Production Structure and Economic Fluctuations

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Abstract

We aim at contributing to the debate on the mechanisms and properties of economic fluctuations. We consider a crucial aspect among many thought to influence this ubiquitous and extremely relevant phenomenon: the interaction structure that characterises the organisation of production, that is, the production relation among sectors of a system.

We build — and simulate — a very simple model representing an input–output system where sectors/firms adapt production and desired levels of stocks. Their output serves both an exogenous final demand and the intermediate demand solicited by the other sectors of the system. Series of simulation runs allow to derive relevant and non–obvious conclusions concerning the levels and, more importantly, the volatility of economic activity, as an outcome of the same, inherent, economic structure.

We claim that the results that we obtain through the highly abstract representation we use, provide useful intuitions on the working of economic cycles, to be later integrated by further studies.

As a by–product of our analysis, we also suggest that the methodology we adopt can provide valuable insights by allowing a detailed analysis of the time path generated in the artificial systems, and therefore assessing with precisions the same mechanisms that affect real–world systems. The natural following step, left for further research, is to investigate how those mechanisms are empirically generated.

Keywords: Production structure, micro- and macro-volatility, simulation models

JEL: E32; E37; C63; C67; D57

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1 Introduction

Business cycles have always been a major concern for economists because of they are a phenomenon of great impact on societies’ welfare, and they have shown to be one of the most constant feature of economies across time. Such is the importance of business cycles that economists seem to have spent more attention in providing normative tools to control them, than in providing a detail explanation of their origin and nature.\footnote{A similar claim is at the base of the research originated in Dosi, Fagiolo, and Roventini (2006).} In broad terms, the general agreement is that business cycles stem from the interaction of two features: comovement of economic variables (including actors’ decisions) on the one hand, and exogenous events, usually random (at micro and/or macro level) on the other hand. Typically, a model meant to explain cycles study how specific stochastic events (i.e. a flow of random shocks) can perturb an economy from its equilibrium state. In some cases, for example, economists suggests that micro-level shocks partially cancel out at aggregate levels, with the fluctuations being due to stochastic excesses not absorbed. Conversely, other works suggest that micro-shocks tend to reinforce each other generating aggregate fluctuations larger than the shocks that originated them. A recent strand of the literature on business cycle has correctly pointed to the central relevance of the production structure. Depending on the way in which the entities composing the supply side of the system interact, they will reinforce or smooth away random shocks, generating or dumping aggregate fluctuations.

In this work we contribute to the debate by removing a number of drawbacks that curtail, in our opinion, the analytical power of most of the literature on this subject. First of all, most of the literature considers jointly the effects of stochastic events and structural features of the economic systems. Indeed, real systems do face a constant flow of non predictable changes impacting on interacting economic agents. However realistic, the joint consideration of the two features of an economic system (structure and stochasticity) prevents the rigorous assessment of their separate contribution to the aggregate phenomenon of fluctuations. We propose to take a different route: we make the highly unrealistic assumption that there is no exogenous flow of shocks, and concentrate on the role of the production structure. Given the simplification obtained by non considering the noise of random events, we will be able to make a detailed assessment of the properties of a realistic representation of an economic system in respect of fluctuations. For similar reasons, we also ignore other aspects proposed as relevant in generating fluctuations, like technological development, price adjustments, lumpy investment, financial constraints, etc. Our approach does not deny the relevance of these factors; we sustain that they are logically, if not practically,
separated. Their contribution must be studied, in order to produce a robust theoretical understanding of the business cycle phenomenon. Indeed, the physical structure through which any micro change (or reaction to a change) is transmitted in the system is the production structure. And the way in which shocks propagate is at least as relevant as the sources of propagation. In the words of Zarnovitz (1977), we investigate a ‘theoretical possibility’ of business fluctuations that, for the very nature of production systems, is also an ‘explanation’ (while not an assessment). The aim of this paper is then to analyse transmission properties of multisectoral production systems.

The structure of the paper is the following. Next section briefly reviews the major contributions of the literature on the subject, highlighting the works more closely related to our approach. In the third section we highlight the main elements that characterise the transmission mechanisms of a production structure. The fourth section describes a very simple model representing a dynamical production system, comprised of an input-output matrix and a few simple behavioural rules governing the actions of economic sectors. Section five then discusses the major results of the model under a few parametrisations. Finally, the last section will draw the conclusions and suggest directions for further research.

# Economic fluctuations

A wide number of reasons may explain why economic aggregates fluctuate, and a wide number of theories attempt to explain short run and long run waves in economic growth. For example, large shocks that affect an entire economic system are likely to fully displace it, changing its long term pattern. Though, while such shocks may be plausible for long run waves, they are less likely to be the cause of short run cycles, given that they do not occur with the same frequency. Moreover, aggregate shocks may affect the various economic entities differently, making it quite difficult to state the final result of a complex combination of reactions. Conversely, shocks at more disaggregate levels of the economy are undoubtedly more frequent. This itself renders the study of the influence of micro shocks on aggregate fluctuations an important piece of analysis, and eventual understanding of business cycles.

It is self–evident that, being economic entities interconnected to each other, they may absorb, linearly transmit, or reinforce the quantitative changes that hit on them — induced by neighbouring entities — to different extents. This can be observed at different levels of aggregation. An economic crisis in one country causes shocks in related countries (e.g. Backus, Kehoe, and Kydland 1995, Head 1995, Kraay and Ventura 1998); a crisis in the financial system causes readjustment in the proximate systems, in both supply and demand (e.g. Stiglitz and Greenwald 2003, Delli Gatti,
the failure of a large company induces readjustments in the same and related sectors; and so on.

Besides, we expect that the more we disaggregate the economic units of analysis, the more shocks’ intensity is likely to reduce and symmetric shocks tend to cancel out. The application of the law of large numbers would suggest that an economy with normally distributed entities (at the same level of aggregation) — or, even better, with representative entities, would not show aggregate fluctuations in the presence of uncorrelated disaggregated shocks (Lucas 1977). This has led to focus most theoretical explanations (or representations) of short business cycles on aggregate shocks (Horvath 2000). Although they are undoubtedly relevant, and have an impact also on micro changes, aggregate interpretations are quite limited in their explanatory power. While having to assume exogenous origin of shocks, they do not provide an understanding of how macro shocks rebound on the economic entities (consumption change, production shift, productivity shifts, investments, etc.).

Indeed, in order for micro shocks to generate aggregate fluctuations, two conditions are necessary: a propagating structure — a connection — between micro economic entities, and a sluggish absorption of changes. In this respect, production structures, made up of sectors connected by input–output relations, represent a good candidate, a part from being the known structure of a basic industrial economy. Two main class of models have been developed to analyse the effect of sectoral shocks on aggregate fluctuations: Real Business Cycles (RBC) and Avalanche (Av) models. Both type of models make attempt to introduce mechanisms that allow for the persistence of micro shocks. In this respect both classes are based either on quite unrealistic behavioural assumption, which determine the conditions of the system, or threshold behaviors, which impose the conditions for shocks persistence. We briefly review them below.

2.1 Mechanisms of shocks propagation in I–O structures

A number of contributions study the phenomenon of generation and propagation of cycles independently from the sectoral structure of the economy. However, the most recent contributions point in the direction of giving this aspect of economic system a strong relevance. Moreover, the two approaches are not incompatible, so that we focus on the literature that explicitly makes use of the I–O structure. In this section we briefly review three strands of this literature, highlighting the elements that inspire our work.

Both cases are easily shown with evidence from the Latin American crisis during the eighties, rooted in the crisis of the Mexican financial system, the financial crisis in Asia during mid nineties, the recent Argentinian financial crisis, etc.
Plain I–O structure

The pioneering RBC model that analyses the diffusion of sectoral shocks is due to Long and Plosser (1983). They use a general equilibrium multi sector model with fully rational, infinitely lived, perfectly informed, homogeneous, etc individuals. In such a system, by assuming a perfectly maximising behaviour (via allocation of production and consumption of resources) and that individuals prefer to smooth savings across time and goods, it is possible to generate persistence of continuous, albeit uncorrelated, shocks in production capacity across sectors. In the words of Long and Plosser [p. 67], “At constant relative prices, this [the assumed maximising behavior] suggests that business-cycle features like persistence and comovement are characteristics of desired consumption plans”.

The necessity to maintain the assumption of perfectly rational agents forces these strand of models to attribute the origin of cycles to external factors only, like exogenous shocks on technology. As such they seem unlikely to provide a consistent explanation (or representation) of aggregate fluctuations. Dupor (1999) shows that under quite general conditions the available multi-sector RBC models produce the same results, in terms of variance convergence, as their respective single sector models. This is shown also under particular conditions of input matrix, or high coefficient of specific inputs. Dupor concludes that [p. 405]: “researchers who wish to use independent sector shocks as a source of aggregate fluctuations must appeal to sector shocks that are large relative to their single sector counterparts, or else consider a different set of models than those discussed in this paper”. In doing so, the author though suggests that another mechanism must be introduced, somehow putting the desired results ahead of the economic issue analysed.

The criticism to this kind of literature suggests that, in order to allow the I–O structure to play a role in the explanation of cycles, we need agents with less then perfect foresight and capacity of adaptation to un–expected conditions. As we will see below, agents that need time to both realize the economic changes of their time, and to introduce the necessary modifications, are sufficient conditions to observe aggregate fluctuations from a single change in one part of the economy. Though being “rational”, in the sense that they make the right decisions, they are limited by lags in the information flow, and physical constraints to the changes that they can apply.

Input matrix incompleteness

Using a similar approach to RBC models, Horvath (1998) and Horvath (2000) focus the attention on the properties stemming from peculiar production structures, as represented by characteristics of the input-output matrix.
The author shows that sectoral shocks will tend to cancel out in case the economic system is sufficiently distributed, that is, when the I–O matrix contains evenly distributed values over all the cells. Conversely, sectoral shocks are not absorbed, and can be reinforced, in case many cells of the matrix are empty. This case indicates that there will be few sectors affecting many other sectors, preventing the possibility of compensating certain shocks.

These conclusions are potentially interesting because point directly to an easy to observe aspect of economic system, for example providing the possibility to test the prediction of the analysis. It is worth to note that this approach draws conclusions that are not neutral to the aggregation level used. In fact, a highly disaggregated matrix is more likely to contain many empty cells than the matrix, for the same system, obtained aggregating sectors.

Dupor (1999) also considers the case of input–output matrices with different input coefficients, finding them irrelevant for the results. But he does not include matrices with empty cells.

The cited contributions, and those inspired to the same approach, suggest that the type of interaction, as expressed by input-output coefficients, can be a relevant factor in determining the existence and dimension of fluctuations. However, this literature is not able, yet, to draw conclusive results.

**Limited interactions and production constraints**

A different kind of mechanisms are assumed and shown to play a role in the Av models: they add an analysis on firms production (technological) possibilities, to the production structure. They are inspired by, and similar to Jovanovic (1987) model, in which the limitation in the number of interactions upon which each player’s decision depends, generates shock cascades and aggregate persistence. Bak, Chen, Scheinkman, and Woodford (1993) and Scheinkman and Woodford (1994) show that, when the interaction structure between sectors is constrained to a completely rigid system, coupled with non convexities in the production function, independent shocks to different sectors do not cancel in the aggregate. In other words the authors place a set of restrictions to the set of possible actions of sectors. First, structuring the production networks with a fixed invariable lattice produces a rigidity that do not allow producers to change (or add) inputs (or suppliers), or increase the produced quantity. This is combined with the structure of the avalanche that propagates through the unchanged network. Second, a production function that is maximised only when a discrete number of units of a good is produced, coupled with a fixed (discrete) maximum amount of inventories that can be stored by the representative firm. Such a structure determines production oscillation and shocks avalanches, due to the fact that firms are constrained in their production opportunities.
Using a very similar production structure, with limited interactions, Weisbuch and Battiston (2005) play on the bankruptcies generated on downstream firms by production failures occurring in upstream firms (reducing the flow of inputs). Stochastically generated failures to produce in a firm causes production constraints in its downstream clients, hence lower investment, and eventually bankruptcies. A lag between the period in which a firm goes bankrupt and a new firm appears, generates a lack of supply, which propagates on the downstream sectors.

Due to the original literature on avalanches, these contributions share two common features that are ill-adapted to economic systems. First, they assume very peculiar interaction structures of one-directional production relations between sectors, which amount to assume one half of the input-output matrix empty, and are not applicable to the general case with cyclical production structure, or full I–O matrices. Second, the strongly rely on quite rigid thresholds determining a mechanicist behaviour by agents. Basically, agents have only two options available (i.e. produce or not produce), and the choice is made on the basis of crude decisional mechanism.

Notwithstanding the limitations, this approach provides a quite sensible mechanism of the origin and transmission of shocks through a system. Our proposal can be interpreted as an extension of this approach by relaxing the strong assumptions on the interaction structure and allowing agents for a more fluid decisional process, as we will see in the next section.

3 The basic elements of a production structure

Our overall hypothesis is that the very production structure of an economy may generate persistent fluctuations even in the absence of continuous unrelated shocks, and rigid thresholds that determine non linearities. The aim is to provide a generalised interpretative framework to explain the occurrence of short term fluctuations in the economy, simply as an outcome of the division of labour in a number of sectors related through trade. In its extreme interpretation, any production system which is not fully integrated is bound to produce business cycles. As we will show, a number of factors influence their extent.

Quantitative adjustments in production in any one sector require adjustments in related sectors. When the structure of the economy is truly Input–Output (not a directed linear supply chain as in the Av models), input flows are cyclical, and adjustment to long run production equilibrium is likely to take a long — when not infinite — time. Our work departs from

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3 A similar model by Battiston, Delli Gatti, Gallegati, Greenwald, and Stiglitz (2005) relates bankruptcies to price changes, buyers payment timing, and the credit market (a central bank).

4 Nirei (2005) is an attempt to generalise such models, though maintaining the assumption of threshold behavior.
both RBC and Av models, with respect to structure, modelling criteria, analysis, and methodological approach. Indeed, our model may be easily reconduced (or restricted) to both. We are much in line with a recent series of papers by Helbing, Witt, Lämmer, and Brenner (2004) that address the same question, although with a number of differences that are discussed later on.\footnote{We thank Matteo Richiardi for pointing out these similar attempts. See also Helbing and Lämmer (2005) and Helbing, Lämmer, Witt, and Brenner (2004)}

We aim to analyse to what extent the structure of input–output relations causes aggregate fluctuations by itself. The main idea is to show that aggregate fluctuations may derive from simple micro imbalances, without the need for continuous exogenous shocks, not even at the firm or sectoral level. In other words, normal production activities, including lags, technological adjustments, information asymmetries, circularity of the input–output system, and so on, may be enough to explain part of the macro volatility, in a closed system.\footnote{An open system entails many other factors, including price factors, of both inputs and outputs — terms of trade, labour cost, external investment, etc. As with exogenous shocks, our analysis does not deny the importance of these factors, but simply separate them from the structural explanation of cycles}

Our assumptions draw on the existing literature, but try to avoid the major difficulties highlighted above. Firstly, we avoid to impose either market clearance assumptions, or crude threshold behaviours. Rather, we adopt an assumption close to a Simonian approach: agents in an economy do their best within the informational and physical constraint they are subject to. We represent decision makers as responding to a single piece of information they obtain: the quantities demanded by their clients. Crucially, their actions are inspired to a conservative approach: an increase in the demand generates a smaller (desired, see below) sudden increment of production. This assumption stems from the fact that decision makers are aware of the instability of their environment, and therefore want to avoid getting permanently caught with over production following a temporary spike of demand. In other words, firms attempt to smooth business cycle, generating a more stable (or less uncertain) environment, as the economic literature would suggest. Notice that this representation, in line with a routinized representation of organizations (Nelson and Winter 1982), does not contradict market equilibrium. Rather, in a stable environment such decisional procedure generates an asymptotic pattern to the equilibrium level, in the absence of feed–backs.

A second assumption concerns the technological possibilities available to the firms. When firms realise the need for a change in production levels, say a desired increase of production, they face a complex and costly organisational transformation of the production capacity, risky investments, an so on. As Bresnahan and Ramey (1994, p. 622) suggest, inducing from a detailed analysis of the automobile industry in the US, “adjusting production
is a more complicated process than simply “changing Q” or choosing the mix of capital and labor”. And this is not only a matter of non convexities in production technologies. The inconsistency of time reversibility in the technological choice, is accompanied by the need to operate on the ‘available technological frontier’ (David 1975). If a firm plans a given production activity for a short period of one week, even though the next day there is an unexpected fall in the demand, it is unlikely that it is followed by a similar downturn in production. The case is evident when we think at production planning that involve hiring of workers, an capital investments (both involving sunk costs). Therefore, any desired change generates a pattern of small modifications, approaching slowly the target level. Moreover, these changes are irreversible in the short term, since only a prolonged change of the environment triggers the modification of previous desired levels.

Resting on those simple conjectures, we thus analyse a model in which we only allow for sluggish adaptation of production due to: conservative (adaptive) behaviour of firms, and the physical constraints faced when varying the production levels. Notice that these assumptions are contrary to the generation of instability of a system. In fact, a firm behaving as described above reduces, if not eliminates, any disturbance affecting its own state, if acting in isolation. Other factors, like new technologies, extremely long term perspective, price changes, etc. are likely to be sources of volatility, therefore contributing to generate aggregate system fluctuations (as for example in Acemoglu and Scott 1997). Our neglect of these and other factors, however unrealistic, serves the purpose of testing whether a volatility reducing micro–behaviour turns into volatility generating aggregate result, two widely observed dynamics. Quite interestingly (and encouraging), Dosi, Fagiolo, and Roventini (2006) have concomitantly proposed a model which incorporates detailed representation of both sluggishness, robustly based on empirical evidence. Heterogeneous firms, which undergo lumpy investment, and slowly adapt their expectation on changes of the future demand, induce comovement of macroeconomic variables, qualitatively similar to the ones observed in the real world. To pursue our line of research, we first need to concentrate on how sectors/firms interact in the system, in the effort to complement those promising results.

Therefore, the second element we consider is meant to represent a generalized economic system. We allow for the circularity of input–output systems, a crucial feature in determining feedback mechanisms, and a principal

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7See also the discussion in Acemoglu and Scott (1997) where the authors argue that “[w]hile the presence of fixed costs can account for the discreteness of economic turning points, it does not naturally lead to persistence because once an individual undertakes an action they are less likely to do so in the near future. […] [A]lthough the presence of fixed costs leads to increasing returns, these are intratemporal; the full extent of economies of scale arising from fixed costs can be exploited within a period.” [p. 502, authors’ emphasis]
component of business cycle. It may be true that most backward linkages concern capital goods (Weisbuch and Battiston 2005), but a rapid glance to an input–output matrix would show that technical coefficients are non null in both directions, and “most commodities are inputs to the production process of other commodities” (Horvath 2000, p. 70). The higher the sectoral aggregation — observed or assumed — the more the relevance of intermediate supply holds true (as mentioned above with respect to matrix incompleteness). We claim that the existence of backward and forward linkages are a crucial element to determine the persistence of shocks in the economy. A perfectly linear (one directional) system, needs much less adaptation of firms and sectors, in order to stabilise the economy. Nonetheless, circularity does not imply that inputs are perfectly substitutable among them, in order to determine the maximising allocation, even at high level of disaggregation.

By using a fixed coefficients production function, we both maintain the mid term rigidity of production processes, and we do not impose limited interactions. We claim that the circular property of the production structure is by itself sufficient to produce shock persistence, even when the input matrix is full (although with heterogeneous input coefficients) — all sectors shop in the remaining \( n - 1 \) sectors.

In conclusion, we represent a model for an economic system made only of: 

- \( i \) production level decisions and
- \( ii \) input-output relations. Clearly, such a model lacks many features present in real economic systems, likely, we are convinced, to affect the propensity to generate cycles. As such, we cannot aim at reproducing realistic data, so our results cannot be tested comparing time series generated by the model with real ones. Instead, given the paucity of the elements comprising the model, we can investigate in detail to which extent the (few) features present in our model affect the cyclical behaviour of the system. As we will see, we will be able to show quite a number of result in which aggregate fluctuations are generated, whereas the individual, micro structure of the model aims at monotonous adaptation. Before discussing the results, next section describes in detail the implementation of the simple model.

4 The model

We model an economic system in which a number of sectors \( i = (1, \ldots, N) \) are potentially connected via (intermediate) market relations. A \( N \times N \) Input–Output matrix determines the \( m_{i,j} \) units of each input \( j = (1, \ldots, N) \) — bought from the \( N - 1 \) residual sectors — necessary to produce one unit of output \( i \). For simplicity, given the focus of this paper on the I–O structure as a generator of aggregate fluctuation, we abstract from market

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8I–O coefficients are therefore physical coefficients.
interactions within sectors: each sector is a production unit. In fact, for our purposes it is not relevant which firm produces the sectoral outcome, but only the total amount produced by each sector.\(^9\) Moreover, besides making the analysis more neat, this assumption allows also to enjoy a large freedom in determining the aggregation level, since we don’t need to assess the effects of between sectors competitiveness, which, for example, is likely to increase for increasing levels of disaggregation. Our economic system is therefore composed of \(N\) production units, each of which may buy and provide inputs from and to any other sector, depending on the I–O structure of the economy.

The demand for each sector \(i\) is determined by a constant consumers demand \(D_i(E)\), and the input needs of other sectors.

\[
D_i = D_i(E) + \sum_{j \neq i \in C_i} m_{ji}Q_j \tag{1}
\]

where \(C_i\) is the set of \(j\) intermediate clients of \(i\) \((\sum j \in C < N)\), \(m_{ji}\) is the technical coefficient of firm \(j\) for input \(i\), i.e. the amount of \(i\) required to produce one unit of \(j\). \(Q_j\) is the actual output produced by sector \(j\).

Concerning the simultaneous production level of each sector in time \(t\), we make two ‘realistic’ assumptions. First, from the technical viewpoint, a firm, like a tanker, cannot abruptly change its course. Therefore, changes to the production level can be introduced only gradually, through capital (dis)investment and job h(f)iring (see discussion in Section 3). Second, we assume that firms do not decide the production level directly on the basis of the observed demand level, but, conscious of the uncertainty of demand, plan their production in order to maintain invariant a desired level of stocks, and avoid stockout.\(^{10}\)

Quantity produced by sector \(i\) in period \(t\) is computed as follows:

\[
Q_{i,t} = \alpha_i Q_{t-1} + (1 - \alpha_i) \left( S_{i,t-1}^* - S_{i,t-1} \right) \tag{2}
\]

where \(\alpha_i\) measures the physical constraint in adapting the production capacity to changes in the required output, \(S_{i,t-1}^*\) are firm’s \(i\) desired stocks, and \(S_{i,t-1}\) its actual stocks. Notice that both variables related to stocks appear with a lag. This is due to two reasons. First, it is a modelling necessity in order to allow the simultaneous determination of production levels for all sectors. Secondly, as frequently happens, modelling necessities reveal sensible, if not generally considered, aspects of the modelled system. In fact, production decisions concern management in the production plant, while

\(^9\)Adding heterogeneous firms within sectors would definitely enrich the causal explanation put forward, and parametrised in this paper (see below). For example to analyse the effect of firms constraints. It is left over for future research, as in the present work we opt for the analysis of the main structural parameters that affect aggregate fluctuations.

\(^{10}\)See also discussion in Schuh (1996).
demand, through sales, is observed by the commercial staff. It is therefore logical that the information from demand reaches plant managers with some lag.

The actual stock level is trivially computed subtracting from the previous heap of stock the amount sold, and adding the new production.

\[ S_{i,t} = S_{i,t-1} + Q_{i,t} - D_{i,t}. \] (3)

Concerning the desired level of stocks, instead, we introduce the conservative behaviour mentioned above. Firms ideally would keep an amount of stocks proportional to the demand they receive, say a multiple \( \sigma \) of that amount. However, when demand varies, they adjust the desired level smoothly, preventing sudden changes to the variable representing the goal of the firm.

\[ S^*_{i,t} = S^*_{i,t-1} + \alpha_s \left( \sigma D_{i,t} - S^*_{i,t-1} \right) \] (4)

where \( \alpha_s \) measures the adaptation to demand changes.

In summary, the dynamics of the model consists of only the equations above. Firstly, at the beginning of a time step, firms determine the quantity to produce, as a function of past imbalances in stock levels and the past production level. These levels define the demand for all sectors, determined by the technical coefficients applied to all produced quantities. Finally, actual and desired stocks can be updated. Figure 1 represents visually this dynamics, highlighting the decisional components of the model in respect of the “mechanics” of trade.

5 Results: production structure and economic fluctuations

In this section we describe some of the most relevant results concerning the effects of the production structure to determine aggregate fluctuations. Given the unusual analytical instrument chosen (at least concerning this topic), we start with a brief methodological introduction, meant to clarify the nature of the results we claim to obtain. Next, we describe a general setting for the model, which is the (extremely) limited area of the potential parameters’ space of the model that we will explore. In the rest of the section we discuss the results from the model.

5.1 Methodological considerations

Real economic systems are constantly subject to a continuous flow of shocks external to the production system, besides changes within the system itself. In order to appreciate the systemic properties we need to abstract from all
the non–relevant disturbances, and concentrate on how the system endoge-
nously contributes to the observed dynamics. Therefore, we will generate
highly abstract patterns, whose analysis will provide insights on the system
properties that would be otherwise lost in the noise generated by all the
elements of a real system.

We are not trying to fit the model to a specific empirical data set, nor we
are interested in studying the complete behaviour of the model for areas of
the parameter’s space that make no economic sense (e.g. negative or infinite
production levels). Therefore, the values of initial settings and results matter
only in relative and not in absolute terms.

We are going to describe a simulation model, with which we will produce
several simulation runs and, we claim, obtain results relevant for the debate
discussed above. Rigorously speaking, we may claim only certainty for the
validation of the results, that is, that the model does actually generate the
results presented, and for the reasons we explain. We limit this part to the
presentation of graphs and verbal explanations that, we believe, are rather
uncontroversial.\footnote{Interested readers can request the model code and and simulation data for replication and extensions of the results. We can guarantee that such possibility does not require} Concerning the relevance of our studies for real systems
we simply avoid to pretend that our model is a quantitative approximation of real economies (which? in which period?). Our opinion is that the study of the model allows us to make general considerations, non-obvious and relevant for applied debates, supported by strong logical arguments, which can be integrated, but not reverted, by arguments concerning more and more elements of the economic world.

5.2 Model setup

We go through the model properties analysing an economic system made up of ten sectors ($N = 10$).\[12] Each sector shops inputs from the remaining $N - 1$, and sells to them part of its own production as intermediate goods. This means that the input matrix is complete, except for the diagonal ($m_{ij} > 0 \ \forall j \neq i; \ m_{ii} = 0$). The sum of all $N - 1$ coefficients for a single sector is given by $M_i$, and its benchmark value is kept at a plausible medium level (unless differently specified) to avoid distortions from highly demanding sectors. Each input coefficient is randomly drawn from a uniform distribution.

Sectors start at an equilibrium level: their supply matches intermediate and final demand, and in each period the same quantity of output must be produced to compensate for the used stocks, and keep the desired stocks unchanged. In their turn, stocks must be ten times the value of total sectoral demand ($\sigma$). Eventually, technological adjustment of production is rather slow, and producers are quiet refractory to follow sudden changes in the demand: in general a smoothing behaviour strongly prevails ($\alpha$ and $\alpha^s$).

Under those general preconditions, in the following sections we analyse the dynamics of the economic system in relation to model parameters. We mainly focus on the cyclical behaviour of systems, and their speed of convergence to the asymptotic equilibrium.\[13]

The very abstract assumption, and the homogeneous initialisation, do not allow for the multiple equilibria that are likely to rise in complex systems with feedbacks. Once more, such simplification allows to have a better understanding of the crucial impacts of the production structure on business fluctuations. Departing from the related literature discussed above,\[14] we consider the effects of a single shock, rather than a continuous flow, which in our model is superfluous in order to obtain fluctuation persistence.\[15] To

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\[12\]You can think of a high level of aggregation in the statistical observation. The complete initialisation and parameters values are available in table [1] in the Appendix.

\[13\]Notice that sectors only approach a final equilibrium level of production, which can be pre-determined under static conditions.

\[14\]Included the more proximate work of Helbing, Witt, Lämmer, and Brenner (2004).

\[15\]An exception are the last results presented in Section 5.8 which we see more as an opportunity for further research than a closing of the paper.
start with, next section (5.3) shows the aggregate cyclical behaviour of a standard setting.

5.3 Single shock analysis

We generate a simulated history following the dynamics of the model’s variables when the system is set to an equilibrium level, and a single shock modifies one of the exogenous elements, namely the external demand for sectors. Modifications across sectors are random and not correlated, and may increase or reduce external demand by at most 10%.

![Industrial Production](image)

Figure 2: Total industrial production fluctuations following a shock from external demand.

Figure 2 shows the dynamics generated by the total industrial production of the system — that is also a proxy for the system’s GDP fluctuations. The new external demand changes the equilibrium level, but the system cannot reach the new equilibrium with a monotonous pattern, and converges to the new level with a fluctuating pattern. Each change in the external demand induces also modifications of the intermediate demand, and sectoral adjustments in the quantity produced typically overshoot the aggregate pattern. As it can be appreciated from the figure, it takes time before the system is able to converge to the new static equilibrium conditions.

We study two features of this pattern: absolute level of the new equilibrium, and volatility induced by the shock.

5.4 Technical coefficients and industrial output

Production level

The equilibrium levels of industrial output depend on two factors only. Obviously, the levels of external demand determine how much of the total pro-
duction needs to be supplied to consumers, exogenous from the production system. Second the technical coefficients also influence strongly the level of activity of the system, defining the intermediate demand to each sector.

![Figure 3: The relevance of input coefficients on output level](image)

Figure 3 shows the equilibrium level of industrial output reached by systems with identical external demand where we fixed the sum of input coefficients for each sector of the economy ($M_i$) to different values\textsuperscript{16}. The higher the sum, the higher is the total production sustained by the system.

This result depends on the very production structure represented by the input–output matrix. In fact, the higher is the level of production required by intermediate sectors, the higher is the total production, given a constant level of external demand.

Notice that the level of production does not depend on the distribution of the technical coefficients. As long as the sum of the coefficients is identical, the same production level is obtained irrespective of their distribution (other things being equal).

**Output shock absorption**

Input coefficients have a relevant role also on the output stability of the system. Starting from an output equilibrium level, we hit the system with a single shock on the external demand for all sectors, uncorrelated among sectors\textsuperscript{17} modifying (positively or negatively) the demand for all sectors by

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\textsuperscript{16}Within this limit, the actual value of the coefficients are randomly drawn.

\textsuperscript{17}In each sector the external demand changes by $\pm X\%$, where $X$ is a uniformly distributed percentage between 1 and 10.
at most 10%. As shown in Figure 2, the economic systems produce a lengthy smoothing oscillatory behaviour. We compute the relative deviation of the aggregate production (industrial output), as the ratio between its standard deviation and its mean: \( \delta = \frac{\sigma_Y}{\mu_Y} \), where \( Y = \sum Q_i \). Figure 4 shows the behaviour of \( \delta \) as a function of the sum of input coefficients for each sector, 80 periods after the shock. As time goes by, the level of \( \delta \) reduces for all \( M_i \), but the relation with the sum of input coefficient is unchanged.

![Aggregate output deviations for different technical coefficients](image)

Figure 4: The relevance of input coefficients on output stability

In sum, input coefficients determine the extent of overshooting behaviour. The higher the coefficients, the higher is the backlash, even when shocks could cancel out through sectors.

### 5.5 Shock persistence and adjustment coefficients

We now consider the effect of micro smoothing behaviours on the volatility of the production system. In particular, we are interested in testing the effects of two parameters of the model on the volatility of the production system. First, we consider the effects of the \( \alpha \)'s, representing the “stickiness” of the production technology to desired changes of \( Q \) aimed at filling the gap between actual and desired stocks. The higher is \( \alpha \), the slower is the reaction to stock unbalances. Concerning its effect, referring to the technical possibilities available, i.e. production capital and organization, we may put forward two opposite hypothesis relevant to system’s volatility:

**Hypothesis 1:** Slower production adjustment (high \( \alpha \)) reduces volatility, because unbalances in demand of one sector, being compensated slowly, slow
Hypothesis 2: Slower production adjustment (high $\alpha$) increase volatility, because the longer lasts the mismatch between actual and equilibrium production level the deeper will become the mismatch between actual and desired stocks.

Secondly, we consider the parameters $\alpha^S$ representing the speed of adjustment of desired stock to observed demand. The higher this parameter, the faster sectors revise the desired levels of stocks to current levels of observed demand. This parameter concerns sectors’ behavioral decision process. Higher $\alpha^S$ represent a stronger belief that the most recently observed demand level is a reliable indicator of future demand levels, while lower levels of the parameter represent a more conservative approach, where managers require a long observation of demand levels different from the past ones before adjusting their expectations. Also concerning the effect of this parameter on the volatility induced by an exogenous shock we may make two hypothesis:

Hypothesis 1: Faster adaptation of desired stocks to new levels of demand (higher $\alpha^S$) reduce volatility because changes in demand are rapidly translated in adjusted levels of production.

Hypothesis 2: Faster adaptation of desired stocks to new levels of demand (higher $\alpha^S$) increase volatility because temporary unbalances of demand in...
one sector are transmitted faster to other sector reinforcing the feedback mechanism of fluctuations.

To test which of these hypotheses are confirmed by the model we run simulation runs with identical systems (e.g. same technical coefficients, same levels of demand, same shocks), but for the values of the $\alpha$’s and $\alpha^S$’s, testing all combinations of these parameters over a range of values (each run used the same parameters for all the sectors of the system). For each simulation (i.e. each couple of values of the parameters) we computed two indicators of volatility. One measures the maximum distance of total production between the highest and the lowest peak, that is the maximum width of the oscillations registered in the production time series. The second indicator measures the relative deviation $\delta = \sigma_Y / \mu_Y$. Figures 5 and 6 report these results.

The results confirms unequivocally both hypotheses 2 and reject hypotheses 1. In a sense, these hypotheses confirm the nature of the economic system represented in the model as a complex system, where the interactions among sectors play a far more important role than the individual sector behaviour. In fact, in both cases we may interpret the results that the stronger the efforts to mitigate the effects of shocks (i.e. slow production’s and quick expectations’ adjustment), the opposite result is actually obtained: the fluctuations become even more accentuated. It is also worth to note that the stronger impact is generated by the behavioural parameters rather than the technical one, as indicated by the stronger effects of the $\alpha^S$ in respect of the $\alpha$. This result seems to suggest that, ceteris paribus, information matters more than technical constraints. Though the model is, by any means, in-

![Relative deviation](image-url)

Figure 6: Relative deviation of total production for different values of $\alpha$ and $\alpha^S$. 

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adequate to discuss normative aspects because of the naive ways strategic decision making is represented, it can anyway suggest the effects of errors in management decisions (i.e. wrong desired levels of stocks) as opposed to the technical constraints preventing sudden adjustment of production levels. Stretching a bit the interpretation of these results, we may suggest that efforts to reduce volatility should better be oriented to improve the transmission of information, in order to coordinate future production plans (i.e. identifying quickly the desired levels of stocks, and therefore the new equilibrium production level), rather than improving the flexibility of production methods (reducing $\alpha$).

5.6 Volatility and stock levels

In our model stock levels are maintained to buffer a sector’s sale against unexpected changes in demand. Therefore, a system made of risk adverse managers (keeping higher levels of stocks) may be thought to lead to a smoother absorption of shocks, in respect of a system composed of firms keeping a short supply to cushion unexpected events. To test this hypothesis we tested the behaviour of the model for different values of $\sigma$, the levels of stock represented as a multiple of demand. For each level of $\sigma$ we run a simulation using all other initialization identical, as usual representing an economic system “almost” at equilibrium, but for a single small shock in demand, uncorrelated across sectors.$^{18}$

![Stocks and production fluctuations](image_url)

Figure 7: Fluctuations persistence on systems with different stock strategies ($\sigma$)

$^{18}$Demand changes by $\pm 1\text{-}10\%$
Figure 7 shows the value of the relative deviation of output production ($\delta$) generated by heterogeneous uncorrelated shocks in the external demand on identical systems where firms store a different multiple of their demand as stocks. We obtain actually the opposite result that we may have expected: fluctuation become larger and more persistent the higher the level of stocks. In some way, a strategy that, for the individual firm, is meant to reduce the fluctuations (using stocks as buffers), ends up by actually increasing the volatility at aggregate levels.

The result is though explainable, both at the theoretical and empirical level. In fact, if no stocks were maintained, production would need to adjust for changes in demand only in case of a shock. That is, the firm may move monotonically from the old to the new production levels, sending consistent signals to the rest of the system (i.e. its own suppliers). Instead, if the same shock affects a firm maintaining large stocks of output, over the same period production needs to adjust to the new demand and to align stocks with the new demand. This generates, besides the adjustment to the demand, a potentially inconsistent signal to the suppliers, since the firm will have a temporary level of production not meant to continue in equilibrium (adjusting stocks).

From the empirical viewpoint, the last few decades have seen the almost universal diffusion of production system meant to keep stocks to a minimum, if at all. Lean production methods, just–in–time, etc. are solutions to the problems of the costs necessary to maintain stocks. Over the same period, economic fluctuations have been strongly reduced, and few empirical assessments have pointed to the relations between the two phenomena. For example, concerning the U.S., “changes in inventory behavior have played a direct role in reducing real output volatility.” (Kahn, McConnell, and Perez-Quiros 2002, p. 183)\textsuperscript{19}

5.7 Irrelevance of technical coefficients distribution

Horvath (1998) and Horvath (2000) suggest that systems with sparse matrices, i.e. input–output matrices with many empty cells, tend to be more volatile in respect of systems with more evenly distributed matrices. Notice that this claim risk, if not adequately qualified, to generate an illogical result. In fact, matrices sparseness increases as a function of sectoral disaggregation; and comparing the same system’s fluctuations using input–output matrices at different levels of aggregation needs to produce the same type of volatility. Indeed, this is not the case if all remaining parameters stand equal, and do not adjust to the different level of analysis considered. If aggregate patterns of adjustment speed are maintained unchanged, the higher the sectoral disaggregation, the higher the system’s volatility, \textit{ceteris paribus}. But,

\textsuperscript{19}Similar results are found in Blanchard and Simon (2001).
for example in our model, reducing the aggregation also requires a change in the adaptation coefficients: the more micro we undergo the analysis, the higher is the speed of adjustment of economic objects. If, for example, we consider sectors, we are evaluating the adaptation of an entire set of firms, which pose questions on how to aggregate their dynamics. Things change when we consider the response of a single firm.

Going back to Horvats’ argument, highly disaggregated matrices are likely to contain many empty cells, so that we may forecast different levels of volatility depending on the aggregation used for the same system, which, as mentioned, is not caused by sparseness, but to a failed adjustment in reaction mechanisms. Results from our model\(^\text{20}\) show that volatility is completely unaffected by input coefficients distribution. Given their sectoral sum — through rows of the matrix — the value of each coefficient is irrelevant. The same occurs in the case of very skewed distributions that present a high number of empty cells (zero input coefficients).

### 5.8 Micro– and Macro–volatility

As we have seen above, one of the major discussion in the literature concerns the relations between micro– and macro–volatility. Put it simply, one side of the literature considers that uncorrelated micro–shocks cancel each other out, reducing their effects at macro–level. Conversely, other researchers consider that micro–shocks are multiplied at macro–level, generating higher macro–volatility than that we can observe at micro–level. The debate is of obvious importance to determine the sources of business cycles, to predict their effects, and determine the more effective policies to mitigate their negative effects.

Our model is only a partial representation of real–world economic system, and is also overly simplistic. However, based on these limitations, it also offers very robust results, allowing reliable conclusions on the (limited) elements included in the model. In particular, we consider our model a reliable representation of the basic structures of the production interactions among sectors or firms. Thus, we can use the model to provide an answer to the debate concerning the contributions to volatility from the production interactions only. Other contributions (e.g. from financial markets, price adjustments, technological innovation, etc.) may re–inforce or counter the forces we analyse, but they need to be considered as additional elements (and, we opine, also generally less relevant), not alternative to our analysis. We consider a necessity, in order to apply a rigorous methodological approach, to be able to identify with certainty the results provided by partial studies, in order to eventually obtain, by gradual extensions of the analysis, more and more detailed representations of realistic economic systems.

\(^{20}\)Available from the authors.
The results we obtain in this paper concerning this micro–macro debate are meant as a first, relevant step to a more extended analysis.

Figure 8: Values of $\delta$ (relative deviation) for aggregate demand and aggregate output in respect of different levels of variance for sector specific, uncorrelated external demand shocks.

The flows of shocks affecting the firms of a system can be distinguished along two dimensions: strength of the shock (e.g. amplitude of the changes in demand) and frequency of the shock (rare or frequent). Concerning the first aspect, we initialized, as usual, a system to equilibrium levels (e.g. production quantities equal to external and intermediate demand for all sectors, constant level of stocks equal to the desired level). We then ran the simulation generating the external demand for all sectors as a random variable with constant (equilibrium) mean level and varying values of variance.

Figure 8 shows unequivocally that, as far as our model has properties similar to real systems, the hypothesis that macro–fluctuations have larger volatility than their sectoral level shocks is fully confirmed. At any level of variance, the aggregate volatility of demand, as captured by the relative deviation index, is sensibly lower than the volatility shown by aggregate output, that is, the total production of the system. This result is not surprising: throughout all tests presented in the previous sections, every result suggested that the system ‘over–reacts’ to any individual micro–level change, even in the case these changes were meant to contrast disturbances. Therefore, when we move from considering one single shock to a flow of shocks, we are not surprised to obtain that the system shows higher volatility than that implied by the micro–level disturbances.

We now consider the second dimension defining a flow of shocks: their frequency. The question is whether, for the same level of demand shocks
Figure 9: Differences of values of $\delta$ for aggregate output minus those for aggregate demand in respect of different levels of variance of demand and of frequency of the shocks.

(i.e. same variance), more frequent changes trigger a higher or lower system volatility than rare disturbances. The results, shown in figure 9, suggest a rather elaborated answer, which partly vindicates (though in a particular sense), the arguments in favour of compensation of micro shocks. The figure shows the difference between aggregate output volatility and aggregate demand volatility. The independent variables are, as before, the volatility of demand at micro–level$^{21}$ and the time intervals in between two shocks. Aggregate demand volatility does not change for the frequency of shocks, at least when computed over a long period. But output volatility does, in a non–monotonous way. For high frequency of shocks the difference is still positive, and increasing for higher variance, indicating that the “over–reaction” argument is valid even in these cases, but it is rather low. While the frequency of shocks slows down, aggregate volatility of output increases, showing that time is needed for the micro–shock to unfold their full effects at the macro–level. However, after a certain threshold, less and less frequent shocks generate a lower volatility of the aggregate level of output.

This result is quite sensible, though not obvious at first sight, and, once again, shows the power of such a simple model to generate and explain complex properties. For frenetic levels of demand modifications, the system has no time to adjust, but part of the job to align actual and desired production for constantly changing demand is performed by the varying demand itself. In other terms, the shocks cancel out through time, if not through their quan-

$^{21}$Beware of the scale of variance. For reasons of visibility, the scale is inverted.
itative effects on the system. While frequency decreases, the disturbances of the shocks not compensated by other (unlikely) shocks, have the time to filter through the production system, generating higher and higher volatility. In a sense, any system has a particular frequency of shocks that make it maximally “resonating”, with the amplification of micro-shocks. When the rate of shocks gets even rarer, the system can enjoy periods of longer and longer quietness after each shock is absorbed and before a new one starts again a cycle of initial local disturbance, propagation of the disturbances to other sectors, and settlement to the new equilibrium.

6 Discussion and extensions

We presented a simple model of production composed by interacting sectors. The main (quit realistic) assumption of the model is that the production processes in any sector of the economy make use of input from any other sector. Moreover, we assumed that sectors (represented as a single firm), adjust slowly both the desired level of production (smoothed revision of expectation) and the actual level (smoothed modification of plant utilization). The model voluntarily leaves aside a large number of aspects of real-world economic systems (e.g. financial markets, price adjustments, growth, etc.). The reason is that the model can be easily and reliably tested in order to extract its properties. The possibility to transfer the (certain) model result to a real context depend, of course, on an ample ceteris paribus clause. However, we are confident that the reported results, concerning the production structure, play a relevant role in real world systems. Therefore our results, and particularly their motivations being accessible because of the relative simplicity of the model, will not be diminished by the integration of our considerations with other aspects of economic systems.

Our model is, in essence, a representation of an input-output table interpreted in physical terms, that is, as the amount of inputs required for one unit of output. Besides this, we include two straightforward assumptions. Firstly, production levels cannot be suddenly changed, but require time to scale up or down the utilization rate, that is, the levels of production. Stocks are meant to make up for the difference between sales and production. The second assumption concerns the intentional behaviour of producers. We implement this aspect by assuming that exists a level of stocks desired for each level of demand. However, a change of demand does not translate suddenly in a new level of desired stock, but, again, only a long time at a different level of demand will convince managers to update the desired levels of stocks.

We show that the model generate, under a wide number of settings, oscillatory patterns converging toward the equilibrium level, where each sector generate exactly the quantity demanded by final consumers and by other sectors as input. The oscillations are provoked by over- and under-shooting
generated by the difficulty of coordination among sectors that, by assumption, “communicate” only via variation of demand to their direct input suppliers. We also show that the total production levels generated by sectors (sum of the physical units) produced at equilibrium depends (for constant final demand) on the total sum of the technical coefficients per sector, irrespective of their distribution. This result is motivated by the impossibility of input substitution, and offers interesting suggestions for the analysis of the study of input-output tables of real-world systems.

We then consider systems at equilibrium facing a single, small change of the external demand for sectors. These simulations allow to study the way the system reacts to shocks. We see that the sum of the coefficients affects, besides the levels, also the volatility of aggregate output: the larger a system, the stronger is the impact of a given shock. Less obviously, we find that, for systems with the same dimension (i.e. same sums of coefficients), the aggregate volatility increases with, both, the inertia to change of production level, and the speed of adjustment of desired stocks. This result is somewhat counter-intuitive, since both the cited variables are associated to a more conservative behaviour that, may be supposed, should smooth away spikes in the pattern from an old to a new equilibrium. However, we show that the opposite actually holds: given the complex interactions among sector, individual-sector attitude intended to smooth shocks actually increase their aggregate impact.

In the same vein, we show that systems maintaining high levels of stocks are more volatile that systems with reduced amounts of stocks. Again, this is a counter-intuitive result reverting the goal of individual producers and the aggregate result. Interestingly, this result finds strong support from the evidence of the recent changes in production methods (i.e. low or no stocks) and diminished cycles.

In relation to specific issues discussed in the literature, we find no support to the suggestion that particular distributions of input-output coefficients modify the volatility of a system. As said, this depends, in case, on the sums of these coefficients, but not on their distribution. Further, we can provide an answer to a long and hotly debated questions: whether micro-levels volatility generates stronger or lower aggregate volatility. Our model shows unequivocally that micro-volatility (e.g. variance of final demand) persists, and generates stronger volatility at aggregate level. Again, we can identify the reason on the complex production structure that exalts micro-level shocks by generating ‘wrong’ messages among producers. Moreover, we can also point to a usually neglected aspect of volatility; we show that the frequency of shocks is a very relevant aspect, besides their dimension, in determining the volatility of aggregate output.

Our work can be developed along two, complementary, directions. Firstly, we can continue to study the properties of the model by extending the elements considered (e.g. pricing), analysing the effects on the present results,
and generating more analytical evidence. Of particular relevance is, in our opinion, the problem of aggregation. Representations of the same systems with different codification systems (e.g., different number of SIC digits), must not modify overall system properties, for example, the volatility observed. Given that our result show that the sums of coefficients matter for the levels and volatility of the system, and that these sum do depend on the aggregation chosen, it should be possible to induce the reaction coefficients that, we know, affect the volatility only.

Secondly, we can find an application of our results to real world evidence. In fact, there are many data sets available for the data we deal with, basically input-output tables. Our model is readily adapt to be parametrized along any number of sector and coefficient necessary, so that we may replicate past series in order to adapt the unobservable parameters, and use the resulting complete model to explain business fluctuations and provide intuitions on future patterns.
References


### Table 1: Parameters values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of sectors/units $i$ of production</td>
<td>10</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Number of $j$ buyer sectors for sector $i$ (IO matrix sparseness)</td>
<td>$N - 1$</td>
</tr>
<tr>
<td>$D_i (E)$</td>
<td>Value of the external demand</td>
<td>10</td>
</tr>
<tr>
<td>$m_{ij}$</td>
<td>Physical input coefficient of input $j$ for sector $i$</td>
<td>$U(0,1)$</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Sum of the $m_{ij}$ input coefficients $j$ for the production of $i$</td>
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<tr>
<td>$\alpha_i$</td>
<td>Friction to changes of production levels / Degree of lock-in on production technology</td>
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</tr>
<tr>
<td>$\alpha_s^*$</td>
<td>Speed of adaptation to demand changes</td>
<td>0.04</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Desired multiple of production to stock</td>
<td>10</td>
</tr>
</tbody>
</table>

*a*Into parenthesis the number of lags of the initial value for lagged variables.*