Validity of Using the Sheffield Algorithm for the Sussex EIT MK4
Xiaolin Zhang¹, Tabassum Qureshi¹, Chris Chatwin¹ and Wei Wang²
¹University of Sussex, Brighton, UK, xz68@sussex.ac.uk
²Micro Image UK Ltd, UK, w97wang@yahoo.co.uk

Abstract: This paper introduces the image reconstruction algorithm from Sheffield group and the validity of this algorithm to the Sussex MK4.

1 Introduction

The Sussex MK4 electrical impedance mammography (EIM) is developed for breast cancer detection[1][2]. This paper is focusing on the validity analysis of using the Sheffield algorithm for the MK4.

2 Methods

The widely used equation to explain the relationship between the change of the conductivity and the change of the boundary voltage measurements is:

\[ \Delta V = V_m - V_{ref} = S(C - C_{ref}) = SC \]

where \( S \) is the Jacobian matrix, \( \partial V_j / \partial C_i = S_{ij} \). Vector \( V_m \) denotes the real voltage measurement corresponding to the real conductivity. Vector \( V_{ref} \) denotes the reference voltage measurements corresponding to the reference conductivity \( C_{ref} \). \( S \) is a function of \( C \). As \( C \) changes, \( S \) changes. Eq. (1) is based on the assumption that the changes of \( C \) are small, so that the changes of \( S \) can be ignored. However, Eq. (1) was proven by us to have a poor noise tolerance for the MK4, thus the Sheffield method using the voltage ratio rather than the difference is employed [3]: (For details, please read [3], Page 368-371)

\[ \Delta \ln V = F \Delta \ln C \] (2)

where \( \Delta \ln V_i = \ln(V_{mi}/V_{ref}) \), \( \Delta \ln C_i = \ln(C_i/C_{ref}) \).

\[ \frac{\partial \ln(V_j)}{\partial \ln(C_i)} = \frac{\partial V_j}{\partial C_i} = S_{ij} \]

The image reconstruction algorithm is:

\[ \{ \Delta \ln(C) = (F^T F + \alpha^2 I)^{-1} F^T \{ \ln(V_m) - \ln(V_{ref}) \} \}

\[ \ln(C_{ref}) = \ln(C) + \Delta \ln(C) \] (3)

where \( \alpha \) is the regularization parameter, \( I \) is the identity matrix. Let’s see Eq. (3). As \( V \) and \( S \) are both determined by \( C \), basically \( F \) is determined by \( C \). As \( C \) changes, \( F \) changes. So this algorithm is based on the assumption that the changes of \( F \) are ignored when the changes of the conductivity are sufficiently small. However how much changes of the conductivity will make the assumption invalid? According to Eq. (3), if \( V_j \) is equal or close to 0, \( F_{ij} \) will go to infinity, which will make the algorithm unavailable. In practice, the measurements from the 0.5 mS/cm saline are used as the reference measurements and in each excitation, we only use at most 12 strongest measurements which are collected parallel to the electric field [1][2], therefore \( V_j \) won’t be close to 0. However if there are big changes of conductivity, \( V_j \) may be close to 0, then Eq. (2)-(4) becomes invalid.

This section discuss how much changes of the conductivity will make Eq. (2)-(4) invalid. See Figure 1. The positive pole of the current source is at the yellow dot \( S^+ \) and the negative pole of the current is at the yellow dot \( S^- \). The electric potential at P1, P2, P3, P4 is denoted by: \( \emptyset_1, \emptyset_2, \emptyset_3, \emptyset_4 \) and the voltage measurements between P2 and P1, P4 and P3 are denoted by \( V_{21}, V_{43} \). For an uniform field (0.5 mS/cm Saline), according to our study, \( V_{21}, V_{43} \) are approximately 300mv when the tank height is 4.5cm.

For a significant changes of the field, \( \emptyset_1 \) and \( \emptyset_3 \) may get close to \( \emptyset_2 \) and \( \emptyset_4 \), which means \( V_{21}, V_{43} \) may become 0 and then Eq. (2)-(4) will be invalid. Thus, we conclude that the changes of the conductivity which cause \( V_{21} \leq 0 \) or \( V_{43} \leq 0 \) will make Eq. (2)-(4) invalid. If \( V_{21} \leq 0 \), there must be a high conductivity path between \( S^+ \) and P1, so that most of the current flows through this path and brings up \( \emptyset_1 \). See Figure 1. The most likely condition to make \( V_{21} \leq 0 \) or \( V_{43} \leq 0 \) is that the high conductivity path needs 1) the shortest distance between \( S^+ \) and P1 without covering P2; 2) a big volume of the conductivity path needs 1) the shortest distance between \( P_1 \) and \( S^+ \) is 4.5 cm.

\[ \begin{align*}
3.5\text{cm} \leq d &\leq 7.5\text{cm} \\
\sigma_1 &\leq \sigma_2 \\
V_{21} &\leq 0
\end{align*} \]

Figure 1: Voltage measurements reverse analysis

3 Conclusions

The Sheffield algorithm is not valid for every condition. For the MK4 system, it is available, for a real breast is too far from the conditions which make the boundary voltage measurements close to 0, hence invalidating Eq. (2)-(4).

References

