanoid introduced in [30] and shown in Fig. 3(a). In these experiments, the human partner holds a plate mounted on Nancy’s left wrist, where there is an ATI mini-40 force/torque sensor, as shown in Fig. 3(b). Nancy’s left wrist is moved by the human partner towards his intended position. The actual position and velocity of the left wrist is provided by Maxon’s EPOS2 70/10 dual loop controller and the torque from the human partner is measured by the force/torque sensor. An industrial PC is used to process the collected data and implement the developed method. Because the human partner’s motion intention cannot be measured in the experiment, we can only understand the experiment results in an indirect way. In particular, a small external torque indicates a small error between the actual trajectory and the motion intention. This has been discussed when developing the intention estimation method in Section III.

Two cases of different motion intentions are considered. In the first case, the human partner aims to move the wrist to a fixed angle and thus the intended motion is a point-to-point movement. In the second case, the human partner aims to move the wrist forward and back between two target angles, and the intended motion is a time-varying trajectory. In both cases, impedance control with zero stiffness is implemented for the comparison purpose. Impedance parameters in (7) are \( M_x = 0.01 \), \( C_x = 0.8 \) and \( G_x = 0 \). The number of NN nodes is \( p = 10 \), and the other parameters of NN in (11) are \( \mu_i = 0 \) and \( \eta_i = 1 \) for \( i = 1, 2, \ldots, 10 \). The adaptation ratio in (18) is \( \alpha = 0.01 \). Other values of the above parameters can be chosen to improve the control performance.

The results in the first case are shown in Figs. 4 and 5. In Fig. 4, the wrist angles with impedance control and the proposed method are shown. The “target angle” in the figure stands for the position that the human partner intends to move the robot arm to. It is found that the response with the proposed method is faster than that with impedance control, which indicates that the wrist with the proposed method follows human partner’s motion intention more “actively”. The NN estimation performance is also illustrated in Fig. 4 by showing the estimated motion intention. While Fig. 4 illustrates that the wrist with two methods is moved to roughly the same angle (the target angle), it is clearly found in Fig. 5 that much less torque is needed with the proposed method. When the target angle is reached, the torque from the human partner becomes zero with both impedance control and the proposed method. Based on these results, it can be concluded that much less effort is required from the human partner with the proposed method, although both impedance control and the proposed method can be employed for human-robot collaboration in the case of point-to-point movement.

Instead of point-to-point movement in the first case, a more common scenario in practice is to move the robot arm along a time-varying trajectory. In the second case, Nancy’s wrist is firstly moved toward a prescribed target position, and back to the other target position. The results in this case are shown in Figs. 6 and 7. The “target angle 1” and “target angle 2” in Fig. 6 stand for the target positions in the forward motion and in the back motion, respectively. Similarly as in Fig. 4, a faster
response is achieved with the proposed method as shown in Fig. 6. In Fig. 7, it is found that an external torque of about 0.4Nm is needed to move Nancy’s wrist so the robot is a load to the human partner, as discussed before. Two ways can be considered to achieve the better performance with impedance control. One is to choose smaller impedance parameters $M_d$ and $C_d$, and make the robot arm “softer”. Unfortunately, it has been proved that the desired inertia cannot be chosen to be arbitrarily small [7] and a large damping is required to stabilize the whole system in practical implementations [31]. The other one requires the human partner to stiffen his limb and make the limb impedance dominate the impedance of the coupled system, but more control effort from the human partner is the cost and it is not achievable when the robot arm has a large weight (and thus a large inertia). In this regard, to make the robot arm actively follow human partner’s motion in the case of time-varying trajectory cannot be achieved by impedance control with a fixed rest position. Compared to impedance control, the proposed method requires a much smaller external torque, which is less than 0.1Nm as also shown in Fig. 7. The above results indicate that Nancy’s wrist can be moved to the target positions with much less effort under the proposed method, even if the human partner changes his motion intention. They have also well justified the validity of Assumption 3, where it is assumed that the motion intention of the human partner can be estimated based on the interaction force, position and velocity, in such a specific collaborative task.

During the experiments, we note that the human partner may change his motion intention according to robot trajectory. This is an interesting issue but is not considered in the proposed method. In this paper, we assume implicitly that the human motion intention is stationary with respect to the actual robot trajectory, i.e., the adaptation of the robot trajectory has no effect on the human motion intention. However, human motion is also an output of the neuromuscular control system, so the dynamic interaction with the robot could well result in concurrent adaptations in the human motion intention. This makes the problem more tricky and it will be further investigated in the future work. Besides, in the discussion throughout this paper, human partner and robot are considered to be two separated subsystems. Particularly, the motion intention of the human partner is estimated by considering the human limb dynamics, then the estimated motion intention is integrated to impedance control of the robot arm. The performance of the whole coupled collaboration system is yet to be rigorously analyzed, which will be also considered in the future work.

VI. Conclusion

In this work, human-robot collaboration has been investigated, in which the motion intention of the human partner has been observed by employing the human limb model and estimating the desired trajectory. A NN method has been proposed to cope with the problem of unknown human limb model. The estimated motion intention has been integrated into impedance control of the robot arm, such that it actively follows its human partner. Experiment results have been provided to verify the validity of the proposed method.
REFERENCES


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