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Search for Invisible Decays of a Higgs Boson Produced in Association with a Z Boson in ATLAS

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A search for evidence of invisible-particle decay modes of a Higgs boson produced in association with a Z boson at the Large Hadron Collider is presented. No deviation from the standard model expectation is observed in 4.5 fb\(^{-1}\) (20.3 fb\(^{-1}\)) of 7 (8) TeV \(p\bar{p}\) collision data collected by the ATLAS experiment. Assuming the standard model rate for \(ZH\) production, an upper limit of 75%, at the 95% confidence level is set on the branching ratio to invisible-particle decay modes of the Higgs boson at a mass of 125.5 GeV. The limit on the branching ratio is also interpreted in terms of an upper limit on the allowed dark matter-nucleon scattering cross section within a Higgs-portal dark matter scenario. Within the constraints of such a scenario, the results presented in this Letter provide the strongest available limits for low-mass dark matter candidates. Limits are also set on an additional neutral Higgs boson, in the mass range 110 < \(m_H\) < 400 GeV, produced in association with a Z boson and decaying to invisible particles.

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Some extensions of the standard model (SM) allow a Higgs boson [1–3] to decay to a pair of stable or long-lived particles [4–18] that are not observed by the ATLAS detector. For instance, the Higgs boson can decay into two particles with very small interaction cross sections with SM particles, such as dark matter (DM) candidates. Collider data can be used to directly constrain the branching ratio of the Higgs boson to invisible particles. Similarly, limits can be placed on the cross section times branching ratio of any additional Higgs bosons decaying predominantly to invisible particles. LEP results [19] put limits on an invisibly decaying Higgs boson, produced in association with a Z boson, for Higgs masses below 120 GeV.

This Letter presents a search for invisible decays of a Higgs boson produced in association with a Z boson. A Higgs boson in the mass range 110 < \(m_H\) < 400 GeV is considered. The distribution of the missing transverse momentum (\(E_T^{\text{miss}}\)) in events with an electron or a muon pair consistent with a Z boson decay is used to constrain the \(ZH\) production cross section times the branching ratio of the Higgs boson decaying to invisible particles, over the full mass range. For the newly discovered Higgs boson, a constraint could be placed on the branching ratio to invisible particles. In this case the mass of the Higgs boson is taken to be \(m_H = 125.5\) GeV, the best-fit value from the ATLAS experiment [20], and the \(ZH\) production cross section is assumed to be that predicted for the SM Higgs boson. This assumption implies that the hypothesized unobserved particles that couple to the Higgs boson have sufficiently weak couplings to other SM particles to not affect the Higgs boson production cross sections. The total cross section for the associated production of a SM Higgs boson, with \(m_H = 125.5\) GeV, and a Z boson, calculated to next-to-next-to-leading order in QCD [21] and including next-to-leading-order (NLO) electroweak corrections [22,23], is 331 fb at \(\sqrt{s} = 7\) TeV and 410 fb at \(\sqrt{s} = 8\) TeV [24]. The SM branching ratio of the Higgs boson decaying to invisible particles is 1.2 \(\times\) 10\(^{-3}\), arising from the \(H \rightarrow ZZ^{(*)} \rightarrow 4\nu\) decay. The present search is not sensitive to the low branching ratio for this decay, but instead searches for enhancements in the decay fraction to invisible particles due to physics beyond the standard model (BSM).

The search uses 4.5 fb\(^{-1}\) of data recorded with the ATLAS detector in 2011 at \(\sqrt{s} = 7\) TeV and 20.3 fb\(^{-1}\) of data recorded in 2012 at \(\sqrt{s} = 8\) TeV. The ATLAS detector has been described elsewhere [25]. Simulated signal and background event samples are produced with Monte Carlo (MC) event generators, passed through a full GEANT4 [26] simulation of the ATLAS detector [27] and reconstructed with the same software as the data.

The signal samples are generated with HERWIG++ [28] and its internal POWHEG method [29,30]. The SM \(ZZ\) and WZ backgrounds are taken from simulation, since they have limited statistics in the control regions that would allow us to estimate these backgrounds with data. All the other background processes to this search are determined from data. In these cases, simulated samples are only used as cross-checks for the obtained background estimates. POWHEG [29–31] interfaced with PYTHIA8 [32] is used to model SM ZZ and WZ production [33]. The production of WW is modeled using HERWIG [34] and SHERPA [35].
for the 7 and 8 TeV data, respectively. A separate sample simulated with gg2VV [36] interfaced with JIMMY [37] accounts for WW/ZZ production through quark-box diagrams, which are not included in the above mentioned samples. The MC@NLO [38] generator interfaced with JIMMY is used to model $t\bar{t}$, $Wt$, and $s$-channel single top-quark production. AERMC [39] interfaced with PYTHIA [40] models $t$-channel single top-quark production. Inclusive $Z/p_T$ production is simulated with ALPGEN [41] interfaced with JIMMY or PYTHIA for the 7 or 8 TeV data, respectively. Inclusive $W$ production is simulated with ALPGEN interfaced with JIMMY. Contributions to this search from the $H \rightarrow WW^{(*)} \rightarrow \ell\ell\ell\nu$ and $H \rightarrow ZZ^{(*)} \rightarrow \ell\ell\ell\nu$ decays of a 125.5 GeV SM Higgs boson are studied using POWHEG [29–31,42,43] interfaced with PYTHIA8 and found to be negligible.

Electron candidates are reconstructed from isolated energy deposits in the electromagnetic calorimeter with a shower shape consistent with electrons or photons, matched to inner detector tracks [44]. The electrons used to form a $Z$ boson candidate are required to have transverse momentum $p_T > 20$ GeV and pseudorapidity $|\eta| < 2.47$ [45]. Electrons with $p_T > 7$ GeV that satisfy less stringent identification criteria on the calorimeter cluster shape, track quality, and track-cluster matching [44] are used to veto events with more than two charged leptons.

Muon candidates are reconstructed combining tracks independently found in the muon spectrometer and inner tracking detector [46]. Muons forming a $Z$ boson candidate are required to have $p_T > 20$ GeV and $|\eta| < 2.4$. Muons with $p_T > 7$ GeV are used to veto events with more than two charged leptons.

Jets are reconstructed using the anti-$k_t$ algorithm [47] with a radius parameter $R = 0.4$. They must have $p_T > 20$ GeV and $|\eta| < 4.5$. To discriminate against jets from additional minimum bias interactions, selection criteria are applied to ensure that most of the jet momentum, for jets with $|\eta| < 2.5$, is associated with tracks originating from the primary vertex, which is taken to be the vertex with the highest summed $p_T^2$ of associated tracks.

To ensure good separation between electrons, muons, and jets, electrons are removed if they are within $\Delta R \leq 0.2$ of an identified muon, and jets are removed if they are within $\Delta R \leq 0.2$ of an identified electron. Remaining electrons and muons are removed if they are within $\Delta R \leq 0.4$ of a remaining jet or if the scalar sum of track momenta, not associated with the lepton, in a cone of $\Delta R < 0.2$ around the lepton direction is greater than 10% of the lepton $p_T$.

The $E_T^{\text{miss}}$ is the magnitude of the negative vectorial sum of the transverse momenta from calibrated objects, such as identified electrons, muons, photons, hadronic decays of tau leptons, and jets [48]. Clusters of calorimeter cells not matched to any object are also included. The analysis also uses a track-based missing transverse momentum ($p_T^{\text{miss}}$) computed from all inner detector tracks with $p_T > 500$ MeV and $|\eta| < 2.5$, that satisfy stringent quality criteria [49] and are consistent with originating from the primary vertex. For the $p_T^{\text{miss}}$ calculation, tracks matched to electrons are discarded and replaced by the transverse energy $E_T$ of the matched cluster measured in the calorimeter to include any photon radiation in the calculation.

Event selection criteria are determined in an optimization procedure, using simulated samples, to maximize the signal significance of the search. Events are required to pass a single-lepton or lepton-pair trigger, with small variations in the applied $p_T$ threshold in different data-taking periods. Events must also have at least one reconstructed vertex with at least three associated tracks with $p_T > 500$ MeV. Data quality criteria are applied to reject events from non-collision backgrounds or events with degraded detector performance [48].

The invariant mass of the selected dilepton system, $m_{ll}$, is required to satisfy $76 < m_{ll} < 106$ GeV to be consistent with leptons originating from a $Z$ boson decay.

Figure 1 shows the $E_T^{\text{miss}}$ distribution in the 8 TeV data sample after the dilepton mass requirement. In this figure the data are consistent with the expected background based on simulated samples for all but the multijet background. The uncertainty band of the expected background is widest in the region dominated by the steeply falling $Z$ boson background. To reject the majority of this background, $E_T^{\text{miss}}$ is required to be greater than 90 GeV. In events where a significant $E_T^{\text{miss}}$ arises from misreconstructed energy in the calorimeter, the vectors of $E_T^{\text{miss}}$ and $p_T^{\text{miss}}$ are likely to have different azimuthal angles. Thus the azimuthal difference of these two vectors, $\Delta \phi(E_T^{\text{miss}}, p_T^{\text{miss}})$, is required to be less than 0.2.

![FIG. 1 (color online). Distribution of $E_T^{\text{miss}}$ for events with the invariant mass of the two leptons 76 < $m_{ll}$ < 106 GeV in the 8 TeV data (dots). The stacked histograms represent the background predictions from simulation. The signal hypothesis is shown by a dotted line and assumes the SM $ZH$ production rate for a $m_H = 125.5$ GeV Higgs boson with BR($H \rightarrow \text{inv.}$) = 1. The inset at the bottom of the figure shows the ratio of the data to the combined background expectations as well as a band corresponding to the combined systematic uncertainties.](image-url)
For the signal, the momentum of the reconstructed \(Z\) boson is expected to be balanced by the momentum of the invisibly decaying Higgs boson. Therefore the azimuthal separation between the dilepton system, where the magnitude of its transverse momentum is defined as \(p_T^\ell\ell\), and the \(E_T^{\text{miss}}\), \(\Delta\phi(p_T^\ell\ell,E_T^{\text{miss}})\), is required to be greater than 2.6. The boost of the \(Z\) boson causes the decay leptons to be produced with a small opening angle. The azimuthal opening angle of the two leptons, \(\Delta\phi(\ell^+,\ell^-)\), is thus required to be less than 1.7. Furthermore \(p_T^\ell\ell\) and \(E_T^{\text{miss}}\) are expected to be similar. Therefore the fractional \(p_T\) difference, defined as \(|E_T^{\text{miss}} - p_T^\ell\ell|/p_T^\ell\ell\), is required to be less than 0.2. Finally, for the majority of the signal no additional high-\(p_T\) jets are expected to be observed in the events, while for the background from boosted \(Z\) bosons and from \(\ell\ell\) pairs one or more jets are expected. Thus, events are required to have no reconstructed jets with \(p_T > 25\) GeV and \(|\eta| < 2.5\).

After the selection requirements, the dominant background is SM ZZ production followed by SM WZ production, as shown in Table I. These backgrounds are simulated using MC samples normalized to NLO cross sections. The simulation of WZ events is validated by comparing them to data events in which the third-lepton veto is replaced by an explicit third-lepton requirement. The theoretical prediction of the ZZ production is in agreement with the ATLAS cross-section measurement at \(\sqrt{s} = 7\) TeV [50].

Background contributions from events with a genuine isolated lepton pair, not originating from a \(Z \rightarrow \ell\ell\) or \(Z \rightarrow \mu\mu\) decay (\(WW, \ell\ell, Wt,\) and \(Z \rightarrow \tau\tau\)), are estimated by exploiting the flavor symmetry in the dilepton final state of these processes. Distributions for events with an \(e\mu\) pair, appropriately scaled to account for differences in electron and muon reconstruction efficiencies, can be used to estimate this background in the electron and muon channels. The difference between the efficiencies for electrons and muons is estimated using the square root of the ratio of the numbers of dimuon and dielectron events in data within the \(m_{\ell\ell}\) window. Events in the \(e\mu\) control region not originating from \(WW, \ell\ell, Wt,\) or \(Z \rightarrow \tau\tau\) backgrounds are subtracted using simulated samples. Important sources of systematic uncertainty are variations in the correction factor for the efficiencies for electrons and muons and uncertainties in the simulated samples used for the subtraction. The combined systematic uncertainty is 23% for both the 7 and 8 TeV data. The estimated background from these sources is consistent with the expectation from the simulation.

The background from inclusive \(Z \rightarrow \ell\ell\) and \(Z \rightarrow \mu\mu\) production in the signal region is estimated from the background in three sideband regions [51]. These sideband regions are formed by considering events failing one or both of the nominal selection requirements applied to \(\Delta\phi(E_T^{\text{miss}},p_T^\ell\ell)\) and the fractional \(p_T\) difference. Contributions from non-\(Z\) backgrounds in the sideband regions are subtracted. The impact from a correlation between the above two variables is determined from the simulation and a correction, of at most 7%, is applied to account for it. The main uncertainties are due to variations in this correction and differences in the shape of the \(E_T^{\text{miss}}\) distribution in the control regions. The overall systematic uncertainty is 52% in the 7 TeV data and 59% in the 8 TeV data.

The small background from events with only one genuine isolated lepton (inclusive \(W\), single-lepton top pairs and single top production) or from multijet events is estimated from data using control samples, selected by requiring two lepton candidates of which at least one fails the full lepton selection criteria. These samples are scaled with a measured \(p_T\)-dependent factor, determined from data as described in Ref. [52]. Systematic uncertainties are determined following the procedures used in Ref. [52], yielding an uncertainty of 40% in the 7 TeV data and 21% in the 8 TeV data.

Systematic uncertainties on the signal and the SM ZZ and WZ backgrounds are derived from the luminosity uncertainty, the propagation of reconstructed object uncertainties, and from theoretical uncertainties on the production cross sections. The luminosity uncertainty is 1.8% for the 7 TeV data-taking period and 2.8% for the 8 TeV data-taking period [53].

Lepton trigger and identification efficiencies as well as the energy scale and resolution are determined from data using large samples of \(Z\) events. After appropriate corrections to the simulation, uncertainties are propagated to the

### Table I

Number of events observed in data and expected from the signal and from each background source for the 7 and 8 TeV data-taking periods. Uncertainties on the signal and background expectations are presented with statistical uncertainties first and systematic uncertainties second.

<table>
<thead>
<tr>
<th>Data period</th>
<th>2011 (7 TeV)</th>
<th>2012 (8 TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ZZ \rightarrow \ell\ell\nu\nu)</td>
<td>20.0 ± 0.7 ± 1.6</td>
<td>91 ± 1 ± 7</td>
</tr>
<tr>
<td>(WZ \rightarrow \ell\ell\ell)</td>
<td>4.8 ± 0.3 ± 0.5</td>
<td>26 ± 1 ± 3</td>
</tr>
<tr>
<td>Dileptonic (t\bar{t}, Wt, WW, Z \rightarrow \tau\tau)</td>
<td>0.5 ± 0.4 ± 0.1</td>
<td>20 ± 3 ± 5</td>
</tr>
<tr>
<td>(Z \rightarrow ee, Z \rightarrow \mu\mu)</td>
<td>0.13 ± 0.12 ± 0.07</td>
<td>0.9 ± 0.3 ± 0.5</td>
</tr>
<tr>
<td>(W + \text{jets, multijet, semileptonic top})</td>
<td>0.020 ± 0.005 ± 0.008</td>
<td>0.29 ± 0.02 ± 0.06</td>
</tr>
<tr>
<td>Total background</td>
<td>25.4 ± 0.8 ± 1.7</td>
<td>138 ± 4 ± 9</td>
</tr>
<tr>
<td>Signal ((m_H = 125.5) GeV, (\sigma_{ZH,SM}, \text{BR}(H \rightarrow \text{inv.}) = 1))</td>
<td>8.9 ± 0.1 ± 0.5</td>
<td>44 ± 1 ± 3</td>
</tr>
<tr>
<td>Observed</td>
<td>28</td>
<td>152</td>
</tr>
</tbody>
</table>

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event selection. These uncertainties contribute typically 1.0%–1.5% to the overall selection uncertainty. Jet energy scale and resolution uncertainties are derived using a combination of techniques that use dijet, photon + jet, and $Z + jet$ events [54,55]. These contribute an uncertainty of between 3% and 6% on the final event selection. The uncertainties on the energy scale and resolution of leptons and jets are also propagated to the $E_T^{miss}$ calculation, and the resulting uncertainty in the latter is included in uncertainties given above. Uncertainties in the pile-up simulation, affecting in particular $E_T^{miss}$, contribute a further 1%–2% uncertainty.

Theoretical uncertainties on the $ZH$ production cross section are derived from variations of the renormalization and factorization scale, $\alpha_s$, and the parton distribution functions (PDFs) [24]. These are combined to give an uncertainty of 3.6%–5.7% on the cross section. This analysis is sensitive to the distribution of the Higgs boson $p_T$ through the $E_T^{miss}$, and uncertainties in the $p_T$ boost of the Higgs boson can affect the signal yield. An additional systematic uncertainty of 1.9% is applied to the normalization [22,23,56], and uncertainties as a function of the Higgs boson $p_T$ are considered as a systematic shape uncertainty.

The cross-section uncertainty on the ZZ background is 5% from varying the PDFs, $\alpha_s$, and QCD scale. The uncertainty on the jet veto for the ZZ background due to the parton showering is estimated to be 6.4% (5.5%) for the 7 (8) TeV data. Because the $E_T^{miss}$ distribution of the final selected sample is used in the limit-setting procedure, the impact of PDFs, $\alpha_s$, and QCD scale uncertainties on the shape of this distribution is also considered. The theoretical uncertainty of the WZ background is considered similarly. The total systematic uncertainty on the SM ZZ background is 8% for both the 7 and 8 TeV data-taking periods, whereas for the WZ background it is 10% (13%) for the 7 (8) TeV data-taking periods.

Event reconstruction and theoretical uncertainties are considered as correlated between the 7 and 8 TeV data, and between the signals and backgrounds estimated from simulation. The systematic uncertainties in methods that determine backgrounds from data using control regions are also assumed to be correlated between the two data sets. The luminosity uncertainty is considered as uncorrelated between the 7 and 8 TeV data.

The numbers of observed and expected events for the 7 and 8 TeV data-taking periods are shown in Table I. Figure 2 shows the $E_T^{miss}$ distribution after the full event selection for the 8 TeV data and the expected backgrounds. The normalization of the backgrounds is extracted from a binned profile maximum likelihood fit in the signal region. Systematic uncertainties are considered as nuisance parameters, and are assumed to be constrained by Gaussian distributions. The signal expectation shown corresponds to a Higgs boson with $m_H = 125.5$ GeV, a SM $ZH$ production rate, and $BR(H \to \text{inv.}) = 1$. No significant excess is observed over the SM expectation.

Limits are set on the cross section times branching ratio for a Higgs boson decaying to invisible particles anywhere in the mass range $110 < m_H < 400$ GeV. The limits are computed using a maximum likelihood fit to the $E_T^{miss}$ distribution following the CL$_S$ (signal confidence level) modified frequentist formalism [57] with a profile likelihood test statistic [58]. Figure 3 shows the 95% C.L. upper limits on $\sigma_{ZH} \times BR(H \to \text{inv.})$ in the mass range $110 < m_H < 400$ GeV for the combined 7 and 8 TeV data. The expectation for a Higgs boson with a production cross section equal to that expected for a SM Higgs boson and $BR(H \to \text{inv.}) = 1$ is also shown.
For the discovered Higgs boson an upper limit of 75% at 95% C.L. (63% at 90% C.L.) is set on the branching ratio to invisible particles. For this the predicted SM $ZH$ production rate with $m_H = 125.5$ GeV, is assumed. The expected limit in the absence of BSM decays to invisible particles is 62% at 95% C.L. (52% at 90% C.L.).

Within the context of a Higgs-portal DM scenario [59], in which the Higgs boson acts as the mediator particle between DM and SM particles, the Higgs boson can decay to a pair of DM particles. In this case the limit on $\text{BR}(H \rightarrow \text{inv})$ for the 125.5 GeV Higgs boson can be interpreted in terms of an upper limit on the DM–nucleon scattering cross section [60]. The formalism used to interpret the $\text{BR}(H \rightarrow \text{inv})$ limit in terms of the spin-independent DM–nucleon scattering cross section is described in Refs. [61,62]. Figure 4 shows 90% C.L. upper limits on the DM–nucleon scattering cross section for three model variants in which a single DM candidate is considered and is either a scalar, a vector, or a Majorana fermion. The Higgs–nucleon coupling is taken as $0.33^{+0.30}_{-0.07}$ [62], the uncertainty of which is expressed by the bands in the figure. Spin-independent results from direct-search experiments are also shown [63–70]. These results do not depend on the assumptions of the Higgs-portal scenario. Within the constraints of such a scenario, however, the results presented in this Letter provide the strongest available limits for low-mass DM candidates. There is no sensitivity to these models once the mass of the DM candidate exceeds $m_H/2$. A search by the ATLAS experiment for DM in more generic models, also using the dilepton + large $E_T^{\text{miss}}$ final state, is presented in Ref. [71].

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[45] ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point in the center of the detector and the z axis along the beam pipe. Polar coordinates \((r, \phi)\) are used in the transverse plane, \(\phi\) being the azimuthal angle around the beam pipe. The pseudorapidity \(\eta\) is defined in terms of the polar angle \(\theta\) as \(\eta = -\ln \tan(\theta/2)\), and \(\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}\).