Cosmological Signatures of a UV-Conformal Standard Model

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(Received 4 April 2014; revised manuscript received 29 July 2014; published 18 September 2014)

Quantum scale invariance in the UV has been recently advocated as an attractive way of solving the gauge hierarchy problem arising in the standard model. We explore the cosmological signatures at the electroweak scale when the breaking of scale invariance originates from a hidden sector and is mediated to the standard model by gauge interactions (gauge mediation). These scenarios, while being hard to distinguish from the standard model at LHC, can give rise to a strong electroweak phase transition leading to the generation of a large stochastic gravitational wave signal in possible reach of future space-based detectors such as eLISA and BBO. This relic would be the cosmological imprint of the breaking of scale invariance in nature.

DOI: 10.1103/PhysRevLett.113.121801

PACS numbers: 11.30.Qc, 04.30.Tv, 12.15.-y, 98.80.Cq

In the standard model (SM), the fact that spontaneous breaking of electroweak (EW) symmetry is driven by a fundamental scalar leads to a puzzle concerning the naturalness of the theory. On the one hand, this scalar should have a mass of the order of the EW scale $v = 246 \text{ GeV}$, recently confirmed by the LHC discovery of a particle consistent with the SM Higgs boson and with mass $m_h \sim 125 \text{ GeV}$ [1,2]. On the other hand, no symmetry in the SM protects $m_h$ from receiving quantum corrections scaling as $\delta m_h^2 \sim M^2$ for every energy scale $M$ to which the Higgs boson is sensitive, so one would expect $m_h$ to be of the order of the largest energy scale at which some new physics enters, e.g., $M_{Pl}$. This “gauge hierarchy problem” hints at the existence of some symmetry at energies above the EW scale which forbids or suppresses the large $\delta m_h^2$ contributions altogether.

In that respect, the idea that nature may be exactly scale invariant at high energies (in the UV) [3] as an explanation for the lightness of the Higgs boson [4,5] has recently attracted renewed interest; see, e.g., [6–9] (exact UV scale invariance has also been widely discussed in the context of asymptotic safety, see [10] for a recent review). Scale invariance would have to be broken at some energy scale $f_c$, above which all quantum corrections to $m_h$ would be forbidden by the symmetry. The mass of the Higgs boson being sensitive to $f_c$ [6], this new scale should be of order of the TeV energy scale. However, it has been recently shown [9] that when the breaking of scale invariance originates in a hidden sector and is mediated to the SM sector via gauge interactions (in analogy to gauge mediation in the context of supersymmetry breaking theories), $f_c$ could be significantly higher. Moreover, gauge mediation of exact scale breaking (GMESB) allows us to compute the effect of breaking of scale invariance on the SM, under general assumptions about the properties of the hidden sector. The Higgs mass $m_h$ would then vanish at tree level and emerge from the breaking of scale invariance via loop corrections involving the SM gauge bosons, naturally explaining the hierarchy $v \ll f_c$ [9]. Due to this hierarchy, GMESB scenarios may be hard to probe at LHC.

In this Letter we show that there are very important differences between the SM and these scenarios from the point of view of electroweak cosmology. In the SM the electroweak phase transition (EWPT) is known to be a smooth crossover [11–13], leaving no trace in the early universe. In contrast, GMESB scenarios generically predict a strongly first order EWPT, due to the form of the scalar potential arising from the breaking of scale invariance. For a considerable fraction of the parameter space, the EWPT gives rise to a large stochastic gravitational wave signal, within reach of planned and future space-based gravitational wave observatories such as eLISA or BBO. Moreover, the requirement of a consistent cosmological evolution allows us to obtain information on $f_c$ and the breaking of scale invariance. Altogether, it is likely that breaking of scale invariance would leave an observable imprint in the early Universe.

Higgs potential from quantum scale invariance.—
We start with a scenario in which scale invariance is broken in a “hidden sector,” interacting with the SM only via SM gauge interactions. GMESB implies that the only terms in the Lagrangian mixing the hidden and visible sectors take the form

$$\mathcal{L} \supset gA_{\mu}^{a}(J_{\text{vis}}^{a} + J_{\text{hid}}^{a}),$$

where $a$ is a gauge group index and $J_{\text{vis}}, J_{\text{hid}}$ are currents in the visible and hidden sectors, respectively.

The full Lagrangian obeys scale invariance only if the tree-level Higgs mass vanishes, so the only free parameter in the Higgs potential is a quartic coupling $\lambda$. A mass is, however, generated by loop corrections once scale invariance is broken. The relevant contributions must be at least of 2-loop order: one via which scale breaking is mediated...
to the SM (through corrections to the gauge boson propagators), and another loop coupling these gauge bosons to the Higgs scalar (additional loops are of course possible, e.g., including the scalar itself; see [9]). Because of this 2-loop suppression we have \( m_h \ll f_c \), with \( f_c \sim O(1-100) \) TeV. The resulting effective potential can be written as [9]

\[
V_{\text{eff}} = \frac{3}{2} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \log \left( p^2 + m_W^2 + g^2 C_{\text{vis}} + g^2 C_{\text{hid}} \right),
\]

with the trace over the gauge group indices, \( m_W \) the gauge boson mass and \( C_{\text{vis}}, C_{\text{hid}} \) parametrizing the corrections to the two-point functions of \( J_{\text{vis}} \) and \( J_{\text{hid}} \), respectively. GMESB ensures that all contributions to the Higgs potential involve \( f_c \), so we restrict ourselves to

\[
\delta V_{\text{eff}} \equiv V_{\text{eff}} - V_{\text{eff}}|_{f_c = 0}.
\]

Each term in the difference has divergences that do not depend on \( f_c \), while \( \delta V_{\text{eff}} \) is well defined.

A major advantage of GMESB scenarios is that we need not know the details of \( C_{\text{hid}} \) to compute the Higgs effective potential, whereas \( C_{\text{vis}} \) takes the same form as in the SM. Following [9], one can expand \( \delta V_{\text{eff}} \) in powers of the gauge couplings, keeping only the dominant, large momentum \( p^2 \gg v^2 \) contributions of the integral in (2), for which case a simple representation of \( C_{\text{vis}} \) exists. The coefficients of the resulting potential will depend on the details of the hidden sector, but requiring the electroweak minimum \( v \) and Higgs mass \( m_h \) resulting from it to have the correct value, one finally arrives at [9]

\[
\delta V_{\text{eff}} \equiv V_0 = -\frac{m_h^2}{4} h^2 \left( 1 + X \log \left[ \frac{h^2}{v^2} \right] \right) + \frac{\lambda}{4} h^4,
\]

where \( X \equiv (2v^2 \lambda/m_h^2) - 1 \). The details of the breaking of scale invariance are thus encoded in the value of \( X \) (or, alternatively, of \( \lambda \)). The SM case is recovered for \( X = 0 \). In general, however, the logarithmic term gives a positive contribution to the potential (in the region \( h < v \)) for \( X > 0 \), giving rise to a potential barrier between the EW symmetric and EW broken minima even at zero temperature, as shown in Fig. 1. As discussed in [9], weakly coupled realizations of the hidden sector (with the assumption that SM gauge interactions do not unify at higher energies) yield \( X > 0 \).

For \( X > 1 \) the symmetric phase is the global minimum of the potential (see Fig. 1), leading to an inconsistent thermal history of the universe, since EW symmetry would never be broken.

The electroweak phase transition.—At finite temperature the Higgs field is surrounded by a thermal plasma of particles. As a result, the free energy of the system is minimized not when the Higgs field is at the minimum of \( V_0 \), but of an effective thermal potential \( V \equiv V_0 + V_T \). The general expression for \( V_T \) at 1-loop [14] (including resummed contributions from bosonic “ring” diagrams [15]) is

\[
V_T = \frac{T^4}{2\pi^2} \sum_a N_a \int_0^\infty dx x^2 \log \left[ 1 + e^{-\sqrt{x^2 + \frac{m_a^2}{T^2}}} \right]
+ \frac{T}{12\pi} \sum_b \bar{N}_b (m_b^2 - m_b^2 + \Pi_b^2(T))^{3/2},
\]

where in the first term we sum over particles coupling to the Higgs boson, with numbers of degrees of freedom \( N_a \) and a \(- (+)\) sign for bosons (fermions). The dominant contributions come from \( a = W, Z, t \), for which \( N_a = (6,3, -12) \). Contributions from the rest of the quarks and leptons are negligible, while those from the Higgs and Goldstone bosons are subdominant and can be safely ignored. The second term in (5) corresponds to the bosonic “ring” contributions, to which the longitudinal component of gauge bosons contribute [\( \bar{N}_b = (2, 1) \) for \( b = W, Z \)]. The field-dependent squared masses \( m_a^2(h) \) and the thermal squared masses \( \Pi_b^2(T) \) are \( m_W^2(h) = g^2 h^2/4 \), \( m_Z^2(h) = (g^2 + g^2)h^2/4 \) and \( \Pi_W^2 = 11g^2T^2/6 \). The hidden sector particles do not contribute to \( V_T \), since their masses are expected to be \( m_{\text{hid}} \sim f_c \gg v \), so their presence in the plasma during the EWPT (when \( T \sim v \)) is strongly Boltzmann suppressed (also, due to GMESB, their contributions would only appear at the 2-loop level).

We may now analyze the dynamics of the EWPT. For \( T \gg v \) the only minimum of \( V \) is at \( \langle h \rangle(T) = 0 \) and EW symmetry is restored. As the Universe expands, temperature decreases and a new local minimum develops away from the origin. As discussed above, if \( X \geq 1 \) this second minimum will never be energetically favored over the symmetric one and EW symmetry breaking will never take place. For \( X < 1 \), a critical temperature \( T_c \) exists at which \( V \) has two degenerate minima, and for \( T < T_c \) it is possible for the Higgs field to tunnel to the broken phase. The rate per unit time and volume of this tunneling process at a given temperature \( T \) is [16,17]

\[
\Gamma \sim T^4 e^{-F_c/T}.
\]
Here, $F_c$ is the free-energy of a critical bubble of true vacuum, i.e., a bubble just large enough for its internal pressure to overcome its surface tension. At $T_c$, a critical bubble has infinite size, so that $F_c \to \infty$, and the phase transition does not proceed. The nucleation temperature $T_n$ is defined as that for which the nucleation probability of a critical bubble within a Hubble volume approaches unity. This happens when $F_c/T \approx 140$, which sets the temperature at which the EWPT effectively starts (for large supercooling, a more accurate procedure is needed to determine $T_n$, see, e.g., [18]).

The EWPT strength $R \equiv v(T)/T$ can be computed both for $T_c$ and $T_n$, as shown in Fig. 2. Since the EWPT effectively starts at $T_n$, its strength is better estimated by $R_n$. A strongly first-order EWPT, avoiding baryon number washout after electroweak baryogenesis, requires $R_n \gtrsim 1$ (see, e.g., [19,20]), which for GMESB scenarios occurs already for $X \gtrsim 0.08$. The logarithmic term in (4) coming from the breaking of scale invariance thus leads to a strong EWPT even for small deviations of $\lambda$ from its SM value. As $X$ grows, a large potential barrier between the minima develops. The amount of supercooling required to tunnel may then be large, and $T_n$ will be substantially lower than $T_c$, resulting in $R_n$ being significantly larger than $R_c$ (as shown in Fig. 2). For $X \gtrsim 0.47$ the symmetric vacuum is actually metastable (with a lifetime longer than the age of the Universe), leading to an inconsistent cosmology.

Once the EWPT starts, bubbles of true vacuum nucleate and expand, filling the entire Universe. The velocity $v_w$ of the expanding bubbles can be computed by solving a set of hydrodynamic equations [18,21], since the plasma friction on the expanding bubble walls is known for the SM [21]. Stationary state bubbles expand either as subsonic deflagrations or as supersonic detonations (see, e.g., [22]), the sound speed of a relativistic plasma being $c_s = 1/\sqrt{3} \sim 0.577$. Subsonic bubbles could potentially lead to baryogenesis for a strong enough EWPT, $R_n \gtrsim 1$. Supersonic bubbles do not allow in general for baryogenesis (see, however, [23]), but collisions of fast moving bubbles at the end of the EWPT can be a powerful source of gravitational waves [24–26]. For very strong phase transitions the bubbles become ultrarelativistic and enter a “runaway” (continuously accelerating) regime [27], leading to very efficient gravitational wave production [22]. For the GMESB scenarios discussed here, we show in Fig. 2 the ranges of $X$ for which deflagrations, detonations, and runaway are realized. We see that $X \gtrsim 0.3$ leads to runaway bubbles, the best possible scenario for gravitational wave production.

Gravitational wave production.—A large stochastic gravitational wave (GW) signal is expected for GMESB scenarios in a sizable portion of the allowed range of $X$, generated when bubbles collide at the end of the EWPT [24–26,28,29], at a temperature $T_s$. The peak amplitude $h^2\Omega_{\text{peak}}$ and peak frequency $f_{\text{peak}}$ of such a GW signal can be estimated as [26]

$$h^2\Omega_{\text{peak}} \approx 10^{-6} \left(\frac{H_s}{\beta}\right)^2 \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \frac{1.84v_w^3}{0.42 + v_w^3} \left(\frac{100}{g_*}\right)^{1/3},$$

(7)

$$f_{\text{peak}} \approx 10^{-2} \text{mHz} \left(\frac{T_s}{100 \text{ GeV}}\right)^2 \frac{1.02}{H_s^{1.8} + v_w^3} \left(\frac{g_*}{100}\right)^{1/6},$$

(8)

where $g_* \approx 107.75$ is the effective number of degrees of freedom in the early Universe, $\alpha$ is the ratio of latent heat to the energy stored in radiation [30], $\kappa(\alpha, v_w)$ is the efficiency in converting this latent heat into kinetic energy that can lead to GW production as computed in [22], and $H_s/\beta$ corresponds to the mean bubble size (normalized to the Hubble radius). Both $\alpha$ and $\kappa$ are computed at $T_s$, while $\beta^{-1}$ estimates the duration of the EWPT and is usually obtained as the leading order term in the expansion of $F_c/T$ in (6) around $T_s$ [31]. This returns an adequate mean bubble size only when the EWPT proceeds much faster than the rate of expansion of the Universe ($H_s/\beta \ll 1$). Close to metastability we compute $\beta$ using instead the procedure described in [30].

For frequencies smaller than $f_{\text{peak}}$ the GW spectrum grows as $f^3$ [25], whereas it falls off as $f^{-1}$ for large frequencies [26]. In Fig. 3 we show the GW spectrum for various values of $X$. For detonations (red spectra) the amplitude seems too small to be observable. For deflagrations the results are even smaller. Note, however, that recently plasma sound waves formed upon bubble collisions have been identified as a strong source of GW [32], possibly leading to an $H_s/\beta$ enhancement of the signal for detonations and deflagrations. For values of $X$ in the region

![Figure 2](color online). Phase transition strength computed at critical (red) and nucleation (blue) temperatures. The dotted horizontal line marks the limit above which the phase transition is strongly first order.
FIG. 3 (color online). \( h^2 \Omega_{GW}(f) \) for various values of \( X \), as indicated in the dashed-dotted red (detonations) and solid blue (runaway) curves. Black dashed lines are the sensitivity curves of eLISA and BBO. The solid-green horizontal line is the GW signal from inflation.

0.29 \( \lesssim X \lesssim 0.47 \), corresponding to runaway (blue spectra), the GW signal would be observable by BBO, and could even be close to the sensitivity curve of eLISA (sound waves can be neglected in this case). This could be a smoking-gun signature of GMESB scenarios in the absence of new physics at LHC. Figure 3 shows that this GW signal would clearly stand out over the GW signal from inflation.

Confined with the particle content of the hidden sector, by considering a (weakly coupled) GMESB scenario with the following generic assumptions: (i) particles in the hidden sector do not couple simultaneously to \( SU(2) \) and \( SU(3) \), resulting in two independent hidden sectors, each with its own breaking scale \( f_{c(n)} \); (ii) the \( N_{B}^{(n)} \) bosons (\( N_{F}^{(n)} \) fermions) belonging to the same hidden sector have a common anomalous dimension \( \gamma_{B}^{(n)} \), which must be negative for consistency [9]. As previously noted, the latter implies \( X > 0 \), so these GMESB scenarios naturally yield an EWPT significantly stronger than that of the SM. Approximate vanishing of the \( 1\)-loop \( SU(n) \) gauge coupling \( \beta \) functions in the UV requires that the hidden sector contribution to the \( \beta \)-function coefficient

\[
\beta_{\text{hid}}^{(n)} = (1 - \gamma_{B}^{(n)} - X_{\text{hid}}^{(n)} )N_{B}^{(n)} - X_{\text{hid}}^{(n)} N_{F}^{(n)} \frac{2N_{F}^{(n)}}{3} - \gamma_{F}^{(n)} \frac{8N_{F}^{(n)}}{3},
\]

approximately cancels the SM one, yielding \( \beta_{\text{hid}}^{(2)} = 19/6 \), \( \beta_{\text{hid}}^{(3)} = 7 \). We can then relate \( f_{c(n)} \) to the particle content of the hidden sector via (see [9] for details)

\[
A_{(n)} = \frac{1}{(16\pi)^{2}} \left( \frac{2b_{\text{hid}}^{(n)} + 4N_{B}^{(n)} N_{F}^{(n)} - 8N_{F}^{(n)}}{\bar{\gamma}_{F}^{(n)}} \right),
\]

where

\[
A_{(2)} = \frac{64\pi^{2} X}{9g_{2}^{2} \bar{Y}} f_{c(2)}^{2},
\]

\[
A_{(3)} = \frac{\pi^{2}}{18g_{2}^{2} \bar{Y}} \left( \frac{1 + 16X}{2\pi^{2} - 3\bar{\lambda}^{2}} \right) m_{h}^{2},
\]

\( g_{h} \) is the \( SU(n) \) gauge coupling, \( \lambda \) is the top-quark Yukawa coupling, and \( Y \equiv 13g_{2}^{2} - 8\lambda \). Preserving perturbativity requires \( -1 \lesssim \gamma_{B,F}^{(n)} < 0 \), resulting in lower and upper bounds on the combination \( N_{B}^{(n)} + 2N_{F}^{(n)} \) and in an upper bound on \( f_{c(n)} \) (automatically saturated in the case of purely fermionic hidden sectors) as a function of \( X \), which decreases with increasing \( N_{B}^{(n)} + 2N_{F}^{(n)} \), as shown in Fig. 4 for various allowed \( (N_{F}^{(n)} , N_{B}^{(n)}) \) combinations. The cosmological bounds on \( X \) derived in previous sections then impose upper bounds on the scale of breaking of scale

FIG. 4 (color online). Upper bound on \( f_{c(2)} \) (upper curves, red) and \( f_{c(3)} \) (lower curves, blue) for various combinations of \( (N_{F}^{(n)} , N_{B}^{(n)}) \).
invariance in the hidden sectors, namely, $f_{c(2)} \lesssim 25$ TeV and $f_{c(3)} \lesssim 7$ TeV. In GMESB scenarios with a common breaking scale $f_c = f_{c(2)} = f_{c(3)}$, Fig. 4 implies $X \ll 1$ and $f_c \sim 1$ TeV, within LHC reach. In contrast, for scenarios with a significant hierarchy between $f_{c(2)}$ and $f_{c(3)}$, large values of $X$ are preferred, together with both $f_{c(2)}, f_{c(3)} \gtrsim 3$ TeV, making the hidden sectors hard to be probed directly at LHC. This highlights the fact that a strong EWPT is anticorrelated with the LHC search prospects for GMESB scenarios.

**Conclusions.**— We have shown that models with gauge mediation of breaking of scale invariance (GMESB scenarios) generically lead to a strong electroweak phase transition. Such setups could easily escape direct detection at LHC, especially if GMESB is dominated by $SU(2)_L$ gauge interactions, where the breaking scale $f_c \gtrsim 5$ TeV. The measurement of a nonstandard cubic Higgs coupling by CLIC combined with the most interesting observation of a large primordial gravitational wave signal from the EWPT, standing out from the inflationary gravitational wave signal observed by BICEP2 in the CMB, is a promising route for testing these GMESB scenarios.

We thank A. Mariotti and S. Abel for very useful discussions and comments on the manuscript. S. J. H. and J. M. N. are supported by the Science Technology and Facilities Council (STFC) under Grant No. ST/J000477/1. G. C. D. is supported by CAPES (Brazil) under Grant No. 0963/13-5.