Whole cell tracking and movement reconstruction through an optimal control problem

Feng Wei Yang
with Anotida Madzvamuse and Chandrasekhar Venkataraman

University of Sussex

F.W.Yang@sussex.ac.uk

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Whole cell tracking through optimal control problem
  - The mathematical model
  - Toy models for proving concepts
  - Real world application
  - 3-D simulation

IBiDi data
  - Our automatic segmentation algorithm
  - Particle tracking results
Objectives

To track the morphology of cells and reconstruct their movements:

V. Peschetola et al. *Cytoskeleton*, 2013
What is our signature

Particle tracking:

- The morphology of cells are not considered
- Manually tracking is slow
- Automatic tracking algorithms are often flawed
  - Segmentation is suboptimal for real data
  - Tracking through pattern recognition is challenging

Pure geometric math models:

- Resolution of the data matters
- Typically no cell-setting is considered
- It is a complicated procedure to obtain the results
- Computational power and advanced numerical methods have to be included for 3-D real-life cell tracking
Our optimal control model

The volume conserved mean curvature flow:

\[
\begin{align*}
\mathbf{V}(\mathbf{x}, t) &= (-\sigma H(\mathbf{x}, t) + \eta(\mathbf{x}, t) + \lambda \mathbf{v}(t)) \mathbf{v}(\mathbf{x}, t) \quad \text{on } \Gamma(t), \ t \in (0, T], \\
\Gamma(0) &= \Gamma^0.
\end{align*}
\]

The phase-field approximation of the above equation - Allen-Cahn:

\[
\begin{align*}
\partial_t \phi(\mathbf{x}, t) &= \Delta \phi(\mathbf{x}, t) - \frac{1}{\epsilon^2} G'(\phi(\mathbf{x}, t)) - \frac{1}{\epsilon} (c_G \eta(\mathbf{x}, t) - \lambda(t)) \quad \text{in } \Omega \times (0, T], \\
\nabla \phi \cdot \mathbf{n} &= 0 \quad \text{on } \partial \Omega \times (0, T], \\
\phi(\cdot, 0) &= \phi^0 \quad \text{in } \Omega.
\end{align*}
\]
Our optimal control model cont.

The objective functional:

\[
J(\phi, \eta) = \frac{1}{2} \int_\Omega (\phi(x, T) - \phi_{\text{obs}}(x))^2 \, dx + \frac{\theta}{2} \int_0^T \int_\Omega \eta(x, t)^2 \, dx \, dt, 
\]

and now we solve the minimisation problem:

\[
\min_\eta J(\phi, \eta), \quad \text{with } J \text{ given above.}
\]
Obtaining solutions

We are using one of the most efficient solution methods, combining most advanced adaptive techniques. Meanwhile, parallelism is employed and the computation has been carried out on large computer cluster with multiple number of computational cores.
Toy model for proving concepts

A circle becomes two ellipses.

Initial data

Desired data
Toy model for proving concepts
A circle becomes 4 children circles.
Toy model for proving concepts cont.
Toy model for proving concepts cont.
Real world application

Two segmented cell images.

Initial data

Desired data
Real world application cont.
Real world application cont.
3-D simulation

(a) Initial shape \( (t=0) \)

(b) Desired shape \( (t=1) \)

(c) Computed solution at \( t=0.05 \)

(d) Computed final shape at \( t=0.1 \)
3-D simulation cont.
IBiDi data cont.
Issues with basic segmentation
Our simple solution to segmentation
Particle tracking
Thank you.