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Research Article

Actuator Fault Diagnosis with Application to a Diesel Engine Testbed

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This work addresses the issues of actuator fault detection and isolation for diesel engines. We are particularly interested in faults affecting the exhaust gas recirculation (EGR) and the variable geometry turbocharger (VGT) actuator valves. A bank of observer-based residuals is designed using a nonlinear mean value model of diesel engines. Each residual on the proposed scheme is based on a nonlinear unknown input observer and designed to be insensitive to only one fault. By using this scheme, each actuator fault can be easily isolated since only one residual goes to zero while the others do not. A decision algorithm based on multi-CUSUM is used. The performances of the proposed approach are shown through a real application to a Caterpillar 3126b engine.

1. Introduction

On-board diagnosis of automotive engines has become increasingly important because of environmentally based legislative regulations such as OBDII (On-Board Diagnostics-II) [1]. On-board diagnosis is also needed to guarantee high-performance engine behavior. Today, due to the legislation, the majority of the code in modern engine management systems is dedicated to diagnosis.

Model-based diagnosis of automotive engines has been considered in earlier papers (see, e.g., [2, 3]), to name only a few. However, the engines investigated in these previous works were all gasoline-fuelled and did not include exhaust gas recirculation (EGR) and variable geometry turbocharger (VGT). Both of these components make the diagnosis problem significantly more difficult since the air flows through the EGR valve, and also the exhaust side of the engine has to be taken into account. An interesting approach to model-based air-path faults detection for an engine which includes EGR and VGT can be found in [4, 5]. By using several models in parallel, where each one is sensitive to one kind of fault, predicted outputs are compared and a diagnosis is provided. The hypothesis test methodology proposed in [4] deals with the multifault detection in air-path system. In [5] the authors propose an extended adaptive Kalman filter to find which faulty model best matches with measured data; then a structured hypothesis allows going back to the faults.

A structural analysis for air path of an automotive diesel engine was developed in order to study the monitorability of the system [6–8]. Other approaches to detect intake leakages in diesel engines based on adaptive observers are proposed in [9, 10] and recently in [11]. Note that, in all these approaches, the leakage size is assumed to be constant. To overcome this limitation, an approach based on a nonlinear unknown input observer (NIUO), for intake leakage detection, is proposed in [12]. No a priori assumption about the leakage size is made. In [13, 14], an interesting method for bias compensation in model-based estimation using model augmentation is proposed. The extended Kalman filter (EKF) is used for estimating the states of the augmented model. Also,
the observability of the augmented model is well discussed. Recently, an automated model-based and data-driven design methodology for automotive engine fault detection and isolation (FDI) is proposed in [15]. This methodology, which combines model-based sequential residual generation and data-driven statistical residual evaluation, is used to create a complete FDI system for an automotive diesel engine.

The problem of designing unknown input observers (UIO) has received great attention in literature. This problem is motivated by certain applications such as fault diagnosis and control system design. If this issue in linear case is well solved, it remains an open problem in the nonlinear case. The first unknown input observers dedicated to linear systems were proposed in [16, 17]. Necessary and sufficient conditions of the existence of the UIO have been well established. New sufficient conditions formulated in terms of linear matrix inequality (LMI) were given by [18]. This result is extended in [12] to cover a wide class of nonlinear systems that cannot be treated by the previous approach as well as nonlinear systems with a large Lipschitz constant.

Our aim is to address the issue of actuator fault detection and isolation for diesel engines. Indeed, actuators (EGR and VGT) fault diagnosis is necessary and crucial to guarantee its healthy operation. In this work, a fault detection and isolation (FDI) system is developed. The proposed FDI system is composed of two parts: residual generation and decision system. A multiobserver strategy is used for residual generation. A mean model of diesel engine is exploited for the design of a set of nonlinear unknown input observers. These observers are designed in order to estimate the states behavior without any knowledge of the unknown inputs. The sufficient conditions of the existence of the NUIO are given in terms of linear matrix inequalities (LMIs). The advantage of this method is that no a priori assumption on the unknown input is required and also can be employed for a wider class of nonlinear systems. To achieve fault detection and isolation, a decision system based on a statistical approach, multi-CUSUM (cumulative sum), is used to process the resulting set of residuals.

This paper is organized as follows. The experimental setup is described in Section 2. The diesel engine model and its validation are described in Section 3. A nonlinear unknown input observer is presented in Section 4. Section 5 describes the residual generation system while the decision system is presented in Section 6. The experimental results and discussion are presented in Section 7. Finally, conclusion and future works are given in the last section.

The notations used in this paper are quite standard. Let $\mathbb{R}$ denote the set of real numbers. The set of $p$ by $q$ real matrices is denoted as $\mathbb{R}^{p \times q}$. $A^T$ and $A^{-1}$ represent the transpose of $A$ and its left inverse (assuming $A$ has full column rank), respectively. $I_r$ represents the identity matrix of dimension $r$. $(\ast)$ is used for the blocks induced by symmetry. $\| \cdot \|$ represents the usual Euclidean norm. $L^2$ denotes the Lebesgue space. $f_{a,i}$ denotes the $i$th component of the vector $f_a$.

## 2. Experimental Installation

The testbed is built with a Caterpillar 3126b truck engine coupled to a SCHORCH dynamometer controlled by CP Cadet Software. The Caterpillar engine is presented in Figure 1. The front and back view of the testbed are shown by the pictures numbered (1) and (2), respectively. Engine's components are inlet manifold (3), encoder for measuring of the engine speed (4), intake air flow meter (5), exhaust manifold (6), GT3782VA variable geometry turbocharger, (7) and exhaust gas valve (8). In order to enable a transient control of the EGR and VGT, a dSPACE MicroAutoBox 1401/1501 real-time controller is connected (see Figure 2). Apart from the standard OEM electronic sensors built in for ECU and dynamometer control, additional sensors and actuators have
been wired in specifically for the MicroAutoBox. These sensors and actuators are listed in Table 1.

The data and control signal flow are illustrated in Figure 2. The engine is connected with two control platforms, which are CP Cadet and dSPACE Control Desk. The engine tests are conducted and monitored by CP Cadet Platform while the EGR and VGT valve positions can be adjusted through dSPACE Control Desk in real time. Testing data can be collected from both platforms and used for data analysis purpose.

### Table 1: Sensors and actuators list.

<table>
<thead>
<tr>
<th>Sensors</th>
<th>Actuators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet temperature sensor</td>
<td>EGR valve drive actuator</td>
</tr>
<tr>
<td>Inlet air flow meter</td>
<td>VGT vanes drive actuator</td>
</tr>
<tr>
<td>Inlet pressure sensor</td>
<td></td>
</tr>
<tr>
<td>Pre-turbo exhaust pressure sensor</td>
<td></td>
</tr>
<tr>
<td>Acceleration pedal position sensor</td>
<td></td>
</tr>
<tr>
<td>Engine speed sensor</td>
<td></td>
</tr>
<tr>
<td>Inlet manifold oxygen sensor</td>
<td></td>
</tr>
<tr>
<td>Exhaust manifold oxygen sensor</td>
<td></td>
</tr>
<tr>
<td>Exhaust opacity sensor (AVL Opacimeter 439)</td>
<td></td>
</tr>
<tr>
<td>Exhaust emission sensor (Testo 350 Engine test kit)</td>
<td></td>
</tr>
<tr>
<td>EGR position feedback sensor</td>
<td></td>
</tr>
<tr>
<td>VGT position feedback sensor</td>
<td></td>
</tr>
</tbody>
</table>

The specification of the Caterpillar 3126b midrange truck engine is given in Table 3.

### 3. Engine Model and Validation

The considered diesel engine is a six-cylinder engine with a high-pressure EGR and VGT. A principle illustration scheme of the air-path system is shown in Figure 3. It consists of two parts: the turbocharger and exhaust gas recirculation. The turbocharger is a turbine driven by the exhaust gas and connected via a common shaft to the compressor, which
compresses the air in the intake. The exhaust gas recirculation (EGR) allows for recirculating gas from the exhaust manifold to the intake manifold. First, the mixture of air coming from the compressor and exhaust gas coming through the EGR valve enters the intake manifold before injecting it into the cylinders. Then, the fuel is injected directly in the cylinders and burned, producing the torque on the crank shaft. Exhaust gases are expelled into the exhaust manifold.

As shown in Figure 3, part of exhaust gas comes out from the exhaust manifold through the turbine and the other part is recirculated through EGR valve. We noted that the temperature of gases (compressed air and EGR gas) entering the intake manifold is reduced using the intercooler and the EGR cooler.

The mean value engine modeling approach is one of the most considered approaches in the literature [19]. It uses temporal and spatial averages of relevant temperatures, pressures, and mass flow rates. The engine model is derived based on the laws of conservation of mass and energy and also on the ideal gas law (see, for instance, [5, 20]). As an example, the pressure dynamics in the intake manifold is obtained by differentiating the ideal gas law equation \( PV = mRT \).

The complete considered mean value model is expressed as follows [5]:

\[
\dot{P}_{\text{Inlet}} = \frac{1}{V_{\text{Inlet}}} \left( R_{\text{Air}} \dot{V}_{\text{Air}} + R_{\text{EGR}} \dot{V}_{\text{EGR}} \right) + \frac{R_{\text{Inlet}} \dot{V}_{\text{Inlet}}}{c_{v,\text{Inlet}}} W_{\text{EGR}} T_{\text{EGR}}
\]

\[
T_{\text{EGR}} = \left( \frac{P_{\text{Inlet}}}{P_{\text{Exh}}} \right)^{\frac{\gamma_{\text{Exh}}-1}{\gamma_{\text{Exh}}}} T_{\text{Exh}}
\]

\[
W_{\text{Exh}} = W_{\text{Inlet}} + W_{\text{Fuel}}
\]

\[
T_{\text{Exh}} = T_{\text{Inlet}} + \frac{Q_{\text{LHV}} h(w_{\text{Fuel}} N_{\text{Eng}})}{c_{p,\text{Exh}} (W_{\text{Inlet}} + W_{\text{Fuel}})}
\]

\[
p_{\text{Exh}} = \frac{m_{\text{Exh}} R_{\text{Exh}} T_{\text{Exh}}}{V_{\text{Exh}}}
\]

\[
W_{\text{Turb}} = \frac{P_{\text{Exh}}}{\sqrt{T_{\text{Exh}}}} \left( \frac{P_{\text{Exh}}}{P_{\text{Arm}}}, u_{\text{VXT}} \right)
\]

\[
R_{\text{Inlet}} = \frac{R_{\text{Air}} m_{\text{Air}} + R_{\text{EGR}} m_{\text{EGR}}}{m_{\text{Air}} + m_{\text{EGR}}}
\]

\[
c_{v,\text{inlet}} = c_{v,\text{fuel}} + c_{v,\text{EGR}} m_{\text{EGR}}
\]

\[
c_{p,\text{Exh}} = c_{v,\text{Exh}} + R_{\text{Inlet}}
\]

\[
A_{\text{EGR}} = \frac{A_{\text{EGRmax}} f_{\text{EGR}} (u_{\text{EGR}})}{}
\]

where \( P_{\text{Inlet}} \) is the pressure in intake manifold. \( m_{\text{Air}} \) and \( m_{\text{EGR}} \) are, respectively, the mass of air and EGR-gas in intake manifold. \( m_{\text{Exh}} \) represents the mass of exhaust gas in exhaust manifold. The other model variables and constant parameters with their value are listed in Table 4. The temperatures \( T_{\text{EGR}} \) and \( T_{\text{Exh}} \) are assumed to be constant and equal to 329.436 K and 837 K, respectively. \( A_{\text{EGRmax}} \) is the maximum opening area for the EGR. The EGR valve (VGT) is closed when \( u_{\text{EGR}} = 0 \% \) \((u_{\text{VXT}} = 0)\) and open when \( u_{\text{EGR}} = 100\% \) \((u_{\text{VXT}} = 100)\). The static functions, \( f_{\text{vol}}, f_{\text{EGR}}, h, \) and \( \tau \), are represented as interpolation in lookup tables. All these parameters are estimated using weighted least squares optimization approach. It is worth noting that the compressor and the CAC are not considered in the model as in [3] since the mass flow and the temperature after the charge-air cooler (CAC) are known variables because they are measured by the production sensors. The considered control inputs are the VGT vane position \( u_{\text{VXT}} \) and the EGR valve position \( u_{\text{EGR}} \). Notice that the following variables: \( T_{\text{CAC}}, W_{\text{HFM}}, N_{\text{Eng}}, W_{\text{Fuel}}, \) and \( T_{\text{Exh}} \) are considered as measurable signals. The measured outputs are the inlet and exhaust pressures.

We draw the reader’s attention that another general mean value model composed of eight states is developed, parameterized, and validated in [21]. The obtained model describes the gas flow dynamics including the dynamics in the manifold pressures, EGR, turbocharger, and actuators.

The mean value model described in the previous section was simulated and compared with real measurements obtained from the Caterpillar testbed. The results are depicted in Figures 4–7. The input variables used for the validation purpose are illustrated in Figure 4. The measured and modeled outputs \( (P_{\text{Inlet}} \) and \( P_{\text{Exh}} \)) are presented in Figure 5. The absolute value of the difference between the real and estimated variables is shown in Figure 6. One can conclude that the used model can reproduce engine dynamic behavior.
Figure 4: Input variables: (a) EGR position [%], (b) VGT position [%], and (c) engine speed [min⁻¹].

Figure 5: Modeled (blue color) and measured (red color) inlet and exhaust pressures.
with good accuracy since the obtained average error is below 3%. Besides, the output measurements are very noisy as shown in Figure 5. Furthermore, the air and exhaust gas mass flows \( m_{\text{Air}} \) and \( m_{\text{EGR}} \) are also presented in Figure 7.

### 4. Nonlinear Unknown Input Observer Design

In this section, we will present briefly a nonlinear unknown input observer developed in [12]. It was proposed for a class of Lipschitz nonlinear systems with large Lipschitz constant. The sufficient existence conditions for this observer are formulated in terms of LMIs.

Obviously, engine model (1) can be put into the form (3a) and (3b). One can see that this form contains three parts: one linear parameter varying part, a nonlinear term with known variables, and a nonlinear state-dependent part. By considering the form (3a) and (3b), this will allow us to tackle our synthesis problem based on Lyapunov theory for LPV systems. Thus, let us consider the general class of nonlinear systems described by the following equations:

\[
\dot{x} = \sum_{j=1}^{n_u} \rho_j A_j x + B_g (v, y, u) + f(x, u) + B_f f_a + B_m \overline{w},
\]

\[
y = Cx + D_m \overline{w},
\]

where \( x \in \mathbb{R}^{n_x} \) is the state vector, \( u \in \mathbb{R}^{n_u} \) is the control input vector, \( f_a \in \mathbb{R}^{n_x} \) represents the actuator faults assimilated as unknown inputs, \( y \in \mathbb{R}^{n_y} \) is the output vector, \( \overline{w} \in \mathbb{R}^{n_w} \) is the vector of disturbances/noises, and \( v \in \mathbb{R}^{n_v} \) is the vector of measurable signals. Notice that \( v \) contains any other
measured signals which have no link with the system output (\( y \)) (e.g., temperature measurement, air mass flow (\( W_{\text{AM}} \)), etc.). \( A_{ij} \) for \( j = 1, \ldots, n_{p} \), \( B_{p} \), \( B_{f} \), \( B_{\omega} \), \( C \), and \( D_{\omega} \) are constant matrices with appropriate dimensions. Without loss of generality, the matrix \( B_{f} \) is assumed to be of full column rank. The functions \( g(\cdot, y, u) \) and \( f(x, u) \) are nonlinear. No assumption is made about the function \( g(\cdot, y, u) \). However, the function \( f(x, u) \) is assumed to be once differentiable with large Lipschitz constant. The weighting functions \( \rho_{j} \) are assumed to be known and depend on measurable variables. They verify the convex sum property:

\[
\sum_{j=1}^{n_{p}} \rho_{j} = 1, \quad \rho_{j} \geq 0, \quad \forall j \in \{1, \ldots, n_{p}\}. \tag{4}
\]

Let us first rewrite \( B_{f} f_{a} \) as follows:

\[
B_{f} f_{a} = \sum_{i=1}^{n_{f}} B_{f,i} f_{a,i}, \tag{5}
\]

where \( B_{f,i} \) is the distribution matrix of the fault \( f_{a,i} \).

The aim is to construct an EUIO-based residual generator which is insensitive to only one actuator fault (e.g., \( B_{f,i} f_{a,i} \)). By choosing the unknown input (scalar) \( d = f_{a,i} \), the system (3a) and (3b) can be rewritten as

\[
\dot{x} = \sum_{j=1}^{n_{p}} \rho_{j} A_{j} x + B_{g} g(u, y, u) + f(x, u) + B_{d} d + B_{w} w, \tag{6a}
\]

\[
y = C x + D_{\omega} w, \tag{6b}
\]

where \( B_{d} = B_{f,i} \), \( B_{\omega} = [B_{\omega} \quad B_{f}] \), \( D_{\omega} = [D_{\omega} \quad 0] \), and \( w = [\bar{w} \quad \bar{f}] \) with \( \bar{f} \) the actuator fault vector \( f_{a} \) without the \( i \)th component. \( B_{f,i} \) is the matrix \( B_{f} \) without the \( i \)th column.

It is worth noting that the necessary condition for the existence of a solution to the unknown input observer design is the following ([18, 22] for a more explanation): \( \text{Rank}(CB_{d}) = \text{Rank}(B_{d}) \), where \( B_{d} \) is a matrix of full column rank as is a column of the matrix \( B_{f} \) which is assumed before to be full column rank.

The residual generator for the system (6a) and (6b) is given by

\[
\dot{z} = N(\rho) z + G f(\bar{x}, u, \dot{u}) + M \bar{f} + L(\rho) y, \tag{7a}
\]

\[
\dot{\bar{x}} = z - E y, \tag{7b}
\]

\[
r = \Pi_{r}(y - C \bar{x}) \tag{7c}
\]

with \( N(\rho) = \sum_{j=1}^{n_{p}} \rho_{j} N_{j} \) and \( L(\rho) = \sum_{j=1}^{n_{p}} \rho_{j} L_{j} \). \( \Pi_{r} \) is a known matrix. \( \bar{x} \) represents the state estimation vector. Matrices \( N, G, M, L, \) and \( E \) are the observer gains and matrices which must be determined such that \( \bar{x} \) converges asymptotically to \( x \). Notice that the index \( \rho \) is omitted where it is not necessary to simplify the notations.

By defining the state estimation error as \( e(t) = \bar{x}(t) - x(t) \), the error dynamics can be expressed as

\[
\dot{e} = Ne + (G - MB_{p}) g(\cdot, y, u) + (NM + LC - MA)x - MB_{d} d + M \bar{f} + (KD_{\omega} - MB_{w}) w - ED_{\omega} \dot{w} \tag{8}
\]

with \( \bar{f} = f(\bar{x}, u) - f(x, u) \) and \( K = L + NE \). Now, if the following matrix equations are satisfied:

\[
N(\rho) = MA(\rho) - K(\rho) C, \quad \text{with each } N_{j} \text{ stable}, \tag{9a}
\]

\[
L(\rho) = K(\rho) \times (I + CE) - MA(\rho) E, \tag{9b}
\]

\[
G = MB_{p}, \tag{9c}
\]

\[
M = I + EC, \tag{9d}
\]

\[
MB_{d} = 0, \tag{9e}
\]

\( e(t) \) goes to zero asymptotically if \( w = 0 \) and is invariant with respect to the unknown input \( d(t) \). The notation \( I \) stands for the identity matrix.

Conditions (9d) and (9e) are equivalent to \( E C B_{d} = -B_{d} \). One necessary condition to have for \( E C B_{d} = -B_{d} \) is that \( CB_{d} \) is of full column rank since \( B_{d} \) is of full column rank. If \( CB_{d} \) is of full column rank, then all possible solutions of \( E C B_{d} = -B_{d} \) can be expressed as follows [18]:

\[
E = U + YV \tag{10}
\]

with \( U = -B_{d}(CB_{d})^{-1} \) and \( V = (I - CB_{d}(CB_{d})^{-1}) \) where \( Y \) can be any compatible matrix and \( X^{+} = (X^{T}X)^{-1}X^{T} \).

Then, the error dynamics becomes

\[
\dot{e} = (MA - KC) e + M \bar{f} + (KD_{\omega} - MB_{w}) w - ED_{\omega} \dot{w}. \tag{11}
\]

In order to minimize the effect of disturbances on the observer error, the \( H_{\infty} \) performance criterion can be used. However, the presence of the term \( \dot{w} \) makes the task difficult because it should be discarded from the derivative of the Lyapunov function as we will see later. Another solution is to add a negative term depending on \( \dot{w}^{T} \dot{w} \) as proposed in [23]. This solution needs to modify the classical \( H_{\infty} \) criterion. In this work, the modified \( H_{\infty} \) criterion presented in [23] is used.

The modified \( H_{\infty} \) estimation problem consists in computing the matrices \( N \) and \( L \) such that

\[
\lim_{t \to \infty} e(t) = 0 \quad \text{for } w(t) = 0, \tag{12a}
\]

\[
\|e\|_{\infty} \leq \gamma \|w\|_{1,2} \quad \text{for } w(t) \neq 0; \quad e(0) = 0, \tag{12b}
\]

where \( \| \cdot \|_{h,p} \) represents the Sobolev norm (see [23]).

Then to satisfy (12a)-(12b), it is sufficient to find a Lyapunov function \( Y \) such that

\[
\Gamma = \dot{Y} + e^{T} e - \gamma^{2} \dot{w}^{T} \dot{w} - \gamma^{2} \dot{w}^{T} \dot{w} < 0, \tag{13}
\]
where \( Y = e^T P e \) with \( P \) a positive definite symmetric matrix.

From (II), \( \Gamma \) is given by
\[
\Gamma = e^T \left( (MA - KC)^T P + P (MA - KC) \right) e
+ e^T PM \bar{f} + (M \bar{f})^T Pe
+ e^T P (KD_w - MB_w) w + w^T (KD_w - MB_w)^T Pe
- \bar{e}^T PED_u \bar{w}
- w^T (ED_u)^T Pe + e^T e - \gamma^2 w^T w - \gamma^2 \bar{w}^T \bar{w}.
\] (14)

Before introducing our main result, let us present the modified mean value theorem [24] for a vector function.

**Theorem 1** (see [24]). Let the canonical basis of the vectorial space \( \mathbb{R}^l \) for all \( s \geq 1 \) be defined by
\[
E_s = \left\{ e_s(i) \mid e_s(i) = \left( 0, \ldots, 0, \frac{i}{s}, 0, \ldots, 0 \right)^T, \; i = 1 \cdots s \right\}.
\] (15)

Let \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \) be a vector function continuous on \([a, b] \in \mathbb{R}^n \) and differentiable in convex hull of the set \((a, b)\). For \( s_1, s_2 \in [a, b] \), there exist \( \delta_{ij}^{\max} \) and \( \delta_{ij}^{\min} \) for \( i = 1 \cdots n \) and \( j = 1 \cdots n \) such that
\[
f(s_2) - f(s_1) = \left( \sum_{i,j=1}^{n,n} H_{ij}^{\max} \delta_{ij}^{\max} \right) + \left( \sum_{i,j=1}^{n,n} H_{ij}^{\min} \delta_{ij}^{\min} \right)
(s_2 - s_1)
= \left[ \sum_{i,j=1}^{n,n} \left( H_{ij}^{\max} \delta_{ij}^{\max} \right) + \sum_{i,j=1}^{n,n} \left( H_{ij}^{\min} \delta_{ij}^{\min} \right) \right] (s_2 - s_1)
\] (16)

where

(i) \( H_{ij}^{\max} \geq \max(\partial f_\alpha/\partial x_i, \gamma_\alpha) \) and \( H_{ij}^{\min} \leq \min(\partial f_\alpha/\partial x_i, \gamma_\alpha) \).

(ii) \( H_{ij}^{\max} = e_n(i) e_n(j) h_{ij}^{\max} \) and \( H_{ij}^{\min} = e_n(i) e_n(j) h_{ij}^{\min} \).

The proof of this theorem is given in [24].

In our case, the nonlinear function \( f \) depends on the state vector \( x \) and also on the known input \( u \). Although the previous theorem is applicable in our case, \( u \) is bounded.

Now, we can give a sufficient condition under which the observer given by (7a), (7b), and (7c) is a NUO. Thus, the negativity of \( \Gamma \) is ensured by the following theorem.

**Theorem 2** (see [25]). The observer error \( e(t) \) converges asymptotically towards zero if there exist matrices \( \overline{K}_k, \overline{Y} \), a positive definite symmetric matrix \( P \), and a positive scalar \( \mu \) such that the following LMIs are satisfied:
\[
P > 0,
\]
\[
\begin{bmatrix}
\Sigma_{i,j=1}^{n,n} \left( \delta_{ij}^{\max} + \delta_{ij}^{\min} \right) = n \times n - n_0, \\
\sum_{i,j=1}^{n,n} \left( \delta_{ij}^{\max} + \delta_{ij}^{\min} \right) = 1,
\end{bmatrix}
\] (19)

where \( n_0 \) is the number of terms in \( \partial f_\alpha/\partial x_j \) that equal zero.
The procedure to design the NUO parameters is summarized by the following algorithm.

Algorithm 3. NUO design is as follows:

1. compute $U$ and $V$ from (10),
2. determine matrices $Y$ and $K_k$, with $k = 1, \ldots, n_p$, from the LMI sets (17a)-(17c),
3. compute the observer matrices $E, M, G, N,$ and $L$ from (10), (9d), (9c), (9b), and (9a), respectively.

5. Residual Generation

This section addresses the problem of actuator faults isolation based on a bank of residual generators (see Figure 8). Each residual on the proposed scheme is a NUO based on model (6a) and (6b). Notice that the system (1) can be easily rewritten in a state space form as (3a) and (3b) (see Appendix C). Furthermore, each residual is designed to be insensitive to only one fault. Thus, the actuator faults can be easily isolated since only one residual goes to zero while the others do not. Therefore, for our application, the combination of two observers is sufficient to detect and isolate any faulty actuator. It is also assumed that only a single actuator fault can occur at one time.

To construct the bank of residual generators we have to

(i) design an NUO which generates the residual $r_1$ insensitive to $f_{a,1}$ by taking

$$B_d = \begin{bmatrix} R_{\text{Exh}}T_{\text{EGR}}A_{\text{EGRmax}}c_{\text{p,Exh}} \cr V_{\text{Inlet}}c_{\gamma,\text{Exh}} \cr 0 \cr A_{\text{EGRmax}} - A_{\text{EGRmax}} \end{bmatrix}^T,$$  \hspace{1cm} (20)

(ii) design an NUO which generates the residual $r_2$ insensitive to $f_{a,2}$ by taking

$$B_d = [0 \ 0 \ 0 \ 1]^T.$$  \hspace{1cm} (21)

To obtain $B_d$ as in (20), it suffices to replace in the original system the input vector $u$ by $u + f_a$, where $f_a$ represents the actuator fault vector. Thus, $B_d$ can be obtained easily by gathering the terms multiplying the vector $f_a$ in one matrix.

Each residual, $r_i(t)$ ($i = 1, 2$), computed from (7a), (7b), and (7c), is a 1-dimensional vector associated with one of the actuators $u_{\text{EGR}}$ and $u_{\text{XVNT}}$. By stacking the two vectors, one defines $\bar{r}(t) = [r_1(t), r_2(t)]^T$. In the absence of faults, this vector of residuals has zero mean, while, upon occurrence of a single step-like fault, the effect on each of its components is indicated in Table 2. A “x” is placed when the fault located in the corresponding column affects the mean of the residual component on the corresponding row, and a “0” when the residual component presents very low sensitivity to the fault. The sampled vector $\bar{r}(k)$ can be rewritten as

$$\bar{r}(k) = \bar{r}_0(k) + \sum_{\ell=1}^{2} r_{\ell} \Gamma_k (k) 1_{[\ell \leq k]} \delta_\ell,$$  \hspace{1cm} (22)

where the sample number $k$ corresponds to the time instant $t = kT$, with $T$, denoting the sampling period, $\bar{r}_0(k)$ is the fault-free residual, $\Gamma_k(k)$ is the dynamic profile of the change on $\bar{r}(k)$ due to a unit step-like fault $f_\ell$, and $\delta_\ell$ is the magnitude of fault $\ell$. Model (22) will be used as the basis for the decision system design in the next section.

One can see that residuals are designed in a deterministic framework. However, due to measurement noise and system disturbances, parameters uncertainties, and variations, the computed residuals are stochastic signals. As a consequence, a multi-CUSUM method will be used for the decision step. Another way would have been to consider the noise and stochastic disturbances characteristics to design robust residuals, analyze the stochastic characteristics of the residuals,

![Figure 8: FDI system.](image-url)
Indeed, in model multi-CUSUM, was developed in [27], each of them involving the log-likelihood ratio between hypotheses \( \mathcal{H}_\ell \) and \( \mathcal{H}_0 \), namely,

\[
s_k(\ell,0) = \ln \frac{p_k(\bar{r}(k))}{p_0(\bar{r}(k))},
\]

where \( p_k \) is the probability density function of \( \bar{r}(k) \) under the hypothesis \( \mathcal{H}_\ell \). Under the Gaussian hypothesis, the log-likelihood ratio can be rewritten as

\[
s_k(\ell,0) = (\mu_\ell - \mu_j)^T \Sigma^{-1} \left( \bar{r}(k) - \frac{1}{2} (\mu_\ell + \mu_j) \right).
\]

The CUSUM decision function is defined recursively as

\[
g_k(\ell,0) = \max \left( 0, g_{k-1}(\ell,0) + s_k(\ell,0) \right).
\]

In order to estimate the fault occurrence time, the number of successive observations for which the decision function remains strictly positive is computed as \( N_\ell(k) = N_\ell(k - 1)1_{\{ g_\ell(k) > 0 \}} + 1 \). To decide whether fault \( \ell \) has occurred, one has to check that, on average, all the likelihood ratios \( s_j(\ell, j) \), for \( 1 \leq j \neq \ell \leq 2 \), are significantly larger than zero. Noticing that \( s_j(\ell, j) = s_j(\ell, 0) - s_j(j, 0) \), one can build a CUSUM algorithm to decide between hypotheses \( \mathcal{H}_\ell \) and \( \mathcal{H}_j \) by taking into account the difference between \( g_\ell(\ell, 0) \) and \( g_\ell(\ell, 0) \). Hence a decision that \( \mathcal{H}_\ell \) holds can be issued when the following decision function becomes positive:

\[
\bar{g}_\ell = \min_{0 < j \neq \ell \leq 2} \left( g_k(\ell,0) - g_k(j,0) - h_{\ell,j} \right)
\]

for \( \ell = 1, 2 \) and \( g_k(0,0) = 0 \).

Like in [26, 27], it is advisable to consider only two values for the thresholds \( h_{\ell,j} \), for each \( \ell = 1, \ldots, n_f \), namely,

\[
h_{\ell,j} = \begin{cases} h_{\ell,j}^d, & \text{for } j = 0, \\ h_{\ell,j}^i, & \text{for } 1 \leq j \neq \ell \leq n_f, \end{cases}
\]

where \( h_{\ell,j}^d \) is the detection threshold and \( h_{\ell,j}^i \) is the isolation threshold. Notice that the mean detection delay, the mean time before false alarm, and the probability of a false isolation depend on the choice of these thresholds. Indeed, the thresholds \( h_{\ell,j}^d \) and \( h_{\ell,j}^i \) can be linked to the mean detection delay \( \bar{r} \) for the fault \( \ell \) thanks to the following expression [27] assuming that \( h_{\ell,j}^d = \bar{r}_{\ell,j} \) where \( \bar{r} \geq 1 \) is a constant:

\[
\bar{r} = \max \left( h_{\ell,j}^d, \min_{j > 0, \ell} \left( \bar{r}_{\ell,j} \right) \right) \text{ as } h_{\ell,j}^i \to \infty,
\]

where \( \bar{r}_{\ell,j} \) is the Kullback-Leibler information defined as

\[
\bar{r}_{\ell,j} = \frac{1}{2} (\mu_\ell - \mu_j)^T \Sigma^{-1} (\mu_\ell - \mu_j).
\]

and design the decision algorithm with respect to these characteristics. However, this would be too complex to do for our application and would lie anyway on given assumptions on the stochastic disturbances.

### 6. Decision System

In this section, we will present a decision system based on a statistical approach proposed in [26]. This approach, named multi-CUSUM, was developed in [27] and refined in [28]. Indeed, in model (22), it is seen that an actuator fault induces a change in the mean of the residual vector \( \bar{r}(k) \). Thus, the problem amounts to detecting and isolating a step-like signal within a white Gaussian noise sequence. Therefore, the change detection/isolation problem can be stated as the following hypothesis testing:

\[
\mathcal{H}_0 : \mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_0, \Sigma), \quad i = 1, \ldots, k,
\]

\[
\mathcal{H}_\ell : \mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_0, \Sigma), \quad i = 1, \ldots, k_\ell - 1,
\]

\[
\mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_\ell, \Sigma), \quad i = k_\ell, \ldots, k,
\]

where \( k_0 \in [1, k] \), and \( \mu_\ell = \mu_0 + \nu_\ell \Gamma_\ell \) is the mean value of the residual sequence when the \( \ell \)th fault has occurred, for \( \ell \in \{1, 2\} \). The decision system is based on a combination of CUSUM decision functions [27], each of them involving the log-likelihood ratio between hypotheses \( \mathcal{H}_\ell \) and \( \mathcal{H}_0 \), namely,

\[
\mathcal{H}_0 : \mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_0, \Sigma), \quad i = 1, \ldots, k,
\]

\[
\mathcal{H}_\ell : \mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_0, \Sigma), \quad i = 1, \ldots, k_\ell - 1,
\]

\[
\mathcal{L}(\bar{r}(i)) = \mathcal{N}(\mu_\ell, \Sigma), \quad i = k_\ell, \ldots, k,
\]
<table>
<thead>
<tr>
<th>Symb.</th>
<th>Quantity</th>
<th>Value/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{\text{inlet}} )</td>
<td>Pressure in intake manifold</td>
<td>Pa</td>
</tr>
<tr>
<td>( p_{\text{atm}} )</td>
<td>Atmospheric pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>( W_{\text{turb}} )</td>
<td>Exhaust mass flow past the turbine</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( \alpha_{\text{VNT}} )</td>
<td>Position of VNT vanes</td>
<td>( % )</td>
</tr>
<tr>
<td>( n_{\text{eng}} )</td>
<td>Engine speed</td>
<td>min(^{-1})</td>
</tr>
<tr>
<td>( W_{\text{fuel}} )</td>
<td>Mass flow of injected fuel</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( Q_{\text{LHV}} )</td>
<td>Lower heating value</td>
<td>J kg(^{-1})</td>
</tr>
<tr>
<td>( A_{\text{EGR}} )</td>
<td>Effective area of EGR valve</td>
<td>m(^2)</td>
</tr>
<tr>
<td>( R_{\text{inlet}} )</td>
<td>Gas constant in intake manifold</td>
<td>J (kg K)</td>
</tr>
<tr>
<td>( W_{\text{inlet}} )</td>
<td>Mass flow into engine inlet ports</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( T_{\text{inlet}} )</td>
<td>Temperature in the intake manifold</td>
<td>K</td>
</tr>
<tr>
<td>( m_{\text{air}} )</td>
<td>Mass of air in intake manifold</td>
<td>kg</td>
</tr>
<tr>
<td>( m_{\text{EGR}} )</td>
<td>Mass of EGR gas in intake manifold</td>
<td>kg</td>
</tr>
<tr>
<td>( W_{\text{exh}} )</td>
<td>Exhaust mass flow into exhaust manifold</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( m_{\text{exh}} )</td>
<td>Mass of exhaust gas in exhaust manifold</td>
<td>kg</td>
</tr>
<tr>
<td>( p_{\text{exh}} )</td>
<td>Pressure in exhaust manifold</td>
<td>Pa</td>
</tr>
<tr>
<td>( c_{\text{p,inlet}} )</td>
<td>Specific heat at const. pres. in intake manifold</td>
<td>J (kg K)</td>
</tr>
<tr>
<td>( c_{\text{p,exh}} )</td>
<td>Specific heat at const. vol in intake manifold</td>
<td>J (kg K)</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Ratio of specific heats</td>
<td>( c_{\text{p,inlet}} / c_{\text{p,exh}} )</td>
</tr>
<tr>
<td>( W_{\text{HFM}} )</td>
<td>Air mass flow past the air mass flow sensor</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( W_{\text{EGR}} )</td>
<td>EGR mass flow into intake manifold</td>
<td>kg s(^{-1})</td>
</tr>
<tr>
<td>( V_{\text{inlet}} )</td>
<td>Volume of intake manifold</td>
<td>1.8 × 10(^{-3}) m(^3)</td>
</tr>
<tr>
<td>( R_{\text{air}} )</td>
<td>Gas constant of air</td>
<td>288.2979 J (kg K)</td>
</tr>
<tr>
<td>( c_{\text{p,air}} )</td>
<td>Specific heat at const. pres. of air</td>
<td>1067.4 J (kg K)</td>
</tr>
<tr>
<td>( c_{\text{v,air}} )</td>
<td>Specific heat at const. vol. of air</td>
<td>779.1021 J (kg K)</td>
</tr>
<tr>
<td>( R_{\text{exh}} )</td>
<td>Gas constant of exhaust gas</td>
<td>290.155 J (kg K)</td>
</tr>
<tr>
<td>( c_{\text{p,exh}} )</td>
<td>Specific heat at const. pres. of exhaust gas</td>
<td>1288.08 J (kg K)</td>
</tr>
<tr>
<td>( c_{\text{v,exh}} )</td>
<td>Specific heat at const. vol. of exhaust gas</td>
<td>997.9250 J (kg K)</td>
</tr>
<tr>
<td>( T_{\text{c}} )</td>
<td>Temperature of the air after the charge-air cooler</td>
<td>322.5 K</td>
</tr>
<tr>
<td>( T_{\text{EGR}} )</td>
<td>Temperature of EGR gas flow into the i.m.</td>
<td>322 K</td>
</tr>
<tr>
<td>( T_{\text{exh}} )</td>
<td>Temperature in exhaust manifold</td>
<td>837 K</td>
</tr>
<tr>
<td>( V_{\text{eng}} )</td>
<td>Engine displacement</td>
<td>7.239 × 10(^{-3}) m(^3)</td>
</tr>
<tr>
<td>( V_{\text{exh}} )</td>
<td>Volume of exhaust manifold</td>
<td>9.4552 × 10(^{-3}) m(^3)</td>
</tr>
</tbody>
</table>

When considering \( h_{t,d} = h_{t,t} = h_{t,c} \), (31) is used instead of (27):

\[
g_{c}^{*} = \min_{0 \leq j \leq 2} \left( g_{k}(\ell, 0) - g_{k}(j, 0) \right) \tag{31}
\]

and an alarm is generated when a \( g_{c}^{*} \geq h_{c} \).

The reader can refer to [26, 27] for more detailed information.

7. Experimental Results and Discussion

The proposed fault detection and isolation approach has been tested on the caterpillar 3126 engine located at Sussex University, UK. The aim is to detect and isolate any additive actuator fault. The detection delay \( T_{d} \) should be lower than 0.01 s in average. This delay is defined as the difference between the alarm time and the actual fault occurrence time.

7.1. Design Parameters. For the design of the residual generators, the gain matrices given by (9a)–(9e) should be determined. Unfortunately, we cannot present their values due to pages limitation. Notice that the LMIs (17a)–(17c) are solved by using the software YALMIP toolbox, which is a toolbox for modeling and optimization in Matlab. Matrix \( \Pi_{r} \) for the two residual generators is chosen as \( \Pi_{r} = [0, 1] \). The sampling period \( T_{s} \) is set as \( 10^{-3} \) s.

Remark 4. It is worth noting that no feasible solution is obtained with the approach presented in [18] due to the Lipschitz constant value of the nonlinear function \( f \) (see (C.7)). In fact, the Lipschitz constant value is bigger than \( 10^{10} \) in our case. Other methods can be used, or extended to our case, as that proposed in [23] for estimating the state and unknown input vectors. Unfortunately, these methods are computationally demanding since the number of LMIs,
in each residual generator, that should be solved is equal to $N_{LMI} = 2^{n_f}$, where $n_f$ is the number of nonlinearities in the system. So, in our case $N_{LMI} = 2^{4 \times 4} = 65536$. In addition, it is only proposed for the standard systems with a linear time-invariant part $(Ax)$. We know that the number of LMIs will be strongly increased if this method is extended to our LPV case.

The covariance matrix $\Sigma$ and the mean $\mu_0$ used for designing the multi-CUSUM algorithm are estimates of the variance and the mean of $\bar{r}$ obtained from a set of simulation data generated in healthy operating conditions. Assuming that all considered faults should be detected and isolated with a mean detection delay $\bar{\tau} \leq 0.1$ s, the threshold is selected as $h_\ell = 1000$.

7.2. Validation Step. For illustrating the performance of this approach, the following scenario is considered. First, a positive step-like change in the EGR actuator appears when $t \in [6.5, 8]$ s. Next, a single negative step-like fault in the VGT actuator is introduced in the time interval $[8.9, 11.5]$ s.

The experiment is performed with engine average speed $N_{Eng} = 1800$ rpm, where the minimum and maximum value of $N_{Eng}$ ($N_{Eng}$ and $N_{Eng}$) are 1700 rpm and 1900 rpm, respectively. The initial conditions for the two observers are randomly chosen as follows:

$$
\begin{align*}
\hat{x}_1 &= [1202 \times 10^{-7} 10^{-10}]^T, \\
\hat{x}_2 &= [1202 \times 10^{-7} 10^{-10}]^T.
\end{align*}
$$

The experimental results are shown in Figure 9. First, the actuators (uEGR and uVGT) behavior is illustrated in Figure 9(a). Then, the normalized residuals $r_1$ and $r_2$ are shown in Figure 9(b). Finally, the decision functions resulting from the multi-CUSUM algorithm are presented in Figure 9(c). It is clear from the last figures (Figure 9(c)) that only the exact decision function $g_2^*$, corresponding to an occurred fault in uVGT, crosses the threshold. It means that this fault is correctly detected and isolated. The obtained mean detection delay $\bar{\tau}$, given by (29), is equal to 0.0890 s. As expected, this fault is detected and isolated with a mean detection delay $\bar{\tau} \leq 0.1$ s.
8. Conclusion

The problem of actuator fault detection and isolation for diesel engines is treated in this paper. The faults affecting the EGR system and VGT actuator valves are considered. A bank of NU1O has been used. Each residual is designed to be insensitive to only one fault. By using this kind of scheme and by assuming that only one single fault can occur at one time, each actuator fault can be easily isolated since only one residual goes to zero while the others do not. A multi-CUSUM algorithm for statistical change detection and isolation is used as a decision system. Fault detection/isolation is achieved within the imposed timeslot. Experimental results are presented to demonstrate the effectiveness of the proposed approach.

Appendices

A. Engine Characteristics

See Table 3.

B. Nomenclature

See Table 4.

C. Diesel Engine Model

The system (1) can be easily rewritten in state space form as (3a) and (3b). Indeed, it suffices to extract the linear part by doing product development and to separate the nonlinear part with known or measurable variables (\(v, y, u\)). The rest is gathered in the general nonlinear part (\(f(x, u)\)). The state, known input and output vectors, and the variables \(\rho\) and \(\nu\) are defined as

\[
x = [P_{\text{Inlet}}, m_{\text{Air}}, m_{\text{EGR}}, m_{\text{Exh}}]^T, \\
u = [u_{\text{EGR}}, u_{\text{XVNT}}]^T, \\
y = [P_{\text{Inlet}}, P_{\text{Exh}}]^T, \\
\rho = N_{\text{Eng}}, \\
\nu = [T_{\text{CAC}}, W_{\text{HFM}}, W_{\text{Fuel}}]^T.
\]

The matrices \(A_1\) and \(A_2\) are given by

\[
A_1 = \begin{bmatrix}
-\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 & 0 & 0 \\
0 & -\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 & 0 \\
0 & 0 & -\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 \\
0 & f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}} & f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}} & 0
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
-\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 & 0 & 0 \\
0 & -\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 & 0 \\
0 & 0 & -\frac{f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}}}{120 V_{\text{Inlet}}} & 0 \\
0 & f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}} & f_{\text{vol}} N_{\text{Eng}} V_{\text{Eng}} & 0
\end{bmatrix}
\]

where \(N_{\text{Eng}}\) and \(\bar{N}_{\text{Eng}}\) are, respectively, the minimum and maximum value of the measurable variable \(N_{\text{Eng}}\). \(\rho_1\) and \(\rho_2\) are defined as

\[
\rho_1 = \frac{N_{\text{Eng}} - \bar{N}_{\text{Eng}}}{N_{\text{Eng}} - \bar{N}_{\text{Eng}}}, \quad \rho_2 = \frac{N_{\text{Eng}} - N_{\text{Eng}}}{N_{\text{Eng}} - \bar{N}_{\text{Eng}}},
\]

The matrices \(B_y\), \(C\), and \(B_{m}\) are expressed as

\[
B_y = \begin{bmatrix}
R_{\text{Air}} c_{\text{Air}} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & R_{\text{Exh}} m_{\text{Exh}}
\end{bmatrix}, \quad B_{m} = [0 \ 0 \ 0]^T,
\]

where \(m_{\text{Exh}}\) is the mean value of measurable variable \(T_{\text{Exh}}\).

The matrix \(B_f\) and the vector \(f_a\) are chosen as follows:

\[
B_f = \begin{bmatrix}
R_{\text{Exh}} T_{\text{EGR}} A_{\text{EGRmax}} c_{\text{Exh}} \\
V_{\text{Inlet}} c_{\text{Exh}} \\
A_{\text{EGRmax}} \\
-A_{\text{EGRmax}}
\end{bmatrix},
\]

\[
f_a = \begin{bmatrix}
P_{\text{Exh}}/\sqrt{R_{\text{Exh}} T_{\text{Exh}}} T_{\text{Exh}} (P_{\text{Inlet}}/P_{\text{Exh}}) f_{\text{EGR}}(u_{\text{EGR}}) \\
P_{\text{Exh}}/\sqrt{T_{\text{Exh}}} T_{\text{Exh}} (P_{\text{Exh}}/P_{\text{Atm}}) u_{\text{XVNT}}
\end{bmatrix}.
\]

Finally, the functions \(g\) and \(f\) are given by

\[
g = [W_{\text{HFM}}, T_{\text{CAC}}, W_{\text{HFM}}, W_{\text{Fuel}}]^T,
\]

\[
f = \begin{bmatrix}
R_{\text{Inlet}} P_{\text{Inlet}}/c_{\text{Inlet}} \\
A_{\text{EGR}} m_{\text{Exh}}/\sqrt{R_{\text{Exh}} T_{\text{Exh}}} \\
R_{\text{Exh}} T_{\text{Exh}} m_{\text{Exh}}/\sqrt{R_{\text{Exh}} T_{\text{Exh}}} P_{\text{Exh}}/P_{\text{Atm}} u_{\text{XVNT}}
\end{bmatrix}.
\]
As mentioned in Section 3, the parameters $f_{\text{vol}}, f_{\text{EGR}}, \tau,$ and $h$ are computed by interpolation in lookup tables. However, in this work, we are interested in the diagnosis in the steady state. So, we choose these parameters constant at each operating point to validate our approach. Furthermore, the proposed approach can take easily the parameter variation into account. For example, if we choose $f_{\text{vol}}$ as variable parameter, the variable $\rho$ can be chosen as $\rho = f_{\text{vol}}N_{\text{Eng}}$ instead of $\rho = N_{\text{Eng}}$. By doing this, there is no need to change the design approach since the engine model has always the form (3a) and (3b).

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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