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GOVERNMENT SPENDING, MIGRATION, AND HUMAN CAPITAL: IMPACT ON ECONOMIC WELFARE AND GROWTH- THEORY AND EVIDENCE

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Summary

The purpose of this dissertation is to analyze the effects of public policies on rural-urban migration and human capital expansion, and to examine the role of human capital (among other domestic and external factors) in the long-term economic growth of developing countries. Human capital expansion and labor migration from villages to cities are two aspects of the structure of labor markets in poor countries that are continuously influenced by public policies—policies that are often either ineffective or have unintended adverse consequences. For example, while much of human resource policy in developing countries is directed toward increasing the supply of educated labor, inter-sectoral in-country migration and unemployment have become more pronounced, requiring new thinking on policy responses. This dissertation analyzes the outcomes of such policies and offers insights into how they might be improved.

Chapter 2 extends a two-sector, general-equilibrium model of rural-urban migration to include government spending. Provision of public goods acts as a productivity-enhancing input in private production that results in external economies of scale. This approach is generalized by introducing an unbalanced allocation of public expenditure in rural and urban sectors due to political economy considerations, differential sector output elasticities with respect to government input, and distortionary taxation. The chapter studies the effects of an increase in public spending and taxation on sectoral outputs, factor prices, urban unemployment, and welfare. Of particular concern here is to study the effect of an unbalanced allocation of government spending between rural and urban areas.

Chapter 3 studies the effects of selected education policies on the size of the educated labor pool and on economic welfare using the “job ladder” model of education, which is relevant
to liberal arts education in developing countries. The policies considered are (1) increasing the
teacher-student ratio, (2) raising the relative wage of teachers, and (3) increasing the direct
subsidy per student. In addition, the chapter analyzes the impact of wage rigidities in the skilled
or modern sector on the size of the educated labor force. The analysis consists of five major
sections. First, it reformulates the Bhagwati-Srinivasan job ladder model to make it amenable to
analyzing the comparative static results of the effects of selected policies. Second, since higher
education is mostly publicly financed, the analysis extends the job ladder model to incorporate
public financing of the education sector. It then examines that model along with the effects of
changes in policy parameters. Third, the analysis develops another extension of the job ladder
model to include private tuition practices by teachers that are prevalent in many developing
countries. Fourth, to analyze the impact of wage rigidities in a less-restrictive framework where
individuals can choose education based on ability and cost, the chapter develops an overlapping
generations model of education with job ladder assumptions of wage rigidities in the skilled or
modern sector. The chapter examines the flexible market and fixed-market (with wage rigidities)
equilibrium scenarios, and compares the impact on the threshold level of abilities and the size of
the educated labor force. Finally, using specific functional forms of human capital production,
cost, and ability density functions, the chapter analyzes the equilibrium outcomes. The analysis
shows that in an economy with wage rigidities in the skilled sectors (modern and education
sectors), the result of quality-enhancing policies under the simple job ladder model is an increase
in the total size of the educated labor force. However, under an extended version of the job
ladder model, the result depends on the relative size of the effects of an increase in the cost of
education and the effects of an increase in the expected wage. The overlapping generations/job
ladder model formulation used in the chapter finds that an increase in the present value of the
expected wage and/or an increase in the marginal product of education will increase the demand
for education. The minimum threshold level of ability falls, and more people are encouraged to acquire educational skills.

Chapter 4 estimates the effects of openness, trade orientation, human capital, and other factors on total factor productivity (TFP) and output for a pooled cross-section, time-series sample of countries from Africa and Asia, as well as for the two regions separately. The models are estimated for the level and growth of both TFP and output by using panel fixed effects. The generalized method of moments is also applied to address endogeneity issues. Several variables related to political, financial, and economic risks are used as instruments, together with the lagged values of the dependent and endogenous explanatory variables. The data for this study span 40 years (1972–2011) and are grouped into five-year averages. Several sources were used to obtain the most updated data, including the newly released Penn World Table (Version 8.0). The chapter finds that inducing a greater outward orientation generally boosts TFP, per capita output, and growth. Greater accumulation of human capital has a consistently positive effect on output and TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than interactive terms with openness. Furthermore, inflation does not negatively affect growth, although inflation variability is found to adversely affect TFP and output in Africa.

Chapter 5 concludes the dissertation by providing conclusions, a summary of major results, and possible directions for future research.
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Chapter 1

Introduction

The purpose of this dissertation is to analyze the effects of public policies related to labor and human capital on economic development of low-income countries. The focus is on rural-urban migration and human capital expansion, the two aspects of the labor market structure that are continuously influenced by public policies. Some of these policies are often ineffective or turn out to have unintended adverse consequences. For example, much human resource policy in developing countries is directed toward increasing the supply of educated labor. Yet the policy often leads to rapid migration to cities, making urban unemployment even more pronounced. This dissertation analyzes the outcomes of such policies and offers insights into how they might be improved.

With respect to human capital, in many densely populated low-income countries, such as those in South Asia, expansion of education does not seem closely tied to other key goals in development such as productivity growth. The simple fact of significant unemployment among educated workers may indicate low marginal productivity of resources devoted to education. One representative study by Ansari and Singh (1997) on Indian data for 1950–87, for instance, reveals a causal link between national income and education (causality running from income to education) but not vice versa. In another study, using time-series data for the period from 1966–96, Self and Grabowski (2004) find a relatively strong causal impact of primary education on India’s growth performance. Secondary education, however, has less impact and tertiary education has virtually no impact on growth. Yet, the expansion of education continues to be a major objective of social policy. Human capital, especially acquired in liberal arts education in colleges, has
expanded in response to large government subsidies in much of the developing world. Recent UNESCO (2010) statistics show that in India college enrollment increased from 2.5 million in 1970 to 20.7 million in 2010, and in China it rose from less than 100,000 in 1970 to 30 million in 2010. In the United States, the corresponding figures are 8.5 million in 1970 and 20.4 million in 2010, whereas in other advanced economies college enrollment rose from 4.9 million in 1970 to 23.7 million in 2010. In many developing countries, people are encouraged to acquire higher education to improve their prospects for employment abroad and higher returns, which leads to an increase in the supply of educated workers in the domestic labor market. However, due to the sticky wage-rate—constant demand for educated labor at home—the rising supply of educated workers in the domestic labor market raises involuntary educated unemployment at home (Stark and Fan, 2011; Fan and Stark, 2007a, b; Stark and Wang, 2002; Stark, Helmenstein, and Prskawetz, 1997, 1998; Bhagwati and Hamada, 1974; Fields 1974). For example, in India in 2011–12 the educated unemployment rates for post-graduates and graduates were 10 percent and 9.4 percent, respectively, compared to 5.4 percent and 1.2 percent for those with secondary and primary education (India Labor Bureau, 2012). In Kerala, the state in India with the highest literacy rates, the unemployment rate among university graduates in 2003 was 36 percent, compared to 12 percent among secondary-incompletes and 1.7 percent among those with primary education (Mathew, 1997; Zachariah and Rajan, 2005). In 2011, the educated unemployment rate among recent college graduates was 9.5 percent (13 percent among graduates with a fine-arts major, and lowest among engineering majors at 7.5 percent) (MyCos Institute, 2011). In 2007, a report by the Arab Labor Organization found that “the trend of higher unemployment among literates applies to countries throughout the Arab world, with a ratio of 5:1 in Morocco and 3:1 in Algeria,” and that in Egypt, “the rate of unemployment is ten times higher in the educated section of the population than among illiterates” (quoted in the Egyptian
newspaper *Al-Masri al-Yawm*). Boudarbat (2006) found that in Morocco in 2000, the unemployment rate of university graduates was about four times that of individuals with less than six years of schooling.¹

Despite educated unemployment and underemployment, the sustained high demand for educated workers is translated into financing of higher education largely by states in many developing countries. For example, in sub-Saharan Africa, the governments provided over 90 percent of the cost for higher education (Teferra, 2007). In Papua New Guinea, the government provided more than 95 percent of university recurrent financing in 1988 (McGavin, 1991).

Although these expenditures on public education might appear to benefit all segments of the population equally, the role of education as a vehicle for the equalization of economic opportunity and income redistribution is seriously in doubt. Jallade (1973) shows that education in many developing countries generally provides a net transfer from low- to high-income families even after accounting for higher marginal taxes on higher-income groups. Analyzing educational policies implemented in Chile from 1987–98, Espinoza (2008) found that access to higher education was disproportionately greater among upper- and upper-middle-income students compared to lower-, lower-middle, and middle-income groups. Evidence in Egypt (Fahim and Sami, 2011) and Tunisia (Abdessalem, 2011) suggests that even though educational reforms aim to increase access for poor students, most of the public spending on higher education goes to students in the richest quintiles. Thus, the demand for higher education policies in many of these countries seems to stem from many socio-political considerations, without much regard for the adverse impact of those policies on the welfare of non-elites.

¹ In many developed countries demand for education in humanities has declined over time. In the United States, only 7 percent of college graduates nationally majored in humanities in 2010, compared to 14 percent in 1966. The U.S. unemployment rate is 9.6 percent among recent graduates with humanities majors, compared to 5.8 percent among graduates with a science major (Levitz and Belkin, 2013).
Concomitant with the expansion of education, a heavy emphasis in government planning is placed on urban development. To raise productivity in capital-intensive manufacturing in urban areas, large public investments are made in physical and social infrastructure such as roads, electricity, and transportation. Belief in a huge potential for productivity growth in modern sectors leads governments to allocate a disproportionately large fraction of their budgets to infrastructure in and around cities. This “urban bias” in government expenditure has created a wide disparity in public services between the rural and urban regions in most countries.

Figure 1.1 shows the rural share in the total population by broad regions and the share of agriculture in total employment. Sub-Saharan Africa and South Asia are regions where over 60 percent of the population is rural. This is followed by East Asia and the Pacific, led by China, with almost half of the population rural. A closer look at the left and right panels of Figure 1.1 shows that poverty and agricultural employment are positively related. About three-fifths of the population in sub-Saharan Africa and a half in South Asia are engaged in agricultural pursuits. For more prosperous regions like the member countries of the

Figure 1.1: Rural Population and Employment in Agriculture

Organization for Economic Cooperation and Development (OECD), we see smaller percentages of population that are rural and employed in agriculture.

In sub-Saharan Africa, poverty is largely a rural phenomenon. But since that region is relatively land-abundant and sparsely populated, provision of adequate public services to the rural poor remains costly. Migration has made the largest cities in sub-Saharan Africa overpopulated, while simultaneously enabling migrants to earn higher incomes relative to nonmigrants (IMF, 2013). In South Asia, however, migration seems to occur more to smaller towns near migrants’ homes. They are more attached to their agricultural incomes and fail to escape from poverty, as poverty in smaller towns is much higher than in large cities. This shows that there is value to addressing poverty with joint policies on land and labor rather than labor alone (IMF, 2013).

Table 1.1 presents the latest statistics on the availability of basic public services in developing countries. The table clearly shows a wide gap between rural and urban areas in most indicators.
Availability of public services seems to have expanded with the growth of cities as centers of modern economic activity. While 71 percent of the population in developing countries live in rural areas, only 30 percent of the rural population has access to sanitation; in contrast, about half of those who live in towns have such access. Improved water supply, for example, reaches 83 percent of urban residents compared to only 56 percent of rural residents. Electricity is available to 76 percent of those in cities, but to only 33 percent of people in villages. Increased budgetary allocation and public service provision in urban areas in consonance with industrial expansion as reflected in the statistics presented here have contributed to significant flows of labor migration to cities, swelling the ranks of the urban population, including the urban poor.

Looking at the numbers reported above for South Asia and sub-Saharan Africa, we find South Asia to have been slightly less urbanized than Africa (31 percent versus 36 percent of population in urban areas). However, rural Africa lags far behind rural South Asia in improved water supply (48 percent versus 88 percent) and electricity (8 percent versus 25 percent). In
Africa, access to sanitation is marginally less than in South Asia (23 percent versus 28 percent). However, the rural-urban differences in under-5 mortality rates in India and Ghana are indicative of the large rural-urban disparities in health facilities, both in South Asia and Sub-Saharan Africa.

A major pull factor for labor migration to cities is the incidence of lower poverty in urban areas generally and large cities in particular. Eastwood and Lipton (2004) find that urban-rural income ratios range from 1.3 to 1.8 in many Asian countries. Using Demographic and Health Surveys data from a comparable measure of inequality and migration for 65 countries, Young (2013) finds that 40 percent of mean country inequality and much of the cross-country variation is accounted for by the urban-rural gap. IMF (2013, p. 86) shows a clear trend of urbanization in most developing countries being associated with higher incomes and lower poverty. Indeed, the report presents evidence to show that the larger the population in cities, the lower the rate of urban poverty. IMF (2013) indicates that, except for some countries in sub-Saharan Africa such as the Central African Republic, Mali, Swaziland, and Malawi, the poor are disproportionately concentrated in smaller towns and villages, whereas in those sub-Saharan countries poverty is mostly concentrated in rural areas.

Higher labor productivity in modern sectors, partly as a result of the contribution from public services, is associated with relatively high wages in manufacturing and urban service jobs. Indeed, public policy has shifted in favor of the urban sector because of the concentration of political and economic power in cities (Lipton, 1976; Harbeson and Rothschild, 1995; IMF, 2013). Instead of leading structural transformation of poor economies, cities have now become, according to the World Bank (1999), “part of the cause and a major symptom of the economic and social crises that have enveloped the continent.” The probability of high-paying jobs,
consumption externalities from public services, and production externalities from infrastructure development can attract rural migrants beyond a sustainable point. A natural consequence of urban growth interacting with pro-urban government policies has been a high, and in many cases rising, rate of unemployment among all types of labor, including educated labor. Rural areas have mostly been neglected economically due to this urban bias in development (Todaro and Smith, 2012).

Also, information asymmetries regarding the availability and allocation of public services between rural and urban areas provide urban residents a distinct advantage. This also acts as an incentive to the governments in many developing countries to maintain the urban bias. Greater wealth and higher education imply that urban dwellers have better access to, and ability to assimilate, the available information. For example, in Nepal, access to the press for citizens in the countryside is constrained by greater poverty, low literacy, and inadequate transport (World Bank, 2002), only partly alleviated now by expansion of mobile communications. Media coverage tends to favor wealthier and educated urban clients, while information externalities are also more positive in urban areas due to high population density. All these factors tend to generate an urban bias in the allocation of public services and external aid.

Examination of the policies that would be optimal from the perspective of a balance between urban and rural development is a topic that has attracted much research since Lewis (1954) and Todaro (1969). Based on state-level data on India for 1970–93, Fan et al. (2000) suggest that “in order to reduce rural poverty, the Indian government should give highest priority to additional investments in rural roads and agricultural research. These types of investments not only have much larger poverty impacts per rupee spent than any other government investment, but also generate higher productivity growth.”
Proponents of balanced economic development suggest that high agricultural growth prevents urban expansion with large slum populations in developing countries. Rapidly growing agriculture and rural industry will boost employment and alleviate poverty (Mellor and Desai, 1985; Bezemer and Heady, 2008; Christiansen et al., 2006; Diao et al. 2007, 2008; Thirtle, 2003), and foster a diffused pattern of urbanization (Hardoy and Satterthwaite, 1986; Tacoli and Satterthwaite, 2003; Renkow, 2007). To achieve such outcomes, however, it may be necessary to engage “middle farmers” to adopt new technologies (Jayne, Mather, and Mghenyi, 2006). But that requires adequate supply of infrastructure (Ahmed et al., 2007; Haggblade, Hazell, and Dorosh, 2007). Rural road networks link areas of surplus production to major marketing centers. Electrification enhances efficiency in the production of nonfarm goods and services, as in the case of handlooms in Ethiopia (Zhang et al., 2011), and improves the quality of life. Communications networks that provide mobile phones or land lines facilitate information about prices and marketing opportunities for producers of surplus output. What is needed is a significant increase in public and private investments in roads, electricity, and telecommunications.

Recent country studies show that pro-urban development promoted by strongly centralized governments induces large migration flows to the capital cities. To alleviate lopsided regional economic growth, more effective ways to develop less-developed regions will be necessary (van Lottum and Marks, 2012).

In addition to placing heavy emphasis on urban development, a large number of developing countries allocate resources to higher education beyond what is economically justified. Tan and Mingat observed in their 1992 study that university education in developing countries has received a much higher rate of public subsidy relative to the rate of return than
have lower education levels. An important reason for this has been the perceived need for improving access to education rather than increasing its depth or quality. As noted earlier, Self and Grabowski (2004) find that the rate of return from education is observed to fall progressively with the rise in the level of education.

This dissertation explores selected aspects of human resource development in economic growth. The amount of resources a country would optimally devote to the expansion of human capital through education depends substantially on its contribution to economic development. This dissertation develops models to analyze the effects of public policies on migration and human capital and, in turn, examine the role of human capital in economic growth and national welfare.

Chapter 2 presents a theoretical work that links government spending to rural-urban migration and urban unemployment in a dual-economy framework, a link that previous research on rural-urban migration in a dual-economy tradition has largely ignored. The models in Chapter 2 come close in spirit to Krichel and Levine (1999), who study rural-urban migration in the Harris-Todaro tradition by including a substantive role for the government. Within a government budget constraint, Krichel and Levine examine the effect of policies on migration under the assumption that the urban sector displays agglomeration effects.

Chapter 2 extends the migration model in a slightly different direction to include government spending, following a public goods model of productive government services (Barro and Sala-i-Martin, 2003). In such a model, the government invests in physical and social infrastructure that enhances the productivity of private inputs. The result is increasing returns to scale in the production function that would display constant returns without such investments. Chapter 2 generalizes this approach by introducing unequal proportions of budgetary allocations
for the rural and urban sectors, unequal output elasticities with respect to the government input, and a tax on urban output to finance government expenditure. The focus of the chapter is on the effects of expenditure and tax policies on sectoral outputs, factor prices, urban unemployment, and national economic welfare. Of particular concern is the effect of an unbalanced allocation of government spending between rural and urban areas. Rural taxation is ruled out mainly because of the real world evidence that such a tax does not make a large direct contribution to government revenue in much of the developing world. However, because the rural population pays indirect taxes such as sales and trade taxes, the assumption is made largely to keep the model relatively simple to analyze, rather than to represent complete reality.

Chapter 2 starts with a neoclassical general equilibrium model of two sectors with full employment of labor in each sector. The second model (presented in Section 2.6) retains external economies of scale in public expenditure but augments the original model to consider unemployment in the urban sector. Migration from overpopulated villages to cities occurs as a response to a wage differential and stops when the rural wage equals the (expected) urban wage after adjusting for urban unemployment.

Each model analyzes two important cases. The first examines what happens to outputs, employment, and urban unemployment when the government varies the fractions of total expenditure going to the two sectors while staying within the budget constraint. A basic assumption of the model is that the urban sector uses more capital per unit of labor than does the rural sector; that is, the urban output is relatively capital-intensive.

In the full-employment version of the model, the effects of an increase in the rural share of the budget provide an interesting case study. Significant evidence exists to show a clear bias in budgetary allocations for urban areas, while an enormous potential to raise agricultural
productivity remains untapped for lack of government support. Raising the rural share of public expenditure leads to three distinctly possible results. First, assuming that public goods contribute more to the productivity of capital and labor in rural than in urban areas, rural output increases and so does national output when the urban sector is not very large. Second, if employment and output in the urban sector are large enough, the reduction of the tax base that follows from a decrease in the urban share of public expenditure and a decrease in urban output can lead to a decline in national output as well. Third, the last result may not hold (and rural and total output can still rise) if government services contribute sufficiently to the change in rural output than to urban.

The welfare analysis reveals that raising the rural share of government spending increases rural output and employment and lowers the migration flow to cities. There is a direct negative impact on urban output as infrastructure spending slows in the urban area. Such a decline, however, is at least partly offset by increased rural demand for urban output, the increase being made possible by higher incomes following greater rural spending by the government. Given that urban centers in most developing countries have a hard time coping with huge demand for infrastructure, this public policy can serve as a mechanism to improve the urban-rural population balance. Again, the result depends on specific values of the parameters in the equations that represent the welfare analysis. For some realistic values of these parameters, the result does indeed seem to hold.

These broad results about the rural-urban balance as a result of reorienting government spending come about through micro changes in the rural as well as urban sector. Better transportation in villages helps link rural production with urban marketing, which raises output prices for farmers. Better communications networks that use modern technology also improve
marketing efficiency by ensuring that information flows faster. Electrification directly assists nonfarm production, facilitating rural employment. A stunning example of all this is China, although there are other cases as well. As China was becoming a manufacturing powerhouse in the 1990s, its share in world manufacturing employment reached a high of 15.6 percent. Over the next couple of decades, the share continued to decline until it reached 13 percent by 2009, while manufacturing production continued its rapid pace. More importantly, within China the urban share of manufacturing was a phenomenal 66 percent in 1990. Over time, however, as the government tried to boost the relatively sluggish rural economy by spending on infrastructure and placing restrictions on migration, the employment scenario completely reversed itself. By 2009, for example, the rural share in manufacturing employment had risen to 65 percent with the concomitant fall in the urban share. This shows that a substantial part of manufacturing may have moved to the rural areas.

A second policy analyzed is an increase in the tax on urban output. An urban tax increase, holding expenditure shares constant, will reduce urban output but will raise both rural and urban expenditures. Thus, rural output invariably increases unless government services yield a marginal productivity of zero. This leads to a reduction in outmigration from rural to urban areas, and may cause reverse migration from urban to rural areas. The effect on urban output is, however, ambiguous. Under the condition that scale economies of greater public expenditure to urban output dominate the contractionary effect of the tax increase, urban manufacturing will expand. Meanwhile, several mixes of parameters are found that yield a loss of urban output, a reduction in the tax base, and an overall decline in total output and welfare in the country. The result on the direction of migration similarly depends on the values of parameters in the model.
The second model in Chapter 2 is the Harris-Todaro version augmented in terms of public expenditures. One main result here is that an increased allocation of public spending to the rural sector can still increase the proportion of urban population employed. This results under the condition that the negative returns-to-scale effect on urban output due to expenditure cut is more than offset by increased labor productivity in urban employment when the level of urban employment falls. The positive employment effect in the urban sector arises from the positive output effect in the rural sector when rural spending is greater. This leads to a rise in rural wages and induces reverse migration from the urban sector. If this rising urban productivity more than offsets the output effect of the public expenditure decline, the urban employment rate increases and the unemployment rate falls. The welfare effects in the unemployment model are subject to a combined influence of a set of elasticities and go different ways depending on the relative influence of composite parameters of the model.

Chapter 3 analyzes another dual-economy model that has a distinct education sector as well as a production sector. This model is appropriate for liberal arts education in less-developed countries in South Asian and sub-Saharan Africa. Traditionally, in inward-looking economies of these regions, the types of education acquired have not been strongly aligned with the needs of industry. Before many countries opened up to trade in the 1990s, the prevailing wage structures also created socioeconomic conditions in which “high wages for educated elites have been set by fiat, legislation, or unionization. The phenomenon of high wages, accruing to employed educated elites, created a strong demand for higher education” (Bhagwati and Hamada, 1974).

Chapter 3 considers a model to examine the effects of policies on the size of the educated labor force and on national welfare. The policies analyzed are increases in education subsidies to students, raising educators’ compensation, and increasing the teacher-student ratio. The model
shows that each of these policies increases the supply of skilled labor, but that in each case the overall national welfare declines. One of the important features of the education system in South Asia and sub-Saharan Africa is that either education is officially free or the students pay a nominal subsidized fee at schools and colleges. On the other hand, the students pay a high “private tuition cost” (roughly five to ten times the subsidized tuition at schools) for private tutoring, sometimes by the same teachers who teach at the schools.

For example, a report published by United Nations Educational, Scientific and Cultural Organization (UNESCO, 1999), shows that the proportions of students receiving tutoring in Sri Lanka in 1990 were 62 percent for arts students, 67 percent for commerce students, and 92 percent for science students. In a more recent study containing a survey of 23 countries, Dang and Rogers (2008) find that about 25 to 90 percent of students receive private tutoring at certain levels. The alarming rate has raised questions among policymakers as to whether this imposes heavy costs on households (Psacharopoulos and Papakonstantinou, 2005) and exacerbates social inequalities. A simple extension incorporating these “private coaching earnings” shows that, despite a substantial education subsidy, the size of the educated labor pool is smaller than intended.

Most recent UNESCO data (EdStats, 2013) show that many sub-Saharan African countries—Malawi, Lesotho, Tanzania, Niger, Burundi, Swaziland, Botswana, Chad, Burkina Faso, and Mauritania—spend from 2 to 18 times GDP per capita on each tertiary student, and yet the gross enrollment rates were less than 8 percent. In tertiary education, public expenditure per student as a share of GDP per capita in 2009 was 149 percent, 88 percent, and 75 percent in Ghana, Benin, and India, respectively. This chapter analyzes an extended version of the Bhagwati and Srinivasan (1974) job ladder model in which the expected qualification for a given
job rises over time. The chapter offers a simple but major extension to incorporate the public financing aspect of higher education prevalent in almost all developing countries. It also makes another important extension of the model to include private tuition practices by the teachers. The goal is to analyze the effects of the educational policies on the size of the educated labor force and on economic welfare in the reformulated original model and in its extended versions.

The policies considered are (1) an increase in the teacher-student ratio, (2) a rise in teachers’ wages, and (3) an increase in the direct subsidy per student. In addition, the chapter analyzes the impact of wage rigidities in the modern sector on the size of the educated labor force. An overlapping generations model of education has been developed with the job ladder assumption of wage rigidities in the skilled sector. The impact on the threshold level of abilities and the size of the educated labor force is compared in different versions of the model. In addition, the equilibrium outcomes are analyzed using specific functional forms of the human capital production function, the cost function, and the ability density function. Policies that enhance the quality of education are found to increase the total size of the educated labor force in the simple model. But in the extended versions, the results depend on the relative sizes of the cost-enhancing impact and the effect on the expected wage of an educated worker.

Chapter 4 examines the role of human capital, as well as other domestic and external economic factors, in long-term economic growth. Empirical evidence suggests that poor but growing nations can rely on accumulation of capital per worker for significant periods of time while they are on the path to their steady-state growth. Once they get closer to their production frontier, however, much of the growth in per capita income must depend on technological progress, or total factor productivity (TFP) growth, which summarizes changes in a large number
of economic and noneconomic factors in addition to direct improvements in production technology.

TFP accounts for a large fraction of output growth in those parts of the world that have experienced modest to high growth. The accumulation of the basic factors of production such as capital per worker contributes significantly to output growth as well. Yet, while factors such as the rate of savings are commonly understood to affect capital accumulation, there is much less consensus about the exact factors that influence TFP. This has led to a burst of economic research over the last 20 years into causes underlying TFP growth.

Availability of a comparable set of cross-country income data, particularly the Penn World Table, has facilitated such cross-country growth exercises. Most such datasets have lacked capital stock data in the past for many developing countries. Consistent series for variables that seem closely related to TFP in theory have also been difficult to obtain for long enough time periods, particularly for developing countries. With the availability of Version 8.0 of the Penn World Table, a more consistent and accurate capital stock series can now be used, as has been done in Chapter 4.

Chapter 4 starts with a discussion of the main empirical literature on TFP as a base case and makes three specific improvements. First, many papers have studied TFP for large panels of countries by including in the same pool of developing and developed countries and countries from all geographic regions. Chapter 4 focuses on developing countries alone because pooling developing and developed countries in a single dataset is equivalent to making an erroneous assumption that all countries share the same technology. Second, the model estimation strategy

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2 Miller and Upadhyay (2002) find that the assumption of a common technology for countries at all stages of the development ladder is not verified. Also, as explained in Chapter 4, the inclusion of country fixed effects does not fully address the cross-country or cross-region differences in technology.
extends other studies by adding observations and updating the dataset up to and including 2011, which gives a relatively long period of data for 40 years (grouped into eight periods of five years each for all countries in the sample). Third, a comparative assessment is also made of findings through the use of alternative methods, in particular the panel fixed effects and the generalized method of moments, to examine whether endogeneity problems really mar the fixed-effects results.

Since growth of TFP cannot be directly observed, economists employ alternative ways to estimate its contribution to the overall growth of an economy. One approach first estimates aggregate output as a function of the basic factors of production, namely labor ($L$) and physical capital ($K$). Another method, called growth accounting, uses fixed shares of capital and labor in output. After the contribution of these factors has been determined, the resulting unexplained part of output is TFP, which can be studied further by examining its determinants. Several papers get significant results for TFP based on a panel data model of a large sample of countries that includes human capital and outward orientation.

Chapter 4 uses data for 20 countries—10 each from Africa and Asia—but as mentioned earlier extends the empirical model of others by including more observations per country and more variables. The chapter examines the differences in growth rates between countries in Africa with those in Asia by estimating the contribution of TFP to economic growth and relating the TFP differences to their underlying causes. A focus on Africa can also help in understanding why this region has failed to move on the path of convergence in per capita income with the rest of the world.

The effect of openness and trade liberalization on economic growth remains a subject of debate. A greater body of empirical evidence, however, supports a positive correlation between
openness and growth. Trade forces domestic producers to compete with the rest of the world, and induces the adoption of more efficient technology that causes TFP to grow faster. Larger exports can relax the foreign exchange constraint, permitting imports of key intermediate and capital goods in production. As reviewed in the chapter, there are many empirical studies that show a strong trade liberalization-growth nexus. In a different vein, Rodriguez and Rodrik (2000) present a fairly comprehensive discussion of the topic to claim that no convincing empirical evidence exists to prove a positive trade-growth relationship. Warner (2003), however, makes strenuous efforts to refute the claims by Rodriguez and Rodrik, and reasserts the existence of a close direct relationship between growth and outward orientation.

Chapter 4 first calculates a measure of TFP from an aggregate production function in which output depends on physical capital, human capital, and labor. It then explores factors that influence the level and growth of TFP, especially openness, trade orientation, terms of trade, and external indebtedness, among external factors, as well as other factors such as education and health-related variables (namely, human capital, life expectancy, and child mortality); inflation; a demographic factor (the dependency ratio); and governance indicators such as corruption, government accountability, and stability. Results of this setup are then compared with those from an alternative but popular framework—growth accounting—following Bernanke and Gurkaynak (2001) and Crafts (1999), where the share of physical capital in output, $\alpha$, is assumed, as in much of the related literature, to be 30 percent for all countries in the sample.

Another approach to the study of TFP is to directly estimate the per capita income growth by including basic inputs in production as well as other factors that influence growth. These other factors are assumed to affect income growth through TFP growth. Chapter 4 estimates the effects of human capital, openness, and trade orientation, among other factors, on the level and
growth of TFP and also the level and growth of output for a pooled cross-section, time-series sample of countries from Africa and Asia, and for the two regions separately.

The models for the level and growth of both TFP and output are estimated by using panel fixed effects and by applying generalized method of moments to address possible problems with endogenous explanatory variables. Several feasible instruments are identified to address endogeneity. These include variables related to political, economic, and financial riskiness of a country together with the lagged values of the dependent and endogenous explanatory variables. This procedure generates a large number of moment conditions relative to the parameters being estimated. The resulting overidentifying restrictions can be tested through the Hansen $J$-statistics. The chapter finds the results of such an exercise to be reasonably satisfactory and the instruments used fairly valid.

The main results show that inducing a greater outward orientation generally increases TFP. Most of the middle- and low-income countries instituted trade policy reforms during the sample period to make their economies more open and raise productivity. The significance of a positive influence of outward orientation on growth of TFP has important policy implications.

Among variables other than those that measure outward orientation, greater accumulation of human capital is found to have a positive effect on TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than interactively with openness. Furthermore, inflation, another domestic economic variable, does not have a significant negative effect on TFP in the full sample or subsamples, although inflation variability is found to adversely affect TFP and output in Africa.
Finally, life expectancy, the social development indicator in the model, positively and significantly affects TFP in Africa. Most poor countries are expected to improve their life expectancy over time, and yet many countries in sub-Saharan Africa experienced an absolute decline during portions of the sample period. There is a strong evidence to suggest that a rise in this indicator will raise TFP in Africa.

While the data compilation for Chapter 4 has been a result of much painstaking search into various sources, there is significant scope for further improvement of data quality and the updating of statistics. Further, even meticulous research into growth factors hardly seems to offer conclusive evidence about how best to accelerate economic growth. It is important to remember what Deaton (2010) said about the difficulty in identifying specific policies, including about openness, that could help poor countries to achieve faster growth: “[E]mpiricists and theorists seem further apart now than at any period in the last quarter century. Yet reintegration [into the world economy] is hardly an option because without it there is no chance of long-term scientific progress” (as quoted by Lin, 2012, p. 31).
Chapter 2

The Role of Government Spending in a Dual Economy Model: A Theoretical Analysis

“The rural child seldom gets even half the town child’s chance of an education....[This] compels many bright children to urbanize if they seek adequate secondary or even primary education, and this is often the first step towards permanent settlement in the city.”


“All developing economic life depends on city economies...”

– Jane Jacobs (1984, p. 132)

Abstract

This chapter extends a two-sector, general-equilibrium model of rural-urban migration to include government spending. Provision of public goods acts as a productivity-enhancing input in private production that results in external economies of scale. This approach is generalized by introducing an unbalanced allocation of public expenditure in rural and urban sectors due to political economy considerations, differential sector output elasticities with respect to government input, and distortionary taxation. The chapter studies the effects of an increase in public spending and taxation on sectoral outputs, factor prices, urban unemployment, and welfare. Of particular concern here is to study the effect of an unbalanced allocation of government spending between rural and urban areas.
2.1. Introduction

The literature on the trends and determinants of rural-urban migration in dual economies is extensive. More than four decades ago, Harris and Todaro (1970) modeled the coexistence of rural migration and urban unemployment by introducing labor market dualism in terms of two main postulates: (1) that a rural-urban wage differential is created by an institutionally mandated minimum wage or other such frictions, and (2) that in making migration decisions, rural workers balance the prospects of high-wage urban employment against the probability of urban unemployment.

Since then, quite a few extensions and modifications of the Harris-Todaro model have been offered in the development literature to address several substantive questions of interest. Khan (1987) discusses the extensions by Bhagwati and Srinivasan (1974), Corden and Findlay (1975), Calvo (1978), Khan (1980), and Neary (1981). Other extensions incorporate the urban informal sector, reflecting the limited formal safety net for the unemployed in developing countries (see the three-sector models by Quibria, 1988; Chandra, 1991; Khan, 1992; Chandra and Khan, 1993; and Gupta, 1993). Hazari and Sgro (1991) extend the original model to include nontraded goods. Datta-Chaudhury and Khan (1984) and Upadhyay (1994) use the Harris-Todaro model to explain educated unemployment prevalent in many developing countries. Krichel and Levine (1999) examine the effect of policies on migration under the assumption that the urban sector displays agglomeration effects. Introducing the necessary and sufficient conditions of wage inequality to the Harris-Todaro model, Temple (2005) shows how wage inequality is reduced through technical progress in agriculture. Rapanos (2007) uses the Harris-Todaro model to analyze environmental taxation where urban sector activity pollutes the rural sector. Chaudhuri (2008) examines the effects of international factor mobility on skilled-
unskilled wage inequality in a small open dual economy using the Harris-Todaro approach. Yabuuchi (2011) examines the effects of outsourcing in an economy with Harris-Todaro type unemployment, and shows that effects on the rural-urban wage differential and urban unemployment depend on whether or not the outsourced factor is produced domestically. In a more recent paper in a dynamic equilibrium framework (the model used here), Mourmouras and Rangazas (2013) show how a “redistributive” urban bias designed to increase urban welfare must take the form of restricting migration to the city. The paper examines how introduction of an import tariff on manufactured goods affects urban unemployment and the environment in a less-developed country. Nakamura (2013) shows that in the Harris-Todaro model framework, when there is no environmental pollution, introduction of an import tariff causes an increase in urban unemployment. And in an empirical study using regional data from Poland, Ghatak et al. (2008) analyze the determinants of migration, incorporating human capital, housing stock, health care, road provision, and other public services into the Harris-Todaro framework.

One of the main features of most of these models is that the urban wage is fixed at a higher level than the rural wage, and this leads to rural-urban migration. Migration equilibrium is achieved through unemployment in the urban sector. Government policies to reduce urban unemployment in these models can end up actually increasing it because of the incentive the policies give rural residents to migrate for better paying jobs. Regarding the urban informal sector, empirical evidence shows that informal employment ranges between 30 and 70 percent of total urban employment (Bhattacharya, 2011, 1993; Banerjee 1983). Regardless of the reasons for such a high share for informal employment, it is likely that the rural-urban wage differential is one of the main driving forces for migration to cities. The provision of urban amenities and services could indeed be a part of the story as well. In any case, empirical studies seem to be in line with the Harris-Todaro model and show that the urban informal sector, rural-urban wage
differential, and prospects of getting absorbed in the urban formal sector after a period of working in the informal sector are important factors in understanding rural-to-urban migration.

Drawing on various recent reviews and articles on dual-economy models (Todaro and Smith, 2012; Lall et. al, 2006; Temple, 2005), this study has added a literature review of theoretical and empirical studies in this area of dual economy models in Appendix 2.1.

None of the papers except Krichel and Levine (1999), who model rural-urban migration in the Harris-Todaro tradition, include a substantive role for the government. Krichel and Levine include such a role, and an associated government budget constraint, in examining the effect of policies on migration under the assumption that the urban sector displays agglomeration effects. The agglomeration effect (or the urban scale economy) considered by Krichel and Levine requires a higher urban employment subsidy than, for instance, that in Bhagwati and Srinivasan (1974), where optimal outcome is possible under equal rural and urban subsidies. In Krichel and Levine (1999), a rise in the employment subsidy necessary to generate the first-best effects can require too high a tax on the output of the rural sector.

This chapter presents a theoretical work linking government spending to rural-urban migration and urban unemployment in a dual-economy framework—a link that previous research in the two-sector, rural-urban migration, or dual-economy tradition has largely ignored. Unlike Krichel and Levine (1999), this chapter models government spending following the public-goods, one-sector-model of productive government services of Barro and Sala-i-Martin (2003). The chapter assumes that government spending contributes to physical and social infrastructure that raises the productivity of private capital and labor. The result is increasing returns to scale in the production function that otherwise displays constant returns in private inputs. To finance spending, government must, however, rely on distortionary taxation that is
assumed to fall on the output of one sector. The model in this chapter therefore also incorporates a government budget constraint explicitly into the dual-economy model in order to analyze the welfare implications of the fiscal policies on relative output, employment, and wage structure.

The welfare analysis reveals that raising the rural share of government spending increases rural output and employment and lowers the migration flow to cities. There is a direct negative impact on urban output as infrastructure spending slows in the urban area. Such a decline, however, is at least partly offset by increased rural demand for urban output, the increase being made possible by higher incomes following greater rural spending by the government. Given that urban centers in most developing countries have a hard time coping with large-scale demand for infrastructure, this public policy can serve as a mechanism to better balance the urban-rural population. Again, the result depends on specific values of the parameters in the equations that represent the welfare analysis presented here. For some realistic values of these parameters, the result does indeed seem to hold (see Section 2.3).

These broad results regarding the rural-urban balance as a result of reorienting government spending come about through micro changes in the rural as well as urban sector. Better transportation in villages helps link rural production with urban marketing, which raises output prices for farmers. Better communications networks that use modern technology also improve marketing efficiency by making information flow faster. Electrification directly assists off-farm production, facilitating rural employment. A stunning example of all this is China, although there are other cases as well. As China was becoming a manufacturing powerhouse in the 1990s, its share in world manufacturing employment reached a high of 15.6 percent. The share gradually declined to 13 percent by 2009, while manufacturing production continued its rapid pace. More importantly, within China, the urban share of manufacturing was a phenomenal
63 percent in 1990. Over time, however, as the government tried to boost the relatively sluggish rural economy by spending on infrastructure and placing restrictions on migration, the employment scenario reversed. By 2009, the rural share in manufacturing employment had risen to 65 percent with the concomitant fall in the urban share\(^3\). This shows that a substantial part of manufacturing might have moved to the rural areas (ILOSTAT, 2013; Banister 2005; Ghose 2008, 2005; BLS 2013).

<table>
<thead>
<tr>
<th>Year</th>
<th>Total (Millions)</th>
<th>Urban (Millions)</th>
<th>Rural (Millions)</th>
<th>Urban Share (%)</th>
<th>Rural Share (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>86.2</td>
<td>53.9</td>
<td>32.3</td>
<td>62.5</td>
<td>37.5</td>
</tr>
<tr>
<td>2005</td>
<td>104.4</td>
<td>44.3</td>
<td>60.1</td>
<td>42.4</td>
<td>57.6</td>
</tr>
<tr>
<td>2008</td>
<td>98.5</td>
<td>34.5</td>
<td>64.0</td>
<td>35.0</td>
<td>65.0</td>
</tr>
<tr>
<td>2009</td>
<td>99.0</td>
<td>34.6</td>
<td>64.4</td>
<td>34.9</td>
<td>65.1</td>
</tr>
</tbody>
</table>

\(^3\) Data do not clearly indicate whether the robust growth of rural employment is due to the rural sector’s attractiveness for new manufacturing firms or to the existing urban firms’ simply relocating to rural areas because of better infrastructure or other incentives. Also not clear is whether the definition of “rural” has changed over time in official statistics. The ambiguity is not resolved by official statistics, for example, in the case of China. As Banister (2005, p. 20) notes, “the breakdown of China’s annual manufacturing employment statistics into the rural classification and the various urban categories is inconsistent with China’s own statistical definitions of urban and rural...” and “Much better data collection and reporting, and much more research, are needed to try to fill in some of the missing information on rural and town manufacturing employment. Reporting is routinely more thorough for city manufacturing units in China.”

However, after fixing discrepancies in Chinese data, Ghose (2008) still shows that the rural share of manufacturing employment has indeed increased from 37 percent to 58 percent between 1990 and 2005. By 2009, according to U.S. Bureau of Labor statistics (International Labor Comparisons, 2013), the rural share in manufacturing employment in China had risen further to 65 percent with the concomitant fall in the urban share. The rural manufacturing employment has thus increased both in absolute terms and relative to urban employment. Since manufacturing productivity has risen faster than farm productivity, China’s rural economy has grown significantly over 1990-2009.

Finally, it is possible that the definitions of rural and urban have changed over time. If, for instance, a firm was operating in a rural area before, that location could now be redefined as urban because it has acquired greater population and more physical amenities. But if so, the rising share of employment in what is still left of rural becomes even more remarkable. As a matter of fact, the data do not make clear whether a rural-urban re-definition has actually occurred.
starts with a full-employment model in which the rural-urban wage differential is exogenously given. The chapter then presents the model in terms of rates of change, relying heavily on Jones (1965), and lays out the basic equations for comparative static analysis. This is followed by an analysis of the stability condition of the system, following the classical contribution of Mayer (1974) and Yabuuchi (1992). The chapter then shows, through comparative static results, the effects of a change in the share of the rural sector in the government budget and a change in the tax rate. This section also analyzes the welfare impact of changes in the allocation and tax policy variables. The chapter then introduces urban unemployment and the Harris-Todaro equilibrium into the two-sector model with government spending. The impact of fiscal policy that changes the share of government expenditure in rural areas is analyzed. The conclusion to the chapter includes suggestions for future research. We have attempted to make one calibration using Cobb-Douglas production functions (like Temple, 2005) and we found that with these results, it is clear that government expenditure in rural areas is more effective in reducing rural-to-urban migration in the Harris-Todaro model than in the full-employment neoclassical model. Sizable urban unemployment gives a stronger incentive for rural labor to stay in agriculture, and also attracts some unemployed labor from the other sector, when public expenditure raises the marginal productivity of labor in rural production. Of course the results are dependent on specific parameter values. We have added these to Appendix 2.5.

2.2. The Model and Assumptions

Consider a general equilibrium model of a small open economy consisting of two sectors, rural and urban, and producing two final goods: an agricultural good (produced by the rural sector), and a manufacturing good (produced by the urban sector). The two goods are traded in world markets where their prices are set. Each sector uses two factors of production—capital and
labor—that are in fixed supply and perfectly mobile across sectors. The production functions in two sectors are given by:

\[ Y_a = G_a F(L_a, K_a) \]
\[ Y_m = G_m H(L_m, K_m) \]

where \( Y_j, L_j, K_j, G_j \) stand for the level of output, employment, capital, and government input of the \( j^{th} \) sector for \( j = a, m \). The subscripts \( a \) and \( m \) stand for rural (agricultural) and urban (manufacturing) sectors, respectively. In each sector, the production function satisfies the standard neoclassical assumptions, such as constant returns to scale in private inputs \( K_j \) and \( L_j \), and positive and diminishing marginal productivity for each factor of production. \( G_j \) represents the contribution to sector \( j \)'s total factor productivity of government spending in that sector. Thus, one can think of government spending on roads, schools, hospitals, security, courts, and other infrastructure as providing a flow of services to producers in each sector that affects the overall sectoral productivity of private factors of production. The government produces this flow of services by appropriating output from the urban manufacturing sector at a constant rate \( \tau \). Thus \( \tau \) acts as the rate of taxation on manufacturing sector output, which is assumed to be the only tax instrument available to the government to fund public services to both the rural and urban sectors. In many developing countries, the rural agriculture sector is not directly taxed, even though governments do raise revenue from it through such methods as pricing boards for agricultural commodities (e.g., the Cocoa Board in Ghana, the Tea Board in India, and Coffee and Cotton boards in Tanzania). In addition, pro-urban monetary, exchange rate, and fiscal policies often result in an implicit tax burden on the rural sector. As discussed in the conclusion...
to this chapter, we would like to incorporate this implicit rural taxation as an extension of the present model in our future research.4

The governmental “technology” for turning units of manufacturing output into a flow of government input into private production in each sector differs across sectors and is given by:

$$G_a = (\mu \tau Y_m)^\delta$$, and

$$G_m = [(1 - \mu) \tau Y_m]^\nu$$, (2.3)

(2.4)

where the allocation of government spending across sectors is given by the ratios $\mu$ and $1-\mu$, respectively. In developing countries, $\mu$ is exogenously set by the government, which typically represents urban interests. This urban bias in government expenditure causes the rural share, $\mu$, to be much lower than the urban share, $1-\mu$. Thus, the choice of $\mu$ in the model reflects sectoral bias because of the political economy element of the government budget.

In other respects, the model here follows the standard literature. Specifically, perfect capital mobility and competitive equilibrium ensure the equality of rental rates of capital in the two sectors equal to the value of their respective marginal products. Thus we have,

$$G_a F_K = r \text{ or, } G_a f(k_a) = r$$, and

$$G_m H_K = r \text{ or, } p(1-\tau) G_m h(k_m) = r$$, (2.5)

(2.6)

where $r$ stands for the rental rate of capital; $p$ stands for the relative price of the manufacturing good in the world economy; $k_j$ is the capital-labor ratio in the $j^{th}$ sector; $f$ is $F/L$; $h$ is $H/L$;

---

4 Another extension could examine the implications of the allocation of foreign aid, which contributes substantially to development spending in many low-income countries.
In both rural and urban sectors, marginal productivity pricing determines the labor allocation. But in the rural sector, the real wage is flexible enough to ensure full employment of labor. Thus one gets:

$$G_a F_L = w_a \quad \text{or} \quad G_a[f - k_a f'(k_a)] = w_a,$$

and

$$p(1 - \tau)G_a H_L = w_m \quad \text{or} \quad p(1 - \tau)G_a[h - k_m h'(k_m)] = w_m,$$

where $F_L$ and $H_L$ are partial derivatives with respect to $L$, and $w_a$ and $w_m$ are the wages in the rural and urban sectors, respectively.

Following Johnson and Meiszkowsky (1970), Jones (1971), and Magee (1973, 1976), it is assumed that labor migrates from the rural to the urban sector because of an exogenously given proportional distortion in the labor market,

$$w_m = \alpha w_a,$$

where $w_m$, the urban wage is an institutionally set minimum wage, $\alpha$ is a given policy parameter, and $\alpha > 1$. $\alpha$ can also be explained as the efficiency wage factor. In many developing countries, this distortion reflects government policies, largely influenced by urban elites and trade unions. Thus, resource allocations between sectors are given by:

$$K_a + K_m = K$$

$$L_a + L_m = L.$$
The model formulation thus far suggests that, given the world price $p$ and exogenous values of $K$, $L$, $\tau$, $\mu$, and $\alpha$, we have 11 equations for solving the equilibrium values of 11 unknowns: $Y_a$, $Y_m$, $K_a$, $K_m$, $L_a$, $L_m$, $G_a$, $G_m$, $r$, $w_a$, and $w_m$.

### 2.3. Workings of the Model: Analysis in Terms of Equations of Change

The above system of equations can be simplified into a smaller set of subsystems. This will help derive the equilibrium condition, in terms of the given technology and other parameters, and the capital-labor ratios. Following the rate-of-change methodology pioneered by Jones (1965), we differentiate equations 2.1 and 2.2, and use equations 2.3 and 2.4 to get the following:

\[
\hat{Y}_a = \delta \hat{Y}_m + \theta_{\tau_a} \hat{L}_a + \theta_{K_a} \hat{K}_a + \delta(\hat{\mu} + \hat{\tau})
\]  
(2.12)

\[
\hat{Y}_m = \nu \hat{Y}_a + \theta_{\tau_m} \hat{L}_m + \theta_{K_m} \hat{K}_m + \nu \left( \hat{\tau} - \frac{\mu}{1 - \mu} \hat{\mu} \right),
\]  
(2.13)

where $\hat{\cdot}$ denotes the relative change of the variable (e.g. $\hat{Y}_j = dY_j / Y_j$). Excluding the effect of public expenditures, $\theta_{ij}$ gives the share of factor $i$ in sector $j$, i.e., $\theta_{\tau_a} = \frac{w_a L_a}{Y_a}$, $\theta_{K_a} = \frac{r K_a}{Y_a}$, $\theta_{\tau_m} = \frac{w_m L_m}{p(1-\tau)Y_m}$, and $\theta_{K_m} = \frac{r K_m}{p(1-\tau)Y_m}$.

Since government spending, which is external to each individual firm, is the source of increasing returns to scale in the model, the average cost pricing holds for each industry. Thus one obtains $w_a L_a + r K_a = Y_a$, and $w_m L_m + r K_m = p(1-\tau)Y_m$. Using these conditions, the following is obtained:

\[
\theta_{\tau_a} \hat{w}_a + \theta_{K_a} \hat{r} + \theta_{L_a} \hat{L}_a + \theta_{K_a} \hat{K}_a = \hat{Y}_a
\]  
(2.14)

\[
\theta_{\tau_m} \hat{w}_m + \theta_{K_m} \hat{r} + \theta_{L_m} \hat{L}_m + \theta_{K_m} \hat{K}_m = \hat{Y}_m + \hat{p} - \frac{\tau}{1 - \tau} \hat{\tau}.
\]  
(2.15)
The assumption of constant returns to scale in private inputs gives us \( \theta_i = 1 - \theta_k \), \( i = a, m \).

Substituting equations 2.14 and 2.15 into 2.12 and 2.13, the following is obtained:

\[
\theta_{t_a} \hat{w}_m + \theta_{t_k} \hat{r} = \nu \hat{Y}_m + \hat{\rho} - \frac{\tau}{1-\tau} \hat{r} + \nu \left( \hat{r} - \frac{\mu}{1-\mu} \hat{\mu} \right) \tag{2.16}
\]

\[
\theta_{t_a} \hat{w}_a + \theta_{t_k} \hat{r} = \delta \hat{Y}_m + \delta (\hat{\mu} + \hat{r}) . \tag{2.17}
\]

Next, the elasticity of substitution in the two sectors is defined as

\[
\sigma_a = (\hat{K}_a - \hat{L}_a) / (\hat{w}_a - \hat{r}) \tag{2.18}
\]

\[
\sigma_m = (\hat{K}_m - \hat{L}_m) / (\hat{w}_m - \hat{r}) , \tag{2.19}
\]

where \( \sigma_a \) and \( \sigma_m \) are both positive. Substituting equations 2.18 and 2.19 into 2.12 and 2.13, the following is obtained:

\[
\hat{L}_m = (1 - \nu) \hat{Y}_m - \sigma_m \theta_{t_m} (\hat{w}_m - \hat{r}) - \nu \hat{r} + \nu \frac{\mu}{1-\mu} \hat{\mu} \tag{2.20}
\]

\[
\hat{K}_m = (1 - \nu) \hat{Y}_m + \sigma_m \theta_{t_m} (\hat{w}_m - \hat{r}) - \nu \hat{r} + \nu \frac{\mu}{1-\mu} \hat{\mu} \tag{2.21}
\]

\[
\hat{L}_a = \hat{Y}_a - \delta \hat{Y}_m - \sigma_a \theta_{t_a} (\hat{w}_a - \hat{r}) - \delta (\hat{\mu} + \hat{r}) \tag{2.22}
\]

\[
\hat{K}_a = \hat{Y}_a - \delta \hat{Y}_m + \sigma_a \theta_{t_a} (\hat{w}_a - \hat{r}) - \delta (\hat{\mu} + \hat{r}) . \tag{2.23}
\]

Differentiating equations 2.10 and 2.11, the following is obtained:

\[
\pi_{t_a} \hat{L}_a + \pi_{t_m} \hat{L}_m = \hat{L} \tag{2.24}
\]

\[
\pi_{k_a} \hat{K}_a + \pi_{k_m} \hat{K}_m = \hat{K} , \tag{2.25}
\]

where \( \pi_{ij} \) is the share of the endowment of factor \( i \) used in industry \( j \), (e.g., \( \pi_{t_a} = L_a / L \)).

Substituting equations 2.20–2.23 into 2.24–2.25, the following is obtained:
\[ \pi_{L_a} \dot{Y}_a + [\pi_{L_a} (1-\nu) - \pi_{L_a} \delta] \dot{Y}_m \]
\[ = \pi_{L_a} \theta_{K_a} \sigma_a (\dot{w}_a - \dot{r}) + \pi_{L_a} \sigma_m \theta_{K_a} (\dot{w}_m - \dot{r}) + \left( \pi_{L_a} \delta - \frac{\pi_{L_a} \nu \mu}{1 - \mu} \right) \dot{\mu} + \left( \pi_{L_a} \delta + \pi_{L_a} \nu \right) \hat{\tau} \]
\[ (2.26) \]

\[ \pi_{K_a} \dot{Y}_a + (\pi_{K_a} (1-\nu) - \pi_{K_a} \delta) \dot{Y}_m \]
\[ = - \pi_{K_a} \theta_{K_a} \sigma_a (\dot{w}_a - \dot{r}) - \pi_{K_a} \sigma_m \theta_{K_a} (\dot{w}_m - \dot{r}) + \left( \pi_{K_a} \delta - \frac{\pi_{K_a} \nu \mu}{1 - \mu} \right) \dot{\mu} + \left( \pi_{K_a} \delta + \pi_{K_a} \nu \right) \hat{\tau} \]
\[ (2.27) \]

From equations 2.16 and 2.17, one obtains
\[ \dot{w}_a - \dot{r} = \frac{1}{\theta} \left[ (\delta - \nu) \dot{Y}_m + \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \dot{\mu} + \left( \delta - \nu + \frac{\tau}{1 - \tau} \right) \hat{\tau} - \dot{p} - \theta \delta \right] . \]
\[ (2.28) \]

Substituting equation 2.28 into 2.26 and 2.27 and simplifying, one obtains
\[ \pi_{L_a} \dot{Y}_a + [(1-\nu)\pi_{L_a} - \delta \pi_{L_a} - \Delta_1 (\delta - \nu)] \dot{Y}_m \]
\[ = \left( \pi_{L_a} \delta - \frac{\pi_{L_a} \nu \mu}{1 - \mu} + \Delta_1 \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right) \dot{\mu} + \left( \pi_{L_a} \delta + \pi_{L_a} \nu + \Delta_1 \left( \delta - \nu + \frac{\tau}{1 - \tau} \right) \right) \hat{\tau} \]
\[ - \Delta_1 \dot{p} + \left( \pi_{L_a} \sigma_m \theta_{K_a} - \Delta_1 \theta_{L_a} \right) \dot{\alpha} \]
\[ (2.29) \]

\[ \pi_{K_a} \dot{Y}_a + (\pi_{K_a} (1-\nu) - \pi_{K_a} \delta + \Delta_2 (\delta - \nu)) \dot{Y}_m \]
\[ = \left( \pi_{K_a} \delta - \frac{\pi_{K_a} \nu \mu}{1 - \mu} - \Delta_2 \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right) \dot{\mu} + \left( \pi_{K_a} \delta + \pi_{K_a} \nu - \Delta_2 \left( \delta - \nu + \frac{\tau}{1 - \tau} \right) \right) \hat{\tau} , \]
\[ + \Delta_2 \dot{p} - \left( \pi_{K_a} \sigma_m \theta_{L_a} - \Delta_2 \theta_{L_a} \right) \dot{\alpha} \]
\[ (2.30) \]

where \( \Delta_1 = \frac{1}{\theta} (\pi_{L_a} \theta_{K_a} \sigma_a + \pi_{L_a} \sigma_m \theta_{K_a}) \)
\( \Delta_2 = \frac{1}{\theta} (\sigma_a \pi_{K_a} \theta_{L_a} + \sigma_m \pi_{K_a} \theta_{L_a}) \), and \( |\theta| = (\theta_{L_a} - \theta_{L_a}) \)

Equations 2.29 and 2.30 can be expressed as:
\[
\begin{bmatrix}
\Omega_{L_a} & \Omega_{K_a} \\
\Omega_{K_a} & \Omega_{K_a}
\end{bmatrix}
\begin{bmatrix}
\hat{y}_a \\
\hat{y}_m
\end{bmatrix}
= \begin{bmatrix}
A_1 \phi + A_2 \hat{x} + A_3 \hat{p} + A_4 \hat{\alpha} \\
B_1 \phi + B_2 \hat{x} + B_3 \hat{p} + B_4 \hat{\alpha}
\end{bmatrix},
\]

(2.31)

where,

\[
\Omega_{L_a} = \pi_{L_a}
\]
\[
\Omega_{K_a} = (1 - \nu)\pi_{L_a} - \nu \pi_{K_a} - \Delta_1 (\delta - \nu)
\]
\[
\Omega_{K_a} = \pi_{K_a}
\]
\[
\Omega_{K_a} = \pi_{K_a} (1 - \nu) - \pi_{K_a} \delta + \Delta_2 (\delta - \nu)
\]
\[
A_1 = \pi_{L_a} \delta - \frac{\pi_{K_a} \nu \mu}{1 - \mu} + \Delta_1 \left(\delta + \frac{\mu \nu}{1 - \mu}\right)
\]
\[
A_2 = \pi_{L_a} \delta + \pi_{K_a} \nu + \Delta_1 \left(\delta - \nu + \frac{\tau}{1 - \tau}\right)
\]
\[
A_3 = -\Delta_2
\]
\[
A_4 = \pi_{L_a} \alpha \sigma_m \theta_{K_a} - \Delta_1 \theta_{L_a}
\]
\[
B_1 = \pi_{K_a} \delta - \frac{\pi_{K_a} \nu \mu}{1 - \mu} - \Delta_2 \left(\delta + \frac{\mu \nu}{1 - \mu}\right)
\]
\[
B_2 = \pi_{K_a} \delta + \pi_{K_a} \nu - \Delta_2 \left(\delta - \nu + \frac{\tau}{1 - \tau}\right)
\]
\[
B_3 = \Delta_2
\]
\[
B_4 = -\left(\pi_{K_a} \sigma_m \theta_{L_a} - \Delta_2 \theta_{L_a}\right)
\]
\[
|\Omega| = \left(\pi_{L_a} \pi_{K_a} - \pi_{K_a} \pi_{L_a}\right) (1 - \nu) + \left(\pi_{K_a} \Delta_1 + \pi_{L_a} \Delta_2\right) (\delta - \nu).
\]

The solution of equation 2.31 will give the effects of a change in sectoral allocation of public spending on output. These output responses in turn can be used to evaluate the effects on employment and relative wages. The output-employment responses obviously depend on the signs of \(\Omega_j\)s, \(A_j\)s, and \(B_j\)s of equation 2.31. As is customary in two-sector models, it is assumed that the urban sector is capital-intensive and the rural sector is labor-intensive, which helps determine some of the signs as follows: \(\Omega_{L_a} > 0, \Omega_{K_a} > 0, A_3 < 0, B_3 > 0,\) and \(B_4 < 0.\)
However, one needs to make further assumptions on the relative magnitudes of $\delta$, $\mu$, and $\tau$ to determine the signs of $\Omega_{\kappa_u}$, $\Omega_{\kappa_a}$, $A_1$, $A_2$, $A_4$, $B_1$, and $B_2$.

### 2.4. Adjustment Process and Stability Analysis

In the Harris-Todaro literature, Khan (1980), Neary (1981), Khan and Naqvi (1983), Funatsu (1988), and Yabuuchi (1992, 1998) have discussed stability conditions of a dynamic generalization of the Harris-Todaro model, as well as the relationship between stability conditions and the model’s comparative static properties, invoking Samuelson’s correspondence principle. The main result of this literature, the Khan-Neary condition, states that the Harris-Todaro equilibrium is locally stable if and only if, at that equilibrium, the urban sector is more capital-intensive in employment-adjusted terms than the rural sector. This section applies this approach to our model and derives an equivalent proposition to the Khan-Neary condition.

Following the dynamic adjustment process used by Mayer (1974), Chang (1981), and Yabuuchi (1992, 1998), one can specify the dynamic adjustment mechanism of this system as follows. For the goods market, a Marshallian adjustment process is assumed where quantities adjust when demand price differs from the supply price. In the factor market, assuming fixed endowments of $K$ and $L$, Walrasian adjustment mechanism is assumed where factor returns need to be adjusted. Also, it is assumed that the urban wage adjusts following the efficiency wage mechanism (equation 2.36). Thus one can write:

\[
\dot{Y}_a = d_1 (1 - a_{\kappa_u} w_a - a_{K_a} r) \tag{2.32}
\]

\[
\dot{Y}_m = d_2 \left( p(1 - \tau) - a_{\kappa_a} w_m - a_{K_a} r \right) \tag{2.33}
\]

\[
\dot{w}_a = d_3 (a_{\kappa_a} Y_a + a_{\kappa_a} Y_m - L) \tag{2.34}
\]
\[ \dot{r} = d_4 (a_{i_4} Y_a + a_{i_5} Y_m - K) \]  
(2.35)

\[ \dot{w}_m = d_5 (\alpha w - w_m) \]  
(2.36)

where \( a_{i_j} \) is amount of \( i^{th} \) factor used to produce one unit of output \( j \) (e.g. \( a_{i_4} = \frac{L_u}{Y_a} \)); the positive coefficient \( d_j \) measures the speed of adjustment; and \( \dot{Y} \) is the time derivative of \( Y \).

The necessary condition for local stability of the system, following the Routh-Hurwitz theorem, is that the sign of the determinant of the Jacobian (\( J \)) of the system of equations 2.32–2.36 is \((-1)^j\), where \( j \) is the number of rows (= columns) in the Jacobian matrix. In this case, \( \text{sign}\, |J| = (-1)^j = (-1)^5 < 0 \). It can be shown that that \( |J|<0 \) implies \( |\Omega|>0 \). Since,

\[
|J| = \begin{vmatrix}
-d_4 a_{i_4} & -d_5 a_{i_5} & 0 & d_1 \delta / Y_a & 0 \\
0 & -d_4 a_{i_4} & 0 & d_1 \delta / Y_a & 0 \\
0 & 0 & -d_2 p(1-\tau) \nu / Y_a & -a_{i_5} \\
0 & 0 & 0 & -a_{i_5} \\
d_1 a_{i_4} & d_2 a_{i_5} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -d_4
\end{vmatrix}
\]

As shown in Appendix 2.2, the determinant of the Jacobian matrix is given by

\[
|J| = \frac{-d_2 d_3 d_4 d_5 KLp(1-\tau) |\theta|}{Y_a Y_m \rho}, \text{ and}
\]

\[
|\Omega| = \left( \pi_{i_4} \pi_{k_5} - \pi_{k_4} \pi_{i_5} \right)(1-\nu) + \left( \pi_{k_5} \Delta_1 + \pi_{i_5} \Delta_2 \right)(\delta - \nu), \text{ which can be expressed as}
\]

\[
|\Omega| = \frac{L_u L_m}{KL} (k_m - k_a)(1-\nu) + \left( \pi_{k_5} \Delta_1 + \pi_{i_5} \Delta_2 \right)(\delta - \nu).
\]

Thus, \((-1)^j |J| < 0, j = 5\), when \( |\Omega| > 0 \). The following assumption can now be made:

**Assumption 1:** We assume that the urban sector is more capital-intensive than the rural sector, both in physical and value terms, i.e., \( k_m > k_a \) and \( \theta_{k_m} / \theta_{L_u} > \theta_{k_a} / \theta_{L_u} \).
We can express $\Omega$ as
$$\Omega = \left( \pi_{L_a} \pi_{K_a} - \pi_{K_a} \pi_{L_a} \right)(1-\nu) + \left( \pi_{K_a} \Delta_1 + \pi_{L_a} \Delta_2 \right)(\delta - \nu).$$
This helps to derive the cases listed below under alternative assumptions of efficiency of public spending, when the determinant $|\Omega|$ is positive. The sign of $|\Omega|$ clearly depends on $\delta$ and $\nu$, which are “technological” parameters in the model. Let us consider the following cases:

I. $\delta \approx 0$, $\nu \approx 0$, $|\Omega| > 0$

$$\Omega = \left( \pi_{L_a} \pi_{K_a} - \pi_{K_a} \pi_{L_a} \right) = \frac{L_a L_m}{KL} (k_m - k_a) > 0.$$

This implies that the stability condition in this model reduces to a stability condition in a traditional two-sector model.

II. $\delta = 1$, $\nu = 0$, $|\Omega| > 0$

$$\Omega = \frac{L_a L_m}{KL} (k_m - k_a) + \left( \pi_{K_a} \Delta_1 + \pi_{L_a} \Delta_2 \right) > 0.$$

This shows that the stability condition holds in this case when $(k_m - k_a) > 0$ and $\Delta_1 > 0, \Delta_2 > 0$.

III. $1 > \delta \approx \nu > 0$, $|\Omega| > 0$

$$\Omega = \left( \pi_{L_a} \pi_{K_a} - \pi_{K_a} \pi_{L_a} \right)(1-\nu) = \frac{L_a L_m}{KL} (k_m - k_a)(1-\nu) > 0.$$

Here, for $0 < \nu < 1$, the stability condition holds when $(k_m - k_a) > 0$.

IV. $1 > \delta > \nu > 0$

$$\Omega = \frac{L_a L_m}{KL} (k_m - k_a)(1-\nu) + \left( \pi_{K_a} \Delta_1 + \pi_{L_a} \Delta_2 \right)(\delta - \nu) > 0.$$

Note that for $\delta = 0, \nu = 1$, $|\Omega| < 0$, hence the stability condition is not satisfied. We also omit the assumption $1 > \nu > \delta > 0$, where $|\Omega| > 0$ only if $\left( \pi_{L_a} - \pi_{K_a} \right)(1-\nu)$ is greater than $\left( \Delta_1 \pi_{K_a} + \Delta_2 \pi_{L_a} \right)(\delta - \nu)$. 
The above results can thus be summarized in terms of the following proposition:

**Proposition 1**: If the urban sector is more capital-intensive both in physical and value terms than the rural sector, and the elasticity \( \delta \) of output with respect to public spending in the rural sector is equal to or higher than the elasticity \( \nu \) in the urban sector \( (1 > \delta \geq \nu \geq 0) \), an equilibrium of the adjustment process is locally stable.

### 2.5. Comparative Statics

This section determines (1) how our endogenous variables (in particular, outputs, employment, and factor rewards) respond to a higher budgetary allocation to the rural sector, and (2) how national economic welfare changes as a result of higher rural expenditure. For simplicity, the comparative static analyses will mainly focus on cases I, II, and III (i.e., under alternative assumptions of efficiency of public services as (i) \( \delta = \nu = 0 \), (ii) \( \delta = 1, \ nu = 0 \) and (iii) \( 1 > \delta \approx \nu > 0 \)).

#### 2.5.1. An Increase in the Allocation of Government Spending (\( \mu \)) to the Rural Sector

Assume that the price \( (p) \), efficiency wage ratio \( (\alpha) \), and tax rate \( (\tau) \) remain unchanged, and only government rural spending as a fraction of total spending \( (\mu) \) changes. In equation 2.31, setting \( \hat{p} = \hat{\alpha} = \hat{\tau} = 0 \), and solving for \( \hat{Y}_o \) and \( \hat{Y}_m \), the following is obtained:

\[
\hat{Y}_m / \hat{\mu} = -\frac{1}{\Omega} \left[ (\pi_{K_o} - \pi_{K_m}) \frac{\nu \mu}{1 - \mu} + (\pi_{K_o} \Delta_1 + \pi_{K_o} \Delta_2) \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right]
\]  

\[
\hat{Y}_d / \hat{\mu} = \frac{1}{\Omega} \left[ (\pi_{I_o} - \pi_{I_m}) (1 - \nu - \mu) \frac{\delta}{1 - \mu} \right.
\]

\[ + \frac{1}{\Omega} \frac{\Delta_1}{1 - \mu} \left[ \pi_{K_o} \left( \delta (1 - \mu) + \mu \nu \right) - \delta \nu \right] \]

\[ + \frac{1}{\Omega} \frac{\Delta_2}{1 - \mu} \left[ \pi_{I_o} \left( \delta (1 - \mu) + \mu \nu \right) - \delta \nu \right]
\]  

(2.37)

(2.38)
These effects shall be analyzed under the following alternative assumptions of efficiency of public spending:

I. \( \delta = \nu = 0 \), i.e., public spending does not generate any productive services in either sector:

\[
\hat{Y}_a / \hat{\mu} = 0 \quad \text{and} \quad \hat{Y}_m / \hat{\mu} = 0.
\]

Thus, an increased allocation to the rural sector does not affect the outputs.

II. \( \delta = 1, \nu = 0 \), i.e., public spending is highly effective in the rural sector due to effective rural development programs but not productive in the urban sector:

\[
\hat{Y}_m / \hat{\mu} = -\left( \pi_{K_a} \Delta_1 + \pi_{L_o} \Delta_2 \right) / |\Omega| < 0
\]

\[
\hat{Y}_a / \hat{\mu} = \left( \pi_{L_o} \pi_{K_a} - \pi_{L_o} \pi_{K_r} \right) / |\Omega| + \left( \Delta_1 \pi_{K_a} + \Delta_2 \pi_{L_o} \right) / |\Omega| > 0.
\]

III. \( 1 > \delta \approx \nu > 0 \), i.e., public spending is equally effective both in the urban and rural sectors:

\[
\hat{Y}_m / \hat{\mu} = -\frac{\nu}{|\Omega| (1 - \mu)} \left[ \left( \pi_{L_o} \pi_{K_a} - \pi_{L_o} \pi_{K_r} \right) \mu + \left( \pi_{K_a} \Delta_1 + \pi_{L_o} \Delta_2 \right) \right] < 0
\]

\[
\hat{Y}_a / \hat{\mu} = \frac{1}{|\Omega|} \left( \pi_{K_a} - \nu \right) \left( \pi_{L_o} - \mu + \Delta_1 \right) - \left( \pi_{L_o} - \nu \right) \left( \pi_{K_r} - \mu - \Delta_2 \right) > 0.
\]

when \( \pi_{K_a} > \nu, \pi_{L_o} > \mu \)

IV. \( 1 > \delta > \nu > 0 \), i.e., public services have a higher elasticity of returns to scale in the rural sector than in the urban sector:

\[
\hat{Y}_m / \hat{\mu} = -\frac{1}{|\Omega|} \left( \pi_{L_o} \pi_{K_a} - \pi_{L_o} \pi_{K_r} \right) \frac{\nu \mu}{1 - \mu} + \left( \pi_{K_a} \Delta_1 + \pi_{L_o} \Delta_2 \right) \left( \delta + \frac{\mu \nu}{1 - \mu} \right) < 0
\]
From the above, it is clear that since taxation on urban output is the only source of public spending in both sectors, the effect of higher rural spending on growth of urban output is negative in alternative scenarios. Rural output rises under specific parameter values of \( \delta, \mu, \) and \( \nu \). We find that \( \dot{Y}_a / \dot{\mu} > 0 \) when (i) \( \delta = 1, \nu = 0 \), (ii) \( 1 > \delta \approx \nu > 0 \), and (iii) \( 0 > \delta > \nu > 1 \) and the expansionary effect of the greater rural allocation outweighs the contractionary effect of reduction in the tax base of public spending, i.e., shrinking of the urban output. Thus we can state the following proposition:

**Proposition 2:** Under our assumption of higher capital intensity for the urban sector, an increase in the allocation of public spending to the rural sector leads to a decline in urban output and to an increase in rural output, provided public services are almost equally productive (\( 1 \geq \delta \approx \nu > 0 \)) in the two sectors. When \( 1 > \delta > \nu > 0 \), however, urban output falls, whereas rural output rises only when the expansionary effect of rising rural allocation outweighs the contractionary effect due to the shrinking of the tax base.

These output responses are now put to use to derive the effects on sectoral factor prices and sectoral employment. From equations 2.16–2.17, 2.20–2.23, and equation 2.28, when the price \( (p) \), efficiency wage ratio \( (\alpha) \), and tax rate \( (\tau) \) remain unchanged and only government rural spending as a fraction of total spending \( (\mu) \) changes, setting \( \dot{p} = \dot{\alpha} = \dot{\tau} = 0 \), the equations below are obtained.
From equations 2.16 and 2.17, and using \( \hat{w}_m = \hat{w}_a + \hat{\alpha} \) from (2.9), we can solve for

\[
\frac{\hat{r}}{\hat{\mu}} = \frac{1}{\theta} \left[ (\nu \theta_{a_a} - \delta \theta_{a_m}) \hat{Y}_m / \hat{\mu} - \left( \delta \theta_{a_m} + \theta_{k_a} \frac{\mu \nu}{1 - \mu} \right) \right] \quad (2.38A)
\]

\[
\hat{w}_a / \hat{\mu} = \frac{1}{\theta} \left[ (\delta \theta_{k_a} - \nu \theta_{k_a}) \hat{Y}_m / \hat{\mu} + \left( \delta \theta_{k_a} + \theta_{k_a} \frac{\mu \nu}{1 - \mu} \right) \right]. \quad (2.38B)
\]

Subtracting equation 2.38A from 2.38B, we can then derive

\[
(\hat{w}_a - \hat{r}) / \hat{\mu} = \frac{1}{\theta} \left[ (\delta - \nu) \hat{Y}_m / \hat{\mu} + \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right]. \quad (2.38C)
\]

Since \( \hat{\alpha} = 0 \), we can write \( \hat{w}_a = \hat{w}_o \), or \( \hat{w}_a - \hat{r} = \hat{w}_o - \hat{r} \) and hence,

\[
\hat{w}_m / \hat{\mu} = \frac{1}{\theta} \left[ (\delta \theta_{a_m} - \nu \theta_{a_m}) \hat{Y}_m / \hat{\mu} + \left( \delta \theta_{a_m} + \theta_{k_a} \frac{\mu \nu}{1 - \mu} \right) \right] \quad (2.38D)
\]

\[
(\hat{w}_m - \hat{r}) / \hat{\mu} = \frac{1}{\theta} \left[ (\delta - \nu) \hat{Y}_m / \hat{\mu} + \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right]. \quad (2.38E)
\]

Similarly, from equations 2.22 and 2.20 we can derive

\[
\hat{L}_a / \hat{\mu} = \hat{Y}_a / \hat{\mu} - \delta \hat{Y}_m / \hat{\mu} - \sigma_a \theta_{k_a} (\hat{w}_a - \hat{r}) / \hat{\mu} - \delta \quad (2.38E)
\]

\[
\hat{L}_m / \hat{\mu} = (1 - \nu) \hat{Y}_m / \hat{\mu} - \sigma_m \theta_{k_m} (\hat{w}_m - \hat{r}) / \hat{\mu} + \frac{\nu \mu}{1 - \mu}. \quad (2.38F)
\]

Under alternative scenarios the following is obtained for the effects of changes in \( \mu \) on factor prices and employment:

1. \( \delta = \nu = 0 \), i.e., public spending does not generate productive services both in the rural and urban sectors: \( \hat{w}_a / \hat{\mu} = 0, \hat{r} / \hat{\mu} = 0, (\hat{w}_a - \hat{r}) / \hat{\mu} = 0, \hat{w}_m / \hat{\mu} = 0, \hat{L}_a / \hat{\mu} = 0, \hat{L}_m / \hat{\mu} = 0 \). Thus, an increased allocation to the rural sector has no impact on wages, rental, and employment in either the rural or urban sectors.
II. When $\delta=1$, $\nu=0$, i.e., public spending is highly effective in the rural sector, the following is obtained:

$$\hat{w}_u / \hat{\mu} = \frac{1}{|\theta|} \hat{\theta}_K (\pi_{t_u} - \pi_{K_u}) > 0, \quad \hat{r} / \hat{\mu} = -\frac{\hat{\theta}_m}{|\theta|} (\pi_{t_u} - \pi_{K_u}) < 0,$$

$$(\hat{w}_u - \hat{r}) / \hat{\mu} = \left(\frac{\pi_{t_u} - \pi_{K_u}}{|\theta|}\right) > 0,$$

$$\hat{L}_u / \hat{\mu} = \frac{\pi_{t_u}}{|\theta|} \left[\sigma_e \theta_{K_u} \pi_{K_u} + \sigma_m \theta_{K_u} \pi_{K_u} + \Delta_\theta |\theta|\right] > 0$$

$$\hat{L}_m / \hat{\mu} = -\frac{\pi_{K_u}}{|\theta|} \left[\Delta_\theta |\theta| + \pi_{t_u} \sigma_e \theta_{K_u} + \pi_{t_u} \sigma_m \theta_{K_u}\right] < 0.$$ 

Thus, an increased allocation to the rural sector will increase wages and employment in the rural sector. Given a constant wage margin ($\alpha$), urban wages will rise, and employment will fall. This reduces the out-migration from rural areas.

III. $1 > \delta \approx \nu > 0$: When public spending is equally effective in the urban and rural sectors, the following is obtained:

$$(\hat{w}_u - \hat{r}) / \hat{\mu} = \frac{\nu}{|\theta|(1 - \mu)} > 0, \quad \hat{r} / \hat{\mu} = \frac{\nu}{|\theta|} \left[\theta |\hat{\theta}_m / \hat{\mu} - \left(\theta_{t_u} + \theta_{K_u} \frac{\mu}{1 - \mu}\right)\right] < 0,$$ since $\hat{\theta}_m / \hat{\mu} < 0$

$$\hat{L}_u / \hat{\mu} = \hat{Y}_u / \hat{\mu} - \nu \hat{Y}_m / \hat{\mu} - \frac{\nu \sigma_e}{|\theta|(1 - \mu)} > 0,$$

$$\hat{L}_m / \hat{\mu} = (1 - \nu) \hat{Y}_m / \hat{\mu} - \sigma_m \theta_{K_u} (\hat{w}_u - \hat{r}) / \hat{\mu} + \frac{\nu}{1 - \mu} < 0.$$ 

In the case where government services are equally productive in rural and urban sectors, i.e., $\delta \approx \nu > 0$, $(\hat{w}_u - \hat{r}) / \hat{\mu} > 0$, and $(\hat{w}_m - \hat{r}) / \hat{\mu} > 0$ for any given $\alpha$, $\hat{r} / \hat{\mu} < 0$, $\hat{w}_u / \hat{\mu} > 0$ when
\[
\frac{\nu}{\theta(1-\mu)} > \left[ \hat{Y}_m / \hat{\mu} \right], \ \hat{L}_a / \hat{\mu} > 0, \text{ and } \hat{L}_m / \hat{\mu} < 0. \text{ So urban employment falls for any given } \alpha.
\]

These results can be summarized in terms of the following proposition:

**Proposition 3:** When output elasticity with respect to public spending is high in the rural sector \((\delta = 1)\) or equal in both the rural and urban sectors \((0 < \delta = \nu < 1)\), an increased allocation of development spending to the rural sector will enhance rural employment and reduce urban employment. This will reduce the rural-urban imbalance.

### 2.5.2. An Increase in the Tax Rate \((\tau)\) on Urban Output

We now assume that the price of the urban good \((p)\), the efficiency wage factor \((\alpha)\), and the expenditure allocation \((\mu)\) remain unchanged, and only the output tax rate \((\tau)\) changes. In the equation system (2.31), setting \(\hat{p} = \hat{\alpha} = \hat{\mu} = 0\), and solving for \(\hat{Y}_a\) and \(\hat{Y}_m\), one obtains

\[
\hat{Y}_a / \hat{\tau} = \frac{1}{|\Omega|} \left( \Omega_{k_a} A_2 - \Omega_{\tau a} B_2 \right), \quad \hat{Y}_m / \hat{\tau} = \frac{1}{|\Omega|} \left( -\Omega_{k_a} A_2 + \Omega_{\tau a} B_2 \right).
\]

Simplifying the above expressions, the following is obtained:

\[
\hat{Y}_a / \hat{\tau} = \frac{1}{|\Omega|} \left( \pi_{k_a} (1-\nu) - \pi_{k_a} \delta + \Delta_2 (\delta - \nu) \right) \left( \pi_{\tau a} \delta + \alpha \pi_{l a} \nu + \Delta_1 \left( \delta - \nu + \frac{\tau}{1-\tau} \right) \right),
\]

\[
- \frac{1}{|\Omega|} \left( (1-\nu) \alpha \pi_{l a} - \delta \pi_{\tau a} - \Delta_1 (\delta - \nu) \right) \left( \pi_{\tau a} \delta + \pi_{k_a} \nu - \Delta_2 \left( \delta - \nu + \frac{\tau}{1-\tau} \right) \right). \tag{2.39}
\]

\[
\hat{Y}_m / \hat{\tau} = \frac{1}{|\Omega|} \left[ \pi_{l a} \Delta_2 \left( \delta - \nu + \frac{\tau}{1-\tau} \right) + \pi_{k_a} \Delta_1 \left( \delta - \nu + \frac{\tau}{1-\tau} \right) + \nu \pi_{k_a} - \nu \pi_{l a} \right]. \tag{2.40}
\]

The effects can then be analyzed under the following alternative assumptions regarding the efficiency of public spending:

I. \(\delta = \nu = 0\), i.e., public spending does not generate any productive services in either sector:
II. $\delta = 1, \nu = 0$, i.e., public spending is highly effective in the rural sector due to effective rural development programs, but it is not productive in the urban sector:

$$\hat{Y}_a / \hat{\tau} = \frac{1}{|\Omega|} \left[ (\pi_{K_a} - \pi_{K_u}) + \left( \pi_{L_a} - \pi_{L_u} + \Delta_l \right) \left( \pi_{K_u} - \Delta_l \right) \right] > 0$$

$$\hat{Y}_m / \hat{\tau} = -\frac{1}{|\Omega|} \left[ \pi_{L_a} + \pi_{K_a} \Delta_l \right] \frac{1}{1 - \tau} < 0$$

III. $1 > \delta \approx \nu > 0$, i.e., public spending is equally effective in the two sectors:

$$\hat{Y}_a / \hat{\tau} = \frac{1}{|\Omega|} \left[ (\pi_{K_a} - \pi_{K_u}) + \left( \pi_{L_a} - \pi_{L_u} + \Delta_l \right) \left( \pi_{K_u} - \Delta_l \right) \right] > 0$$

$$\hat{Y}_m / \hat{\tau} = \frac{\nu}{1 - \nu} - \frac{1}{|\Omega|} \left[ \pi_{L_a} \left( \frac{\tau}{1 - \tau} \right) + \pi_{K_a} \Delta_l \left( \frac{\tau}{1 - \tau} \right) \right]$$

We find that when $\delta = \nu = 0$, $\hat{Y}_m / \hat{\tau} < 0$ and $\hat{Y}_a / \hat{\tau} > 0$; and when $1 > \delta \geq \nu > 0$, $\hat{Y}_m / \hat{\tau} > 0$, $\hat{Y}_a / \hat{\tau} < 0$ $(> 0) \left( \pi_{L_a} + \pi_{K_a} \Delta_l \right) \left( \frac{\tau}{1 - \tau} \right) > (<) \frac{\nu}{1 - \nu}$. Thus the effect on $Y_m$ depends on the weights of expansionary $G_m$ relative to the contractionary reduction in the tax base. This leads to the following proposition:

**Proposition 4:** Under our assumption of higher capital intensity for the urban sector, an increase in the urban tax leads to output expansion in the rural sector, when public services yield similar returns $(1 > \delta \approx \nu \geq 0)$ in the two sectors. The effect on urban output is ambiguous $(when 1 > \delta \approx \nu > 0)$, since it depends on the size of expansionary effects of rural allocation relative to the contractionary effects on the tax base.
These output responses are now used to derive the effects on sectoral factor prices, sectoral employment, and the rate of urban unemployment. From equations 2.20–2.23 and 2.28, the following is obtained:

\[
\hat{\rho}/\hat{\tau} = \frac{1}{1\theta} \left[ (\nu\theta_{k_a} - \delta\theta_{k_u}) \hat{Y}_u/\hat{\tau} - \left( \delta\theta_{k_u} - \nu + \frac{\tau}{1-\tau} \right) \right] \\
= -\frac{\tau}{(1-\tau)\theta} \quad \text{when } \delta = \nu = 0 \\
= \left[ \nu\hat{Y}_m/\hat{\tau} + \frac{\nu\theta_{k_u}}{\theta} \right] \\
\]

\[
\hat{\omega}_u/\hat{\tau} = \frac{1}{\theta} \left[ (\delta\theta_{k_u} - \nu\theta_{k_u}) \hat{Y}_m/\hat{\tau} + \delta\theta_{k_u} \right] \\
= 0 \quad \text{when } \delta = \nu = 0 \\
= \left[ \nu\hat{Y}_m/\hat{\tau} + \frac{\nu\theta_{k_u}}{\theta} \right] , \quad \text{when } 1 \geq \delta \approx \nu > 0 \\
\]

\[
(\hat{\omega}_u - \hat{\rho})/\hat{\tau} = \frac{1}{\theta} \left[ (\delta - \nu)\hat{Y}_m/\hat{\tau} + \left( \delta - \nu + \frac{\tau}{1-\tau} \right) \right] \\
= \frac{\tau}{\theta(1-\tau)} \quad \text{when } \delta = \nu = 0 \\
= \frac{\tau}{\theta(1-\tau)} \quad \text{when } \delta \approx \nu > 0 \\
\]

\[
\hat{L}_m/\hat{\tau} = (1-\nu)\hat{Y}_m/\hat{\tau} - \frac{\sigma_a\theta_{k_a} \tau}{\theta(1-\tau)} - \nu < 0 \\
\hat{L}_u/\hat{\tau} = \hat{Y}_u/\hat{\tau} - \delta\hat{Y}_m/\hat{\tau} - \sigma_a\theta_{k_u} (\hat{\omega}_u - \hat{\rho})/\hat{\tau} - \delta \\
= \hat{Y}_u/\hat{\tau} - \nu\hat{Y}_m/\hat{\tau} - \frac{\sigma_a\theta_{k_u} \tau}{\theta(1-\tau)} - \nu \\
\quad \text{when } 1 > \delta \approx \nu > 0 \left| \hat{Y}_u/\hat{\tau} - \nu\hat{Y}_m/\hat{\tau} \right| > \left| \frac{\sigma_a\theta_{k_u} \tau}{\theta(1-\tau)} - \nu \right| \\
\]

These equations show that even when output elasticities are zero with respect to public spending, i.e., \( \delta = \nu = 0 \), an increase in the tax on urban output will have an impact on output and employment. Wages will remain unaffected, but the rental for capital will fall, resulting in
(\hat{w}_u - \hat{r})/\hat{\tau} > 0$. In case of equally productive government services in the rural and urban sectors (i.e., $\delta = \nu > 0$), $(\hat{w}_u - \hat{r})/\hat{\tau} > 0$, and $(\hat{w}_m - \hat{r})/\hat{\tau} > 0$ for any given $\varepsilon$, $\hat{L}_u/\hat{\tau} > 0$, and $\hat{L}_m/\hat{\tau} < 0$.

These results can be stated as follows:

**Proposition 5:** When public spending elasticities of output are equal in the two sectors, a rise in the urban tax will increase rural employment and reduce urban employment for any given efficiency wage margin. This will in turn reduce out-migration from the rural to the urban sector.

### 2.5.3. Welfare Effects of Changes in Policy Variables

It is assumed that the social utility derived from the consumption of two commodities, $Y_u$ and $Y_m$, is represented by $U = U(D_u, D_m)$ where $D_j$ denotes the private consumption of good $j$. The budget constraint of the economy is given by

$$E(p, \tau, U) = I = Y_u + p(1-\tau)Y_m,$$  \hspace{1cm} (2.41)

where $E(p, \tau, U)$ is the expenditure function derived from the minimization of total expenditure subject to the utility constraint. Assuming that our economy is small and open ($dp = 0$), and that $\tau$ is exogenously given ($d\tau = 0$), the following is obtained by totally differentiating (equation 2.41),

$$dW = E_d dU = dY_u + p(1-\tau)dY_m.$$  \hspace{1cm} (2.42)

Equation 2.41 is used to derive the welfare effects of increased allocation to the rural sector ($\mu$) financed by an increased output tax ($\tau$) on the urban sector.
Here we have used a simple welfare analysis of a representative private agent in a small open economy along the lines followed in the conventional Harris-Todaro literature (Chaudhuri, 2013; Yabuuchi, 2005; Temple, 2005; Gupta, 2003; Krichel and Levine, 1999; Chandra and Khan, 1993). Since the economy is small and \( p \) is determined by world prices, welfare of the private agent is measured by the real income at world prices after tax. However, it should be noted that in a developing economy with multiple distortions, the effects of parametric changes on social welfare will depend on the relative magnitudes of these effects.

2.5.3.1. Welfare Effects of Increased Allocation to the Rural Sector (\( \mu \))

From equation 2.42, and from total differentials of equations 2.1 and 2.2, the following can be derived (as shown in Appendix 2.3):

\[
\frac{dW}{d\mu} = \frac{Y_m}{\mu} \left[ \frac{\delta Y_u}{Y_u} - \frac{\nu p(1-\tau)}{(1-\mu)(1-\nu)} \right] + \left( \frac{\delta Y_u}{Y_u} + \frac{\nu p(1-\tau)}{1-\nu} \right) \frac{dY_u}{d\mu} + \frac{(\alpha - 1)}{\alpha} \frac{w_m}{y_m} \frac{dl_m}{d\mu}. \tag{2.43}
\]

Here one finds that the welfare effect consists of three components: the primary growth effect, the returns-to-scale effect, and the employment effect. Setting \( \delta = \nu, and \frac{dl_m}{d\mu} = 0 \) one gets primary growth effects. The returns-to-scale effect will largely depend on relative values of \( \delta, \nu, \tau, \alpha, and \frac{Y_u}{y_m} \).

Alternative scenarios can be calibrated based on different values of the parameters \( \delta, \nu, \tau, \alpha \), under the following alternative assumptions of relative efficiency of public spending:

1. \( \delta = \nu = 0 \), and \( \frac{dW}{d\mu} = 0 \) .
I. \( \delta = 1, \nu = 0 \), i.e., public spending is highly effective in the rural sector due to effective rural development programs but not productive in the urban sector:

\[
\frac{dW}{d\mu} = \frac{Y_a}{\mu} Y_m dY_m + (\alpha - 1) \frac{dL_m}{d\mu}. 
\]

Since \( \frac{dY_m}{d\mu} < 0 \) and \( \frac{dL_m}{d\mu} < 0 \), sign of \( dW/d\mu \) will largely depend on relative size of \( Y_a \).

The welfare impact in terms of factor shares and government expenditure shares can also be expressed as (derivation is in Appendix 2.3):

\[
\frac{dW}{d\mu} = \frac{dY_a}{d\mu} + p(1-\tau) \frac{dY_m}{d\mu} = \frac{p(1-\tau)Y_m \theta_m}{\mu \alpha \pi_{li}} \left[ \frac{\pi_{la} \hat{Y}_a + \pi_{la} \hat{Y}_m}{\theta_a \hat{\mu}} + \frac{\pi_{lm} \hat{Y}_m}{\theta_m \hat{\mu}} \right] = \frac{w_a L \hat{Y}_a}{\mu} \left[ \frac{\pi_{la} \hat{Y}_a + \pi_{lm} \hat{Y}_m}{\theta_a \hat{\mu}} + \frac{\pi_{lm} \hat{Y}_m}{\theta_m \hat{\mu}} \right] (2.43A)
\]

Since factor shares \( (\theta_i) \), elasticities of public spending \( (\delta, \nu) \), and the rural expenditure share \( (\mu) \) are the prime determinants of the sign of \( dW/d\mu \), the welfare effect is ambiguous. However, if it is assumed that the relative shares are such that public goods contribute more to the productivity of capital and labor in rural areas than in urban areas, and the urban sector is not very large, then rural output will increase and so will national output. This leads to the following proposition:

**Proposition 6:** When public spending elasticities of output are equal in the two sectors, an increased allocation to the rural sector is welfare-enhancing only when rural output is sufficiently large.

In future extensions of the model, it would be useful to calibrate it to examine and compare the welfare effects of public spending policies in rural and urban areas. In countries like
China, India, Ghana, and other South Asian and sub-Saharan African countries where rural-urban migration is one of the major characteristics of the urban labor market, such an exercise could provide additional insights into the welfare effects of public expenditure policies.

2.5.3.2. Welfare Effects of an Increase in $\tau$

The welfare effect for an increase in urban output tax can be derived in a similar way:

$$
\frac{dW}{d\tau} = Y_m \left[ \frac{\delta Y_a}{Y_m} + \frac{\nu - \tau}{\tau(1-\nu)} \right] + \left( \frac{\delta Y_a}{Y_m} + \nu p(1-\tau) \right) \frac{dY_m}{d\tau} + \frac{(\alpha - 1)}{\alpha} w_m \frac{dL_m}{d\tau},
$$

(2.44)

which shows that the welfare effect depends on relative shares and sizes of expansionary effects of $\tau$ on $Y_u (\hat{Y}_u / \hat{\tau} > 0)$ and on contractionary effects of $\tau$ on $Y_m (\hat{Y}_m / \hat{\tau} < 0)$. These effects can be analyzed under alternative assumptions of efficiency of public spending. Here again one finds that the welfare effect can go either way. When the above is expressed in terms of relative shares (derivation is shown in Appendix 2.3) as

$$
\frac{dw}{d\tau} = \frac{p(1-\tau)Y_m \theta_{L_u}}{\tau \pi_{L_u}} \left[ \frac{\pi L_u \hat{Y}_a}{\alpha \theta_{L_u} \hat{\tau}} + \frac{\pi L_u \hat{Y}_m}{\theta_{L_u} \hat{\tau}} - \frac{\tau \pi_{L_u}}{(1-\tau) \theta_{L_u}} \right] = \frac{w_m L}{\tau} \frac{\pi L_u \hat{Y}_a + \pi L_u \hat{Y}_m}{\alpha \theta_{L_u} \hat{\tau} + \theta_{L_u} \hat{\tau} - \tau p Y_m},
$$

(2.44)

one sees that the effect depends on the relative values of factor shares ($\theta_{L_u}$), effects on output, the ratio of tax revenue to the wages of urban workers, and the relative size of the effects on urban and rural output. The welfare effect is positive when the expansionary impact of increased expenditure for the rural sector (holding expenditure shares constant) is higher than the contractionary effect of the higher tax rate on urban output.
2.6. A Model with Urban Unemployment

We now extend our two-sector neoclassical model with full employment to consider urban unemployment.

2.6.1. The Basic Structure

The new model retains much of the structure from the public spending augmented model of Section 2.2. The production functions, equations showing the effectiveness of public expenditure, and marginal productivities of capital and labor—all summarized in the first eight equations of the model and reproduced below—still apply. As will be discussed later in this chapter, a major change now occurs in the wage relation between the two sectors. Putting together all these equations one obtains the following model:

\[ Y_a = G_a F(L_a, K_a) \]  \hspace{1cm} (2.6.1)

\[ Y_m = G_m H(L_m, K_m) \]  \hspace{1cm} (2.6.2)

\[ G_a = (\mu \tau Y_m)^{\delta}, \text{ and} \]  \hspace{1cm} (2.6.3)

\[ G_m = [(1 - \mu) \tau Y_m]^\nu. \]  \hspace{1cm} (2.6.4)

\[ G_a F_K = r \text{ or } G_a p f'(k_a) = r \]  \hspace{1cm} (2.6.5)

\[ p(1 - \tau)G_a H_K = r \text{ or } p(1 - \tau)G_m h'(k_m) = r \]  \hspace{1cm} (2.6.6)

\[ G_a F_L = w_a \text{ or } G_a [f - k_a f'(k_a)] = w_a \]  \hspace{1cm} (2.6.7)

\[ p(1 - \tau)G_m H_L = w_m \text{ or } p(1 - \tau)G_m [h - k_m h'(k_m)] = w_m, \]  \hspace{1cm} (2.6.8)

where \( Y_j, L_j, K_j, G_j \) stand for the level of output, employment, capital, and government input, respectively, of the \( j^{th} \) sector for \( j = a, m \). The subscripts \( a \) and \( m \) stand for rural (agricultural)
and urban (manufacturing) sectors, respectively. \( G_j \) represents the contribution of government spending to the total factor productivity of sector \( j \), and \( \tau \) is the rate of taxation on manufacturing sector output. The parameters \( \delta \) and \( \nu \) indicate the technologies that turn output into flows of government input in private production, and the allocation shares of government spending across sectors are given by the ratios \( \mu \) and \( 1 - \mu \). \( r \) stands for rental rate of capital, and \( p \) stands for the relative price of the manufacturing good in the world economy; \( k_j \) is the capital-labor ratio in the \( j \)th sector; \( f \) is \( F/L \); \( h \) is \( H/L \); \( F_K \), and \( H_K \) are partial derivatives with respect to capital; and \( f' \) and \( h' \) are partial derivatives in intensive forms. \( F_L \) and \( H_L \) are partial derivatives with respect to \( L \), and \( w_u \) and \( w_m \) are the wages in the rural and urban sectors, respectively.

Labor migrates from the rural to the urban sector to take advantage of higher urban wages. Migrants are risk-neutral and compare the rural wage \( w_u \) with the expected wage in the urban sector. This expected wage equals \( w_m \) times the probability of finding a job in the urban sector. Thus, migration continues until equilibrium is reached in which the expected urban wage under migration equals the rural wage. The labor market equilibrium is therefore given by:

\[
\lambda w_u = (1 + \lambda)w_u, \quad (2.6.9)
\]

where \( \lambda \), the ratio of unemployed \( (L_U) \) to the employed \( (L_m) \) in the urban sector, is now endogenously determined, and \( w_m \) is the institutionally mandated minimum wage in the urban sector. Resource allocations between the two sectors are given by:

\[
K_u + K_m = K \quad (2.6.10)
\]

\[
L_u + L_m + L_U = L_a + (1 + \lambda)L_m = L_a + \varepsilon L_m, \quad \varepsilon = (1 + \lambda). \quad (2.6.11)
\]
The formulation of the model thus far suggests that, given the world price \( p \), and exogenous values of \( K, L, \tau, \mu, \) and \( \omega_m \), there are 11 equations to solve for 11 unknowns: \( Y_a, Y_m, K_a, K_m, L_a, L_m, G_a, G_m, r, w_a, \) and \( \lambda \).

### 2.6.2. Workings of the Model: Analysis in Terms of Equations of Change

The above system of equations can be simplified into a smaller set of subsystems. This will help to derive the equilibrium condition in terms of the given technology and other parameters, and the capital-labor ratios. Following again the rate-of-change methodology pioneered by Jones (1965), we differentiate equations 2.6.1 and 2.6.2, and use equations 2.6.10 and 2.6.11 to get the following:

\[
\hat{\dot{Y}}_a = \delta \dot{Y}_a + \theta_{L_a} \hat{\dot{L}}_a + \theta_{K_a} \hat{\dot{K}}_a + \delta(\dot{\mu} + \dot{\tau}) \tag{2.6.12}
\]

\[
\hat{\dot{Y}}_m = \nu \dot{Y}_m + \theta_{L_m} \hat{\dot{L}}_m + \theta_{K_m} \hat{\dot{K}}_m + \nu \left( \dot{\tau} - \frac{\mu}{1 - \mu} \dot{\mu} \right), \tag{2.6.13}
\]

where \( \hat{\dot{\cdot}} \) denotes the relative change of the variable (e.g., \( \hat{\dot{Y}}_j = dY_j / Y_j \)). Excluding the effect of public expenditures, \( \theta_{ij} \) gives the share of factor \( i \) in sector \( j \), i.e., \( \theta_{L_a} = \frac{w_a L_a}{Y_a}, \theta_{K_a} = \frac{r K_a}{Y_a} \),

\[
\theta_{L_a} = \frac{w_a L_a}{p(1 - \tau)Y_m}, \text{ and } \theta_{K_m} = \frac{r K_m}{p(1 - \tau)Y_m}. \]

Since government spending is the source of increasing returns to scale that are external to individual firms, the average cost pricing holds for each industry. Thus, we have \( w_a L_a + r K_a = Y_a \), and \( w_m L_m + r K_m = p(1 - \tau)Y_m \). Using these conditions, the following can be obtained:

\[
\theta_{L_a} \hat{\dot{w}}_a + \theta_{K_a} \hat{\dot{r}} + \theta_{L_a} \hat{\dot{L}}_a + \theta_{K_a} \hat{\dot{K}}_a = \hat{ \dot{Y}}_a \tag{2.6.14}
\]
\[ \theta_{\omega} \hat{w}_m + \theta_{\kappa_a} \hat{r} + \theta_{\omega} \hat{L}_m + \theta_{\kappa_m} \hat{K}_m = \hat{Y}_m + \hat{p} - \frac{\tau}{1-\tau} \hat{\tau}. \] (2.6.15)

The assumption of constant returns to scale in private inputs gives us \( \theta_{\kappa_i} = 1 - \theta_{\kappa_i}, i = a, m \)

Substituting equations 2.6.14 and 2.6.15 into 2.6.12 and 2.6.13, the following can be obtained:

\[ \theta_{\omega} \hat{w}_m + \theta_{\kappa_a} \hat{r} = \nu \hat{Y}_m + \hat{p} - \frac{\tau}{1-\tau} \hat{\tau} + \nu \left( \hat{\tau} - \frac{\mu}{1-\mu} \hat{\mu} \right) \] (2.6.16)

\[ \theta_{\omega} \hat{w}_m + \theta_{\kappa_m} \hat{r} = \delta \hat{Y}_m + \delta (\hat{\mu} + \hat{\tau}). \] (2.6.17)

Next, the elasticity of substitution is defined in the two sectors as

\[ \sigma_a = \frac{(\hat{K}_a - \hat{L}_a)}{(\hat{w}_a - \hat{r})} \] (2.6.18)

\[ \sigma_m = \frac{(\hat{K}_m - \hat{L}_m)}{(\hat{w}_m - \hat{r})}, \] (2.6.19)

where \( \sigma_a \) and \( \sigma_m \) are both positive. Substituting equations 2.6.18 and 2.6.19 into 2.6.12 and 2.6.13, one gets

\[ \hat{L}_m = (1 - \nu) \hat{Y}_m - \sigma_m \theta_{\kappa_m} (\hat{w}_m - \hat{r}) - \nu \hat{\tau} + \nu \frac{\mu}{1-\mu} \hat{\mu} \] (2.6.20)

\[ \hat{K}_m = (1 - \nu) \hat{Y}_m + \sigma_a \theta_{\kappa_a} (\hat{w}_m - \hat{r}) - \nu \hat{\tau} + \nu \frac{\mu}{1-\mu} \hat{\mu} \] (2.6.21)

\[ \hat{L}_a = \hat{Y}_a - \delta \hat{Y}_m - \sigma_a \theta_{\kappa_a} (\hat{w}_a - \hat{r}) - \delta (\hat{\mu} + \hat{\tau}) \] (2.6.22)

\[ \hat{K}_a = \hat{Y}_a - \delta \hat{Y}_m + \sigma_a \theta_{\kappa_a} (\hat{w}_a - \hat{r}) - \delta (\hat{\mu} + \hat{\tau}). \] (2.6.23)

Differentiating equations 2.6.8 and 2.6.9, one obtains

\[ \pi_{\kappa_a} \hat{L}_a + \epsilon \pi_{\kappa_m} \hat{L}_m + \epsilon \pi_{\kappa_m} \hat{\epsilon} = \hat{L} \] (2.6.24)

\[ \pi_{\kappa_m} \hat{K}_a + \pi_{\kappa_m} \hat{K}_m = \hat{K}, \] (2.6.25)

where \( \pi_j \) is the share of factor \( i \) used in industry \( j \), (e.g., \( \pi_{\kappa_a} = L_a / L \)).
Substituting equations 2.6.20–2.6.23 into equations 2.6.24–2.6.25, one obtains

\[
\pi_{1u} \hat{Y}_a + [(1 - \nu)\varepsilon \pi_{1u} - \delta \pi_{1u} ] \hat{Y}_m = \pi_{1u} \theta_{1u} \sigma_a (\hat{w}_a - \hat{r}) + \pi_{1u} \varepsilon \sigma_m \theta_{1u} (\hat{w}_m - \hat{r}) \\
+ \left( \pi_{1u} \delta - \frac{\varepsilon \pi_{1u} \gamma \mu}{1 - \mu} \right) \hat{\mu} + \left( \pi_{1u} \delta + \varepsilon \pi_{1u} \nu \right) \hat{\xi} - \varepsilon \pi_{1u} \hat{\varepsilon}
\]  

(2.6.26)

\[
\pi_{2u} \hat{Y}_a + (\pi_{2u} (1 - \nu) - \pi_{2u} \delta) \hat{Y}_m = -\pi_{2u} \theta_{2u} \sigma_a (\hat{w}_a - \hat{r}) - \pi_{2u} \sigma_m \theta_{2u} (\hat{w}_m - \hat{r}) \\
+ \left( \pi_{2u} \delta - \frac{\pi_{2u} \gamma \mu}{1 - \mu} \right) \hat{\mu} + \left( \pi_{2u} \delta + \pi_{2u} \nu \right) \hat{\xi}
\]  

(2.6.27)

Solving equations 2.6.16 and 2.6.17, substituting into equations 2.6.26–2.6.27, and simplifying (as shown in Appendix 2.3), one obtains

\[
\pi_{1u} \hat{Y}_a + [(1 - \nu)\varepsilon \pi_{1u} - \delta \pi_{1u} ] \hat{Y}_m = \frac{\pi_{1u} \theta_{1u} \sigma_a}{\theta_{1u}} (\delta - \nu / \theta_{1u}) \hat{Y}_m + \varepsilon \pi_{1u} \hat{\varepsilon}
\]

\[
= \left[ \frac{\pi_{1u} \theta_{1u} \sigma_a}{\theta_{1u}} \delta + \pi_{1u} \delta + \frac{\mu \nu \pi_{1u} \varepsilon \sigma_m}{(1 - \mu)} + \frac{\mu \nu}{(1 - \mu)} \theta_{1u} \theta_{2u} \theta_{k_a} \right] \hat{\mu}
\]

\[
+ \left[ \pi_{1u} \delta + \varepsilon \pi_{1u} \nu (1 - \sigma_m) + \frac{\pi_{1u} \theta_{1u} \sigma_a}{\theta_{1u} \theta_{k_a}} (\delta \theta_{k_a} - \nu) + \frac{\tau}{(1 - \tau) \theta_{1u} \theta_{k_a}} \right]
\]

\[
\left[ \pi_{1u} \sigma_m \theta_{1u} + \varepsilon \pi_{1u} \sigma_m \theta_{1u} \theta_{k_a} \right] \hat{\xi}
\]

\[
- \frac{1}{\theta_{1u} \theta_{k_a}} \left( \pi_{1u} \theta_{k_a} \sigma_a + \pi_{1u} \varepsilon \sigma_m \theta_{1u} \theta_{k_a} \right) \hat{\rho}
\]

(2.6.28)

\[
\pi_{2u} \hat{Y}_a + \left( \pi_{2u} (1 - \nu) - \pi_{2u} \delta \right) \hat{Y}_m = \left( \pi_{2u} \delta (1 - \sigma_a) - \left( \pi_{2u} \theta_{k_a} + \pi_{2u} \sigma_a + \pi_{2u} \sigma_m \theta_{k_a} \right) \frac{\mu \nu}{\theta_{k_a} (1 - \mu)} \right] \hat{\mu} + \\
\left[ \pi_{2u} \delta (1 - \sigma_a) + \pi_{2u} \nu + \frac{1}{\theta_{k_a}} \left( \pi_{2u} \theta_{k_a} \sigma_m \theta_{1u} + \pi_{2u} \sigma_m \right) \left( \nu - \frac{\tau}{(1 - \tau)} \right) \right]
\]

\[
\left[ \pi_{2u} \sigma_a + \pi_{2u} \sigma_m \theta_{1u} \right] \hat{\xi}
\]

(2.6.29)

Similarly, using equations 2.6.9 and 2.6.16–2.6.17 one can derive
\[
\hat{\theta} + \frac{\delta \theta_{k_a} - \nu \theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \hat{Y}_m = \left( \frac{\nu - \tau}{1 - \tau} - \delta \frac{\theta_{k_a}}{\theta_{k_a}} \right) \hat{\nu} + \theta_{k_a} \left( \frac{\theta_{k_a}}{\theta_{k_a}} \right) \hat{\theta}_{k_a} \left( \frac{\mu \nu}{1 - \mu} + \delta \frac{\theta_{k_a}}{\theta_{k_a}} \right) \hat{\mu} \\
\text{(2.6.30)}
\]

Substituting the value of \( \hat{\epsilon} \) from equation 2.6.30 into 2.6.28, one gets

\[
\pi_{l_a} \hat{Y}_a + [(1 - \nu) \mu \pi_{k_a} - \delta \pi_{l_a} + \nu \varepsilon \pi_{l_a} \sigma_m - \frac{\pi_{l_a} \theta_{k_a} \sigma_a}{\theta_{l_a}} (\delta - \nu / \theta_{k_a}) - \varepsilon \pi_{l_a} \frac{\delta \theta_{k_a} - \nu \theta_{k_a}}{\theta_{l_a} \theta_{k_a}}] \hat{Y}_m = \\
\left[ \frac{\delta (\pi_{l_a} \theta_{k_a} \sigma_a + \varepsilon \pi_{l_a} \sigma_m)}{\theta_{l_a}} + \frac{\mu \nu (\pi_{l_a} \theta_{k_a} \sigma_a - \varepsilon \pi_{l_a} \theta_{l_a} \theta_{k_a} + \varepsilon \pi_{l_a} \theta_{k_a})}{(1 - \mu) \theta_{l_a} \theta_{k_a}} \right] \hat{\mu} \\
\left[ \pi_{l_a} \delta + \frac{\varepsilon \pi_{l_a} \nu}{\theta_{l_a} \theta_{k_a}} (\theta_{l_a} \theta_{k_a} (1 - \sigma_m) - \theta_{k_a}) + \frac{\pi_{l_a} \theta_{k_a} \sigma_a}{\theta_{l_a} \theta_{k_a}} (\delta \theta_{k_a} - \nu) \right] \\
\left[ \frac{\tau (\pi_{l_a} \theta_{k_a} \sigma_a + \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a} + \varepsilon \pi_{l_a} \theta_{k_a}) + \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a}}{(1 - \tau) \theta_{l_a} \theta_{k_a}} + \delta \frac{\varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a}}{\theta_{l_a}} \right] \hat{\nu} \\
- \frac{1}{\theta_{l_a} \theta_{k_a}} \left( \varepsilon \pi_{l_a} \theta_{l_a} \sigma_a \theta_{k_a} + \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a} \right) \hat{\pi}
\]

(2.6.28a)

Equations 2.6.28a and 2.6.29 can be expressed as:

\[
\begin{bmatrix}
\Omega_{l_a} & \Omega_{l_a} \\
\Omega_{k_a} & \Omega_{k_a}
\end{bmatrix}
\begin{bmatrix}
\hat{Y}_a \\
\hat{Y}_m
\end{bmatrix}
= 
\begin{bmatrix}
A_1 \hat{\mu} + A_2 \hat{\epsilon} + A_3 \hat{\pi} \\
B_1 \hat{\mu} + B_2 \hat{\epsilon} + B_3 \hat{\pi}
\end{bmatrix},
\]

(2.6.31)

where

\[
\Omega_{l_a} = \pi_{l_a} \\
\Omega_{k_a} = (1 - \nu) \mu \pi_{k_a} - \delta \pi_{l_a} + \nu \varepsilon \pi_{l_a} \sigma_m - \frac{\pi_{l_a} \theta_{k_a} \sigma_a}{\theta_{l_a}} (\delta - \nu / \theta_{k_a}) - \varepsilon \pi_{l_a} \frac{\delta \theta_{k_a} - \nu \theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \\
\Omega_{k_a} = \pi_{k_a} \\
\Omega_{k_a} = \pi_{k_a} (1 - \nu) - \pi_{k_a} \delta + \pi_{k_a} \sigma_a (\delta - \nu / \theta_{k_a}) - \frac{\nu \pi_{k_a} \sigma_m \theta_{l_a}}{\theta_{k_a}} \\
\left| \Omega \right| = \left( \pi_{l_a} \pi_{k_a} - \varepsilon \pi_{l_a} \pi_{k_a} \right) (1 - \nu) + \pi_{l_a} \pi_{k_a} \delta \left( \delta - \nu / \theta_{k_a} \right) - \frac{\nu \pi_{k_a} \sigma_m \theta_{l_a}}{\theta_{k_a}} - \nu \pi_{k_a} \varepsilon \pi_{l_a} \sigma_m + \varepsilon \pi_{k_a} \pi_{l_a} \frac{\delta \theta_{k_a} - \nu \theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \\
A_1 = \pi_{l_a} \delta + \left( \frac{\pi_{l_a} \theta_{k_a} \sigma_a + \varepsilon \pi_{l_a}}{\theta_{l_a}} \right) \delta + \left( \pi_{l_a} \varepsilon \sigma_m + \frac{\pi_{l_a} \theta_{k_a} \sigma_a}{\theta_{l_a} \theta_{k_a}} - \varepsilon \pi_{l_a} + \frac{\varepsilon \pi_{l_a} \theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \right) \frac{\mu \nu}{(1 - \mu)}
\]
\[ A_2 = \pi_{La} \delta + \varepsilon \pi_{La} \nu \left( 1 - \sigma_m - \frac{\theta_{K_a}}{\theta_{La} \theta_{K_a}} \right) + \pi_{La} \theta_{K_a} \sigma_a \left( \delta \theta_{K_a} + \frac{\tau}{1 - \tau} - \nu \right) + \frac{\varepsilon \pi_{La} \tau}{(1 - \tau)} \left( \sigma_m + \frac{\delta \theta_{K_a}}{\theta_{La} \theta_{K_a}} \right) + \frac{\delta \varepsilon \pi_{La}}{\theta_{La}} \]

\[ A_3 = -\frac{1}{\theta_{La} \theta_{K_a}} \left( \pi_{La} \theta_{K_a} \sigma_a + \pi_{La} \sigma_m \theta_{La} \theta_{K_a} + \varepsilon \pi_{La} \theta_{La} \right) \]

\[ B_1 = \pi_{K_a} \delta (1 - \sigma_a) - \left( \pi_{K_a} \theta_{K_a} + \pi_{K_a} \sigma_a + \pi_{K_a} \sigma_m \theta_{La} \right) \frac{\mu \nu}{\theta_{K_a} (1 - \mu)} \]

\[ B_2 = \pi_{K_a} \delta (1 - \sigma_a) + \pi_{K_a} \nu + \frac{1}{\theta_{K_a}} \left( \pi_{K_a} \sigma_m \theta_{La} + \pi_{K_a} \sigma_a \right) \left( \nu - \frac{\tau}{1 - \tau} \right) \]

\[ B_3 = \frac{\left( \pi_{K_a} \sigma_a + \pi_{K_a} \sigma_m \theta_{La} \right)}{\theta_{K_a}}. \]

The solution of equation 2.6.31 will give the effects of a change in sectoral allocation of public spending on output. These output responses in turn can be used to evaluate the effects on employment and relative wages. The output-employment responses obviously depend on the signs of \( \Omega_{ij} \), \( A_j \), and \( B_j \) of equation 2.6.31. As is customary in two-sector models, it is assumed that the urban sector is capital-intensive and the rural sector is labor-intensive, which helps determine some of the signs as follows: \( \Omega_{La} > 0, \Omega_{K_a} > 0, A_3 < 0, B_3 < 0 \). However, we need to make further assumptions on the relative magnitudes of \( \delta, \mu, \) and \( \tau \) to determine the signs of \( \Omega_{La}, \Omega_{K_a}, A_1, A_2, A_3, B_3 \), and \( B_2 \).

2.6.3. Adjustment Process and Stability Analysis

Following a similar approach to the one in Section 2.4, we derive a proposition about the Khan-Neary condition by specifying a dynamic adjustment process for this system (Mayer, 1974; Chang, 1981; Yabuuchi, 1992, 1998). It is assumed that the urban wage follows a Harris-Todaro mechanism as given in equation 2.6.36 below. Thus we have:

\[ \dot{Y}_a = d_1 (1 - a_{La} w_a - a_{K_a} r) \quad \text{(2.6.32)} \]
\[
\dot{Y}_m = d_2 \left( p(1-\tau) - a_{Lm} w_m - a_{Km} r \right) \\
\dot{w}_a = d_3 (a_{La} Y_a + \varepsilon a_{La} Y_m - L) \\
\dot{r} = d_4 (a_{Ka} Y_a + a_{Km} Y_m - K) \\
d(1+\lambda) / dt = d_5 (w_m - (1+\lambda)w_a),
\]

where \( a_{ij} \) is the amount of \( i^{th} \) factor used to produce one unit of output \( j \) (e.g., \( a_{La} = \frac{L_a}{Y_a} \)); the positive coefficient \( d_j \) is the speed of adjustment; and \( \dot{Y} \) is the time derivative of \( Y \).

The necessary condition for local stability of the system, following the Routh-Hurwitz theorem, is that the sign of the determinant of the Jacobian (\( J \)) of the system of equations 2.6.32–2.6.36 is \((-1)^j\), where \( j \) is the number of rows (= columns) in the Jacobian matrix. In this case, \( \text{sign} |J| = (-1)^5 = (-1)^5 < 0 \). It can be shown that that \( |J| < 0 \) implies \( |\Omega| > 0 \).

Simplifying, we get the determinant of the Jacobian matrix as

\[
|J| = \left| \begin{array}{cccc}
-d_1 a_{Lm} & -d_1 a_{Km} & 0 & d_1 \delta / Y_m \\
0 & -d_2 a_{Km} & 0 & d_2 p(1-\tau) \nu / Y_m \\
d_3 (\varepsilon L_m - L_a \sigma a_{Km}) / w_a & d_3 (L_a \sigma a_{Km} + \varepsilon L_a \sigma a_{Km} / p(1-\tau)) & d_3 a_{La} & d_3 \left( \varepsilon (1-\nu)a_{La} - \delta L_a / Y_a \right) \\
d_4 K_a \sigma a_{La} & d_4 (K_a \sigma a_{La} - K_n a_{La} / \sigma_{La}) / r & d_4 a_{La} & d_4 (1-\nu)a_{La} - \delta K_a / Y_a \\
-d_5 (1+\lambda) & 0 & 0 & -d_5 w_a
\end{array} \right|
\]

Simplifying, we get the determinant of the Jacobian matrix as

\[
|J| = \left. \left| \begin{array}{cccc}
-d_1 d_2 d_3 d_4 d_5 KL p(1-\tau)|\theta| \\
Y_a Y_m r
\end{array} \right| \right| |\Omega|, \quad \text{and}
\]

\[
|\Omega| = \left( \frac{(\pi_d \xi_{Lm} - \varepsilon \pi_d \xi_{La})(1-\nu) + \pi_d \xi_{Km} \sigma a_{La} \delta / \theta_{La} - \nu \pi_d \xi_{Km} \sigma a_{La} \delta / \theta_{La} - \nu \varepsilon \pi_d \xi_{Km} a_{La} \sigma / \theta_{La} + \varepsilon \pi_d \xi_{Km} a_{La} / \theta_{La}}{\theta_{La} \theta_{La}} \right)
\]

Thus, \((-1)^j |J| < 0, j = 5\), when \( |\Omega| > 0 \). We can now make the following assumption.

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Assumption 2: The urban sector is more capital-intensive than the rural sector, both in employment-adjusted terms and in value terms, i.e., \( k_m > \varepsilon k_u \) and \( \theta_{K_m} / \theta_{L_m} > \theta_{K_u} / \theta_{L_u} \).

We can express \(|\Omega|\) as

\[
\frac{L_u L_m (k_m - \varepsilon k_u)(1 - \nu)}{KL} + \frac{\pi L_u \pi_{K_u} \sigma_u}{\theta_{L_u}} (\delta + \nu / \theta_{K_u}) - \frac{\nu \pi L_u \pi_{K_u} \sigma_m \theta_{L_m}}{\theta_{K_u}} - \nu \varepsilon \pi_{K_u} \pi_{L_u} \sigma_m + \varepsilon \pi_{K_u} \pi_{L_u} \frac{\delta \theta_{K_u} - \nu \theta_{K_u}}{\theta_{L_u} \theta_{K_u}}.
\]

This helps to derive the following cases under alternative assumptions of efficiency of public spending, when the determinant \(|\Omega|\) is positive. The sign of \(|\Omega|\) clearly depends on \( \delta \) and \( \nu \), which are “technological” parameters in the model. Let us consider three cases:

I. \( \delta = 0, \nu = 0, |\Omega| > 0 \)

\[
|\Omega| = \left( \pi L_u \pi_{K_u} - \pi_{K_u} \varepsilon \pi_{L_u} \right) = \frac{L_u L_m (k_m - \varepsilon k_u)}{KL} > 0.
\]

This implies that the stability condition reduces to the Khan-Neary stability condition in the traditional Harris-Todaro Model.

II. \( \delta = 1, \nu = 0, \sigma_u \geq 0, |\Omega| > 0 \)

\[
|\Omega| = \frac{L_u L_m (k_m - \varepsilon k_u)}{KL} + \frac{\pi L_u \pi_{K_u} \sigma_u + \varepsilon \pi_{K_u} \pi_{L_u}}{\theta_{L_u}} > 0.
\]

This shows that our stability condition holds when the Khan-Neary condition holds.

III. \( \delta = \nu / \theta_{K_u}, 0 < \nu < 1 \)

\[
|\Omega| \geq 0 \text{ when } k_m - \varepsilon^* k_u > 0, \text{ where } \varepsilon^* = \frac{1 + \delta - \nu (1 - \sigma_u)}{1 - \nu - \nu \sigma_m \theta_{L_u} / \theta_{K_u}}.
\]

Here, for \( 0 < \nu < 1/(1 + \sigma_m \theta_{L_u} / \theta_{K_u}) < 1 \), the stability condition holds when the urban sector is relatively capital-intensive in elasticity, share, and employment-adjusted terms. Note that for
\( \delta = 0, \nu = 1, \quad |\Omega| < 0, \) and the stability condition is not satisfied. We also omit the assumptions \( 1 > \nu > \delta > 0, \) which we intend to examine in future research.

Thus, the above results in this section can be summarized in terms of the following proposition:

**Proposition 7:** If the urban sector is more capital-intensive both in physical and value terms than the rural sector, and the elasticity of output with respect to public spending (\( \delta \)) in the rural sector is higher than the elasticity (\( \nu \)) in the urban sector \((1 \geq \delta > \nu \geq 0),\) an equilibrium of the adjustment process is locally stable. This stability condition also reduces to the Khan-Neary condition when \( \delta = \nu = 0.\)

### 2.6.4. Comparative Statics

This section first determines how our endogenous variables (in particular, outputs, employment, and factor rewards) respond to a higher budgetary allocation to the rural sector. Second, the section examines the comparative static properties of the model when the tax on urban output is varied.

It is clear that when \( \delta = 0, \nu = 0, \) under \( CRS, \Omega = \left( \pi_u \pi_{k_u} - \varepsilon \pi_{k_u} \pi_{l_u} \right) \) which is positive when the Khan-Neary stability condition holds. However, when \( \delta \neq 0, \nu \neq 0 \) (including the case when \( \delta = \nu \)), we get different effects depending on the signs and relative values of elasticities of public spending in both the sectors (\( \delta, \nu \)), and elasticities of substitution in rural and urban sectors (\( \sigma_a, \sigma_m \)). In addition to the stability condition, the following assumption is made following Jones (1968) to control the number of analytical outcomes:
Assumption 3: At constant commodity prices, the demand for each factor of production rises with the expansion of any industry.

For simplicity, the comparative static analyses will mainly focus on cases I, II, and III (i.e., under alternative assumptions of efficiency of public services as (i) \( \delta = \nu = 0 \), (ii) \( \delta = 1, \nu = 0 \) and (iii) \( 1 > \delta > \nu > 0 \), and \( \delta = \nu / \theta_{K_u} \)).

2.6.4.1. An Increase in the Allocation of Government Spending (\( \mu \)) to the Rural Sector

Let us assume that the price (\( p \)), and tax rate (\( \tau \)) remain unchanged, and only government rural spending as a fraction of total spending (\( \mu \)) changes. In equation 2.6.31, setting \( \hat{p} = \hat{\tau} = 0 \), and solving for \( \hat{Y}_a / \hat{\mu} \) and \( \hat{Y}_m / \hat{\mu} \), one obtains

\[
\hat{Y}_a / \hat{\mu} = (\Omega_{K_u} A_1 - \Omega_{L_u} B_1) / |\Omega| \quad \text{and} \\
\hat{Y}_m / \hat{\mu} = - (\Omega_{K_u} A_1 - \Omega_{L_u} B_1) / |\Omega|.
\]  

(2.37)

(2.38)

These effects are analyzed below under alternative assumptions of efficiency of public spending.

I. \( \delta = \nu = 0 \), i.e., public spending does not generate any productive services in either sector.

Since, \( A_1 = 0, B_1 = 0 \) when \( \delta = \nu = 0 \), we get \( \hat{Y}_a / \hat{\mu} = 0 \), and \( \hat{Y}_m / \hat{\mu} = 0 \). Thus, an increased allocation to the rural sector does not affect outputs when public spending is not linked to any productive services.

II. \( \delta = 1, \nu = 0 \), i.e., public spending is highly effective in the rural sector due to effective rural development programs and improved infrastructure investment, but it is not productive in the urban sector:
\[ \hat{Y}_a / \hat{\mu} = \frac{1}{\Omega} \left( \pi_{K_a} - \pi_{K_a} + \pi_{K_a} \sigma_a \right) \left( \pi_{L_a} + \frac{\pi_{L_a} \theta_{K_a} \sigma_a + \epsilon \pi_{L_a}}{\theta_{L_a}} \right) + \frac{\pi_{K_a}}{\Omega} \left( \pi_{L_a} - \epsilon \pi_{L_a} + \frac{\pi_{L_a} \theta_{K_a} \sigma_a + \epsilon \pi_{L_a}}{\theta_{L_a}} \right) \left( 1 - \sigma_a \right) \geq 0 \] (2.6.37*)

\[ \hat{Y}_m / \hat{\mu} = \frac{1}{\Omega} \left[ -\sigma_a \pi_{K_a} \pi_{L_a} - \pi_{K_a} \left( \frac{\pi_{L_a} \theta_{K_a} \sigma_a + \epsilon \pi_{L_a}}{\theta_{L_a}} \right) \right] \leq 0 \] (2.6.38*)

III. $1 > \delta > 0, \delta = \nu / \theta_{K_a}, \sigma_a > 1 - \mu > 0$, i.e., public spending has higher elasticity with respect to output in the rural sector than in the urban sector.

When $\delta \neq 0, \nu \neq 0$ by assumption 3, when signs of $|\Omega|$ and $\Omega_y$ are positive as well as $1 > \delta > 0$. Since $A_i > 0$, and $B_i < 0$, we obtain $\hat{Y}_a / \hat{\mu} > 0$ and $\hat{Y}_m / \hat{\mu} < 0$.

It is to be noted that the above assumption imposes the conditions on the signs and sizes of $\delta$ and $\mu$ for stability and for $|\Omega| > 0$. The second part of assumption (iii) is also discussed in the text on page 59 as a sub-case of assumption 2, where the stability condition holds when the urban sector is relatively capital-intensive in all of the elasticity, share, and employment-adjusted terms. Under this assumption we can show from equation 2.6.16 and 2.6.17 that for given $p$ and $\tau$, $\hat{Y}_a / \hat{\mu}$ and $\hat{Y}_m / \hat{\mu}$ are very close to each other. In Appendix 2.4 on page 93, we have also derived $|\Omega|$ under these conditions.

From the above it is clear that, since taxation on urban output is the only source of public spending in both sectors, the effect of higher rural spending on the growth of urban output is negative in alternative scenarios. Rural output rises under specific parameter values of $\delta, \mu,$ and $\nu$. We find that $\hat{Y}_a / \hat{\mu} > 0$ when (i) $\delta = 1, \nu = 0$, (ii) $1 > \delta > 0$, $\delta = \nu / \theta_{K_a}$, $\sigma_a > 1 - \mu$ and the expansionary effect of the greater rural allocation outweighs the contractionary effect of reduction in the tax base of public spending. Thus the following proposition can be stated:
Proposition 8: Under our assumption of higher capital intensity for the urban sector, with variable elasticities of public spending and unemployment, an increase in the allocation of public spending to the rural sector leads to a decline in urban output, and to an increase in the output of the rural sector, provided public services have higher elasticity in the rural sector than in the urban sector.

These output responses are now used to derive the effects on sectoral factor prices, sectoral employment, and urban unemployment. From equations 2.6.16, 2.6.17, 2.6.20–2.6.23, and equation 2.6.28, the equations below are obtained when \( \hat{p} = \hat{\tau} = 0 \) and \( \hat{w}_m = 0 \).

From equation 2.6.17, one can derive

\[
\hat{\rho} / \hat{\mu} = \frac{1}{\theta_{k_a}} \left[ \nu \hat{Y}_m / \hat{\mu} - \frac{\mu \nu}{(1 - \mu)} \right].
\]  

(2.6.38A)

Using equations 2.6.38A and 2.6.17 one gets

\[
\hat{w}_a / \hat{\mu} = \frac{1}{\theta_{k_m} \theta_{k_a}} \left[ (\delta \theta_{k_m} - \nu \theta_{k_a}) \hat{Y}_m / \hat{\mu} + \frac{1}{\theta_{k_m} \theta_{k_a}} \left( \delta \theta_{k_m} + \theta_{k_a} \frac{\mu \nu}{(1 - \mu)} \right) \right].
\]  

(2.6.38B)

Subtracting equation 2.6.38A from 2.6.38B, one can derive

\[
(\hat{w}_a - \hat{\rho}) / \hat{\mu} = \frac{1}{\theta_{k_a}} \left[ (\delta \theta_{k_m} - \nu) \hat{Y}_m / \hat{\mu} + \left( \delta \theta_{k_m} + \frac{\mu \nu}{(1 - \mu)} \right) \right].
\]  

(2.6.38C)

Similarly, from equations 2.6.20, 2.6.22, 2.6.30, and 2.6.38A–2.6.38C when \( \hat{p} = \hat{\tau} = 0 \) and \( \hat{w}_m = 0 \), one gets

\[
\hat{L}_n / \hat{\mu} = \left( 1 - \sigma_m \nu \right) \hat{Y}_m / \hat{\mu} + \left( 1 - \sigma_m \right) \frac{\mu \nu}{(1 - \mu)}.
\]  

(2.6.38D)

\[
\hat{L}_a / \hat{\mu} = \hat{Y}_a / \hat{\mu} - \delta \hat{Y}_m / \hat{\mu} - \sigma_a \delta_{k_a} (\hat{w}_a - \hat{\rho}) / \hat{\mu} - \delta.
\]  

(2.6.38E)

Also from equation 2.6.30, when \( \hat{p} = \hat{\tau} = 0 \), one gets
\[
\frac{\hat{M} / \hat{\mu}}{\hat{M} / \hat{\mu}} = -\frac{\delta\theta_{k_a} - \nu\theta_{k_a}}{\theta_{k_a}} \hat{Y}_m / \hat{\mu} - \frac{\theta_{k_a}}{\theta_{k_a}(1 - \mu)} \left( \frac{\mu \nu}{(1 - \mu)} + \delta \frac{\theta_{k_a}}{\theta_{k_a}} \right).
\]

(2.6.38F)

Thus, since \(\hat{Y}_a / \hat{\mu} \geq 0\), and \(\hat{Y}_m / \hat{\mu} \leq 0\), from equation 2.6.38D it is clear that the effect of an increase in public rural expenditure on urban employment depends on the values of elasticity of substitution \(\sigma_m\), the allocation ratio \(\mu\), and the elasticity of public spending in the urban sector \(\nu\). Assuming \(0 < \sigma_m < 1\), following proposition can be stated:

**Proposition 9:** An increased allocation of public spending to the rural sector will increase (reduce) urban employment only if the direct output effect is higher (lower) than the productivity effect of public services in the urban sector, and \(0 < \sigma_m < 1\). Also, the unemployment rate will fall if the combined output effects outweigh the effects on the wage-rental differential.

\[
\left| \frac{\hat{Y}_m / \hat{\mu}}{\hat{Y}_a / \hat{\mu}} \right| < (>) \frac{\mu \nu (1 - \sigma_m)}{(1 - \mu)(1 - \nu + \sigma_m \nu)}
\]

Proposition 9 shows that an increased allocation of public spending to the rural sector can still increase the proportion of urban employment. This occurs under the condition that the negative scale effect on urban output due to expenditure cuts is more than offset by increased labor productivity in urban employment when the level of urban employment falls. The positive employment effect in the urban sector arises from the positive output effect in the rural sector when rural spending is greater. This leads to a rise in the rural wage and induces reverse migration from the urban sector. If this rising urban productivity more than offsets the output effect of the public expenditure decline, the urban employment rate increases and the unemployment rate falls.

Furthermore, under alternative scenarios, the following can be obtained for the effects of changes in \(\mu\) on factor prices and employment:
I. \( \delta = \nu = 0 \), i.e., public spending does not generate productive services both in the rural and urban sectors: 

\[
\hat{w}_u / \hat{\mu} = 0, \hat{r} / \hat{\mu} = 0, (\hat{w}_u - \hat{r}) / \hat{\mu} = 0, \hat{L}_u / \hat{\mu} = 0, \hat{L}_m / \hat{\mu} = 0, \hat{\varepsilon} / \hat{\mu} = 0.
\]

Thus, an increased allocation to the rural sector has no impact on wages, rentals, and employment in the rural and urban sectors when public spending is not productive in any sector.

II. When \( \delta = 1, \nu = 0 \), i.e., public spending is highly effective in the rural sector, one obtains

\[
\hat{r} / \hat{\mu} = 0, \quad \hat{w}_u / \hat{\mu} = \frac{1}{\theta_{L_u}} (1 - \pi_{K_u} \pi_{L_u} \sigma_a / \theta_{L_u} \Omega) > 0
\]

\[
(\hat{w}_u - \hat{r}) / \hat{\mu} > 0, \hat{L}_u / \hat{\mu} > 0, \hat{L}_m / \hat{\mu} = (1 - \nu) \hat{Y}_m / \hat{\mu} - \sigma_m \theta_{K_u} (\hat{w}_u - \hat{r}) / \hat{\mu} + \frac{\nu}{1 - \mu}.
\]

An increased allocation to the rural sector will increase wages and employment in the rural sector. The effect on urban employment, and hence urban unemployment, is ambiguous. Depending on the relative magnitudes of \( \sigma_a, \sigma_m, \delta, \) and \( \nu \), urban employment may rise if

\[
\left| (1 - \nu) \hat{Y}_m / \hat{\mu} - \sigma_m \theta_{K_u} (\hat{w}_u - \hat{r}) / \hat{\mu} \right| < \frac{\nu}{1 - \mu}.
\]

This reduces the urban unemployment level, and reduces out-migration from the rural areas.

III. \( 1 > \delta > \nu > 0, \delta = \nu / \theta_{K_u}, \sigma_a > 1 - \mu > 0 \), i.e., public spending has higher elasticity with respect to output in the rural sector than in the urban sector, but equal wage and rental elasticities of output.

From equations 2.6.38A to 2.6.38F, we can derive

\[
\hat{r} / \hat{\mu} = \delta \left( \hat{Y}_m / \hat{\mu} - \frac{\mu}{1 - \mu} \right) < 0,
\]

\[
\hat{w}_u / \hat{\mu} = \delta \hat{Y}_m / \hat{\mu} + \frac{\delta}{\theta_{L_u}} \left( 1 + \frac{\mu \theta_{K_u}}{(1 - \mu)} \right) > 0 \quad \text{when} \quad \left| \hat{Y}_m / \hat{\mu} \right| < \frac{1}{\theta_{L_u}} \left( 1 + \frac{\mu \theta_{K_u}}{(1 - \mu)} \right).
\]
\[
(\hat{w}_a - \hat{r})/\hat{\mu} = \frac{\delta}{\theta_{K_a}(1 - \mu)} > 0 ,
\]
\[
\hat{L}_u/\hat{\mu} = \hat{Y}_a/\hat{\mu} - \delta \hat{Y}_m/\hat{\mu} - \sigma_a / \theta_{K_a}(\hat{w}_a - \hat{r})/\hat{\mu} - \delta > 0
\]
when \[
\hat{Y}_a/\hat{\mu} - \delta \hat{Y}_m/\hat{\mu} > \frac{\delta \sigma_a \theta_{K_a}}{\theta_{K_a}(1 - \mu)} + \delta
\]
\[
\hat{\mu} < 0 \text{ or } > 0 \text{ if } -\frac{\partial \theta_{K_a} - \nu \theta_{K_a} \hat{Y}_m/\hat{\mu} < 0}{\theta_{K_a} \theta_{K_a}} < \frac{\theta_{K_a}}{\theta_{K_a} \theta_{K_a}} \left( \frac{\mu \nu}{(1 - \mu)} + \delta \theta_{K_a} \right)
\]
This shows that when the government services are ineffective (i.e., \(\delta = \nu = 0\)), an increased allocation will have no impact on output, employment, and factor returns. In the case where government services have higher elasticity in the rural than in the urban sector (i.e., \(\delta > \nu\)), \((\hat{w}_a - \hat{r})/\hat{\mu} > 0\), \(\hat{r}/\hat{\mu} < 0\), \(\hat{w}_a/\hat{\mu} > 0\), \(\hat{L}_u/\hat{\mu} > 0\), the sign of \(\hat{L}_m/\hat{\mu}\) is ambiguous. The rate of unemployment may fall or rise depending on the relative magnitudes of expansionary effects of public inputs, and contractionary effects of lower allocation on \(Y_m\). These results can be summarized in terms of the following proposition:

**Proposition 10:** When output elasticity with respect to public spending is high (\(\delta = 1\)) in the rural sector and relatively low in the urban sector (\(0 < \nu < 1\)), an increased allocation of development spending to the rural sector will enhance rural employment. Urban unemployment may rise (fall) if the size of the expansionary impact of productive public services is higher (lower) than the contractionary impact of lower allocation. This reduces out-migration from the rural to the urban sector.
2.6.4.2 An Increase in the Tax Rate ($\tau$) on Urban Output

We now assume that the price of the urban good ($p$), and the expenditure allocation ($\mu$) remain unchanged, and only the output tax rate ($\tau$) changes. In the equation system (2.6.31), setting $\dot{p} = 0$, and $\dot{\mu} = 0$, and solving for $\dot{Y}_a$ and $\dot{Y}_m$, we obtain

$$\dot{Y}_a / \dot{\tau} = (\Omega_{k_u} A_2 - \Omega_{k_u} B_2) / |\Omega|,$$  
and

$$\dot{Y}_m / \dot{\tau} = - (\Omega_{k_u} A_2 - \Omega_{k_u} B_2) / |\Omega|. $$

We can analyze the effects under alternative assumptions regarding the efficiency of public spending.

I. $\delta = \nu = 0$, i.e., public spending does not generate any productive services both in either sector:

$$\dot{Y}_a / \dot{\tau} = \left[ \frac{\tau \pi_{k_u} \left( \frac{\pi_{k_u} \theta_{k_u} \sigma_a}{\theta_{l_u} \theta_{k_u}} + \epsilon \pi_{l_u} \sigma_m \right)}{1 - \tau \left( \frac{\pi_{k_u} \theta_{l_u} \sigma_a}{\theta_{k_u}} + \epsilon \pi_{l_u} \sigma_m \right)} + \frac{\pi_{l_u} \left( \pi_{k_u} \sigma_m \theta_{l_u} + \pi_{k_u} \sigma_a \right) \tau}{\left( 1 - \tau \right)} \right] / |\Omega| > 0 $$

$$\dot{Y}_m / \dot{\tau} = \left[ \frac{\tau \pi_{k_u} \left( \frac{\pi_{k_u} \theta_{k_u} \sigma_a}{\theta_{l_u} \theta_{k_u}} + \epsilon \pi_{l_u} \sigma_m \right)}{1 - \tau \left( \frac{\pi_{k_u} \theta_{l_u} \sigma_a}{\theta_{k_u}} + \epsilon \pi_{l_u} \sigma_m \right)} + \frac{\pi_{l_u} \left( \pi_{k_u} \sigma_m \theta_{l_u} + \pi_{k_u} \sigma_a \right) \tau}{\left( 1 - \tau \right)} \right] / |\Omega| < 0 $$

II. $\delta = 1, \nu = 0$, i.e., public spending is highly effective in the rural sector due to effective rural-development programs, but not productive in the urban sector: $\dot{Y}_a / \dot{\tau} > 0$, $\dot{Y}_m / \dot{\tau} < 0$ and $\dot{Y}_a / \dot{\tau} > 0$.

II. $\delta = 1, \nu = 0$, i.e., public spending does not generate any productive services in urban sector (e.g., building roads connecting villages that produce agricultural products with roads to the market areas will generate high returns than additional roads to heavily congested area in a heavily congested city, as it is in many developing countries).
\[ A_2 = \pi_{l_a} + \frac{\pi_{k_a} \theta_{k_a} \sigma_a}{\theta_{l_a} \theta_{k_a}} \left( \theta_{k_a} + \frac{\tau}{1-\tau} \right) + \frac{\epsilon \pi_{l_a}}{\theta_{l_a} \theta_{k_a}} + \frac{\epsilon \pi_{l_a}}{\theta_{l_a}} \left( \sigma_m + \frac{\theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \right) \]

\[ B_2 = \pi_{k_a} (1 - \sigma_a) - \frac{1}{\theta_{k_a}} \left( \pi_{k_a} \sigma_m \theta_{l_a} + \pi_{k_a} \sigma_a \right) \frac{\tau}{(1-\tau)} \]

\[ \Omega_{k_a} = \pi_{k_a} - (1 - \sigma_a) \pi_{k_a}, \quad \Omega_{l_a} = \left( \pi_{k_a} + \frac{\pi_{l_a} \theta_{k_a} \sigma_a}{\theta_{l_a}} + \theta_{l_a} \theta_{k_a} \right) \]

\[ \hat{Y}_a / \hat{\tau} = (\Omega_{k_a} A_2 - \Omega_{l_a} B_2) / |\Omega| > 0, \text{ since } A_2 > 0, B_2 < 0, \Omega_{k_a} > 0, \Omega_{l_a} < 0 \]

\[ \hat{Y}_m / \hat{\tau} = -(\Omega_{k_a} A_2 - \Omega_{l_a} B_2) / |\Omega| \]

\[ \hat{y}_m / \hat{\tau} = \pi_{l_a} \pi_{k_a} (1 - \sigma_a) - \frac{1}{\theta_{k_a}} \left( \pi_{l_a} \pi_{k_a} \sigma_m \theta_{l_a} + \pi_{l_a} \pi_{k_a} \sigma_a \right) \frac{\tau}{(1-\tau)} - \pi_{k_a} \pi_{l_a} \]

\[ \hat{y}_m / \hat{\tau} = \left[ \frac{\sigma_m \pi_{l_a} \pi_{k_a}}{\theta_{k_a} \theta_{l_a}} \left( \theta_{k_a} + \frac{\tau}{1-\tau} \right) + \frac{\epsilon \pi_{l_a} \pi_{k_a}}{\theta_{l_a}} \left( \sigma_m + \frac{\theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \right) + \frac{\epsilon \pi_{l_a} \pi_{l_a}}{\theta_{l_a}} \right] / |\Omega| < 0 \]

III. \( 1 > \delta > \nu > 0 \), i.e., public spending is equally effective both in the urban and rural sectors:

By assumptions (1) and (2), the signs of \( |\Omega| \) and \( \Omega_y \) are positive as well as \( 1 > \delta > \nu > 0 \).

In order to derive meaningful results, we assume \( 0 < \sigma_a < 1, 0 < \sigma_m < 1, \) and \( 0 < \nu < \frac{\tau}{1-\tau} < 1 \)

which implies that \( \tau < 0.5 \). Thus we get \( A_2 > 0, B_2 < 0, \hat{Y}_a / \hat{\tau} > 0 \) and \( \hat{Y}_m / \hat{\tau} < 0 \). These results can be summarized in terms of the following proposition:

**Proposition 11:** Under our assumption of higher capital intensity for the urban sector, an increase in the tax on urban output leads to an increase in the output of the rural sector, when public services yield higher returns \( (1 > \delta > \nu \geq 0) \) in the rural sector. The effect on urban output
is ambiguous and it depends on the size of expansionary effects of the expenditure allocation relative to the contractionary effects on the tax-base.

We now use these output responses to derive the effects on sectoral factor prices, sectoral employment, and the rate of urban unemployment.

From equation (2.6.16), when \( \hat{p} = \hat{\tau} = 0 \) and \( \hat{w}_m = 0 \):

\[
\hat{r} / \hat{\tau} = \frac{\nu}{\theta_{k_u}} \hat{y}_m / \hat{\tau} + \frac{1}{\theta_{k_u}} \left( \nu - \frac{\tau}{1 - \tau} \right)
\]

\[
= -\frac{\tau}{(1 - \tau) \theta_{k_u}} \leq 0 \quad \text{when} \quad \delta = \nu = 0 \tag{2.6.42A}
\]

\[
= \frac{\nu}{\theta_{k_u}} \left( \hat{y}_m / \hat{\tau} + 1 - \frac{\tau}{\nu(1 - \tau)} \right) \leq 0 \quad \text{when} \quad 1 \geq \delta \approx \nu > 0
\]

since \( \hat{y}_m / \hat{\tau} \leq 0, \) and \( 0 \leq \tau, \nu \leq 1 \)

\[
\hat{w}_a / \hat{\tau} = \frac{\delta \theta_{k_u} - \nu \theta_{k_u} \hat{y}_m / \hat{\tau}}{\theta_{k_u} \theta_{k_u}} - \frac{\theta_{k_u}}{\theta_{k_u} \theta_{k_u}} \left( \nu - \frac{\tau}{1 - \tau} - \frac{\delta \theta_{k_u}}{\theta_{k_u} \theta_{k_u}} \right)
\]

\[
= \frac{\theta_{k_u} \tau}{\theta_{k_u} \theta_{k_u} (1 - \tau)} \geq 0, \quad \text{when} \quad \delta = \nu = 0
\]

\[
= \frac{\nu(\theta_{k_u} - \theta_{k_u})}{\theta_{k_u} \theta_{k_u}} \hat{y}_m / \hat{\tau} + \frac{\theta_{k_u}}{\theta_{k_u} \theta_{k_u}} \left( \frac{\tau}{1 - \tau} + \frac{\nu \theta_{k_u} - \nu}{\theta_{k_u} \theta_{k_u}} \right), \tag{2.6.42B}
\]

when \( 1 \geq \delta \approx \nu > 0 \)

Subtracting (2.6.42A) from (2.6.42B):

\[
\left( \hat{w}_a - \hat{r} \right) / \hat{\tau} = \left( \frac{\delta \theta_{k_u} - \nu}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \right) \hat{y}_m / \hat{\tau} - \frac{\theta_{k_u}}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \left( \nu - \frac{\tau}{1 - \tau} - \frac{\delta \theta_{k_u}}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \right) + \frac{\theta_{k_u}}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \left( \nu - \frac{\tau}{1 - \tau} \right)
\]

\[
= \frac{1}{\theta_{k_u} \theta_{k_u} \theta_{k_u} (1 - \tau)} \geq 0 \quad \text{when} \quad \delta = \nu = 0
\]

\[
= -\left( \frac{\nu \theta_{k_u}}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \right) \hat{y}_m / \hat{\tau} + \frac{\theta_{k_u}}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} \left( \frac{\nu(\theta_{k_u} - \theta_{k_u})}{\theta_{k_u} \theta_{k_u} \theta_{k_u}} + \frac{\tau}{1 - \tau} \right) \geq 0 \quad \text{when} \quad 0 \leq \delta \approx \nu \leq 1
\]

(2.6.42C)
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Also from (2.6.20), (2.6.22) and (2.6.30)

\[
\hat{L}_m / \hat{\tau} = (1-\nu)\hat{Y}_m / \hat{\tau} - \sigma_m \theta_{K_m} (\hat{w}_m - \hat{r}) / \hat{\tau} - \nu \tag{2.6.42D}
\]

\[
\hat{L}_a / \hat{\tau} = \hat{Y}_a / \hat{\tau} - \delta \hat{Y}_m / \hat{\tau} - \sigma_a \theta_{K_a} (\hat{w}_a - \hat{r}) / \hat{\tau} - \delta
\]

\[
= \hat{Y}_a / \hat{\tau} - \nu \hat{Y}_m / \hat{\tau} - \frac{\sigma_a \theta_{K_a} \tau}{\theta(1-\tau)} - \nu \tag{2.6.42E}
\]

when \( 1>\delta \approx \nu > 0 \) \( \left| \hat{Y}_a / \hat{\tau} - \nu \hat{Y}_m / \hat{\tau} \right| > \left| \frac{\sigma_a \theta_{K_a} \tau}{\theta(1-\tau)} - \nu \right| \)

Also from 2.6.30, when \( \hat{\rho} = \hat{\mu} = 0 \), we get

\[
\hat{e} / \hat{\tau} = -\frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{l_a} \theta_{K_a}} \hat{Y}_m / \hat{\tau} + \frac{\theta_{K_a}}{\theta_{l_a} \theta_{K_a}} \left( \nu - \frac{\tau}{1-\tau} - \delta \frac{\theta_{K_a}}{\theta_{K_a}} \right)
\]

\[
= -\frac{\tau}{1-\tau} \frac{\theta_{K_a}}{\theta_{l_a} \theta_{K_a}} < 0, \text{ when } \delta = \nu = 0 \tag{2.6.42F}
\]

\[
= -\frac{\nu(\theta_{K_a} - \theta_{K_a})}{\theta_{l_a} \theta_{K_a}} \hat{Y}_m / \hat{\tau} - \frac{\theta_{K_a}}{\theta_{l_a} \theta_{K_a}} \left( \frac{\tau}{1-\tau} + \frac{\nu(\theta_{K_a} - \theta_{K_a})}{\theta_{K_a}} \right) \text{ when } \delta \approx \nu \leq 1
\]

since by assumption the modern sector is relatively capital-intensive both in physical terms and factor-share-adjusted terms, we get \( \theta_{K_a} - \theta_{K_a} > 0 \). These equations show that even when government services have zero output elasticities to public spending, i.e., \( \delta = \nu = 0 \), an increase in the urban tax will have an impact on output and employment. Urban wages will remain unaffected, but the rental for capital will fall and \( (\hat{w}_a - \hat{r}) / \hat{\tau} > 0 \). In the case of equally productive government services in the rural and urban sectors (i.e., \( \delta \approx \nu > 0 \)), \( (\hat{w}_a - \hat{r}) / \hat{\tau} > 0 \), and \( (\hat{w}_m - \hat{r}) / \hat{\tau} > 0 \), \( \hat{L}_a / \hat{\tau} > 0 \), and \( \hat{L}_m / \hat{\tau} < 0 \). So unemployment falls for any given \( \varepsilon \). Thus we can state the following:

**Proposition 12:** When public spending elasticities of output are equal in the two sectors, an increased tax on urban output will enhance rural employment and reduce urban unemployment.
for any given wage differential. This in turn reduces out-migration from the rural to the urban sector.

2.6.5. Welfare Analysis

In the same way we derived the welfare effects in the full-employment version of the model\(^5\), we can analyze the welfare impact of the changes in the policy variables. These effects are given by:

\[
\frac{dW}{d\mu} = Y_m \left( \frac{\delta Y_u}{\mu Y_m} - \frac{\nu p(1-\tau)}{(1-\nu)(1-\mu)} \right) + \left( \frac{\delta Y_u}{Y_m} + \frac{\nu p(1-\tau)}{1-\nu} \right) \frac{dY_m}{d\mu} - \frac{w_m L_m}{\epsilon} \frac{d\epsilon}{d\mu}
\]  

(2.6.43)

\[
\frac{dW}{d\tau} = \left( \frac{\delta Y_u}{Y_m} + \frac{\nu p(1-\tau)}{1-\nu} \right) \frac{dY_m}{d\tau} + Y_m \left( \frac{Y \delta}{Y_m \tau} + \frac{\nu p(1-\tau)}{\tau(1-\nu)} - p \right) - \frac{w_m L_m}{\epsilon} \frac{d\epsilon}{d\tau}
\]  

(2.6.44)

Thus, welfare effects will depend on the primary growth effect, scale effect, and employment effect. This shows that the welfare effect depends on the relative shares and relative sizes of expansionary effects and contractionary effects of \(\mu\) and \(\tau\) on \(Y_m\). Whereas an increase in \(\mu\) has direct negative effects on \(Y_m\), an increase in \(\tau\) has both an expansionary and contractionary impact on \(Y_m\). An increased allocation to the rural sector, or an increase in output tax, and hence increased rural expenditure, may be welfare enhancing or immiserizing, depending on the relative sizes of the three effects, based on the values of the parameters \(\delta, \nu, \mu, \tau\) and relative sizes of \(Y_m\) and \(Y_u\). In a future extension, following Bhagwati (1974), as well as Krichel and Levine (1999), we would like to calibrate this model to analyze and rank various policies as well as to quantify the welfare implications.

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\(^5\)As mentioned in section 2.5.3, following the conventional Harris-Todaro literature, we use a simple welfare analysis of a representative private agent in a small open economy where \(p\) is determined by world prices, and welfare of the private agent is measured by the real income at world prices after tax. Here we abstract from the heterogenous consumption pattern across rural and urban sectors.
2.7. A Note on the Effects of Foreign Aid

Renewed thinking on the development process has led to a reconsideration of the role of foreign aid in promoting economic development. External aid is no longer considered to be an instrument that just promotes economic development by removing the savings or the foreign exchange gap. The focus also includes poverty alleviation, employment generation, and income distribution (Akramov, 2006). In order to provide direct assistance to the poor, many donor agencies such as the World Bank, the U.S. Agency for International Development, and the UK Department for International Development have started to focus on financing projects in education, health, agriculture, and rural development. Hence, in many developing countries, these significant inflows of foreign aid, if directed toward development expenditure in rural areas, will influence the effects of government expenditure on national output and welfare. It would be interesting to explore their effects in detail as well. Qualitatively one can expect the following:

1) In the dual-economy framework, opening the economy to aid or capital flows would raise demand for labor and wages in the modern sector and increase the urban-rural wage differential, thus accelerating rural-to-urban migration, in the presence of well-established rural-urban linkages.

2) Aid toward the rural sector that is designed to improve the state of rural technology would result in an increase in government services and in wages for both sectors. This will also reduce wage inequality (Mourmouras and Rangazas, 2013). Thus, directing aid projects to the rural sector will benefit households in both sectors.
3) Aid would lessen the burden on urban taxpayers to finance rural development. Hence, productive government services financed by aid would enhance output and employment in both sectors, reduce wage inequality, and decrease migration due to higher rural wages and other development amenities.

4) While foreign aid should help undo the urban bias of domestic spending and contribute to improving rural conditions, the results would be subject to caveats. Urban policymakers are likely to resist the conditions that are generally imposed by aid agencies and would want to divert aid to their own constituencies and for private gain. These issues of incentive compatibility would be interesting to explore.

5) Information asymmetry (Majumdar et al., 2004) on the availability and allocation of foreign aid between rural and urban areas might poses its own problems. Greater wealth and higher education imply that urban residents can better exploit information to take advantage of aid. For example, in Nepal access to the press is constrained by poverty, low literacy, and inadequate transport (World Bank, 2002). Other factors favoring information access for the urban population include biased media coverage that serves wealthier and educated urban clients, and urban bias due to positive information externalities as a result of high population density that can prevent a reduction in the wage differential or migration.

In most developing countries, growth is largely driven by urban areas, and urban-led growth can have a positive and stimulating impact on rural development through increased employment and higher incomes in the presence of well-established rural-urban linkages. However, in many countries poor transportation networks, law and order problems, and the
urban bias of large labor-intensive projects prevent the rural sector from benefiting from such growth. Urban bias in allocation of foreign-aid-funded development projects could then encourage rural-urban migration and lead to urban unemployment, with the associated problem of declines in law and order in cities (DFID, 2006). Recent country studies show that pro-urban development promoted by strongly centralized governments induces large migration flows to the capital cities. To alleviate lopsided regional economic growth, more effective ways to develop less-developed regions will be necessary (van Lottum and Daan, 2012).

2.8. Conclusion

This chapter has presented two dual-economy models of migration between rural and urban sectors of a developing economy with an explicit government budget constraint. In addition to capital and labor, government expenditure serves as an input in the production process that brings forth external economies. The expenditure must, however, be financed through a tax on urban output. Rural taxation is ruled out because of real-world evidence that such taxation does not directly contribute to government revenue in many developing countries, and in order to keep the model sufficiently simple.

The first model is the neoclassical general-equilibrium model of two sectors with full employment of labor in each sector. The second model (presented in Section 2.6) retains external economies of scale in public expenditure but augments the original model to consider positive unemployment in the urban sector. Migration from rural to urban areas occurs as a response to a positive urban wage differential and stops when the rural wage equals the expected urban wage that includes the effect of urban unemployment.
Each of the two models analyzes two important cases. They examine what happens to output, employment, and urban unemployment when the government varies the fraction of total expenditure going to the rural and urban sectors while still satisfying the budget constraint. A basic assumption of the model is that the urban sector uses more capital per unit of labor than does the rural sector—that is, urban output is relatively capital-intensive.

In the full employment version of the model, the effects of increased government expenditure in the rural sector provide an interesting case study. An enormous potential to raise agricultural productivity goes untapped because rural areas are starved of public expenditure as a result of urban bias. This study finds that a higher expenditure share going to the rural sector leads to three distinct possibilities. First, assuming that public goods contribute more to the productivity of capital and labor in rural than in urban areas, rural output increases and so does national output when the urban sector is not very large. Second, if employment and output in the urban sector are large enough, the reduction of the tax base that follows from a decrease in the urban share of public expenditure and hence urban output can lead to a decrease in overall national output as well. Third, the last result may not hold (and rural and total output can still rise) if government services contribute sufficiently more to rural output than to urban output.

Second, the study analyzes the effects of an increase in the tax on urban output. An urban tax increase, holding expenditure shares constant, will reduce urban output but raise both rural and urban expenditures. Thus, rural output invariably increases unless government services yield a marginal productivity of zero. This also leads to a reduction in out-migration from rural to urban areas, and can create reverse migration back to rural areas. The effect on urban output is, however, ambiguous. Under the condition that scale economies of greater public expenditure to urban output dominate the contractionary effect of the tax increase, urban manufacturing will
expand. Meanwhile, the study also finds several mixes of parameters that yield a loss of urban output, a reduction in the tax base, and an overall decline in total output and welfare in the country. The result regarding the direction of migration similarly depends on the values of parameters in the model.

The second model is of the Harris-Todaro type augmented by public expenditures. The first point to note here is that the stability condition (Khan-Neary condition) that shows the stability of the model in employment-adjusted terms in the conventional Harris-Todaro version is more stringent in this augmented version.

A main finding of the study is that increased public spending in the rural sector can still increase urban employment. This follows from the condition that the negative returns-to-scale effect on urban output due to expenditure cuts is more than offset by increased urban labor productivity when urban employment falls. The positive employment effect in the urban sector arises from the positive output effect in the rural sector when rural spending is greater. This leads to a rise in the rural wage and induces reverse migration from the urban sector. If the rising urban productivity more than offsets the output effect of the decline in public expenditure, then the urban employment rate increases and the unemployment rate falls.

The welfare effects in the unemployment model are subject to a combined influence of a set of elasticities and go different ways depending on the relative influence of composite parameters of the model.

Finally, the results of the model indicate several directions for further research, two of which seem particularly promising. First, since poor countries typically have a large rural sector, governments are inclined to impose a tax on this sector in various ways.
Second, significant inflows of foreign aid, if directed toward development expenditure in rural areas, will influence national output and welfare. Productive government services financed by aid should raise the output and employment level in both rural and urban sectors, lessen wage inequality, and reduce the rate of migration due to higher rural wages and other more services in rural areas.

The models can be extended to include dynamics following Mas-Colell and Razin (1973). Also, economic growth can be studied in a dynamic setting. Some estimates on the costs of dualism could be obtained through simple calibration exercise using data on migration, output, and income shares from India, China, Ghana, and other South Asian and sub-Saharan economies where migration is an important aspect of the labor market.

While not attempted here, heterogeneous consumption pattern in rural and urban areas, as observed in practice, could allow the use of a social welfare function of the following form, \( W = \Gamma(U_a, U_m) \), where \( \Gamma_a > 0 \) and \( \Gamma_m > 0 \), as expounded by Atkinson and Stiglitz (1980) in our welfare analysis.

**Appendix 2.1 Survey of Literature on Dual Economy Models**

This appendix surveys the theoretical and empirical studies on labor-related migration, which plays a significant role in the urbanization process and is also viewed as the labor adjustment accompanying the shift in importance from agriculture to manufacturing. The appendix draws on various recent reviews and articles, including Todaro and Smith (2012), Lall et. al (2006), and Temple (2005). The theoretical framework regarding internal migration has been usually classified into three types: (1) Dual-economy models (1950s and 1960s); (2) Harris-Todaro
models (1970s and 1980s); and (3) Microeconomic models. Each of the models is discussed briefly below.

1. **Dual-economy models**: Lewis (1954) can be credited with developing the first theoretical framework involving internal migration. His model assumes that a rural economy is characterized by surplus labor, which has a marginal product of zero and receives wages equal to the average product. This provides the background for an elastic supply of labor to the growing manufacturing sector where wages can be slightly higher to induce migration. This process continues until all surplus labor is absorbed by manufacturing and other modern sectors.

The assumption of zero marginal productivity has been criticized, but what matters is the capacity of the rural sector to provide labor to the urban sector. The model therefore suggests that internal migration is desirable if it facilitates growth. The implication of this framework is that governments in countries experiencing growth should see to it that labor migration is not impeded but rather encouraged. However, the framework is limited because it cannot explain the high levels of unemployment observed in urban areas. The harmful effects of migration are not captured by this model.

2. **Harris-Todaro models**: In these models (Todaro, 1969; Harris-Todaro, 1970) the focus is on the harmful effect of migration, that is, urban unemployment. The models suggest that the elasticity of urban labor supply with respect to the wage differential is greater than the wage differential itself. An increase in job creation will trigger an increase in migration and hence the number of unemployed workers. Todaro (1969) argues that this explains the high unemployment levels of Kenya in 1964 as a result of the government’s initiative to increase employment.
These models imply that policy initiatives should be directed to restricting migration flows and creating rural jobs, which is different from creating jobs in the urban sector. The latter increases the number of high-wage jobs, but at the same time increases inequality and unemployment.

The model has been criticized as static in trying to describe the phenomenon of migration that is fundamentally dynamic. It has also been criticized for not explicitly modeling the subsistence sector that employs uneducated migrants (Cole and Sanders, 1985). Further, the assumption that migration is led by expected income differentials may overlook other important factors, as there is evidence of urban migration despite urban expected income being less than rural income (Katz and Stark, 1986a). In view of these critiques, later models have departed from the Harris-Todaro setting and presented internal migration in a much different light.

3. **Microeconomic models**: These models dwell on the potentially beneficial role of migration and try to address the questions of migration selectivity, that is, who chooses to migrate, when and how to migrate, and the effect of migration on the rural economy.

The question of who migrates has sought answers in the role of information asymmetry, inadequate insurance, inefficiency of credit markets, or relative deprivation. Migration takes place under the condition of information asymmetry where information about skills does not flow freely. According to Katz and Stark (1984), rural household employers have more knowledge about the skills of their workers than urban employers in the host region. Hence skilled workers may not find it beneficial to migrate for fear of receiving lower wages. However,
Katz and Stark (1986b) later suggest that skilled migrants can signal themselves to host region employers by incurring a moderate amount of money.

Another motivation for migration could be an improvement in social status in the home community (Stark, 1984; Katz and Stark, 1986a). The models that include such an incentive predict that desire for social status can lead to migration even without much monetary gain.

On the issue of when and how to migrate, a detailed job search framework is used in which migration and job search are joint decisions. Vishwanath (1991) has developed a model of a continuous lifetime program of search and migrants. His model is based on the assumption that rural individuals have three options: stay home forever, engage in the urban job search from home, or engage in the urban job search after migrating. This model explains why migration can be rational even if the mean urban wage is below rural wage. Sato (2004) develops a model incorporating frictional urban unemployment. His model suggests that a policy that encourages rural-to-urban migration will increase welfare.

The implication of these models is that governments can play a significant role in encouraging migration, helping migrants in the job search, and thereby increasing the efficiency of job matches.

Possibly one of the main contributions of the microeconomic models is that they provide more insight into the role of migration in rural development through remittances.

**Empirical Studies**

Theoretical formalization does not provide any single message regarding internal migration. In the empirical literature, we do not find structural tests of the theories reviewed above, but rather
only partial findings that cannot validate or contradict them. This section reviews some of the prominent empirical studies.

Studies on the determinants of internal migration have identified that migration responds to spatial differences in incomes net of migration costs. Greenwood (1997) provides details of the studies that show estimations of “modified gravity models” of migration. The models consider migration as directly related to the size of the population at both origin and destination, and inversely related to the distance.

The bulk of the literature on internal migration is directed toward assessing the consequences of migration. These studies can be broadly classified based on the consequences of migration for (1) the individual, (2) the rural area of origin, (3) the urban area of destination, and (4) the economy as a whole.

**Consequences for the Individual**

Most studies assess the economic condition of migrants in terms of financial gains by comparing the wages between rural and urban jobs. However, these studies are criticized because migration is seen as taking place even when the destination offers a lower net income (Vishwanath, 1991; Katz and Stark, 1986a). One important reason for a perceived anomaly could be that workers do not mind a short-term loss for long-term gains.

Borjas and Trejo (1992) provide evidence that although the migrants may earn less in the initial years, the difference withers away over time. In her study on migration to Bangkok, Yamauchi (2004) finds that educated migrants are able to raise their wages faster than their uneducated counterparts. Yamauchi and Tanabe (2003), again in relation to migration to Bangkok, show that the job search experience of the previous cohorts of migrants increases the
employment probability of new migrants. However, the probability declines as the size of the migrant population increases, a result the authors ascribe to the congestion effect of migrants.

The policy implication emerging from these studies is that education or social networks can be of help to migrants. Measures to improve the bargaining power of migrants might include information sharing and collective negotiations (Mosse et al., 2002). Migrants may be encouraged to take up entrepreneurship through the provision of credit facilities. Yet, they may nevertheless encounter severe hurdles in their attempts to find satisfactory access to health care and education for themselves and their children. Evidence from China (Shaokang, Zhenwei, and Blas, 2002) shows that rural migrants face strong obstacles in terms of access to health care. If smooth migration is a goal of urban development policy, a congenial environment needs to be created for rural migrants. Measures may also be taken to preserve their rights in the area of origin while they are away.

**Consequences for the Rural Area of Origin**

Several studies have investigated how migration affects the families of migrants and rural areas of origin. Stark, Taylor, and Yitzhaki (1986) examine whether remittances increase or decrease inequality in rural areas. They reveal that in villages with many households having links to internal migrants, remittances from the migrants have an equalizing impact on village income distribution. The findings are similar for countries like Pakistan (Adams, 1994) and the Philippines (Rodriguez, 1998).

Another important question that has been studied in the context of the rural area of origin is how migration can affect production there. According to the Lewis (1954) model, rural production does not fall, as it is the surplus labor that is siphoned off to urban jobs. In his study of migration to South African mines, Lucas (1987) found rural crop production decreases in the
short run but increases over time as investment from remittances begins to yield substantial returns. The findings of Rozelle, Taylor, and DeBrauw (1999) for farm households in northeast China are similar to those of Lucas (1987). In addressing the issue of human capital growth at the place of origin, Kochar (2004) showed that in India, rural schooling decisions are undertaken keeping in mind the possibility of employment in urban areas.

**Consequences for the Urban Area of Destination**

Studies on the effects of migration for the destination areas are rather scarce. The early papers on this tried to discuss the Todaro paradox (Todaro, 1976; Garcia-Ferrer, 1980; Salvatore, 1981; and Lucas, 1985). Wrage (1981) has investigated the effect of internal migration on wages in urban areas of destination. The Todaro model argues that a migration-induced increase in labor supply could depress wages. Another sensitive issue that has been investigated by Knight, Song, and Huaibin (1999) is whether migrants take up the jobs that would otherwise be available to locals. Their results suggest that the migrants of rural origin get the jobs that are shunned by locals. However, investigating the same issue, Roberts (2001) found opposite results, since local workers laid off by state enterprises are observed competing with migrant laborers in the formal sector.
Appendix 2.2. Derivation of Stability Condition in the Neoclassical Model

Differentiating (2.32) and considering (2.17) in the text yields:

\[
-\left(\frac{L_a}{Y_a}\right)dw_a - \left(\frac{K_a}{Y_a}\right)dr - \left[\left(\frac{w_a}{Y_a}\right)dL_a + \left(\frac{r}{Y_a}\right)dK_a\right] - a_{La}dw_a - a_{Ka}dr - \left(\frac{w_aL_a}{Y_a}\right)\hat{L}_a + \left(\frac{rK_a}{Y_a}\right)\hat{K}_a
\]

\[
= -a_{La}dw_a - a_{Ka}dr - \left\{\left(\frac{w_aL_a}{Y_a}\right)\hat{L}_a + \left(\frac{rK_a}{Y_a}\right)\hat{K}_a\right\} - a_{La}dw_a - a_{Ka}dr - \left(\frac{w_aL_a}{Y_a}\right)\hat{L}_a + \left(\frac{rK_a}{Y_a}\right)\hat{K}_a
\]

\[
= -a_{La}dw_a - a_{Ka}dr - \left\{\left(\theta_{La} + \theta_{Ka}\right)\hat{L}_a - \left(\theta_{La} + \theta_{Ka}\right)\hat{L}_a\right\} - a_{La}dw_a - a_{Ka}dr - \left(\frac{w_aL_a}{Y_a}\right)\hat{L}_a + \left(\frac{rK_a}{Y_a}\right)\hat{K}_a
\]

From 2.6.12, we get \(\hat{y}_a = \delta\hat{y}_m + \theta_{La}\hat{L}_a + \theta_{Ka}\hat{K}_a\), when \(\hat{\mu} = 0 = \hat{\tau}\) (2.32A)

Similarly, we have from (2.33):

\[
= -a_{La}dw_m - a_{Ka}dr - \left[p(1-\tau)(\theta_{La}\hat{L}_m + \theta_{Ka}\hat{K}_m) + p(1-\tau)\hat{Y}_m\right] + \left(a_{La}\sigma_a(1-\nu) - \delta L_a / Y_a\right)dY_m - a_{La}dw_m
\]

(2.33B)

Now, differentiating (2.34), and substituting 2.20, 2.22, 2.24, we obtain

we get the following:

\[
= -L_a\sigma_a\theta_{La}\frac{d\sigma_a}{w_a} + \left(L_a\sigma_a\theta_{Ka} + L_m\sigma_m\theta_{La}\right)\frac{dr}{p(1-\tau)r} + a_{La}\frac{dY_m}{\hat{Y}_m} + \left(a_{La}\sigma_m(1-\nu) - \delta L_a / Y_a\right)dY_m - L_a\sigma_a\theta_{La} / w_a dw_m
\]

(2.34B)

We get similar expressions from 2.35 and 2.21 and 2.23, and finally from (2.9) and (2.36) about the dynamics of efficiency wage.

Using equations 2.7, 2.16–2.17, and 2.26–2.27, the determinant of Jacobian matrix for the adjustment process can be written as,

\[
\begin{vmatrix}
-d_{La} & -d_{La} & 0 & d_{La}\delta / Y_m & 0 \\
0 & -d_{La} & 0 & d_{La}p(1-\tau)\nu / Y_m & -a_{La} \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
-d_{La} & d_{La}(\sigma_a\theta_{La} + L_a\sigma_m\theta_{ka} / p(1-\tau)) & d_{La} & d_{La}(1-\nu)\alpha_{La} - \delta L_a / Y_a & -d_{La}L_a\sigma_a\theta_{La} / w_a \\
d_{La}K_a\sigma_{La}a_{La} & -d_{La}(K_a\sigma_{La}\theta_{La} + K_a\theta_{ka}\sigma_{La}) / \hat{\tau} & d_{La} & d_{La}(1-\nu)\alpha_{La} - \delta K_a / Y_a & d_{La}K_a\theta_{La}\sigma_{La} / w_a \\
d_{La} & 0 & 0 & 0 & -d_{La} \\
\end{vmatrix}
\]

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Adding the terms in the 1st column to the corresponding terms in the 5th column, we obtain

\[
\begin{vmatrix}
-\theta_{t_k} & -\theta_{K_k} & 0 & \delta & 0 \\
0 & -\theta_{K_k} & 0 & \nu & -\theta_{t_m} \\
-\pi_{t_k}\sigma_a\theta_{k_k} & \pi_{t_k}\sigma_a\theta_{k_k} + \pi_{t_k}\sigma_m\theta_{k_k} & \pi_{t_k} & (1-\nu)\pi_{t_m} - \delta\pi_{k_m} & -\pi_{t_k}\theta_{k_m}\sigma_m \\
\pi_{K_k}\sigma_a\theta_{t_k} & -\pi_{K_k}\sigma_a\theta_{t_k} + \pi_{K_k}\theta_{t_k}\sigma_m & \pi_{K_k} & (1-\nu)\pi_{K_m} - \delta\pi_{k_m} & \pi_{K_k}\theta_{t_m}\sigma_m \\
0 & 0 & 0 & 0 & -w_m
\end{vmatrix}
\]

Expanding the $|J|$, by 5th row

\[
\begin{vmatrix}
-\theta_{t_k} & -\theta_{K_k} & 0 & \delta & -\theta_{t_m} \\
0 & -\theta_{K_k} & 0 & \nu & -\theta_{t_m} \\
-\pi_{t_k}\sigma_a\theta_{k_k} & \sigma_1 & \pi_{t_k} & (1-\nu)\pi_{t_m} - \delta\pi_{k_m} & -\sigma_1 \\
\pi_{K_k}\sigma_a\theta_{t_k} & -\sigma_2 & \pi_{K_k} & (1-\nu)\pi_{K_m} - \delta\pi_{k_m} & \sigma_2 \\
1 & 0 & 0 & 0 & 0
\end{vmatrix}
\]

Adding the elements of 1st column to the corresponding elements of 4th column,

\[
\begin{vmatrix}
-\theta_{K_k} & 0 & \delta & -1 \\
0 & -\theta_{K_k} & \nu & -1 \\
-\pi_{t_k}\sigma_a\theta_{K_k} & \sigma_1 & \pi_{t_k} & (1-\nu)\pi_{t_m} - \delta\pi_{K_m} \\
\pi_{K_k}\sigma_a\theta_{K_k} & -\sigma_2 & \pi_{K_k} & (1-\nu)\pi_{K_m} - \delta\pi_{K_m} \\
0 & 0 & 0 & 0
\end{vmatrix}
\]

Subtracting the elements of 2nd row from the corresponding elements of 1st row,

\[
\begin{vmatrix}
\theta_{K_k} - \theta_{K_k} & 0 & \delta - \nu & 0 \\
-\theta_{K_k} & 0 & \nu & -1 \\
\pi_{t_k} & \pi_{t_k} & (1-\nu)\pi_{t_m} - \delta\pi_{t_m} & 0 \\
-\pi_{K_k} & -\pi_{K_k} & (1-\nu)\pi_{K_m} - \delta\pi_{K_m} & 0
\end{vmatrix}
\]
Using the elimination process by 4th column,

\[
\begin{vmatrix}
\theta_{k_a} - \theta_{k_s} & 0 & \delta - \nu \\
\pi_{k_a} & (1 - \nu) \pi_{l_a} - \delta \pi_{k_a} \\
-\sigma_2 & (1 - \nu) \pi_{k_a} - \delta \pi_{k_a}
\end{vmatrix}
\]

Simplifying we obtain,

\[
\begin{vmatrix}
1 & 0 & \delta - \nu \\
\Delta_1 & \pi_{k_a} - \delta \pi_{k_a} \\
-\Delta_2 & (1 - \nu) \pi_{k_a} - \delta \pi_{k_a}
\end{vmatrix},
\]

where \( \Delta_1 = \frac{1}{|\theta|} (\pi_{l_a} \sigma_a \theta_{k_a} + \pi_{l_a} \sigma_m \theta_{k_a}) \), and \( \Delta_2 = \frac{1}{|\theta|} (\pi_{k_a} \sigma_a \theta_{l_a} + \pi_{k_a} \theta_{l_a} \sigma_m) \).

Then, adding the elements of 1st column*(-( - )) to the corresponding elements of 3rd column

\[
\begin{vmatrix}
1 & 0 & 0 \\
\Delta_1 & (1 - \nu) \pi_{l_a} - \delta \pi_{k_a} - \Delta_1 (\delta - \nu) \\
-\Delta_2 & (1 - \nu) \pi_{k_a} - \delta \pi_{k_a} + \Delta_2 (\delta - \nu)
\end{vmatrix}
\]

Appendix 2.3. Additional Derivations

We have

\[
\theta_{l_a} \hat{w}_a + \theta_{k_a} \hat{r} = \delta \hat{Y}_m + \delta (\hat{\mu} + \hat{\tau}) \tag{2.16}
\]

\[
\theta_{l_a} \hat{w}_m + \theta_{k_a} \hat{r} = \nu \hat{Y}_m + \hat{\rho} - \frac{\tau}{1 - \tau} \hat{\tau} + \nu \left( \hat{\tau} - \frac{\mu}{1 - \mu} \hat{\mu} \right) \tag{2.17}
\]

Also from (2.7) \( \hat{w}_m = \hat{w}_a + \hat{\alpha} \)

Subtracting (2.17) from (2.16), and using \( |\theta| = \theta_{l_a} - \theta_{k_a} = \theta_{k_a} - \theta_{l_a} \)

\[
\hat{w}_a - \hat{r} = \frac{1}{|\theta|} \left[ (\delta - \nu) \hat{Y}_m + \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \hat{\mu} + \left( \delta - \nu + \frac{\tau}{1 - \tau} \right) \hat{\tau} - \hat{\rho} - \theta_{l_a} \hat{\alpha} \right]
\]
**Derivation of Equations 2.43, 2.44, and 2.43a**

\[ dW = E_i dU = dY_a + p(1-\tau)dY_m \]  

(2.41)

From total differentials of equations 2.1 to 2.4

\[ \ln Y_a^* = \delta \ln(\mu r Y_a^*) + \ln F(K_a, L_a) = \delta \ln \mu + \ln \tau + \ln Y_a^* + \ln F(K_a, L_a) \]

\[ \frac{dY_a}{Y_a} = \frac{\delta}{\mu} d\mu + \frac{\delta}{\tau} d\tau + \frac{\delta}{Y_m} dY_m + \frac{dK_a}{Y_a} r + \frac{dL_a}{Y_a} w_d \]

\[ dY_a = \frac{\delta}{Y_m} Y_a dY_m + \delta Y_a (\hat{\mu} + \hat{\tau}) + r dK_a + w_d dL_a \]

From equations (2.2) and (2.4) we can write

\[ \ln Y_m = \nu \ln((1-\mu)\tau Y_m^*) + \ln H(K_m, L_m) = \nu \ln(1-\mu) + \ln \tau + \ln Y_m^* + \ln H(K_m, L_m) \]

\[ \frac{dY_m}{Y_m} = -\frac{\nu \mu}{(1-\mu)} \hat{\mu} Y_m + \frac{\nu}{(1-\nu)} Y_m \hat{\tau} + \frac{1}{p(1-\tau)(1-\nu)} (r dK_m + w_d dL_m) \]

Substituting (A) and (B) in 2.41

\[ dW = dY_a + p(1-\tau)dY_m \]

\[ dW = \frac{\delta}{Y_m} Y_a dY_m + \delta Y_a (\hat{\mu} + \hat{\tau}) + r dK_a + w_d dL_a - \]

\[ \frac{1}{(1-\nu)} \frac{p(1-\tau)\nu \mu}{(1-\mu)} \hat{\mu} Y_m + \frac{p(1-\tau)}{(1-\nu)} \nu Y_m \hat{\tau} + \frac{1}{(1-\nu)} (r dK_m + w_d dL_m) \]

\[ dW = \frac{\delta}{Y_m} Y_a dY_m - \frac{1}{\alpha} w_d dL_m - r dK_m + Y_a \delta (\hat{\mu} + \hat{\tau}) + \frac{1}{1-\nu} \left[ w_d dL_m + r dK_m + \nu p(1-\tau) Y_m \left( \hat{\tau} - \frac{\mu}{1-\mu} \hat{\mu} \right) \right] \]

\[ dW = \frac{\delta}{Y_m} Y_a dY_m + \frac{1}{(1-\nu)} \left( \frac{\alpha - 1}{\alpha - 1}(1-\nu) \right) w_d dL_m + \nu p(1-\tau) dY_m + Y_a \delta (\hat{\mu} + \hat{\tau}) + \frac{1}{1-\nu} \left[ \nu p(1-\tau) Y_m \left( \hat{\tau} - \frac{\mu}{1-\mu} \hat{\mu} \right) \right] \]

Since \( Y_m \) is linearly homogeneous in \( K_m, L_m, \) \( p(1-\tau)dY_m = w_d dL_m + rdK_m \)

\[ dW = \frac{\delta}{Y_m} Y_a dY_m + \frac{1}{1-\nu} \left( \frac{\alpha - 1}{\alpha} \right) w_d dL_m + \nu p(1-\tau) dY_m + Y_a \delta (\hat{\mu} + \hat{\tau}) + \frac{1}{1-\nu} \left[ \nu p(1-\tau) Y_m \left( \hat{\tau} - \frac{\mu}{1-\mu} \hat{\mu} \right) \right] \]

(2.43*).

Setting, \( \hat{\tau} = 0 \), we can derive \( \frac{dW}{d\mu} \) as

\[ \frac{dW}{d\mu} = Y_m \left[ \frac{\delta Y_m}{\mu Y_m} - \frac{\nu p(1-\tau)}{(1-\mu)(1-\nu)} \right] + \left( \frac{\delta Y_m}{Y_m} + \frac{\nu p(1-\tau)}{1-\nu} \right) \frac{dY_m}{d\mu} + \frac{(\alpha - 1)}{\alpha} \frac{w_d dL_m}{d\mu} \]

(2.43)
To derive the effect of an increase in \( \tau \), we can get the following as well:

\[
dW = E_t dU = dY_a + p(1-\tau)dY_m - pY_md\tau. \quad (2.44*)
\]

Similarly setting, \( \dot{\mu} = 0 \), in the above expression in equation A2.43* and using A2.44* for we can derive \( \frac{dW}{d\tau} \) as

\[
dW = Y_a \left[ \frac{\delta Y_a}{\tau Y_m} + \frac{\nu - \tau}{\tau(1-\tau)} \right] + \frac{\delta Y_a}{Y_m} dY_m + \frac{\nu p(1-\tau)}{1-\nu} \frac{dY_m}{d\tau} + \frac{(\alpha - 1)}{\alpha} w_m \frac{dL_m}{d\tau}. \quad (2.44)
\]

Also one can analyze the effects in terms of shares for both changes in \( \mu \) and \( \tau \)

\[
\frac{dW}{d\mu} = \frac{dY_a}{d\mu} + p(1-\tau) \frac{dY_m}{d\mu} \]

\[
= \frac{Y_a \dot{Y}_a}{\mu \hat{\mu}} + p(1-\tau) \frac{Y_m \dot{Y}_m}{\mu \hat{\mu}} = \frac{Y_m}{\mu \hat{\mu}} \left[ \frac{Y_a \dot{Y}_a}{\mu \hat{\mu}} + p(1-\tau) \frac{Y_m \dot{Y}_m}{\mu \hat{\mu}} \right]
\]

\[
\frac{dW}{d\tau} = \frac{dY_a}{d\tau} + p(1-\tau) \frac{dY_m}{d\tau} - pY_m
\]

\[
= \frac{Y_a \dot{Y}_a}{\tau \hat{\tau}} + p(1-\tau) \frac{Y_m \dot{Y}_m}{\tau \hat{\tau}} - pY_m = \frac{Y_m}{\tau \hat{\tau}} \left[ \frac{Y_a \dot{Y}_a}{\tau \hat{\tau}} + p(1-\tau) \frac{Y_m \dot{Y}_m}{\tau \hat{\tau}} - pY_m \right]
\]

\[
= p(1-\tau)Y_m \left[ \frac{\pi_{i_a} \theta_{i_a} \dot{Y}_a}{\tau \hat{\tau}} + \frac{\pi_{i_m} \theta_{i_m} \dot{Y}_m}{\tau \hat{\tau}} - \frac{\tau}{(1-\tau)} \right] = \frac{w_a L}{\tau \hat{\tau}} \left[ \frac{\pi_{i_a} \theta_{i_a} \dot{Y}_a}{\tau \hat{\tau}} + \frac{\pi_{i_m} \theta_{i_m} \dot{Y}_m}{\tau \hat{\tau}} - \frac{\tau}{(1-\tau)} \right]
\]

\[
= p(1-\tau)Y_m \left[ \frac{\pi_{i_a} \theta_{i_a} \dot{Y}_a}{\alpha \theta_{i_a} \hat{\tau}} + \frac{\pi_{i_m} \theta_{i_m} \dot{Y}_m}{\theta_{i_m} \hat{\tau}} - \frac{\tau}{(1-\tau)\theta_{i_m}} \right] = \frac{w_a L}{\tau \hat{\tau}} \left[ \frac{\pi_{i_a} \theta_{i_a} \dot{Y}_a}{\alpha \theta_{i_a} \hat{\tau}} + \frac{\pi_{i_m} \theta_{i_m} \dot{Y}_m}{\theta_{i_m} \hat{\tau}} - \frac{\tau}{(1-\tau)\theta_{i_m}} \right]
\]

\[
\text{2.44 (a)}
\]
**Appendix 2.4 Derivation of Stability Conditions in the Harris-Todaro Model**

In our Harris-Todaro model set up in section 2.6, the dynamic adjustment process for the supply side of the model is specified as described below.

Using equations 2.6.9, 2.6.16, 2.6.17, 2.6.26a, and 2.6.27 we can write

\[ \dot{Y}_a = d_a \left( 1 - a_{i_a} w_a - a_{K_a} r \right) \]  
\[ \dot{Y}_m = d_z \left( p(1-\tau) - a_{i_m} w_m - a_{K_m} r \right) \]  
\[ \dot{w}_a = d_s (a_{i_a} Y_a + \varepsilon a_{i_m} Y_m - L) \]  
\[ \dot{r} = d_s (a_{K_a} Y_a + a_{K_m} Y_m - K) \]  
\[ d(1+\lambda) / dt = d_z \left( w_m - (1+\lambda) w_a \right). \]

Differentiating (2.6.32) and considering (2.6.17) in the text yield

\[ -(L_a / Y_a) dw_a - (K_a / Y_a) dr - \left[ (w_a / Y_a) dL_a + (r / Y_a) dK_a \right. \] 
\[ \left. - \{(w_a L_a / Y_a^2) dY_a + (r K_a / Y_a^2) dY_a^2 \} \] 
\[ = -a_{i_a} dw_a - a_{K_a} dr - \left[ (w_a L_a / Y_a) \hat{L}_a + (r K_a / Y_a) \hat{K}_a \right. \] 
\[ \left. - \{(w_a L_a / Y_a) \hat{Y}_a + (r K_a / Y_a) \hat{Y}_a^2 \} \] 
\[ = -a_{i_a} dw_a - a_{K_a} dr - \left[ \theta_{i_a} \hat{L}_a + \theta_{K_a} \hat{K}_a - \{(\theta_{i_a} + \theta_{K_a}) \hat{Y}_a \} \right. \] 
\[ \left. = -a_{i_a} dw_a - a_{K_a} dr - \left[ \theta_{i_a} \hat{L}_a + \theta_{K_a} \hat{K}_a - \hat{Y}_a \right. \] 
\[ \left. \right] \] 
From 2.6.12, we get, \( \hat{Y}_a = \delta \hat{Y}_m + \theta_{i_m} \hat{L}_m + \theta_{K_m} \hat{K}_m \) \( \text{when } \hat{\mu} = 0 = \hat{r} \) \[ \]  
\[ = -a_{i_a} dw_a - a_{K_a} dr - (\delta / Y_m) dY_m \]  
\[ \]  
(2.6.32A)

Similarly, we have from (2.6.33) and (2.6.13),

\[ -a_{K_m} dr + (p(1-\tau)\nu / Y_m) dY_m \text{ [Since } w_m \text{ is constant]} \]  
\[ (2.6.33B) \]

Now, differentiating (2.6.34), we obtain

\[ dL_a + (1+\lambda) dL_m + L_m d\lambda = \] 
\[ = dL_a + \varepsilon dL_m + L_m d\varepsilon, \text{ assuming } (1+\lambda = \varepsilon), \]

Substituting \( \hat{L}_a \) and \( \hat{L}_m \) from (2.6.20) and (2.6.22), in (2.6.34) we get the following:
\[
\begin{align*}
&= -\pi_{La} \sigma_a \theta_{Ka} \frac{dW_a}{W_a} + \left( \pi_{La} \sigma_a \theta_{Ka} + \varepsilon \pi_{Lm} \sigma_m \theta_{Lm} \right) \frac{dr}{r} + \pi_{La} \frac{dY_a}{Y_a} + \left( \pi_{La} + (1 - \pi_{La}) \right) \frac{dY_m}{Y_m} + \pi_{Lm} d\varepsilon \\
&\text{We get similar expression from 2.6.35 and 2.6.21 and 2.6.23. And, finally, from} \\
(2.6.30) \text{ and (2.6.36), we get:} \\
&= -w_a \varepsilon \frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{t_a} \theta_{K_a}} dY_m \\
&- w_a d\varepsilon \\
&\text{Thus we get the Jacobian as} \\
|J| &= \frac{d_4 d_4 d_4 d_4 p_{K1} (1 - \tau)}{Y_a Y_a r} \\
&\begin{vmatrix}
-\theta_{L_a} & -\theta_{K_a} & 0 & \delta & 0 \\
0 & -\theta_{L_a} & 0 & \nu & 0 \\
-\pi_{La} \sigma_a \theta_{Ka} & \pi_{La} & \pi_{La} & \varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} \\
\varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} & \pi_{La} & 0 & 0 \\
0 & 0 & 0 & -\Theta \frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{t_a} \theta_{K_a}} & -1 \\
\end{vmatrix} \\
&\text{where } \sigma_1 = \pi_{La} \sigma_a \theta_{Ka} + \varepsilon \pi_{Lm} \sigma_m \theta_{Lm} \text{ and } \sigma_2 = -\left( \pi_{Ka} \sigma_a \theta_{La} + \pi_{Km} \sigma_m \theta_{Lm} \right) \\
&\text{By R3- } \pi_{Ln} * \text{R5 (adding the elements of 5th row multiplied by } \pi_{Ln} \text{) to corresponding elements of} \\
3 \text{rd row} \\
|J| &= \frac{d_4 d_4 d_4 d_4 p_{K1} (1 - \tau)}{Y_a Y_a r} \\
&\begin{vmatrix}
-\theta_{L_a} & -\theta_{K_a} & 0 & \delta & 0 \\
0 & -\theta_{L_a} & 0 & \nu & 0 \\
-\pi_{La} \sigma_a \theta_{Ka} & \pi_{La} & \pi_{La} & \varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} \\
\varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} & \pi_{La} & 0 & 0 \\
0 & 0 & 0 & -\Theta \frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{t_a} \theta_{K_a}} & -1 \\
\end{vmatrix} \\
&\text{Expanding the determinant by C5} \\
|J| &= \frac{-d_4 d_4 d_4 d_4 p_{K1} (1 - \tau)}{Y_a Y_a r} \\
&\begin{vmatrix}
-\theta_{L_a} & -\theta_{K_a} & 0 & \delta & 0 \\
0 & -\theta_{L_a} & 0 & \nu & 0 \\
-\pi_{La} \sigma_a \theta_{Ka} & \pi_{La} & \pi_{La} & \varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} \\
\varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} & \pi_{La} & 0 & 0 \\
\pi_{La} \sigma_a \theta_{Ka} & \pi_{La} & \pi_{La} & \varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} \\
\varepsilon (1 - \nu) \pi_{La} - \delta \pi_{La} & \pi_{La} & \pi_{La} & 0 & 0 \\
\end{vmatrix} \\
&\text{By R1+R2*}(\theta_{K_a} / \theta_{K_a}) \end{align*}
\]
By C4+C2* $\nu / \theta_k$:

$$|J| = \frac{-d_1 d_2 d_3 d_4 \pred K L (1 - \tau)}{Y_a Y_m r} \begin{vmatrix} -\theta_{k_a} & 0 & 0 & \delta - \nu \theta_{k_a} / \theta_{k_a} \\ 0 & -\theta_{k_a} & 0 & \nu \\ -\pi_{k_a} \sigma_{\theta_{k_a}} \sigma_1 & \pi_{k_a} & \epsilon (1 - \nu) \pi_{k_a} - \delta \pi_{k_a} - \epsilon \pi_{k_a} - \frac{\delta \theta_{k_a} - \nu \theta_{k_a}}{\theta_{k_a} \theta_{k_a}} \\ \pi_{k_a} \sigma_{\theta_{k_a}} \sigma_2 & \pi_{k_a} & (1 - \nu) \pi_{k_a} - \delta \pi_{k_a} \end{vmatrix}$$

By C4+C1* $(\delta - \nu \theta_{k_a} / \theta_{k_a}) / \theta_{k_a}$:

$$|J| = \frac{-d_1 d_2 d_3 d_4 K L p (1 - \tau)}{Y_a Y_m r} \begin{vmatrix} -\theta_{k_a} & 0 & 0 & 0 \\ 0 & -\theta_{k_a} & 0 & 0 \\ -\pi_{k_a} \sigma_{\theta_{k_a}} \sigma_1 & \pi_{k_a} & \Omega_{k_a} & \Omega_{k_a} \\ \pi_{k_a} \sigma_{\theta_{k_a}} \sigma_2 & \pi_{k_a} & \Omega_{k_a} & \Omega_{k_a} \end{vmatrix}$$

$$= \frac{-d_1 d_2 d_3 d_4 K L p (1 - \tau) \theta_{l_m} \theta_{k_m}}{Y_a Y_m r} \begin{vmatrix} \Omega_{l_m} & \Omega_{k_m} \\ \Omega_{k_m} & \Omega_{k_m} \end{vmatrix}$$

$$= \frac{-d_1 d_2 d_3 d_4 K L p (1 - \tau) \theta_{l_m} \theta_{k_m}}{Y_a Y_m r} |\Omega|$$

where $\sigma_1 = (\pi_{l_m} \sigma_{\theta_{k_a}} + \epsilon \pi_{l_m} \sigma_{\theta_{k_a}} \sigma_{\theta_{k_a}})$, and $\sigma_2 = -(\pi_{k_m} \sigma_{\theta_{k_a}} + \pi_{k_m} \sigma_{\theta_{k_a}} \sigma_{\theta_{k_a}})$.

$$\Omega_{l_m} = \pi_{l_m}; \hspace{0.5cm} \Omega_{k_m} = \epsilon (1 - \nu) \pi_{l_m} - \delta \pi_{l_m} + \sigma_{\nu} / \theta_{k_m} - \pi_{l_m} \sigma_{\theta_{k_a}} \sigma_{\theta_{k_a}} \left( \delta - \nu \theta_{k_m} / \theta_{k_m} \right) / \theta_{l_m} - \epsilon \pi_{l_m} \frac{\delta \theta_{k_m} - \nu \theta_{k_m}}{\theta_{l_m} \theta_{k_m}}$$

$$\Omega_{k_m} = \pi_{l_m} + (1 - \nu) \pi_{k_m} + \sigma_{\nu} / \theta_{k_m} - \delta \pi_{k_m} + \pi_{k_m} \sigma_{\theta_{k_a}} \sigma_{\theta_{k_a}} \left( \delta - \nu \theta_{k_m} / \theta_{k_m} \right) / \theta_{l_m}$$

$$|\Omega| = (1 - \nu) \pi_{l_m} \pi_{k_m} + \sigma_{\nu} \pi_{l_m} / \theta_{k_m} - \delta \pi_{l_m} \pi_{k_m} + \pi_{k_m} \pi_{l_m} \sigma_{\theta_{k_a}} \sigma_{\theta_{k_a}} \left( \delta - \nu \theta_{k_m} / \theta_{k_m} \right) / \theta_{l_m} + \epsilon \pi_{l_m} \pi_{k_m} \frac{\delta \theta_{k_m} - \nu \theta_{k_m}}{\theta_{l_m} \theta_{k_m}}$$
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\[
\Omega = (1 - \nu) \left( \pi_{k_w} \pi_{k_a} - \pi_{k_w} \pi_{k_a} \right) + \nu \pi_{k_w} \pi_{k_a} \left( \theta_{k_w} - \theta_{k_a} \right) + \pi_{k_w} \pi_{k_a} \sigma \delta - \nu \theta_{k_a} \right) / \theta_{k_a} + \nu \theta_{k_a} \pi_{k_w} - \nu \theta_{k_w} \pi_{k_a} \theta_{k_w} \theta_{k_a}
\]

\[
- \sigma \pi_{k_w} \pi_{k_a} \theta_{k_w} \theta_{k_a} = - \sigma \pi_{k_w} \pi_{k_a} \theta_{k_w} \theta_{k_a} - \nu \theta_{k_a} \theta_{k_w} \pi_{k_a} \pi_{k_w} \theta_{k_w} \theta_{k_a}
\]

\[
\Omega = (1 - \nu) \left( \pi_{k_w} \pi_{k_a} - \pi_{k_w} \pi_{k_a} \right) + \pi_{k_w} \pi_{k_a} \sigma \delta - \nu \theta_{k_a} \right) / \theta_{k_a} + \nu \theta_{k_a} \pi_{k_w} - \nu \theta_{k_w} \pi_{k_a} \theta_{k_w} \theta_{k_a}
\]

\[
\Omega = \left( \pi_{k_w} \pi_{k_a} - \pi_{k_w} \pi_{k_a} \right) (1 - \nu) + \pi_{k_w} \pi_{k_a} \sigma \delta - \nu \theta_{k_a} \right) / \theta_{k_a} + \nu \theta_{k_a} \pi_{k_w} - \nu \theta_{k_w} \pi_{k_a} \theta_{k_w} \theta_{k_a}
\]

Additional Derivations

Derivations of Equations 2.6.28 to 2.6.31

We have

\[
\theta_{l_a} \hat{w}_a + \theta_{k_a} \hat{\tau} = \delta \hat{y}_m + \delta (\hat{\mu} + \hat{\tau})
\]

(2.6.16)

\[
\theta_{l_m} \hat{w}_m + \theta_{k_m} \hat{\tau} = \nu \hat{y}_m + \nu \left( \hat{\tau} - \frac{\mu}{1 - \mu} \hat{\mu} \right)
\]

(2.6.17)

Also from (2.6.9) \( \hat{w}_m = \hat{w}_a + \hat{\epsilon} \) where \( \epsilon = 1 + \lambda \)

From equation A2.6.16:

\[
\hat{w}_a = \frac{\delta}{\theta_{l_a}} \hat{y}_m + \frac{\hat{\delta}}{\theta_{l_a}} (\hat{\mu} + \hat{\tau}) - \frac{\theta_{k_a}}{\theta_{l_a}} \hat{\tau}
\]

Also \( \hat{w}_m = 0 \), since \( w_m \) is the institutionally fixed urban wage

\[
\hat{\tau} = \frac{\nu}{\theta_{k_m}} \hat{y}_m + \frac{\hat{\nu}}{\theta_{k_m}} + \frac{1}{\theta_{k_m}} \left( \frac{\nu - \tau}{1 - \tau} \right) \hat{\tau} - \frac{\mu \nu}{\theta_{k_m} (1 - \mu)} \hat{\mu}
\]

(A)

\[
\hat{w}_a = \frac{\delta}{\theta_{l_m}} \hat{y}_m + \frac{\hat{\delta}}{\theta_{l_m}} \hat{\mu} - \frac{\theta_{k_a}}{\theta_{k_m}} \left[ \frac{\nu}{\theta_{k_m}} \hat{y}_m + \frac{\hat{\nu}}{\theta_{k_m}} + \frac{1}{\theta_{k_m}} \left( \frac{\nu - \tau}{1 - \tau} \right) \hat{\tau} - \frac{\mu \nu}{\theta_{k_m} (1 - \mu)} \hat{\mu} \right]
\]

(B)

Substituting equations 2.20–A2.23 into 2.24–A2.25, one gets
\begin{align*}
\pi_{l_a} \dot{Y}_a + [(1 - \nu) &\varepsilon \pi_{l_a} - \delta \pi_{l_a}] \dot{Y}_m - \varepsilon \pi_{l_a} \dot{\epsilon} + \varepsilon \pi_{l_a} \dot{\epsilon} \\
&= \pi_{l_a} \theta_{k_a} \sigma_a (\hat{w}_a - \hat{r}) + \pi_{l_a} \varepsilon \pi_{m \theta} \theta_{k_a} (\hat{w}_m - \hat{r}) \\
&\quad + \left( \pi_{l_a} \delta - \frac{\varepsilon \pi_{l_a} \nu \mu}{1 - \mu} \right) \dot{\mu} + \left( \pi_{l_a} \delta + \varepsilon \pi_{l_a} \nu \right) \hat{\tau} \\
&\quad (2.6.26)
\end{align*}

\begin{align*}
\pi_{k_a} \dot{Y}_a + (\pi_{k_a} (1 - \nu) - \pi_{k_a} \delta) \dot{Y}_m \\
&= -\pi_{k_a} \theta_{l_a} \sigma_a (\hat{w}_a - \hat{r}) - \pi_{k_a} \sigma_m \theta_{l_a} (\hat{w}_m - \hat{r}) \\
&\quad + \left( \pi_{k_a} \delta - \frac{\pi_{k_a} \nu \mu}{1 - \mu} \right) \hat{\mu} + \left( \pi_{k_a} \delta + \pi_{k_a} \nu \right) \hat{\tau} \\
&\quad (2.6.27)
\end{align*}

Substitution of (A) and (B) will give us 2.6.27 and 2.6.28 will give us

\begin{align*}
\pi_{l_a} \dot{Y}_a + [(1 - \nu) &\varepsilon \pi_{l_a} - \delta \pi_{l_a} + \nu \varepsilon \pi_{l_a} \sigma_m - \pi_{l_a} \theta_{k_a} \sigma_a (\delta - \nu / \theta_{k_a})] \dot{Y}_m + \varepsilon \pi_{l_a} \dot{\epsilon} \\
&= \left[ \pi_{l_a} \theta_{k_a} \sigma_a \delta + \pi_{l_a} \delta + \frac{\mu \nu \varepsilon \pi_{l_a} \sigma_m}{(1 - \mu)} + \frac{\mu \nu}{(1 - \mu) \theta_{l_a} \theta_{k_a}} \left( \pi_{l_a} \theta_{k_a} \sigma_a - \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a} \right) \right] \dot{\mu} \\
&\quad \left[ \pi_{l_a} \delta + \varepsilon \pi_{l_a} \nu (1 - \sigma_m) + \pi_{l_a} \theta_{k_a} \sigma_m \left( \delta \theta_{k_a} - \nu \right) + \frac{\tau}{(1 - \nu) \theta_{l_a} \theta_{k_a}} \left( \pi_{l_a} \theta_{k_a} \sigma_a + \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a} \right) \right] \hat{\tau} \\
&\quad - \frac{1}{\theta_{l_a} \theta_{k_a}} \left( \pi_{l_a} \theta_{k_a} \sigma_a + \pi_{l_a} \varepsilon \pi_{l_a} \sigma_m \theta_{l_a} \theta_{k_a} \right) \hat{p} \\
&\quad (2.6.28)
\end{align*}

\begin{align*}
\pi_{k_a} \dot{Y}_a + \left( \pi_{k_a} (1 - \nu) - \pi_{k_a} \delta + \pi_{k_a} \sigma_a (\delta - \nu / \theta_{k_a}) - \frac{\nu \pi_{k_a} \sigma_m \theta_{k_a}}{\theta_{k_a}} \right) \dot{Y}_m \\
&= \left[ \pi_{k_a} \delta (1 - \sigma_a) - \left( \pi_{k_a} \theta_{k_a} + \pi_{k_a} \sigma_a + \pi_{k_a} \sigma_m \theta_{k_a} \right) \frac{\mu \nu}{\theta_{k_a} (1 - \mu)} \right] \dot{\mu} + \\
&\quad \left[ \pi_{k_a} \delta (1 - \sigma_a) + \pi_{k_a} \nu + \frac{1}{\theta_{k_a}} \left( \pi_{k_a} \sigma_m \theta_{l_a} + \pi_{k_a} \sigma_a \right) \left( \nu - \frac{\nu}{1 - \nu} \right) \right] \hat{\tau} + \left( \pi_{k_a} \sigma_a + \pi_{k_a} \sigma_m \theta_{l_a} \right) \hat{p} \\
&\quad (2.6.29)
\end{align*}

Similarly, from total differential of 2.6.9 and (A) and (B) one can derive

\begin{align*}
\dot{\epsilon} &= -\frac{\delta \theta_{l_a} - \nu \theta_{l_a} \hat{Y}_m}{\theta_{l_a} \theta_{k_a}} + \frac{\hat{p} \theta_{k_a}}{\theta_{l_a} \theta_{k_a}} \left( \nu - \frac{\tau}{1 - \tau} \delta \frac{\theta_{k_a}}{\theta_{k_a}} \right) \hat{\tau} - \frac{\theta_{l_a}}{\theta_{l_a} \theta_{k_a}} \left( \frac{\mu \nu}{1 - \mu} + \frac{\delta \theta_{k_a}}{\theta_{k_a}} \right) \dot{\mu} \\
&\quad (2.6.30)
\end{align*}

Substitution of \( \dot{\epsilon} \) in equation A2.6.28 gives us

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Then, equations 2.6.28a and 2.6.29 can be written as

\[
\begin{bmatrix}
\Omega_{L_a} & \Omega_{L_n} \\
\Omega_{K_a} & \Omega_{K_n}
\end{bmatrix}
\begin{bmatrix}
\ddot{Y}_a \\
\ddot{Y}_n
\end{bmatrix}
= \begin{bmatrix}
A_1 \ddot{\mu} + A_2 \dot{\xi} + A_3 \dot{p} \\
B_1 \ddot{\mu} + B_2 \dot{\xi} + B_3 \dot{p}
\end{bmatrix},
\]

(2.6.31)

where

\[
\Omega_{L_a} = \pi_{L_a}
\]

\[
\Omega_{L_n} = (1-v)e_{L_n} - \sigma_{L_n} + v_{E_{L_n}} \sigma_{m} - \frac{\pi_{L_a} \theta_{K_a} \sigma_{m}}{\theta_{L_a}} (\sigma - v/\theta_{K_n}) - e_{L_n} \frac{\delta \theta_{K_a} - v \theta_{K_a}}{\theta_{L_a} \theta_{K_a}}
\]

\[
\Omega_{K_n} = \pi_{K_n}
\]

\[
\Omega_{K_n} = \pi_{K_n} (1-v) - \pi_{K_a} \delta + \pi_{K_n} \sigma_{m} (\sigma - v/\theta_{K_n}) - \frac{v \pi_{K_n} \sigma_{m} \theta_{L_n}}{\theta_{K_n}}
\]

\[
[\Omega] = \begin{bmatrix}
\pi_{L_a} \tau_{L_a} \sigma_{m} - e_{L_n} \tau_{L_n} \\
\pi_{L_a} \tau_{L_a} \sigma_{L_a} - v_{E_{L_a}} \tau_{L_a}
\end{bmatrix}
\begin{bmatrix}
\frac{\pi_{L_a} \tau_{L_a} \sigma_{m} \theta_{K_a}}{\theta_{L_a}} - \frac{v \pi_{K_n} \sigma_{m} \theta_{L_n}}{\theta_{K_n}} \delta \theta_{K_a} - v \theta_{K_a} \theta_{K_n}
\end{bmatrix}
\]

\[
A_1 = \pi_{L_a} \delta + \left( \frac{\pi_{L_a} \theta_{K_a} \sigma_{m} + e_{L_n} \tau_{L_n}}{\theta_{K_a}} \right) \delta + \left( \frac{v \pi_{K_n} \tau_{K_n} \sigma_{m} - e_{L_n} \tau_{L_n} - \pi_{K_n} \tau_{K_n} \sigma_{L_a} + e_{L_n} \tau_{L_a}}{\theta_{K_n}} \right) \frac{\mu v}{(1-\tau)}
\]

\[
A_2 = \pi_{L_a} \delta + e_{L_n} \tau_{L_n} \left( 1 - \sigma_{m} - \frac{\theta_{K_a}}{\theta_{L_a} \theta_{K_n}} \right) + \frac{\pi_{L_a} \tau_{L_a} \sigma_{m} \theta_{K_a}}{\theta_{L_a} \theta_{K_n}} \left( \delta \theta_{K_n} + \frac{\tau}{1-\tau} - v \right) + \frac{\varepsilon_{L_n} \tau_{L_n}}{(1-\tau)} \left( \sigma_{m} + \frac{\delta \theta_{K_n}}{\theta_{L_a} \theta_{K_n}} \right) + \frac{e_{L_n} \tau_{L_n}}{\theta_{L_a}}
\]

\[
A_3 = -\frac{1}{\theta_{L_a} \theta_{K_n}} \left( \pi_{L_a} \theta_{K_a} \sigma_{m} + \pi_{L_n} \varepsilon_{L_n} \theta_{L_a} \theta_{K_n} + \pi_{L_n} \theta_{K_a} \sigma_{m} \theta_{K_n} + e_{L_n} \theta_{L_a} \theta_{K_n} \right)
\]

\[
B_1 = \pi_{K_n} \delta (1-\sigma_{n}) - \left( \pi_{K_n} \theta_{K_n} + \pi_{K_n} \sigma_{m} + \pi_{K_n} \sigma_{m} \theta_{L_n} \right) \frac{\mu v}{\theta_{K_n} (1-\mu)}
\]

\[
B_2 = \pi_{K_n} \delta (1-\sigma_{n}) + \pi_{K_n} \sigma_{m} + \frac{1}{\theta_{K_n}} \left( \pi_{K_n} \sigma_{m} \theta_{L_n} + \pi_{K_n} \sigma_{m} \theta_{K_n} \right) \left( \frac{v}{1-\tau} \right)
\]
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\[ B_3 = \frac{(\pi_K \sigma_a + \pi_K \sigma_m \theta_m)}{\theta_m} \]

**Derivation of \(|\Omega|\) When \( \delta = \nu / \theta_{K_a} \)**

Substituting, \( \delta = \nu / \theta_{K_a} \), and using \( \theta_{K_i} = 1 - \theta_{K_i}, i = a, m; \pi_{K_i} = 1 - \varepsilon \pi_{K_i} \) and \( \pi_{K_a} = 1 - \pi_{K_a} \)

\[ \hat{e} = -\frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{K_a}} \hat{y}_m + \hat{p} \theta_{K_a} + \frac{\theta_{K_a}}{\theta_{K_a}} \left( \nu - \frac{\tau}{1 - \tau} - \delta \frac{\theta_{K_a}}{\theta_{K_a}} \right) \hat{\epsilon} - \frac{\theta_{K_a}}{\theta_{K_a}} \left( \frac{\mu \nu}{(1 - \mu)} + \delta \frac{\theta_{K_a}}{\theta_{K_a}} \right) \hat{\mu} \]

Substituting, \( \delta = \nu / \theta_{K_a} \), and using \( \theta_{K_i} = 1 - \theta_{K_i}, i = a, m; \pi_{K_i} = 1 - \varepsilon \pi_{K_i} \) and \( \pi_{K_a} = 1 - \pi_{K_a} \)

\[ |\Omega| = (\pi_L \pi_K - \varepsilon \pi_K \pi_m)(1 - \nu) + \frac{\pi_K \pi_{K_a} \sigma_a}{\theta_{K_a}} (\delta - \nu / \theta_{K_a}) - \frac{\nu \pi_L \pi_{K_a} \sigma_m \theta_m}{\theta_{K_a}} \]

\[ |\Omega| = -u e \pi_K \pi_{K_a} \sigma_a e \pi_K \pi_m \]

\[ |\Omega| = \pi_K \pi_{K_a} - (1 - \delta \sigma_m \theta_m) - (1 - \delta - \nu \sigma_m) e \pi_K \pi_m \]

**Derivation of \( \hat{Y}_a / \hat{\mu} \)**

\[ \hat{Y}_a / \hat{\mu} = (\Omega_{K_a} A_i - \Omega_{K_a} B_i) / |\Omega| \] (2.37)

\[ \hat{Y}_a / \hat{\mu} = -(\Omega_{K_a} A_i - \Omega_{K_a} B_i) / |\Omega| \] (2.38)
\[ B_i = \pi_k \delta (1 - \sigma_y) - \left( \pi_k \theta_{k_a} + \pi_k \theta_{k_l} + \pi_k \sigma_a + \pi_k \sigma_l \right) \frac{\mu \nu}{\theta_k (1 - \mu)} \]

\[ \hat{Y}_m / \hat{\mu} = \frac{1}{|\Omega|} \left( \pi_k - \pi_k \sigma_a - \pi_k \sigma_l \right) \left( \pi_k + \frac{\pi_k \theta_k \sigma_a + \epsilon \pi_k}{\theta_k} \right) = \frac{1}{|\Omega|} \left( \epsilon \pi_k - \pi_k \frac{\epsilon}{\theta_k} \right) \left( \pi_k (1 - \sigma_y) \right) \]

\[ \hat{Y}_m / \hat{\mu} = \frac{1}{|\Omega|} \left[ -\sigma_k \pi_k \frac{\epsilon}{\theta_k} \left( \pi_k \theta_k \sigma_a + \epsilon \pi_k \right) \right] \]

**Derivation of \( \hat{Y}_m / \hat{\mu} \)**

\[ \hat{Y}_m / \hat{\mu} = -\frac{1}{|\Omega|} \left[ \pi_k A_i - \pi_k B_i \right] \]

\[ = -\frac{1}{|\Omega|} \left( \frac{\nu}{1 - \mu} \right) \left[ \pi_k \left( \pi_k \mu + \Delta_l \right) - \pi_k \left( \pi_k \mu - \Delta_l \right) \right] \]

\[ = -\frac{1}{|\Omega|} \left( \frac{\nu}{1 - \mu} \right) \left[ \mu \left( \pi_k \mu - \pi_k \right) + \Delta_l \pi_k \right] < 0 \text{ since, } \pi_k > \pi_k > 0 \]

\[ \hat{Y}_m / \hat{\mu} = -\frac{1}{|\Omega|} \left[ \left( \pi_k \mu - \epsilon \pi_k \pi_k \right) \frac{\nu}{1 - \mu} + \left( \pi_k \Delta_l + \pi_k \Delta_l \right) \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \right] \]

\[ = -\frac{\mu \nu}{(1 - \nu)(1 - \mu)} - \frac{1}{|\Omega|} \left( \frac{\nu}{1 - \mu} \right) \left( \pi_k \Delta_l + \pi_k \Delta_l \right) \text{ when } \delta \approx \nu \]

When

\[ |\Omega| \hat{Y}_m / \hat{\mu} = \left( \pi_k (1 - \nu) - \pi_k \delta + \Delta_l (\delta - \nu) \right) \left( \frac{\nu}{1 - \mu} \right) + \left( \frac{\mu \nu}{1 - \mu} \right) \]

\[ + \left( -\nu \epsilon \pi_k + \delta \pi_k + \Delta_l (\delta - \nu) \right) \left( \pi_k \delta - \frac{\pi_k \nu}{1 - \mu} \right) - \Delta_l \left( \delta + \frac{\mu \nu}{1 - \mu} \right) \]
Derivation of Effects of Change in $\tau$

$$\hat{Y}_a / \hat{t} = (\Omega_{K_a} A_2 - \Omega_{L_a} B_2) / |\Omega|$$

$$\hat{Y}_m / \hat{t} = -(\Omega_{K_a} A_2 - \Omega_{L_a} B_2) / |\Omega|$$

$$A_2 = \pi_{K_a} \delta + \varepsilon \pi_{L_a} \nu \left(1 - \sigma_m \frac{\theta_{K_a}}{\theta_{L_a} \theta_{K_a}}\right) + \pi_{K_a} \theta_{K_a} \sigma_m \left(\delta \theta_{K_a} + \frac{\tau}{1 - \tau} - \nu \right) + \frac{\varepsilon \pi_{L_a} \tau}{\theta_{L_a} \theta_{K_a}} \left(\sigma_m + \frac{\delta \theta_{K_a}}{\theta_{L_a} \theta_{K_a}}\right) + \delta \frac{\varepsilon \pi_{L_a}}{\theta_{L_a}}$$

$$B_2 = \pi_{K_a} \delta (1 - \sigma_m) + \pi_{K_a} \nu \left(\frac{\pi_{K_a} \sigma_m \theta_{L_a} + \pi_{K_a} \sigma_a}{\theta_{L_a}}\right) \left(\nu - \frac{\tau}{1 - \tau}\right)$$

$$\Omega_{K_a} = \pi_{K_a} (1 - \nu) - \pi_{K_a} \delta + \pi_{K_a} \sigma_a (\delta - \nu / \theta_{K_a}) - \frac{\nu \pi_{K_a} \sigma_a \theta_{L_a}}{\theta_{K_a}}$$

$$\Omega_{L_a} = (1 - \nu) \varepsilon \pi_{L_a} - \delta \pi_{L_a} + \nu \varepsilon \pi_{L_a} \sigma_m - \frac{\pi_{L_a} \theta_{K_a} \sigma_m}{\theta_{L_a}} (\delta - \nu / \theta_{K_a}) + \varepsilon \pi_{L_a} \frac{\delta \theta_{K_a} - \nu \theta_{K_a}}{\theta_{L_a} \theta_{K_a}}$$

$$\Omega_{K_n} = \pi_{K_n}, \Omega_{L_n} = \pi_{L_n}$$

1. $\delta = \nu = 0$, i.e., public spending does not generate any productive services in either sector:

$$A_2 = \frac{\pi_{K_a} \theta_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} \frac{\tau}{1 - \tau} + \frac{\varepsilon \pi_{L_a} \tau \sigma_m}{(1 - \tau)}, B_2 = - \frac{1}{\theta_{K_a}} \left(\pi_{K_a} \sigma_m \theta_{L_a} + \pi_{K_a} \sigma_a\right) \frac{\tau}{(1 - \tau)}$$

$$\Omega_{K_a} = \pi_{K_a}, \Omega_{L_n} = (1 - \nu) \varepsilon \pi_{L_a}$$

$$\hat{Y}_a / \hat{t} = \left[\frac{\varepsilon \pi_{L_a}}{1 - \tau} \left(\frac{\pi_{L_a} \theta_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} + \varepsilon \pi_{L_a} \sigma_m\right) + \frac{\pi_{L_a} \theta_{K_a} \sigma_a \theta_{L_a} + \pi_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} \frac{\tau}{(1 - \tau)}\right] / |\Omega| > 0 ,$$

$$\hat{Y}_m / \hat{t} = - \left[\frac{\varepsilon \pi_{L_a}}{1 - \tau} \left(\frac{\pi_{L_a} \theta_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} + \varepsilon \pi_{L_a} \sigma_m\right) + \frac{\pi_{L_a} \theta_{K_a} \sigma_a \theta_{L_a} + \pi_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} \frac{\tau}{(1 - \tau)}\right] / |\Omega| < 0$$

1. $\delta = 1, \nu = 0$, i.e., public spending does not generate any productive services in either sector:

$$\hat{Y}_a / \hat{t} = (\Omega_{K_a} A_2 - \Omega_{L_a} B_2) / |\Omega| > 0, since A_2 > 0, B_2 < 0, \Omega_{K_a} > 0, \Omega_{L_a} < 0$$

$$\hat{Y}_m / \hat{t} = - \left[\frac{\varepsilon \pi_{L_a}}{1 - \tau} \left(\frac{\pi_{L_a} \theta_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} + \varepsilon \pi_{L_a} \sigma_m\right) + \frac{\pi_{L_a} \theta_{K_a} \sigma_a \theta_{L_a} + \pi_{K_a} \sigma_a}{\theta_{L_a} \theta_{K_a}} \frac{\tau}{(1 - \tau)}\right] / |\Omega| < 0$$
Appendix 2.5 A Simple Calibration Exercise Using Cobb-Douglas Production Functions

A2.5.1 The Model with Cobb-Douglas Form of Production Functions

This appendix sets the stage for calibration exercise with specific functional forms. Here is an attempt to make a rough estimate. However, the intent is to make a full calibration exercise in our future research.

The general model analyzed in the main text postulates that the influence of government spending on output in each sector is represented by a power function akin to TFP. The main equations in sections 2.1 and 2.6 for both neo-classical and Harris-Todaro models, viz. [equations (2.1) - (2.4), and also equations (2.6.1) - (2.6.4)], can be re-written as

\[ Y_a = G_a F(L_a, K_a) = (G_a^*)^\delta F(L_a, K_a) \]
\[ Y_m = G_m H(L_m, K_m) = (G_m^*)^{\nu} F(L_m, K_m) \]
\[ G_a = (\mu \tau Y_m)^{\delta} = (G_a^*)^{\delta}, \text{ where } \delta \neq 1-\alpha \]
\[ G_m = ((1-\mu) \tau Y_m)^{\nu} = (G_m^*)^{\nu}, \text{ where } \nu \neq 1-\beta \]

For a calibration exercise, in Appendix 2.5 we consider a special case of the model in which the influence of government spending on output is similar to human capital accumulation as expounded by Lucas (1988) and others. In this special case, \( L_a \) and \( L_m \) are units of raw labor and \( G_a^* \) and \( G_m^* \) are spending on education/ job spending. Thus, in Appendix 2.5 the model used for the simple calibration exercise can be considered as a special case when \( \delta = 1-\alpha \), and \( \nu = 1-\beta \). Indeed, government spending acts as labor-augmenting technological progress. In the calibration exercise, many of the parameter constraints derived from our general model will not hold. Nonetheless, the special case of \( \delta = 1-\alpha \), and \( \nu = 1-\beta \) will provide us some insights into the effects of the allocation of government spending among rural and urban sectors.
For simplification, let us assume Cobb-Douglas forms of production functions. The general equilibrium structure given above is then written as follows:

\[ Y_a = A_a L_a^{1-\alpha} K_a^{\alpha} G_a^{1-\alpha} \]  
(A2.12)

\[ Y_m = A_m L_m^{1-\beta} K_m^{\beta} G_m^{1-\beta} \]  
(A2.13)

\[ \alpha Y_a / K_a = r \]  
(A2.14)

\[ p(1-\tau)\beta Y_m / K_m = r \]  
(A2.15)

\[ (1-\alpha)Y_a / L_a = w_a \]  
(A2.16)

\[ p(1-\tau)(1-\beta)Y_m / L_m = w_m \]  
(A2.17)

\[ w_a = w_m \]  
(A2.18)

\[ K_a + K_m = K \]  
(A2.19)

\[ L_a + L_m = L \]  
(A2.20)

\[ \mu r Y_m = G_a \]  
(A2.21)

\[ (1-\mu)r Y_m = G_m \]  
(A2.22)

In our model, given the world price \( p \), and exogenous values of \( K, L, \tau, \) and \( \mu \), we have 11 equations for solving the equilibrium values of 11 unknowns: equilibrium values of 11 unknowns: \( Y_a, Y_m, K_a, K_m, L_a, L_m, G_a, G_m, r, w_a, \) and \( w_m \).

**A2.5.2 Workings of the Model**

The above system of equations can be simplified into a smaller set of subsystems. This will help us to derive the equilibrium condition in terms of given technology parameters, capital, and labor ratios. For simplification we use the following notations:
Using the above notations for shares, we can get the employment of the urban sector as

\[ L_m = (1 - n)L \quad \text{(A2.23)} \]

With our assumptions of perfect mobility of capital and labor, constant returns to scale, exogenous tax rate, and Cobb-Douglas production functions, we can derive the technology parameters using only an assumption about labor share in the economy (\( \theta \)) and data on rural sector employment share (\( n \)), which are easily available for different countries. Our long-run equilibrium condition (A2.18), \( w_a = w_m \), can be re-written as:
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\[(1-\alpha)\frac{Y_a}{nL} = w_m = \frac{p(1-\beta)(1-\tau)Y_a}{(1-n)L}\]

Alternatively \(\frac{Y_a}{pY_m} = \left(\frac{1-\beta}{1-\alpha}\right)^n \frac{n}{1-n} (1-\tau)\) \hspace{1cm} (A2.24)

Since the capital is perfectly mobile between the two sectors, the identical rental rates in equations (A2.13) and (A2.14) give us

\[\frac{\alpha}{K_a} = p\beta(1-\tau)\frac{Y_a}{K_m}\] or, \[\frac{Y_a}{pY_m} = \frac{\beta(1-\tau)K_a}{\alpha K_m} = \frac{\beta(1-\tau)}{1-z} \hspace{1cm} (A2.25)\]

From (A2.24) and (A2.25), we get, the equilibrium condition as

\[\frac{z}{1-z} = \left(\frac{\alpha}{\beta}\right)^n \frac{n}{1-n}. \hspace{1cm} (A2.26)\]

In fact, equation (A2.26) simplifies sub-system of equations (A2.14)–(A2.20) of our general equilibrium system. Similarly, by methods of substitution we can simplify equations (A2.13) and (A2.22) as

\[Y_m = A_m L_m^{1-\beta} K_m^{\beta} \Rightarrow G_m^{1-\beta} = A_m L_m^{1-\beta} K_m^{\beta} (1-\mu)^{1-\beta} \tau^{1-\beta} Y_m^{1-\beta}\]

\[Y_m = A_m^{1/\beta} L_m^{(1-\beta)/\beta} K_m^{(1-\mu)^{(1-\beta)/\beta}} \tau^{(1-\beta)/\beta} \hspace{1cm} (A2.27)\]

Using (A2.27), (A2.13), (A2.22) and (A2.26), we get another simplified form as

\[\left(\frac{1-\beta}{1-\alpha}\right)^n \frac{n}{1-n} (1-\tau) = \frac{Y_a}{pY_m} = \frac{A_n z^\alpha K_a^{\mu^{1-\alpha}} L^{1-\mu} \tau^{1-\alpha} Y_m^{1-\alpha}}{p A_m (1-z)^\beta K_m^{(1-n)-\beta} (1-\mu)^{-\beta} \tau^{-\beta} Y_m^{-\beta}}\]

\[\left(\frac{1-\beta}{1-\alpha}\right)^n \frac{n}{1-n} (1-\tau) = \frac{A_n z^\alpha n^{1-\beta} \mu^{1-\beta} \tau^{1-\beta} (K/L)^{1-\beta} Y_m^{1-\beta}}{p A_m (1-z)^\beta (1-n)^{-\beta} (1-\mu)^{-\beta}} \hspace{1cm} \text{which gives}\]

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Hence, equations (A2.26) and (A2.28) give us two equations in terms of two unknowns, \( n \) and \( z \).

From (A2.26), substituting the value of \( \frac{z}{1 - z} \) into equation (A2.28), we get an expression for \( n \) in terms of technology parameters and given exogenous variables as:

\[
(1-n)^{(\alpha-\beta)/\beta} = \left(\frac{\alpha}{\beta}\right)^{\alpha} \left(\frac{1-\beta}{1-\alpha}\right)^{(1-\beta)/\beta} \cdot \frac{A_u^\alpha \tau^{(\beta-\alpha)/\beta} (L)^{(\beta-\alpha)/\beta}}{pA_m^\alpha \beta (1-u)^{\alpha(1-\beta)/\beta} (1-\mu)^{\beta(1-\beta)/\beta} (1-\tau)} \tag{A2.29}
\]

Further simplification yields

\[
n = 1 - \frac{1}{L} \left(\frac{\alpha}{\beta}\right)^{(\alpha\beta)/\beta} \left(\frac{1-\beta}{1-\alpha}\right)^{(1-\beta)/\beta} \cdot \frac{A_u^\beta \tau^{(\beta-\alpha)/\beta} (L)^{(\beta-\alpha)/\beta}}{p\left(\frac{A_m}{A_m^\alpha \beta (1-u)^{\alpha(1-\beta)/\beta} (1-\mu)^{\beta(1-\beta)/\beta} (1-\tau)}\right)^{(\alpha-\beta)/\beta}} \tag{A2.30}
\]

Hence, we get the values of \( L_a \) and \( L_m \) as the following:

\[
L_a = nL = \left(\frac{\alpha}{\beta}\right)^{(\alpha\beta)/\beta} \left(\frac{1-\beta}{1-\alpha}\right)^{(1-\beta)/\beta} \cdot \frac{A_u^\beta \tau^{(\beta-\alpha)/\beta} (L)^{(\beta-\alpha)/\beta}}{p\left(\frac{A_m}{A_m^\alpha \beta (1-u)^{\alpha(1-\beta)/\beta} (1-\mu)^{\beta(1-\beta)/\beta} (1-\tau)}\right)^{(\alpha-\beta)/\beta}} \tag{A2.31}
\]

\[
L_m = (1-n)L = \left(\frac{\alpha}{\beta}\right)^{(\alpha\beta)/\beta} \left(\frac{1-\beta}{1-\alpha}\right)^{(1-\beta)/\beta} \cdot \frac{A_u^\beta \tau^{(\beta-\alpha)/\beta} (L)^{(\beta-\alpha)/\beta}}{p\left(\frac{A_m}{A_m^\alpha \beta (1-u)^{\alpha(1-\beta)/\beta} (1-\mu)^{\beta(1-\beta)/\beta} (1-\tau)}\right)^{(\alpha-\beta)/\beta}} \tag{A2.32}
\]

Similarly substituting values of \( n \) and \( 1-n \), from (15) we get
Thus, given the world prices, technology parameters, and government policy variables ($\mu$ and $\tau$), we can derive the capital and labor shares in the economy. Substituting these values, we get the values of output, employment, and relative wage structure from the general equilibrium system of equations. Thus, $K_a$, $K_m$, and $Y_m$ are given by

$$K_a = \frac{\alpha(1-\beta)L - \alpha(1-\beta)\left(\frac{\alpha}{\beta}\right)\left(1-\beta\right)\left(1-\alpha\right)\left(1-\beta\right)\left(1-\alpha\right)\mu}{\alpha(1-\beta)L - (\alpha - \beta), \left(\frac{\alpha}{\beta}\right)\left(1-\beta\right)\left(1-\alpha\right)\left(1-\beta\right)\left(1-\alpha\right)\mu}$$

$$K_m = \frac{\alpha(1-\beta)L - \alpha(1-\beta)\left(\frac{\alpha}{\beta}\right)\left(1-\beta\right)\left(1-\alpha\right)\left(1-\beta\right)\left(1-\alpha\right)\mu}{\alpha(1-\beta)L - (\alpha - \beta), \left(\frac{\alpha}{\beta}\right)\left(1-\beta\right)\left(1-\alpha\right)\left(1-\beta\right)\left(1-\alpha\right)\mu}$$

For simplification, writing

$$D = \left(\frac{\alpha}{\beta}\right)\left(1-\beta\right)\left(1-\alpha\right)\left(1-\beta\right)\left(1-\alpha\right)\mu$$

(A2.33)
we may write \( Y_a, Y_m, K_a, K_m, L_a, L_m, G_a, G_m, r, w_a, \) and \( w_m \) in terms of our technology parameters and exogenous variables. We will use these for our comparative static analysis.

### A2.5.3 The Comparative Statics

#### A2.5.3.A A Change in \( \mu \):  
It would now be useful to the analysis to observe the results of setting different specifications regarding the change in the relative structure of rural and urban sector. Let us assume that rural and urban sectors have different proportions of total government spending allocated for development and infrastructure. Then the rural sector gets an increase in the share of government spending, with all other things remaining unchanged. The resulting changes in \( n, z, Y_a / pY_m, \) and \( Y \) will then be given by

\[
\frac{\partial n}{\partial \mu} = \frac{\beta (1-n)}{\beta - \alpha} \left[ \frac{1-\alpha}{\mu} + \frac{\alpha (1-\beta)}{\beta} \cdot \frac{1}{1-\mu} \right] > 0, \text{ iff } 1 > \beta > \alpha > 0, \text{ given } 0 < \mu, n < 1 \quad (A2.37)
\]

\[
\frac{\partial z}{\partial \mu} = (1-z)^2 \frac{\alpha (1-\beta)}{(1-\alpha)(1-n)} \cdot \frac{1}{\beta - \alpha} \left[ \frac{1-\alpha}{\mu} + \frac{\alpha (1-\beta)}{\beta} \cdot \frac{1}{1-\mu} \right] \text{ iff } 1 > \beta > \alpha > 0, \text{ given } 0 < \mu, n < 1 
\]

and,

\[
\frac{\partial (Y_a / pY_m)}{\partial \mu} = \frac{(1-\beta)(1-x)}{(1-\alpha)(1-n)^2} \frac{\partial n}{\partial \mu} > 0, \text{ iff } 1 > \beta > \alpha > 0. \quad (A2.39)
\]

The results contained in these equations can be summarized under the following proposition:

**Proposition A1:** An increase in the share of government expenditure in the rural sector (\( \mu \)) raises the employment in this sector, increases its capital intensity, and also increases rural output relative to urban output, provided the urban sector continues to be relatively capital intensive.
An increase in the proportion of government expenditure allocated to rural sectors raises the productivity of rural capital and labor, causing producers to hire more of these factors. This raises rural employment, reduces urban employment, and lowers urban output. Since agriculture is relatively capital-scarce, the productivity of capital increases faster than the productivity of labor. Thus, the capital labor ratio rises in agriculture although manufacturing remains relatively capital-intensive.

The assumption of capital and labor mobility ensures that the increased productivity and higher employment of the two factors of production in agriculture will raise the share of agricultural output in the economy. Since the government raises revenue by taxing only the urban output, however, the total tax revenue declines even as the government allocates a greater fraction of the budget to rural areas.

**A2.5.3B. Change in \( \tau \):** We now turn to the other fiscal policy variable \( \tau \). Holding the relative allocations to rural and urban sectors \( \mu \) and \( 1-\mu \) constant, the effects of an exogenous change in the urban tax \( \tau \) on \( n, z, \) and \( (Y_a / pY_u) \) are given by the following:

\[
\frac{\partial n}{\partial \tau} = (1-n) \left[ \frac{1}{\tau} + \frac{\beta}{\beta - \alpha} \cdot \frac{1}{1-\tau} \right] > 0, \text{ iff } \beta > \alpha > 0, \text{ given } 0 < \tau, n < 1, \quad (A2.40)
\]

\[
\frac{\partial z}{\partial \tau} = (1-n)^2 \cdot \frac{\alpha(1-\beta)}{\beta(1-\alpha)(1-n)} \left[ \frac{1}{\tau} + \frac{\beta}{(1-\tau)(\beta - \alpha)} \right] > 0, \text{ iff } 1 > \beta > \alpha > 0, \quad (A2.40a)
\]

and,

\[
\frac{\partial (Y_a / pY_u)}{\partial \tau} = \frac{(1-\beta)}{(1-\alpha)(1-n)} \left( \frac{1-\tau}{\tau} + \frac{n\alpha + (1-n)\beta}{\beta - \alpha} \right) > 0, \text{ iff } 1 > \beta > \alpha > 0, \text{ given, } 0 < n, \tau < 1 \quad (A2.40b)
\]

These results can be summarized under the following proposition:
**Proposition A2:** For any given allocation of government budget between urban and rural sectors, an increase in urban tax raises government spending in rural areas only if urban output falls proportionately less than the tax increase. In that case, agriculture experiences an increase in employment and output, and a rise in capital intensity.

An increase in the tax rate on income of the urban sector has a direct positive effect on tax revenue but, to the extent it discourages urban production, it causes the tax base to shrink and could well reduce the overall tax collection in equilibrium. If, however, the elasticity of urban output with respect to the tax rate is less than one, the overall budget will increase. For a given percentage share of budgetary expenditure in rural areas, the total rural spending goes up, providing externality to the rural output. The result is an increase in the productivity of capital and labor, causing rural employment and output to go up. Again, the relatively low capital intensity of agriculture raises the productivity of capital more and makes the sector more capital-intensive, although we continue to assume that, among the two, manufacturing remains the more capital-intensive sector.

The increased productivity and higher employment of the two factors of production in agriculture will raise the share of agricultural output in the economy. Since the government raises revenue by taxing only urban output, however, the total tax collection increases even as the government allocates a constant fraction of the budget to rural areas.

**A2.5.4. The Harris-Todaro Model with Government Spending**

In order to introduce the labor market specification following Harris and Todaro (1970), we assume that the urban wage is set above the market-clearing wage that would hold in flexible wage model or first-best equilibrium. As a result of this urban wage set-up, there is
unemployment in the urban labor market. Thus our equation (A2.20) in the flexible wage model is replaced by $L_u + L_m + L = L$, where, $L_u$ is labor unemployed in the urban sector. We assume that the migrants are risk neutral and compare the rural wage $w_a'$ with the expected wage in the urban sector after migration. This expected wage is the fixed urban wage $w_m'$ times the probability of finding a job in the urban sector. Thus migration continues until equilibrium is reached in which the expected wage following migration equals the rural wage. This replaces the labor market equilibrium condition (A2.18) by $w_a' = (1-u)w_m'$, where $u = L_u/(L_u + L_m)$, and $(1-u)$ is the probability of finding a job in the urban sector. Thus, our general equilibrium system with Harris-Todaro assumption can be written as:

\begin{align}
Y_a' &= A_a L_a^{1-\alpha} K_a^{\alpha} G_a^{1-\alpha} \\
Y_m' &= A_m L_m^{1-\beta} K_m^{\beta} G_m^{1-\beta} \\
\alpha Y_a' / K_a' &= r' \\
p(1-\tau)\beta Y_m' / K_m' &= r' \\
(1-\alpha)Y_a' / L_a' &= w_a' \\
p(1-\tau)(1-\beta)Y_m' / L_m' &= w_m' \\
w_a' &= (1-u)w_m' \text{ where } u = L_u/(L_u + L_m) \\
K_a' + K_m' &= K \\
L_a + L_m + L_u &= L \\
\mu r Y_a' &= G_a' \\
(1-\mu)\tau Y_m' &= G_m' 
\end{align}

If $u = 0$ we get the neo-classical or first-best solution where unemployment and wage differentials are eliminated. In our model, given the world price $p$, and exogenous values of $K, L,$
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\[ w'_m, \tau, \text{ and } \mu, \text{ we have 11 equations for solving the equilibrium values of 11 unknowns: } Y'_a, Y'_m, \]
\[ K'_a, K'_m, L'_a, L'_m, G'_a, G'_m, r', w'_a, \text{ and } u. \]

In order to compare the results of the Harris-Todaro model with those of the flexible-wage model, we use the following additional notations for equations under Harris-Todaro equilibrium:

\[ Y'_j = \text{Output of the } j\text{th sector} \]
\[ K'_j = \text{Capital used in the } j\text{th sector} \]
\[ L'_j = \text{Labor employed in the } j\text{th sector} \]
\[ G'_j = \text{Level of government spending in the } j\text{th sector} \]
\[ w'_j = \text{wage in the } j\text{th sector} \]
\[ r' = \text{rental rate of capital} \]
\[ j = a, m \]
\[ n_h = L'_a / L = \text{Share of rural labor to total labor force in economy} \]
\[ z_h = K'_a/K = \text{Capital used in agriculture} \]
\[ 1 - z_h = K'_m/K = \text{Capital used in manufacturing} \]

Using the above notations for shares, we can get the employment of the urban sector as

\[ L'_m = (1-u)(1-n_h)L \quad \text{(A2.52)} \]

With our assumptions of perfect mobility of capital and labor, constant returns to scale, and exogenous tax rate, we can derive the technology parameters using only an assumption about labor share in the economy (\( \theta_h \)) and data on the rural sector employment share (\( n_h \)),

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which are easily available for different countries. Using (A2.52) our long-run equilibrium condition \( w'_a = (1-u)w'_m \), can be re-written as:

\[
(1-\alpha) \frac{Y'_a}{n_h L} = (1-u)w'_m = \frac{p(1-u)(1-\beta)(1-\tau)Y'_m}{(1-n_h)(1-u)L},
\]

alternatively,

\[
\frac{Y'_a}{pY_m} = \left( \frac{1-\beta}{1-\alpha} \right) \frac{n_h}{1-n_h}(1-\tau)
\]

(A2.53)

Since the capital is perfectly mobile between the two sectors, the identical rental rates in equations (A2.43) and (A2.44) give us

\[
\frac{Y'_a}{K_a} = p\beta(1-\tau) \frac{Y'_m}{K_m} \quad \text{or,} \quad \frac{Y'_a}{pY_m} = \frac{\beta(1-\tau)}{\alpha} \frac{Y'_m}{K_m} = \frac{\beta(1-\tau)}{\alpha} \frac{z_h}{1-z_h}
\]

(A2.54)

From (A2.53) and (A2.54), we get, the equilibrium condition as

\[
\frac{z_h}{1-z_h} = \left( \frac{\alpha}{\beta} \right) \left( \frac{1-\beta}{1-\alpha} \right) \frac{n_h}{1-n_h}
\]

(A2.55)

In fact, equation (A2.55) simplifies the sub-system of equations (A2.43) - (A2.49) of our general equilibrium system. Similarly, by methods of substitution we can simplify equations (A2.42) and (A2.51) as

\[
Y'_a = A_m L_m^{1-\beta} K_m^{1-\beta} G_m^{1-\beta} = A_m L_m^{1-\beta} K_m^{1-\beta} (1-\mu)^{1-\beta} \tau^{1-\beta} Y_m^{1-\beta}
\]

\[
Y'_m = A_m L_m^{1-\beta} K_m^{1-\beta} (1-\mu)^{1-\beta} \tau^{1-\beta} Y_m^{1-\beta}
\]

(A2.56)

Using (A2.41), (A2.42), (A2.53)-(A2.56), we get another simplified form as:

\[
\frac{Y'_a}{pY_m} = \frac{A_m z_h^a K^a n_h^{1-a} L^{1-a} \mu^{1-a} \tau^{1-a} Y_m^{1-a}}{pA_m (1-z_h)^{\beta} K^{\beta} (1-n_h)^{1-\beta} (1-u)^{1-\beta} L^{1-\beta} (1-\mu)^{1-\beta} \tau^{1-\beta} Y_m^{1-\beta}}
\]
\[
\left( 1 - \beta \right) \frac{n_h}{1 - n_h} (1 - \tau) = \frac{A_z z_h^\alpha n_h^{1-\alpha} \mu^{1-\alpha} \tau^{(1-\alpha)\beta} (K/L)^{(1-\alpha)\beta} Y_m^{\beta} \beta}{p A_m^{\alpha}} (1 - z_h)^\beta (1 - n_h)^{(1-\beta)} (1 - \tau) (1 - \mu)^{(1-\beta)}.
\]

i.e., \[
\left( 1 - \beta \right) \frac{n_h}{1 - n_h} (1 - \tau) = \left( \frac{z_h}{1 - z_h} \right)^\alpha \frac{A_z n_h^{1-\alpha} \mu^{1-\alpha} \tau^{(1-\alpha)\beta} (L)^{(1-\alpha)\beta} \beta}{p A_m^{\alpha \beta} (1 - u)^{\alpha(1-\beta)\beta} (1 - \mu)^{\alpha \beta}}.
\]

(A2.57)

Here, for any observed \( L, L_m, \mu \), and \( \tau \), we can derive the \( u \) in terms of observables. Alternatively, for any observed data on \( u \), equations (A2.55) and (A2.57) can give us \( n_h \) and \( z_h \) in terms of technology parameters.

From (A2.55), substituting the value of \( \frac{z_h}{1 - z_h} \) into equation (A2.57), we get an expression for \( n_h \) in terms of technology parameters and given exogenous variables as:

\[
(1 - n_h)^{(\alpha - \beta)\beta} = \left( \frac{\alpha}{\beta} \right)^{\alpha} \frac{1 - \beta}{1 - \alpha}^{\alpha - 1} \frac{A_z \mu^{1-\alpha} \tau^{(1-\alpha)\beta} (L)^{(1-\alpha)\beta} \beta}{p A_m^{\alpha \beta} (1 - u)^{\alpha(1-\beta)\beta} (1 - \mu)^{\alpha \beta} (1 - \tau)}.
\]

(A2.58)

Further simplification yields

\[
n_h = 1 - \frac{1}{L} \left( \frac{\alpha}{\beta} \right)^{\alpha \beta} \frac{1 - \beta}{1 - \alpha}^{(\alpha - 1)\beta} \frac{A_z \beta \mu^{1-\alpha} \tau^{(1-\alpha)\beta} (L)^{(1-\alpha)\beta} \beta}{\tau p^{\alpha \beta} A_m^{\alpha \beta} (1 - u)^{\alpha \beta} (1 - \mu)^{\alpha \beta} (1 - \tau)}.
\]

(A2.59)

where \( 1-u \) is given by

\[
(1 - u) = \frac{\beta^{\beta - 1} (1 - \beta)^{\beta - 1} (1 - \tau)^{\beta - 1} A_g^{\beta - 1} (1 - n_h)^{\beta - 1} (1 - \alpha)^{\beta - 1} (1 - \mu)^{\beta - 1} \tau^{\beta - 1}}{w_m (1 - n_h) L (\beta (1 - \alpha) + n_h (\alpha - \beta))^{\beta - 1}}.
\]

(A2.60)

Hence, we get the values of \( L \) and \( L_m \) as the following:
Thus, for any set of observed values of $L_a$, $L_m$, $\mu$, and $\tau$, and given technology parameters, we can easily calibrate the Harris-Todaro model of dual economy, in order to analyze the impact of the allocation of government expenditure on output, employment, relative wages and rural to urban migration. It should be mentioned that despite the endogeneity of $u$ as determined by equation (A2.47) on page 105, viz., $w_a = (1-u)w_m$, where $u = (L_m + L_n)/L_a$, and reflected by the reduced form expression of $u$ in terms of other technology parameters and observables derived in equation A2.60, our simple calibration exercise in section A2.5.6 following Temple (2002) assumes and uses observed $u$, together with technology parameters. A fuller and more rigorous comparative static analysis of the impact of $\mu$ and $\tau$ would indeed incorporate the endogeneity of $u$.

**A2.5.4.2 The Comparative Statics**

**A2.5.4.2A Change in the Government Budget Share of Rural Development Expenditure**

In this setup, starting from equilibrium shares of labor and equilibrium output in the two sectors, we can analyze the effects of an increase in the rural share of government spending on $n_h$, $u$, $z_h$, $Y_u / p Y_m$, and $Y^*$ using the following results:
The results contained in these equations can be summarized under the following proposition:

**Proposition A3:** An increase in the share of government expenditure in the rural sector ($\mu$) raises the employment in this sector, increases its capital intensity, and also increases rural output relative to urban output, provided the urban sector continues to be relatively capital-intensive. Further, even if labor shares in the two sectors were left unchanged, the increase in $\mu$ would reduce urban unemployment.

An increase in the share of rural sector in government expenditure raises the productivity of capital and labor there causing producers to hire more of these factors. This raises rural employment, reduces urban employment, and lowers urban output. Since agriculture is relatively capital-scarce, the productivity of capital increases faster than the productivity of labor. Thus, the capital labor ratio rises in agriculture, although manufacturing remains relatively capital intensive.

The assumption of capital and labor mobility ensures that the increased productivity and higher employment of the two factors of production in agriculture will raise the share of agricultural output in the economy. Since the government raises revenue by taxing only the urban output, however, the total tax revenue declines even as the government allocates a greater fraction of the budget to rural areas.
Proposition A4 (Change in $\tau$): If an increase in the tax on urban output raises urban unemployment, the percentage of the labor force employed in agriculture increases, whereas if the unemployment rate declines, the effect on the share of rural labor is ambiguous, depending primarily on the shares of capital in manufacturing and labor in agriculture.

A change in $\tau$ leads to the following results:

$$\frac{\partial n_h}{\partial \tau} = \frac{(1-n_h)}{(\beta - \alpha)} \left[ \frac{1}{\tau} + \frac{\beta}{1 - \tau} + \frac{\alpha(1 - \beta) \partial u}{1 - u} \right]$$ \hspace{1cm} (A2.65)

$$\frac{\partial}{\partial \tau} \left( \frac{Y}{pY_m} \right) = \frac{(1 - \beta)(1 - \tau)}{(1 - \alpha)(1 - n_h)^2} \frac{\partial n_h}{\partial \tau} - \frac{1 - \beta}{1 - \alpha} \frac{n_h}{1 - n_h}. \hspace{1cm} (A2.66)$$

A2.5.5 Comparisons Between Flexible-Wage-Model Results and Harris-Todaro-Dual Economy Results

Now we will compare the results of the flexible wage model (or first-best equilibrium) with those derived under dualistic equilibrium under Harris-Todaro assumption. This will help explain the costs of dualism. It would be helpful to know these aggregate costs of rural-urban wage differentials, even though the rural-urban wage differential in the real world may not reflect the economy of Harris-Todaro equilibrium condition.

Using the results derived under equations (A2.28) and (A2.58), we can derive an expression for $n$, the employment of the rural sector in the flexible wage model or first-best economy as:

$$n = 1 - (1 - n_h)(1 - u)^{\alpha(1 - \beta)/(\alpha - \beta)}.$$ \hspace{1cm} (A2.67)
It would also be useful to compare the relative outputs under these two equilibrium conditions. In order to compare these, we need to derive the ratio of output in the flexible wage model to the output in dual-economy. The ratio is given by

\[
\Pi_y = \frac{Y_a + pY_m}{Y_a' + pY_m'} = \frac{pY_m \left(1 + \frac{Y_a}{pY_m}\right)}{pY_m' \left(1 + \frac{Y_a'}{pY_m'}\right)} = \frac{Y_m \left(1 + \frac{1 - \beta}{1 - \alpha} \cdot \frac{n}{1 - n_h}\right)}{Y_m' \left(1 + \frac{1 - \beta}{1 - \alpha} \cdot \frac{n_h}{1 - n_h}\right)}.
\]

(A2.68)

Using the production functions for \(Y_m\) and \(Y_m'\), and the values of \(z\) and \(z_h\) from equations (A2.26) and (A2.55), we get

\[
\Pi_y = \frac{(1 - z)}{(1 - z_h)} \left(\frac{1 - n}{1 - n_h}\right)^{(1 - \beta)/\beta} \left(1 + \frac{1 - \beta}{1 - \alpha} \cdot \frac{n}{1 - n_h}\right) (1 - u)^{(\beta - 1)/\beta}.
\]

Also from (A2.26) and (A2.55) we find

\[
\frac{(1 - z)}{(1 - z_h)} = \frac{(1 - n) \cdot \beta(1 - \alpha)(1 - n_h) + \alpha n_h (1 - \beta)}{(1 - n_h) \cdot \beta(1 - \alpha)(1 - n) + \alpha n (1 - \beta)}.
\]

Substituting this in the above expression we get

\[
\Pi_y = \left(\frac{1 - n}{1 - n_h}\right)^{(1 - \beta)/\beta} \left(\frac{\beta(1 - \alpha)(1 - n_h) + \alpha n_h (1 - \beta)}{\beta(1 - \alpha)(1 - n) + \alpha n (1 - \beta)}\right) \left(\frac{1 - \beta}{1 - \alpha} \cdot \frac{n}{1 - n_h}\right) (1 - u)^{(\beta - 1)/\beta}.
\]

(A2.69)

When \(\beta > \alpha\), from (A2.67), we get by introspection, \(n < n_h\). This shows us that, if the manufacturing or modern sector is relatively capital-intensive, agricultural employment is lower in the flexible-wage model than in the dual-economy model with Harris-Todaro equilibrium. This result is consistent with the results derived by Corden and Findlay (1975).
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Thus in our analysis, the key equation is,

\[
\left(\frac{1 - \beta}{1 - \alpha}\right) \frac{n_h}{(1 - n_h) (1 - \tau)} = \left(\frac{z_h}{1 - z_h}\right)^\alpha \frac{A_n n_h^{1 - \alpha} \mu^{1 - \alpha} \tau^{(\beta - \alpha)/\beta}}{p A_m^{\alpha/\beta} (1 - n_h)^{\alpha (1 - \beta)/\beta} (1 - u)^{\alpha (1 - \beta)/\beta} (1 - \mu)^{\alpha (1 - \beta)/\beta}}.
\]

(A2.69A)

A2.5.6 Calibration of the Model

In this section we calibrate the model to find the equilibrium values of a few endogenous variables based on realistic values of key parameters. The main variable of interest, the one that reflects the relative size of the rural sector, is the share of this sector \( n \) in total employment. In addition, in the Harris-Todaro model, the urban unemployment rate \( u \) is another variable determined together with \( n \) that determines the rate of migration.

In our calibration exercise, for simplification we assume that our observed world corresponds to Harris-Todaro equilibrium. Hence, the observed data on shares of agriculture in output and employment give us the equilibrium values for an economy. This assumption is common in most computable general equilibrium (CGE) and calibration exercises. It helps us to avoid arbitrary disequilibrium in the model.

In addition to notations and equations used in our model, we use the following notations in our calibration exercises.

\[\theta_{ah} = \frac{w^a_L}{Y^a} = \text{Share of labor income in agriculture}\]
\[\theta_{mh} = \frac{w^m_L}{pY^m} = \text{Share of labor income in manufacturing}\]
\[\theta_h = \frac{(w^a_L + w^m_L)}{Y^a} = \text{Labor income share in national income}\]
\[s_h = \frac{Y^a}{Y'} = \text{Share of agriculture in total output}\]
\[Y' = Y^a + p(1 - \tau)Y^m + prY^m = \text{Aggregate output of the economy}\]
Given our production functions are Cobb-Douglas, the technology parameters in both the agricultural and manufacturing sectors can be expressed in terms of aggregate labor share ($\theta_a$), agricultural output share ($s_a$), and agricultural employment share ($n_a$):

$$\theta_{ah} = \frac{w_a L_a / Y_a}{(w' L / Y') \alpha (L'_a / L) (Y'_a / Y')} = \frac{\theta_a n_a}{s} \quad (A2.70)$$

$$1 - \alpha = \theta_{ah} = \frac{\theta_a n_a}{s} \quad (A2.71)$$

$$1 - \beta = \theta_{ah} = \frac{\theta_a (1 - n_a)}{(1 - s_a)} \quad (A2.72)$$

Data on these variables are easily available for many developing countries. Thus, we can use the above expressions (A2.70)–(A2.72), in addition to our key equations (A2.24), (A2.30), (A2.53), and (A2.59).

We set the parameters $\alpha$ and $\beta$, which are the shares of capital in agriculture and manufacturing, respectively. Manufacturing represents the more modern segment of the economy and is naturally the more capital-intensive sector. Agriculture in most developing countries is the mainstay of the rural economy and tends to be highly labor-intensive. Following studies by Knight (1998), World Bank (2003), and Krichel and Levine (1999), we choose $\alpha$ to be 0.2, 0.3, or 0.5 and $\beta$ to be 0.8, 0.7, or 0.6.

We set the urban tax rate $\tau$ at 0.4. This is somewhat on the high side, but in our model the only source of financing government expenditure is the tax on urban income. In a related study with urban agglomeration, Krichel and Levine (1999) show that migration from rural to urban areas without a government employment subsidy is too low compared to the welfare-
maximizing outcome, which occurs under a subsidy-supported migration. To arrive at the optimal solution, they claim, the tax rate must be raised substantially from the existing rates.

Finally, we vary the share of public expenditure in the rural area, \( \mu \) from 0.5 to 0.8. This is the rural share in total expenditure rather than in expenditure per capita. Given that rural employment is a large fraction of labor force in most developing economies, a smaller per capita expenditure still requires a higher fraction of total government expenditure.

**A2.5.6.1 Calibration Results from Neo-Classical (Flexible-Wage) Model**

The calibration of the neoclassical model with no unemployment is based on equation (A2.29) when \( u = 0 \) and of the Harris-Todaro model with unemployment on (A2.29) and (A2.69A). We start with the full employment model.

The results show that for \( \alpha = 0.2 \) and \( \beta = 0.8 \), a rural expenditure share of \( \mu = 0.6 \) is associated with a 32 percent employment share \( n \) in the rural area. A rise in \( \mu \) leads to a slowdown of migration and leaves a greater fraction of labor force in the rural sector. The change in \( n \) is, however, a declining function of \( \mu \). For the case where technology yields a higher share of capital in rural output, rural labor as a fraction of total labor declines first and then rises for higher values of \( \alpha \). This nonlinear pattern is seen from the result for \( \alpha = 0.3 \) for which a 60 percent rural share in public expenditure leaves 31 percent of labor force in the rural sector (down from 32.3 percent for \( \alpha = 0.2 \)) and \( \alpha = 0.5 \) for which a 60 percent rural share in public expenditure is associated with 49 percent of labor force in the rural sector.
A2.5.6.2 Calibration Results from Harris-Todaro Model

The calibration of the Harris-Todaro model requires holding constant either wage differential or unemployment rate. From the studies of wage differential and observed rates of unemployment in economies with a variety of rural-urban settings, we have chosen a 30 percent rate for urban unemployment while we examine the response of other endogenous variables.

Once again starting from $\alpha = 0.2$ and $\beta = 0.8$, we find a rural expenditure share of $\mu = 0.6$ to be associated with a 34 percent employment share $n$ for the rural sector. A rise in $\mu$ leads to a slowdown of migration to urban areas, leaving a greater fraction of the labor force in the rural areas. The change in $n$ is, however, a declining function of $\mu$. For economies with higher shares of capital in rural output, rural labor as a fraction of total labor remains constant up to $\alpha = 0.3$ and then rises for higher values of $\alpha$. The nonlinearity in the response of $n$ is seen from the result for $\alpha = 0.3$ for which a 60 percent rural share in public expenditure leaves 33.9 percent of labor force in the rural sector (the same as for $\alpha = 0.2$) and $\alpha = 0.5$ for which a 60 percent rural share in public expenditure is associated with 54.4 percent of labor force in the rural sector.

From these results, it is clear that government expenditure in rural areas is more effective in reducing rural-to-urban migration in the Harris-Todaro model than in the full-employment neoclassical model. A sizable urban unemployment gives a stronger incentive for rural labor to stay in agriculture, and may also attract some unemployed labor from the other sector, when public expenditure raises the marginal productivity of labor in rural production.
Chapter 3

Expansion of Education and Economic Welfare: A Theoretical Analysis

Abstract

This chapter studies the effects of selected education policies on the size of the educated labor pool and on economic welfare using the “job ladder” model of education, which is relevant to liberal arts education in developing countries. The policies considered are (1) increasing the teacher-student ratio, (2) raising the relative wage of teachers, and (3) increasing the direct subsidy per student. In addition, the chapter analyzes the impact of wage rigidities in the skilled or modern sector on the size of the educated labor force. The analysis consists of five major sections. First, it reformulates the Bhagwati-Srinivasan job ladder model to make it amenable to analyzing the comparative static results of the effects of selected policies. Second, since higher education is mostly publicly financed, the analysis extends the job ladder model to incorporate public financing of the education sector. It then examines that model along with the effects of changes in policy parameters. Third, the analysis develops another extension of the job ladder model to include private tuition practices used by teachers that are prevalent in many developing countries. Fourth, to analyze the impact of wage rigidities in a less-restrictive framework where individuals can choose education based on ability and cost, the chapter develops an overlapping generations model of education with job ladder assumptions of wage rigidities in the skilled or modern sector. The chapter examines the flexible market and fix-market (with wage rigidities) equilibrium scenarios, and compares the impact on the threshold level of abilities and the size of the educated labor force. Finally, using specific functional forms of human capital production, cost, and ability density functions, the chapter analyzes the equilibrium outcomes. The analysis shows that in an economy with wage rigidities in the skilled sectors (modern and education sectors), the result of quality-enhancing policies under the simple job ladder model is an increase in the total size of the educated labor force. However, under an extended version of the job ladder model, the result depends on the relative size of the effects of an increase in cost of education, and effects of an increase in the expected wage. The overlapping generations/job ladder model formulation used in the chapter finds that an increase in the present value of the expected wage and/or an increase in the marginal product of education will increase the demand for education. The minimum threshold level of ability falls, and more people are encouraged to acquire educational skills.
3.1. Introduction

One of the important factors distinguishing developing from developed countries is the nature of the wage structure. In many developing countries, despite some affirmative action programs, traditional discrimination (based on caste, race, region, etc.) and traditional (colonial) wage structures have led to socioeconomic conditions in which “high wages for educated elites are set by fiat, legislation, or unionization. The phenomenon of high wages, accruing to employed educated elites, creates a political demand for higher education (Bhagwati and Hamada, 1974).”\(^6\)

In addition, in many developing countries, people are encouraged to acquire higher education to improve their prospects for employment abroad and higher returns, which leads to an increase in the supply of educated workers in the domestic labor market. However, due to the sticky wage-rate—constant demand for educated labor at home—the rising supply of educated workers in the domestic labor market raises involuntary educated unemployment at home (Stark and Fan, 2011; Fan and Stark, 2007a, b; Stark and Wang, 2002; Stark, Helmenstein, and Prskawetz, 1997, 1998; Bhagwati and Hamada, 1974; Fields 1974). For example, in India in 2011–12, the educated unemployment rates for post-graduates and graduates were 10 percent and 9.4 percent, respectively, compared to 5.4 percent and 1.2 percent for those with secondary and primary education (India Labor Bureau, 2012). In China, the unemployment rate among recent college graduates was 9.5 percent in 2011 (13 percent among graduates with fine-arts major, and lowest among engineering majors at 7.5 percent) (MyCos Institute, 2011). In Kerala, the state in India with the highest literacy rates, the unemployment rate among university graduates in 2003 was 36 percent, compared to 12 percent among secondary-incompletes and 1.7 percent among those with primary education (Mathew, 1997; Zachariah and Rajan, 2005). In 2007, a report by the Arab Labor Organization found that “the trend of higher unemployment among literates applies

\(^6\) Dumont (1969), Sunkel (1971), and Seers and Jolly (1972), among others, analyze the implications of discriminatory wage structures.
to countries throughout the Arab world, with a ratio of 5:1 in Morocco and 3:1 in Algeria,” and that in Egypt, “the rate of unemployment is ten times higher in the educated section of the population than among illiterates” (quoted in the Egyptian newspaper Al-Masri al-Yawm). Boudarbat (2006) found that in Morocco in 2000, the unemployment rate of university graduates was about four times that of individuals with less than six years of schooling.

Despite educated unemployment and underemployment, the sustained high demand for educated workers is translated into financing of higher education largely by states in many developing countries. For example, in sub-Saharan Africa, the governments provided over 90 percent of the cost for higher education (Teferra, 2007). In Papua New Guinea, the government provided more than 95 percent of university recurrent financing in 1988 (McGavin, 1991).

Although these expenditures on public education might appear to benefit all segments of the population equally, the role of education as a vehicle for the equalization of economic opportunity and redistribution of income is seriously in doubt. Jallade (1973) shows that education in many developing countries generally provides a net transfer from low- to high-income families even after accounting for higher marginal taxes on higher-income groups. Analyzing the educational policies implemented in Chile from 1987–98, Espinoza (2008) found that access to higher education was disproportionately greater among upper- and upper-middle-income students compared to lower-, lower-middle, and middle-income groups. Evidence in Egypt (Fahim and Sami, 2011) and Tunisia (Abdessalem, 2011) suggests that even though educational reforms aim to increase access for poor students, most of the public spending on higher education goes to students in the richest quintiles. Thus, the demand for higher education policies in many of these countries seems to stem from many socio-political considerations, without much regard for their adverse welfare impact on non-elites. This chapter analyzes the
effects of selected education policies on the size of the educated labor force and on economic welfare in a theoretical framework. The policies considered are (1) increasing the teacher-student ratio, (2) raising the relative wage of teachers, and (3) increasing the direct subsidy per student.

Among the existing models of education—human capital (Becker, 1974), screening (Arrow, 1973; Spence, 1973) and the job ladder (Bhagwati and Srinivasan, 1977)—the job ladder model seems to be the most relevant to higher education, especially liberal arts education in developing countries such as India, Bangladesh, Pakistan, Sri Lanka, Malaysia, Thailand, and many sub-Saharan African countries. The model examines the social inefficiency properties of the education system under a general equilibrium framework. Jobs are specified and ranked by wage in a job ladder framework. Therefore, when wages are sticky, an excess supply of educated labor filters down from the best job to the next best job in the ladder. The hiring principle is then to prefer “more educated” to “less educated” or “uneducated.” This is to preserve sociological “fairness” in the sense that the educated have put in more to get the job, even when both educated and uneducated are equally productive.\(^7\)\(^8\) This “fairness-in-hiring” principle makes education an instrument of competition for jobs, yielding a divergence between private and social returns to education. Thus, access to higher education appears to be an important determinant of the distribution of income in a way that is not directly related to the productivity of workers.

The empirical findings on the effectiveness of these policies in developed and developing countries vary widely and are fraught with methodological problems. Primarily, it is unclear

\(^7\) In India, examples of fairness in hiring are found very often. In many jobs, the required qualification is stated as a high school diploma, yet the availability of a large number of applicants leads the selection committee to consider only the applicants with a bachelor’s or master’s degree.

\(^8\) The obverse of fairness is prejudice. When jobs are scarce, discrimination in hiring may be on the basis of race (whites vs. blacks), caste (upper castes vs. lower castes), etc.
what unit of production to use (individual pupil, school, classroom), and whether the relevant objective is to maximize academic achievement or some other output, such as economic payoff, national income, or welfare. There is some evidence that increasing school inputs (teacher quality, teacher-student ratio, average length of a school term, or teachers’ salaries) may have a higher economic payoff (measured by increased earnings of school-leavers) in middle-income and developed countries (Carnoy, Sack, and Thias, 1977; Behrman and Birdsall, 1983; Card and Krueger, 1992). In a review of several studies on developing countries, Fuller (1986) found that neither the teacher-student ratio nor the salary of teachers had much influence on student achievement. In recent natural experiments, Case and Deaton (1999) found that a reduction in class sizes in South Africa raised academic achievement on reading tests (but not math tests) among black students but not among white students, who have smaller class sizes, thus proving the diminishing returns of the ratio. In studies using Israeli data, Angrist and Lavy (1999, 2002) found that reduction in class size raised reading scores and (less often) math scores, but that providing computers had no effect on academic performance. Urquiola (2006) found a positive impact of lower class size in a case study of Bolivia. Finally, in a review of studies, Hanushek and Luque (2003) found that though class size has positive effects on learning, in many of these studies the effects were often imprecisely measured.

With respect to subsidies, public subsidies to education are already very high in developing countries. For example, using figures for the mid-1980s, Tan and Mingat (1992) report that in some Asian countries the public subsidy per student amounts to 11 percent of per capita GNP at the primary level, 17 percent at the secondary level, and as high as 191 percent at the higher-education level. Most recent UNESCO data show that many sub-Saharan African countries (Malawi, Lesotho, Tanzania, Niger, Burundi, Swaziland, Botswana, Chad, Burkina Faso, and Mauritania) spend from 2 to 18 times GDP per capita on each tertiary student, and that
despite the spending, gross enrollment rates were less than 8 percent.\footnote{ UNESCO Institute for Statistics in EdStats, February 2013. Data are available at www.worldbank.org/education/edstats/} In tertiary education, public expenditure per student as a share of per capita GDP in 2009 was 149 percent in Ghana, 88 percent in Benin, and 75 percent in India.

The extended job ladder model presented here offers a simple but major extension to incorporate this public financing aspect of higher education prevalent in almost all developing countries. In addition, another extension of the job ladder model has been made here to include private tuition practices by teachers. The aim is to analyze the effects of educational policies on the size of the educated labor force and on economic welfare using a reformulated original version and extended versions of the job ladder model.

In addition to considering a job ladder model of education to analyze the effects of government education policies on the size of the educated labor force and economic welfare, this chapter analyzes the impact of wage rigidities in the skilled or modern sector on the size of the educated labor force. The analysis consists of five major sections. First, it reformulates the Bhagwati-Srinivasan job ladder model to make it amenable to analyze the comparative static results of the effects of selected policies. Second, since higher education is mostly publicly financed, the chapter extends the job ladder model to incorporate public financing of the education sector. It then analyzes the model along with the effects of changes in policy parameters. Third, the chapter develops another extension of the job ladder model to include private tuition practices by teachers that are prevalent in many developing countries. Fourth, in order to analyze the impact of wage rigidities in a less-restrictive framework in which individuals can choose education based on ability and cost, the chapter develops an overlapping generation model of education with a job ladder assumption of wage rigidities in the skilled or...
modern sector. The chapter analyzes the flexible market and fix-market (with wage rigidities) equilibrium scenarios, and compares the impact on the threshold level of abilities and the size of the educated labor force. Finally, using specific functional forms of the human-capital production, cost, and ability density functions, the chapter analyzes the equilibrium outcomes.

The analysis shows that in an economy with wage rigidities in the skilled sectors (modern and education sector), the result of the quality-enhancing policies is an increase in the total size of the educated labor force under the simple job ladder model. However, in extended versions of the job ladder model, the results depend on the relative sizes of the cost-enhancing impact and the effect on the expected wage of an educated worker.

This chapter is organized as follows. In Section 2, the Bhagwati-Srinivasan job ladder model is reformulated to make it suitable for the analysis here. Section 3 derives the comparative static effects of changes in the teacher-student ratio, teacher’s wage, and subsidy/expenditure per student on the size of the educated labor force and economic welfare. Sections 4 and 5 discuss a flexible wage model and a social optimum model, respectively, and demonstrate the inefficiency of different systems. Section 6 discusses growing inequality with expansion of education under the fairness-in-hiring principle within a job ladder framework. Section 7 offers a simple but major extension to incorporate this public financing aspect of higher education common in almost all developing countries. Section 8 makes another extension of the job ladder model to include private-tuition practices by teachers. In order to analyze the impact of wage rigidities in a less-restrictive framework in which individuals can choose education based on ability and cost, Section 9 develops an overlapping generation model of education with job ladder assumption of wage rigidities in the skilled or modern sector. The section analyzes the flexible market and fix-market (with wage rigidities) equilibrium scenarios, and compares the impact on the threshold level of abilities and the size of the educated labor force. In addition, using specific functional
forms of human capital production, cost, and ability density functions the section analyzes the equilibrium outcomes. Section 10 puts forth conclusions and ideas for future extensions in light of the empirical survey of the literature on financing of higher education, as discussed in Appendix 3.1.

3.2. The Job Ladder Model

The model assumes two non-education sectors (Sectors 1 and 3) and one education sector (Sector 2). In Sector 1, output is a function of employed educated workers ($L^E_1$):

$$Q_1 = f_1(L^E_1),$$  \hspace{1cm} (3.1)

where $f_1$ is strictly concave, $f_1' > 0 \forall L^E_1 > 0$, $f_1' = \infty$ when $L^E_1 = 0$, and $f_1' = 0$ when $L^E_1 = \infty$.

Sector 2, the education sector, employs educated labor ($L^E_2$) as teachers to educate students ($S$) at an exogenously specified teacher-student ratio ($\varepsilon$). Here, the teaching cost is the only direct cost of education and equals the teacher-student ratio times the teacher’s wage. No value is placed on education as a consumption good. The education technology is simply given by:

$$S = \frac{1}{\varepsilon} L^E_2.$$  \hspace{1cm} (3.2)

Finally, Sector 3 employs both educated ($L^E_3$) and uneducated labor ($L^U$) at an identical wage $w$, set at marginal value product in terms of the output of Sector 1. Thus, output in Sector 3 is a linear function of the labor employed:

$$Q_3 = w(L^E_3 + L^U).$$  \hspace{1cm} (3.3)

Assuming the economy is small and open, the relative price of output of Sector 1 to that of Sector 3 is taken as exogenous and equal to 1. The job ladder consideration is incorporated into the model by assuming that wages ($\lambda_1 w$ and $\lambda_2 w$, $\lambda_1 > \lambda_2 > 1$) in Sectors 1 and 2 are sticky and
the supply of educated labor in excess of its employment in Sectors 1 and 2 spills over into Sector 3. The equilibrium size of the educated labor force is determined by the principle of present value equalization. That is, the present value of an educated worker’s income stream net of any education costs equals the present value of that worker’s income stream if the worker had not educated him or herself but was employed as an uneducated worker. This condition can be written as:

\[
\frac{\bar{w}}{r} \int_{0}^{T} e^{-rt} \, dt - c \int_{0}^{t} e^{-rt} \, dt = \frac{w}{r} \int_{0}^{T} e^{-rt} \, dt
\]

or,

\[
\gamma \bar{w} = w + (1 - \gamma)c
\]

(3.4)

where,

\[
\gamma = \frac{\int_{0}^{T} e^{-rt} \, dt}{\int_{0}^{T} e^{-rt} \, dt}.
\]

(3.5)

In the equations, \(T\) is the total working life, of which first \(t\) periods are spent in school if the worker chooses to be educated, \(r\) is the non-negative rate of discount, \(c\) is the education cost charged to the worker per period in school, and \(\bar{w}\) is the wage per period of an educated worker. \(\bar{w}\) is the expected average wage of an educated worker, which equals the weighted average of the wages in non-education sectors and the education sector, with weights being equal to the employment shares of educated workers in each sector.

Assuming a steady state, the total labor force is set at unity. \(L^E\) is the portion of total labor force educated. Thus, for a given working life \(T\) and the schooling period \(t\), the number of students (in steady state) will be given by:

\[
S = \frac{t}{T-t}L^E = \delta L^E,
\]

(3.6)
where \( \delta = \frac{t}{T-t} \). For an exogenously given teacher-student ratio \( \varepsilon \), the number of teachers is given as \( L^E_2 = \varepsilon \delta L^E \) from equations 3.2 and 3.6. Employment of educated workers in Sector 3 \( (L^E_3) \) and the size of the pool of uneducated workers \( (L^U) \) who are employed only in Sector 3 are given by:

\[
L^E_3 = L^E - L^E_1 - L^E_2 = (1 - \varepsilon \delta)L^E - L^E_1, \quad \text{and} \quad (3.7)
\]

\[
L^U = 1 - L^E - S = 1 - (1 + \delta)L^E. \quad (3.8)
\]

The equilibrium values of only \( L^E \) and \( L^E_1 \) still need to be determined. Given \( f_1 \) and \( \lambda_1 w, L^E_1 \) is determined by the marginal productivity rule:

\[
f'_1 (L^E_1) = \lambda_1 w \quad \text{or} \quad L^E_1 = g (\lambda_1 w), \quad (3.9)
\]

where \( \bar{w} \), the average wage of the educated worker, is the weighted average of wages \( \lambda_1 w, \lambda_2 w, \) and \( w \) in Sectors 1, 2, and 3, respectively. The weights are the proportions of the educated labor force employed in the respective sectors. The average wage can be written as:

\[
\bar{w} = \frac{\lambda_1 w L^E_1}{L^E} + \frac{\lambda_2 w \varepsilon \delta L^E}{L^E} + \frac{(1 - \varepsilon \delta)L^E - L^E_1)w}{L^E}. \quad (3.9a)
\]

Substitution of \( \bar{w} \) in equation 3.4 gives \( L^E \) as:

\[
L^E = \frac{\gamma w(\lambda_1 - 1) L^E_1}{(1 - \gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon \delta}. \quad (3.10)
\]

The specification of the model is now complete. The parameters are \( w, (t, T, r, c, \lambda_1, \lambda_2, \delta, \varepsilon, f_1 \) and 1 (the total labor force). There are 10 equations for 10 unknowns: \( L^E_1, L^E_2, L^E_3, L^U, S, L^E, \gamma, \bar{w}, Q_1 \) and \( Q_3 \).
3.2.1. Equilibrium Values

In this model for given costs, wages, the discount rate, working and schooling periods, and the exogenous teacher-student ratio, all other variables are trivially determined once $L^E$ and $L^E_1$ are determined. These are given by above equations 3.9 and 3.10. This solution is meaningful if $(1 - \varepsilon \delta)L^E \geq L^E_1$ and $(1 + \delta)L^E \leq 1$. The first condition states that the size of the educated labor force net of teachers is at least as large as employment in Sector 1. The second condition implies that the educated labor force and the student body do not exhaust the total labor force. Since $\lambda_i \geq 1$, it is clear from equation (10) that the first condition will be satisfied, provided:

\[(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1) \varepsilon \delta \geq 0 \quad (3.11)\]

or, \[w + (1 - \gamma)c \geq \gamma w(\lambda_2 \varepsilon \delta + 1 - \varepsilon \delta) \quad (3.11')\]

and \[\gamma w(\lambda_1 - 1)(1 - \varepsilon \delta) \geq (1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1) \varepsilon \delta \quad (3.12)\]

or, \[\gamma w[\lambda_1 (1 - \varepsilon \delta) + \lambda_2 \varepsilon \delta] \geq w + (1 - \gamma)c. \quad (3.12')\]

Equation 3.11’ implies that the opportunity cost of education (which equals the sum of income forgone and the discounted sum of the charges for education) must be higher than the expected minimum income of an educated worker (which equals the discounted sum of income of an educated worker if that worker gets a job either in the education or subsistence sector). Equation 3.12’ shows that the expected maximum income of an educated worker (which equals the discounted sum of income of an educated worker if that worker gets a job either in Sector 1 or the education sector) must be higher than the opportunity cost of education. These two conditions together, from equations 3.11’ and 3.12’, imply that the opportunity cost of getting educated in this model lies between the expected minimum (when an educated worker gets a job
only in the education sector and subsistence sector) and the expected maximum income of an educated worker (none of the educated workers are employed in the subsistence sector).

\[
\gamma \nu (\lambda_1 (1 - \varepsilon \delta) + \lambda_2 \varepsilon \delta) \geq w + (1 - \gamma) c \geq \gamma \nu (\lambda_2 \varepsilon \delta + (1 - \varepsilon \delta)).
\]  

(3.13)

\{\text{discounted expected maximum income}\} \{\text{opportunity cost}\} \{\text{discounted expected minimum income}\}

Finally, since education has no value as a consumption good, the national welfare is given by:

\[
W^M = f_1 (L^E) + w (1 - L^E - \varepsilon \delta L^E - \delta L^E).
\]  

(3.14)

3.3. Comparative Static Results: Changes in Policy Parameters

This section discusses the effects of changes in the policy parameters: teacher-student ratio, teacher’s wage level, and education subsidy (which affect the cost of education). The analysis assumes that these quality indices are changed exogenously and do not influence the period of schooling. The results from the job ladder or minimum wage model are then compared with those from the flexible-wage and social-optimum models. To distinguish the results, \( M \) is used for the job ladder model, \( F \), for the flexible wage model, and \( S \) for the social optimum model.

3.3.1. Policy 1: Increase in Teacher-Student Ratio (\( \varepsilon \))

In the job ladder model, the supply of educated labor in excess of employment in Sectors 1 and 2 spills over into Sector 3. An exogenous increase in the teacher-student ratio gives us the following results:

1. The opportunity cost of education, \( w + (1 - \gamma) c \), does not change, as wages and costs are independent of \( \varepsilon \).

2. The discounted sum of the expected minimum income \( W^E_{\min} \) (when no educated worker is employed in the highest paying sector) of an educated worker rises, but the discounted
sum of expected maximum income $W^E_{\max}$ (when no educated worker is employed in the lowest paying sector) falls, i.e.:

$$\frac{dW^E_{\min}}{d\varepsilon} = \frac{d}{d\varepsilon} (\gamma w(\lambda_2 \varepsilon \delta + (1 - \varepsilon \delta))) = \gamma w(\lambda_2 - 1)\delta > 0 \quad \text{as} \quad \lambda_2 > 1$$

$$\frac{dW^E_{\max}}{d\varepsilon} = \frac{d}{d\varepsilon} (\gamma w(\lambda_2 \varepsilon \delta + \lambda_1 (1 - \varepsilon \delta))) = \gamma w(\lambda_2 - \lambda_1)\delta < 0 \quad \text{as} \quad \lambda_2 < \lambda_1.$$  

(3) The last result is stated in the proposition that follows below.

**Proposition 1:** When the period of schooling and the wage rates are independent of schooling quality measured in terms of the teacher-student ratio, an exogenous increase (decrease) in the teacher-student ratio will increase (decrease) the total size of the educated labor force, while employment in Sector 1 will remain unchanged and the welfare level will fall (rise).

Proof: Differentiating equations 3.9 and 3.10 with respect to ($\varepsilon$) results in:

$$\frac{dL^E_{L}}{d\varepsilon} = 0, \quad \frac{dL^E_{M}}{d\varepsilon} = \frac{\gamma w\delta(\lambda_2 - 1)L^E_{L}}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1)\varepsilon \delta} > 0$$

as $\lambda_2 > 1$ and $(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1)\varepsilon \delta > 0$  \hspace{1cm} (3.15)

$$\frac{dW^M}{d\varepsilon} = -w\delta(L^E_{L} + (1 + \varepsilon)dL^E_{L} / d\varepsilon) < 0$$

$$\frac{dW^M}{d\varepsilon} = -w\delta L^E_{L} \left[ \frac{(1 - \gamma)(w + c) + \gamma w\delta(\lambda_2 - 1)}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1)\varepsilon \delta} \right] < 0.$$ \hspace{1cm} (3.16)

An increase in the teacher-student ratio raises the expected wage, $\bar{w}$, of an educated worker. This rise in $\bar{w}$ expands the proportion of educated labor in the labor force due to the increasing number of students and teachers. This in turn lowers the proportion of the labor force employed in Sector 3, which is a full employment sector. Since employment, productivity, and output of
Sector 1 remain unaffected, and education has no value as a consumption good, the welfare level of the economy falls due to a fall in the output of Sector 3.

### 3.3.2. Policy 2: Increase in Teacher’s Wage Level ($\lambda_2$)

1. **An increase in $\lambda_2$, with $\lambda_1$ remaining unchanged:** This will produce similar effects on the size of the educated labor force and welfare of the economy. The results are given by:

   $$\begin{align*}
   \frac{dL_{EM}}{d\lambda_2} &= 0, \quad \frac{dL_{EM}}{d\lambda_2} = \frac{\gamma w_e \delta L_{EM}}{(1-\gamma)(w+c) - \gamma w(\lambda_2-1)\delta} > 0 \\
   \frac{dW^M}{d\lambda_2} &= -w(1+\varepsilon)\delta \frac{dL_{EM}}{d\lambda_2} < 0.
   
   \end{align*}$$

   An increase in the salary of teachers raises the expected average wage of an educated worker, which increases the size of the educated labor force. The employment in Sector 1 does not change. Only a fraction of the larger pool of educated workers gets employed as teachers and the remainder goes back to Sector 3, where educated labor has no extra productivity beyond the productivity of uneducated labor. Since teachers and students are withdrawn from Sector 3, welfare will be reduced by the amount the educated workers could have produced in Sector 3.

2. **When both $\lambda_1$ and $\lambda_2$ are increased:** If wages in both Sectors 1 and 2 are increased, the effect on the size of the educated labor force and welfare will depend on $\xi$, the elasticity of demand for labor in Sector 1 with respect to the wage rate. This is verified by the following expression:

   $$\begin{align*}
   \frac{dL_{EM}}{d\lambda_1} &= \frac{L_{EM}}{\lambda_1-1}(1+\lambda_1-1\xi), \quad \text{where} \quad \xi = \frac{g'}{g} \lambda w.
   \end{align*}$$

   If $|\xi| < \frac{\lambda_1}{\lambda_1-1}$, i.e., demand for labor in Sector 1, is not very elastic with respect to the wage rate, $\lambda_1$ and $\lambda_2$ act in the same direction and the size of the educated labor force will rise due to an
increase in the expected wage of the educated worker. However, if demand for labor in Sector 1 is sufficiently elastic, the increase in $\lambda_1$ will reduce the size of the educated labor force. The increase in $\lambda_2$, on the other hand, tends to increase this size and hence the net effect is ambiguous.

**Figure 3.1. The Adjustment Process**

The adjustment mechanism can be interpreted easily from equations 3.4 and 3.9a. Substituting

$$
\bar{w} = \frac{\lambda_1 w L^E_1}{L^E} + \frac{\lambda_2 w \varepsilon \delta L^E}{L^E} + \frac{(1 - \varepsilon \delta)L^E_1 - L^E_1)w}{L^E}
$$

(3.9a) in equation 3.4 $\gamma \bar{w} = w + (1 - \gamma)c$, we get

$$
\frac{(\lambda_1 - 1)w L^E_{ML}}{L^E_{ML}} + (\lambda_2 - 1)w \varepsilon \delta + w = \frac{w + (1 - \gamma)c}{\gamma}
$$

The left-hand-side expression, i.e., the expected wage of an educated worker, is shown by the downward sloping line AA in Figure 3.1. The right-hand side expression, i.e., the discounted opportunity cost of education, is shown by line C for given c, w, and $\gamma$ (since $r$, $t$, and $T$ are given). Clearly, an increase in $\varepsilon$ or $\lambda_2$ (or both) raises the average expected wage of the educated worker (shifts A to A’) without affecting the opportunity cost. Since $L^E_{ML}$ does not change, in equilibrium the equality is brought through an increase in $L^E$ to $L^E'$. 
3.3.3. Policy 3: Increase in Education Expenditure or Subsidy

If the education expenditure or subsidy is increased, which lowers the direct costs charged for education ($c'$), we get similar results again, i.e., an increase in the size of the educated labor force at the cost of a welfare loss to the economy. To show the effect of a higher rate of subsidy we have:

$$c' = \beta c = (1 - \mu)c = (1 - \mu)L\lambda_2 w\varepsilon,$$

where $\beta = 1 - \mu$, (3.20)

where $\mu$ is the rate of subsidy per student per schooling period. An increase in $\mu$, i.e., a decrease in $c'$, lowers the opportunity cost of education without directly affecting the average expected wage of an educated worker. This increases the size of the educated labor force. Since the output of Sector 1 remains unchanged, the withdrawal of workers causes a welfare loss measured in terms of loss of output in Sector 3. The results are:

$$L_{EM} = \frac{\gamma w(L_{EM} - 1)}{(1 - \gamma)(w + c') - \gamma w(L_{EM} - 1)\varepsilon\delta} = \frac{\gamma w(L_{EM} - 1)}{(1 - \gamma)(w + c(1 - \mu)) - \gamma w(L_{EM} - 1)\varepsilon\delta},$$

when $\mu \geq 0$, $L_{EM} \geq L_{EM}$, $L_{EM}' = L_{EM}$, when $\mu = 0$. Since the size of the educated labor force is different due to lower private costs of education, we have used $L_{EM}'$ to differentiate it from $L_{EM}$.

$$\frac{dL_{EM}}{d\mu} = 0, \quad \frac{dL_{EM}'}{d\mu} = \frac{(1 - \gamma)L\lambda_2 wE L_{EM}'}{(1 - \gamma)(w + c') - \gamma w(L_{EM} - 1)\varepsilon\delta} > 0$$

(3.21)

$$\frac{dW^M}{d\mu} = -w(1 + \varepsilon)\delta \frac{dL_{EM}'}{d\mu} < 0.$$  (3.22)

This also shows that public subsidies toward the cost of education boost the size of the educated labor force in the job ladder framework by lowering the private opportunity cost of an educated worker. An increase in a subsidy further enhances the size of the educated labor force, with a reduction in welfare in this framework as well.
3.3.4. A Comparison of the Effects of Changes in ε and λ₂

The comparison between the effects of a change in these policy parameters, ε and λ₂, becomes equivalent to comparing γδ(λ₂ - 1) and γδε. For a non-negative rate of discount, i.e., when r ≥ 0, (1 - γ) ≥ γδ, we get the cases (a), (b), and (c):

(a) \( \frac{dL^E_M}{d\varepsilon} > \frac{dL^E_M}{d\lambda_2} \) if \( \lambda_2 - 1 > \varepsilon \). \hspace{1cm} (3.23)

Therefore, for an increase in the teacher-student ratio, the size of the educated labor force will be higher than in an increase in the wages in the education sector. Also, comparing the costs associated with each policy parameter results in the following:

(b) \( \frac{dL^E_M}{d\lambda_2} / \frac{dc}{d\lambda_2} = \frac{\gamma\delta L^E_M}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \) \hspace{1cm} (3.24)

(c) \( \frac{dL^E_M}{d\varepsilon} / \frac{dc}{d\varepsilon} = \frac{[\gamma\delta(\lambda_2 - 1)/\lambda_2]L^E_M}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \). \hspace{1cm} (3.25)

Here, clearly, \( \frac{dL^E_M}{d\varepsilon} / \frac{dc}{d\varepsilon} < \frac{dL^E_M}{d\lambda_2} / \frac{dc}{d\lambda_2} \), since, \( 0 > \lambda_2 > 1 \), \( 0 < (\lambda_2 - 1)/\lambda_2 < 1 \). This shows that the cost of increasing the size of the educated workers through an increase in the teacher-student ratio, and thus attracting more students and teachers to the education sector, is also the least cost-effective way to increase the size of the educated labor force.

However, the economic welfare loss is at a minimum if the teachers’ wage is raised instead.

\[ W^M = f(L^E_1) + w(1 - L^E_1 - \varepsilon\delta L^E - \delta L^E) \]

\[ \frac{dW^M}{d\varepsilon} = -w\varepsilon L^E + (1 + \varepsilon)\frac{dL^E}{d\varepsilon}, \quad \frac{dW^M}{d\lambda_2} = -w\varepsilon\delta(L^E + 1)\frac{dL^E}{d\lambda_2} \].

Since, \( \frac{dL^E}{d\varepsilon} \geq \frac{dL^E}{d\lambda_2}, \quad \left| \frac{dW^M}{d\varepsilon} \right| \geq \left| \frac{dW^M}{d\lambda_2} \right| \), welfare loss is higher for change in ε.
Hence, if the objective is to increase the size of the educated labor force for different long-run non-pecuniary benefits of education (reduced fertility, improved sanitation, and improved quality of life, for instance), then increasing the teacher-student ratio is the most cost-effective method. Obviously, under the present conditions the combination of these three policies will lead to a substantial increase in the educated labor force at the cost of economic welfare.

\[
\frac{dW^M}{dc} = -\delta L^EM \left( 1 - \gamma(w + c) + \gamma w \delta(\lambda_2 - 1) \right) < 0
\]

\[
\frac{dW^M}{d\lambda_2} = -\delta L^EM \left( 1 - \gamma(w + c) - \gamma w(\lambda_2 - 1) \delta \epsilon \right) < 0
\]

3.3.5. Effects on Income Distribution

The share of educated labor in total wage income \( \alpha^M = \left( \frac{\bar{w}L^EM}{\bar{w}L^EM + w(1 - (1 + \delta)L^EM)} \right) \) increases for an increase in \( \epsilon \) and \( \lambda_2 \). The results can be written as:

\[
\frac{d\alpha^M}{d\epsilon} = \frac{w\alpha^M dL^EM}{DL^EM d\epsilon} > 0, \text{ i.e., } \left| \frac{\xi^M_{\epsilon}}{D} \right| > 0 \tag{3.26}
\]

\[
\frac{d\alpha^M}{d\lambda_2} = \frac{w\alpha^M dL^EM}{DL^EM d\lambda_2} > 0, \text{ i.e., } \left| \frac{\xi^M_{\lambda_2}}{D} \right| > 0, \tag{3.27}
\]

where \( \xi^M_{\epsilon} \), \( \xi^M_{\lambda_2} \), \( \xi^UL_{\epsilon} \), and \( \xi^UL_{\lambda_2} \) are the elasticities, and \( D = \bar{w}L^EM + w(1 - (1 + \delta)L^EM) \) (derivations are shown Appendix 3.2). It can also be shown that \( \frac{d\alpha^M}{d\mu} > 0 \). So an exogenous increase in these policy parameters will cause a rise in the share of educated labor’s wage bill in total wage income. Thus, expansion of education actually raises the shares of income of the educated workers in the economy.
3.3.6. **Effects on the Marginal Product of Education**

One of the main arguments to reduce public subsidies to higher education emerged from the fact that estimated social rates of return were found to be consistently lower than private rates of return to education. Hence it was recommended that people could be asked to pay for their education (Barr, 2004; Bloom and Sevilla, 2004; Tilak, 2004; Psacharopoulos, 1986, 1994; World Bank, 1994). This section therefore analyzes the effects of changes in policy instruments on the difference between the private and social marginal products of education.

In this job ladder model of education, since the schooling period $t$ is fixed for any educated worker, education cannot be varied at the margin. However, alternative methods such as the flow approach and the stock approach can be used to evaluate the contribution of education to an individual and society. Under the flow approach, the difference in wages between educated and uneducated workers is attributed as the private marginal product of education, $\frac{w+(1-\gamma)c}{\gamma} - w$, or $\frac{1-\gamma}{\gamma}(w+c)$, from equation 3.4. Under the stock approach, on the other hand, the difference in the sum of discounted lifetime earnings of an educated worker and that of an uneducated worker after schooling is attributed as the private marginal product of education. Given that the working and schooling periods are the same, and the same discount rates apply in all cases in the steady state, both the approaches therefore yield identical results with respect to the difference between private and social returns to education. Now, the social marginal product of education can be easily derived from the social optimum model, where educated labor is chosen to maximize welfare $[W = f(L^E) + w(1-(1+\delta)L^E)]$ in such a way that the educated labor force is absorbed in Sector 1 and the education sector. The maximization process gives us the wage of the educated worker as
\[ f'_i(L_E^i) = w \frac{1+\delta}{1-\varepsilon\delta}, \] where \( L_E^i = (1-\varepsilon\delta)L_E^n = g(w \frac{1+\delta}{1-\varepsilon\delta}). \) Thus the social marginal product of education is the difference between the wage of an educated worker and an uneducated worker

\[
\frac{w(1+\delta)}{(1-\varepsilon\delta)} - w \delta (1+\varepsilon) \quad \text{or}, \quad \frac{w\delta(1+\varepsilon)}{(1-\varepsilon\delta)}.
\]

Thus, the difference between the private and social marginal products is given by:

\[
R = \frac{w+(1-\gamma)c}{\gamma} - \frac{w(1+\delta)}{(1-\varepsilon\delta)}, \quad (3.28)
\]

or, rearranging terms, by:

\[
R = \left( \frac{w}{\gamma} + \frac{1-\gamma}{\gamma}c \right) - \left( w(1+\delta) + \delta \frac{w(1+\delta)}{(1-\varepsilon\delta)} \right) \varepsilon. \quad (3.29)
\]

The term \( \varepsilon w(1+\delta)/(1-\varepsilon\delta) \), which represents the social cost of schooling per student year, is the product of the teacher-student ratio \( \varepsilon \) and the social wage of the teacher \( w(1+\delta)/(1-\varepsilon\delta) \).

Since for \( r \geq 0, \frac{1}{\gamma} \geq (1+\delta) \), as long as \( c \) is at least as much as it costs socially to educate the student, \( R > 0 \). This is true even when \( c \) is below the social cost but \( r \) is sufficiently high. If \( c \) is smaller and \( r \) is not sufficiently high, then \( R < 0 \). The effects of an increase in \( \varepsilon \) and a decrease in \( c \) appear below:

\[
\frac{dR}{d\varepsilon} = -\frac{w\delta(1+\delta)}{(1-\varepsilon\delta)^2} < 0, \quad \text{and} \quad \frac{dR}{dc} = \frac{1-\gamma}{\gamma} > 0, \quad \frac{dR}{dc} \frac{dR}{d\varepsilon} = -\frac{\gamma w\delta(1+\delta)}{(1-\gamma)(1-\varepsilon\delta)^2} < 0.
\]

(3.30)

This leads to the following proposition:

**Proposition 2:** A rise in the teacher-student ratio raises the social marginal cost and hence reduces the gap between the private and social marginal products of education. A reduction in
the education costs charged also lowers the opportunity cost and increases the difference between the private and social marginal costs of education.

3.4. The Flexible Wage Model

This section relaxes the assumption of higher wages in Sector 1 and uses the flexible-wage assumption for educated workers, so that the educated worker gets the same wage across Sectors 1 and 2. Hence \( \lambda_1 = \lambda_2 = \lambda \) is now endogenous in the flexible-wage model, and the educated labor force gets fully absorbed in Sectors 1 and 2, resulting in zero spillover to Sector 3.

3.4.1. Increase in the Teacher-Student Ratio \((\varepsilon)\)

In the flexible wage model, there can no longer be any difference in the wages of educated workers by sector of employment. Hence there is no spillover of the educated labor force into employment in Sector 3. As in the job ladder model, employment in Sector 1 remains unchanged, the proportion of the educated labor force increases, and social welfare falls. So in this model we get equations 3.9 and 3.10 modified as:

\[
f'(L_{EF}) = \bar{w} \quad \text{or,} \quad L_{EF} = g(\bar{w}) = g\left(\frac{w+(1-\gamma)c}{\gamma}\right) \quad (3.9')
\]

and

\[
L_i^{EF} = L_{EF} - L_i^{EF} = (1-\varepsilon\delta)L_{EF}, \quad \text{i.e.,} \quad L_i^{EF} = 0. \quad (3.10')
\]

The solution is meaningful if \(1 \geq (1+\delta)L_{EF} \geq 0\). The comparative static result for a change in the teacher-student ratio is now given by:

\[
\frac{dL_{EF}^F}{d\varepsilon} = \frac{dL_{EF}}{d\varepsilon} = \frac{\delta}{1-\varepsilon\delta}L_{EF}^F > 0 \quad (3.31)
\]

and

\[
\frac{dW_{EF}^F}{d\varepsilon} = -w \frac{1+\delta}{1-\varepsilon\delta}L_{EF}^F < 0. \quad (3.32)
\]
There is no spillover of educated labor force in Sector 3. Neither productivity nor employment in Sector 1 changes. Thus, the increased educated labor force is completely absorbed in the education sector, and the welfare level of the economy falls due to a decrease in the proportion of the labor force that would have been employed in Sector 3.

3.4.2. Decreases in Costs of Education

An increase in the subsidy to education leads to an increase in employment in Sector 1 and in the size of the educated labor force. The welfare impact, however, depends on the difference between the cost charged to individuals and the social cost of education, as well as on the private discount rate. The relevant comparative statics are given by the following expressions:

\[
\frac{dL^E_F}{dc} = \frac{1-\gamma}{\gamma} g' < 0 \quad (3.33)
\]

\[
\frac{dL^E_F}{dc} = \left(\frac{1-\gamma}{\gamma} g'\right)/(1-\varepsilon\delta) < 0, \text{ and} \quad (3.34)
\]

\[
\frac{dW^F}{dc} = \left(\frac{w+1-\gamma}{\gamma} - w\frac{1+\delta}{1-\varepsilon\delta}\right) \frac{dL^E_F}{dc} \leq 0, \quad (3.35)
\]

according as \(\frac{w+(1-\gamma)c}{\gamma} \geq w\frac{1+\delta}{1-\varepsilon\delta} = w(1+\delta) + \varepsilon \frac{w(1+\delta)}{1-\varepsilon\delta} \delta\). \quad (3.36)

Also, note that when \(r \geq 0\), \(\frac{1-\gamma}{\gamma} \geq \delta\). Thus it can further be asserted that:

(a) \(\frac{dW^F}{dc} < 0\) when \(c \geq \varepsilon \frac{w(1+\delta)}{1-\varepsilon\delta}\) and \(r > 0\) \quad (3.37)

(b) \(\frac{dW^F}{dc} = 0\) when \(c = \varepsilon \frac{w(1+\delta)}{1-\varepsilon\delta}\) and \(r = 0\) \quad (3.38)

(c) \(\frac{dW^F}{dc} = 0\) when \(c < \varepsilon \frac{w(1+\delta)}{1-\varepsilon\delta}\) and \(r\) is suitably high \quad (3.39)
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(d) \( \frac{dW^F}{dc} > 0 \) when \( c < \varepsilon \frac{w(1+\delta)}{1-\varepsilon \delta} \) and \( r \) is sufficiently low. \hspace{1cm} (3.40)

In most developing countries, the cost charged for education is much lower than its actual social cost and \( r \) is sufficiently low as well. As in case (d), the reduction in \( c \) reduces welfare in the economy.

3.5. The Social Optimum Model

In the social optimum model, the implicit assumption is that all educated workers are used either as teachers or as production workers in Sector 1, and the educated labor force is chosen to maximize the welfare of the economy. Hence a change in \( \varepsilon \) affecting the social cost affects employment in Sector 1 and the effect on the total educated labor supply is ambiguous. We choose \( L^{ES} \) to maximize \( W^S = f(L^{ES}) + w(1-(1+\delta)L^{ES}) \). Therefore, in the social optimum model equations 3.6–3.9 are modified as:

\[
\frac{f'}{(1-\varepsilon \delta)L^{ES}} = w \frac{1+\delta}{1-\varepsilon \delta} \quad \text{or} \quad L^{ES}_1 = (1-\varepsilon \delta)L^{ES} = g(w \frac{1+\delta}{1-\varepsilon \delta}) \quad (3.9'')
\]

and \( L^{ES}_1 = L^{ES} - L^{ES}_2 = (1-\varepsilon \delta)L^{ES} \), i.e., \( L^{ES}_2 = 0 \). \hspace{1cm} (3.10'')

The solution is meaningful if \( 1 \geq (1+\delta)L^{ES} \geq 0 \). The comparative static results for a change in the teacher-student ratio are now given by:

\[
\frac{dL^{ES}}{d\varepsilon} = g' \frac{w\delta(1+\delta)}{(1-\varepsilon \delta)^2} < 0 \quad \text{as} \quad g' < 0 \hspace{1cm} (3.41)
\]

\[
\frac{dL^{ES}}{d\varepsilon} \geq 0 \quad \text{according as} \quad \left| \frac{dL^{ES}}{d\varepsilon} \right| \leq \delta L^{ES}_1. \hspace{1cm} (3.42)
\]

The change in welfare can go in either direction. Thus we get the following proposition:
Proposition 3: In the social optimum model, an exogenous increase in the teacher-student ratio raises the marginal social cost and lowers the employment of educated labor in Sector 1. How the total supply of educated labor changes depends on the relative strength of the changes in Sectors 1 and 2, while the effect on welfare is ambiguous.

Note that the educated worker earns the social wage in both Sectors 1 and 2. The education cost $c$ does not affect the equilibrium values of employment or the size of the educated labor force.

3.6. The Job Ladder Model with the Fairness-in-Hiring Principle: Rising Inequality with Educational Expansion

Under the fairness-in-hiring concept, the employer contracts educated labor to preserve a sense of fairness. As long as the size of the educated labor force exceeds the amount of labor required in Sector 1, the previous analysis of the job ladder or minimum wage model holds. Since educated and uneducated labor are perfect substitutes, but educated workers are hired because they have put in more effort and education has resource costs, the educated labor supply will fall to zero in both the flexible wage and social optimum models. That is, $L_{EF}^E = L_{ES}^E$ and $f'(L_{ES}^E) = w$, and $(1 - L_{ES}^E)$ will be employed in Sector 3. Thus, when education has no productivity, but an educated worker is hired due to the fairness-in-hiring principle, the job ladder model or the minimum wage model, along with the fairness-in-hiring rule, leads to creation of educated labor force that does not add to any welfare of the economy. As a result of the above-mentioned policies, the share of educated labor in the total wage bill in wage income rises. Thus, in the minimum wage model, under the fairness-in-hiring principle, educated-uneducated income inequality rises along with the loss of welfare.
3.7. Job Ladder Model with the Government Sector

This section makes a simple but important and major extension of the job ladder model to capture the “public financing” of the education sector that is common at schools and colleges in developing countries. In most developing countries, financing of higher education is largely done by the government through tax financing, with very little or zero costs shared by students. For example, in sub-Saharan Africa, the governments provided over 90 percent of the cost for higher education (Teferra, 2007), and in Papua New Guinea the government provided more than 95 percent of university recurrent financing in 1988 (McGavin, 1991). Since the user fee charged per student is very low, the government meets the expenses of the education sector from the tax revenue generated from Sector 1 (say, the modern sector). For the sake of simplicity of analysis, it is assumed here that both the education sector and the subsistence sector are not directly taxed.

The basic structure of the original job ladder model is maintained, i.e., the model assumes two non-education sectors—Sector 1 (the modern sector) and Sector 3 (the subsistence sector)—and one education sector (Sector 2). An asterisk is used to distinguish the results of this extended job ladder model with the government sector from the original job ladder model in outlined in Section 3.1.

In Sector 1, output is a function of employed educated workers ($Q_1$):

$$
Q_1 = f_1(L_1^{E*}),
$$

(3.7.1)

where $f_1$ is strictly concave, $f_1' > 0 \forall L_1^{E*} > 0$, $f_1' = \infty$ when $L_1^{E*} = 0$, and $f_1' = 0$ when $L_1^{E*} = \infty$.

Sector 2, the education sector, employs educated labor ($L_2^{E*}$) as teachers to educate students ($S$) at an exogenously specified teacher-student ratio ($\varepsilon$). Here, the teaching cost is the only direct cost of education and equals the teacher-student ratio times the teacher’s wage. No
value is placed on education as a consumption good. The education technology is simply given by:

\[ S = \frac{1}{\varepsilon} L_2^{E*} \]  
(3.7.2)

Finally, Sector 3 employs both educated \((L_3^{E*})\) and uneducated labor \((L_3^{U})\) at an identical wage \(w\), set at the marginal value product in terms of the output of Sector 1. Output in Sector 3 is a linear function of the labor employed:

\[ Q_3 = w(L_3^{E*} + L_3^{U*}) \]  
(3.7.3)

It is assumed that \(G_R\) is the tax revenue generated from Sector 1 to meet the part of the expenses of the education sector. The government revenue is given by

\[ G_R = t_s \lambda_1 w L_2^{E*} \]  
(3.7.4a)

where, \(t_s\) is the tax rate levied on Sector 1.

The cost of education sector per student can be expressed as

\[ c = \lambda_2 w L_2^{E*} / S \]  
(3.7.4b)

where, \(S\) is the total number of students.

Thus the educated worker pays only \(c'\), a part of the cost of education, and the remaining cost is met by government tax revenue generated from Sector 1. Thus,

\[ c' = c - G_R / S = \frac{\lambda_2 w L_2^{E*}}{S} - t_s \lambda_1 w L_1^{E*} / S \]  
(3.7.4c)

Alternatively,

\[ c' = c - \sigma \]  
(3.7.4d)

where \(\sigma\) is tax revenue spent per student.

Assuming the economy is small and open, the relative price of output of Sector 1 to that of Sector 3 is taken as exogenous and equal to 1. The job ladder consideration is incorporated
into the model by assuming that wages ($\lambda_1 w$ and $\lambda_2 w$, $\lambda_1 > \lambda_2 > 1$) in Sectors 1 and 2 are sticky and the supply of educated labor in excess of its employment in Sectors 1 and 2 spills over into Sector 3. The equilibrium size of the educated labor force is determined by the principle of present value equalization. That is, for an educated worker, the present value of the worker’s income stream net of any education costs equals the present value of the worker’s income stream if the worker had not educated him or herself but was employed as an uneducated worker. This condition will be now given by:

$$\bar{w} \int_T^T e^{-rt} d\tau - c' \int_0^T e^{-rt} d\tau = \bar{w} \int_0^T e^{-rt} d\tau,$$

or, $\gamma \bar{w} = w + (1 - \gamma)c'$, \hspace{1cm} (3.7.5)

where, $\gamma = \frac{\int_T^T e^{-rt} d\tau}{\int_0^T e^{-rt} d\tau}$, \hspace{1cm} (3.7.6)

where $T$ is the total working life of which first $t$ periods are spent in school if the worker chooses to be educated, $r$ is the non-negative rate of discount, $c'$ is the portion of education cost that the worker pays per period in school, and $\bar{w}$ is the expected average wage of an educated worker, which equals the weighted average of the wages in non-education sectors and the education sector, with weights being equal to the employment shares of educated workers in each sector.

Assuming a steady state, the total labor force is set at unity. $L^{s\ast}$ is the portion of the total labor force that is educated. Thus, for a given working life $T$ and the schooling period $t$, the number of students (in steady state) will be given by:

$$S = \frac{t}{T - t} L^{s\ast} = \delta L^{s\ast}, \hspace{1cm} (3.7.7)$$
where, $\delta = \frac{t}{T-t}$. For an exogenously given teacher-student ratio $\varepsilon$, the number of teachers is given as $L_{2}^{E} = \varepsilon \delta L^{E}$ from equations 3.7.2 and 3.7.6. Employment of educated workers in Sector 3 ($L_{3}^{E}$) and the size of the pool of uneducated workers ($L^{U}$) who are employed only in Sector 3 are given by:

$$L_{3}^{E} = L^{E} - L_{1}^{E} - L_{2}^{E} = (1-\varepsilon \delta)L^{E} - L_{1}^{E}$$  \hspace{1cm} (3.7.8)

and $$L^{U} = 1 - L^{E} - S = 1 - (1+\delta)L^{E}.$$  \hspace{1cm} (3.7.9)

The equilibrium values of only $L^{E}$ and $L_{1}^{E}$ still need to be determined. Given $f_{1}$ and $\lambda_{1}w$, $L_{1}^{E}$ are determined by the marginal productivity rule $f_{i}^{'}(L_{i}^{E}) = (1-t_{i})\lambda_{i}w$ or

$$L_{1}^{E} = g(t_{i}, \lambda_{i}w) \text{ or } L_{1}^{E} = h(\lambda_{i}w) \text{ for any exogenously given } t_{i},$$  \hspace{1cm} (3.7.10)

where, $\bar{w}$, the average wage of the educated worker, is the weighted average of the wages $(1-t_{i})\lambda_{i}w$, $\lambda_{2}w$, and $w$ in sectors 1, 2, and 3, respectively. The weights are the proportions of the educated labor force employed in the respective sectors. The average wage can be written as:

$$\bar{w} = \frac{(1-t_{i})\lambda_{i}wL_{i}^{E} + \lambda_{2}w(1-\varepsilon \delta)L^{E} - L_{1}^{E})w}{L^{E}}.$$  \hspace{1cm} (3.7.11)

where $\bar{w}$ is the expected average wage of an educated worker, which equals the weighted average of the wages in non-education sectors and the education sector, with weights being equal to the employment shares of educated workers in each sector.

Substitution of $\bar{w}$ in equation 3.7.4' and substitution of a balanced budget condition for the education sector gives us $L_{1}^{E}$ as:

$$L_{1}^{E} = \frac{\gamma w(\lambda_{1}-1)+t_{i}\lambda_{i}w((1-\gamma)/\delta - \gamma)}{(1-\gamma)(w+c) - \gamma w(\lambda_{2}-1)\delta L_{1}^{E}}.$$  \hspace{1cm} (3.7.12)

Clearly, the following can be obtained from above:
(1) **No public financing in education:** When $c' = c$, i.e., there is no public financing of education, $\sigma = 0$, and hence $t_x = 0$. The size of the educated labor force $L^E$ in our extended model is the same as $L^{EM}$ in the original job ladder model.

(2) **Education is fully publicly financed:** When the education sector is fully publicly financed, $c' = 0$, i.e., there is no user fee or private cost of education, $c = \sigma$, $t_x = \lambda_2 wL_2^E / \lambda_1 wL_1^E$. The tax rate for a fully subsidized education system in this model will equal the ratio of the wage bill in the education sector to the modern sector. Substitution of equilibrium values from equation 3.7.10’ gives the tax rate in terms of the parameters as $t_x = \frac{\gamma (\lambda_1 - 1) \lambda_2 \varepsilon \delta}{\lambda_1 (1 - \gamma + \gamma \varepsilon \delta)}$.

### 3.7.1. Equilibrium Values in the Job Ladder Model with Public Financing

In this extended original job ladder model, given wages, the discount rate, working and schooling periods, the exogenous teacher-student ratio, and all other variables are trivially determined once $L^E$ and $L_i^E$ are determined for any tax rate $t_x$. These are given by equations 3.7.9 and 3.7.10. This solution is meaningful if $(1 - \varepsilon \delta)L^E \geq L_1^E$ and $(1 + \delta)L^E \leq 1$. The first condition states that the size of the educated labor force net of teachers is at least as large as the employment in Sector 1. The second condition implies that the educated labor force and the student body do not exhaust the total labor force. The first condition will be satisfied provided that:

\[
w + (1 - \gamma)c' \geq \gamma [\lambda_2 \varepsilon \delta + (1 - \varepsilon \delta)] \tag{3.7.13}
\]

\[
\gamma [\lambda_1 (1 - t_x) (1 - \varepsilon \delta) + \lambda_2 \varepsilon \delta] \geq w + (1 - \gamma)c'. \tag{3.7.14}
\]
Equation 3.7.13 implies that the opportunity cost of education (which equals the sum of income foregone and the discounted sum of the charges for education) must be higher than the expected minimum income of an educated worker (which equals the discounted sum of income of an educated worker if the worker gets a job either in the education or subsistence sector).

Equation 3.7.14 shows that the expected maximum income of an educated worker (which equals the discounted sum of income of an educated worker if the worker gets a job either in Sector 1 or in the education sector) must be higher than the opportunity cost of education. These two conditions together, from equations 3.7.13 and 3.7.14, imply that the opportunity cost of getting educated in this model lies between the expected minimum income (when an educated worker gets a job only in the education and subsistence sectors) and expected maximum income of an educated worker (none of the educated workers are employed in the subsistence sector). This can be written as:

$$\gamma w [\lambda_T (1-t_s)(1-\epsilon\delta) + \lambda_E \epsilon \delta] \geq w + (1-\gamma) c' \geq \gamma w (\lambda_E \epsilon \delta + (1-\epsilon \delta)).$$  \hspace{1cm} (3.7.15)

Finally, since education has no value as a consumption good, the national welfare is given by:

$$W^{*} = (1-t_s) f_t(L_t^{E^*}) + w(1-L_t^{E^*} - \epsilon \delta L_t^{E^*} - \delta L_t^{E^*}).$$  \hspace{1cm} (3.7.16)

### 3.7.2. Comparative Static Results: Changes in Policy Parameters

This subsection discusses the effects of changes in the policy parameters: the teacher-student ratio, teacher’s wage level, and the tax rate (all of which affect the private cost and subsidy composition of the education sector). First, it is assumed that these are changed exogenously and do not influence the period of schooling. The results from the job ladder or minimum wage model will be compared with those from the flexible wage and social optimum models. To
distinguish the results, the superscript $M$ is used for the job ladder model, superscript $F$ for the flexible wage model, and superscript $S$ for social optimum model.

**Policy 1: Increase in the Teacher-Student Ratio ($\varepsilon$)**

In the job ladder model, the supply of educated labor in excess of employment in Sectors 1 and 2 spills over into Sector 3. An exogenous increase in the teacher-student ratio gives the following results:

1. The opportunity cost of education, $w + (1-\gamma)(\lambda_2 w e - t_e \lambda_1 w / L^*_t)$ rises by $(1-\gamma)(\lambda_2 w)$ for any given $t_e$.

2. The discounted sum of expected minimum income $\bar{W}^{E}_{\min}$ (when no educated worker is employed in the highest paying sector) of an educated worker rises, but the discounted sum of expected maximum income $\bar{W}^{E}_{\max}$ (when no educated worker is employed in the lowest paying sector) falls, i.e.:

   \[
   \frac{d\bar{W}^{E}_{\min}}{d\varepsilon} = \frac{d}{d\varepsilon} [\gamma w(\lambda_2, \varepsilon \delta + (1-\varepsilon \delta))] = \gamma w(\lambda_2 - 1) \delta > 0 \quad \text{as} \quad \lambda_2 > 1
   \]

   \[
   \frac{d\bar{W}^{E}_{\max}}{d\varepsilon} = \frac{d}{d\varepsilon} [\gamma w(\lambda_t (1-t_e)(1-\varepsilon \delta) + \lambda_2 \varepsilon \delta))] = \gamma w(\lambda_2 - \lambda_t (1-t_e)) \delta < 0 \quad \text{as} \quad \lambda_t < \lambda_t (1-t_e)
   \]

3. For any given $t_e$, if the increased cost of education due to an increase in the teacher-student ratio is shifted to the students, then $\frac{dL^{E*}}{d\varepsilon} \leq 0$:

   \[
   \frac{dL^{E*}}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1) \delta - (1-\gamma)\lambda_2 w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta} L^{E*} \leq 0, \quad (3.7.17)
   \]

   since, $\gamma(\lambda_2 - 1) \delta - (1-\gamma)\lambda_2 \leq 0$ Given $0<\gamma<1$, $0<\delta<1$, $\lambda_2 > 1$, $\gamma \delta (\lambda_2 - 1) \leq (1-\gamma)\lambda_2$. 

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(4) Even if the \( t_x \) is adjusted in such a way that the increase in subsidy per student is just to cover the additional cost, \( \frac{dc'}{dE} = 0, \frac{d\sigma}{dE} = \frac{dc}{dE} = \lambda_xw \) due to (a fall in the size of \( L^E_x \) for an increase in \( t_x \), (b) an increase in the opportunity cost of education, and (c) a decline in the expected maximum income of an educated worker, as a result of an increase in the teacher-student ratio:

\[
\frac{dL^E_x}{dE} = \frac{\gamma w(\lambda_x - 1)\frac{dL^E_x}{dE} - \delta}{[(1-\gamma)(w+c') - \gamma w(\lambda_x - 1)\sigma + \gamma \sigma \delta]} \leq 0, \text{ since, } \frac{dL^E_x}{dE} \leq 0
\]

(5) The size of the educated labor force will rise as a result of the increase in the teacher-student ratio only if the tax rate is adjusted in such a way that the subsidy per student is high enough to reduce the private cost of education and the decline in the employment in Sector 1 is offset by an increase in the number of additional teachers in Sector 2, Thus, \( \frac{dL^E_x}{dE} \geq 0 \) if:

\[
(1-\gamma-\gamma \delta)\frac{d\sigma}{dE} + \gamma w(\lambda_x - 1)\frac{dL^E_x}{dE} \geq \gamma \delta w + (1-\gamma - \gamma \delta)\lambda_xw. \quad (3.7.17a)
\]

(6) From the welfare function, it can be derived that:

\[
\frac{dW^M}{dE} = [f'(L^E_x) - w] \frac{dL^E_x}{dE} - w\delta[L^E_x + (1+\varepsilon)] \frac{dL^E_x}{dE}.
\]

Thus, the following result with regard to changes in welfare as a result of a change in \( \varepsilon \):

(a) For any given \( t_x \), \( \frac{dW^M}{dE} \geq 0 \), since \( \frac{dL^E_x}{dE} = 0 \), and \( \frac{dL^E_x}{dE} \leq 0 \).

(b) If \( t_x \) is adjusted in such a way that the increase in subsidy per student is just to cover the additional cost, \( \frac{dc'}{dE} = 0, \frac{d\sigma}{dE} = \frac{dc}{dE} = \lambda_xw \), \( \frac{dW^M}{dE} \geq 0 \), since \( \frac{dL^E_x}{dE} = 0 \), and \( \frac{dL^E_x}{dE} \leq 0 \).

(c) When, \( \frac{dW^M}{dE} = [(1-t_x)\lambda_xw - w]h' \Delta t_x - w\delta[L^E_x + (1+\varepsilon)] \frac{dL^E_x}{dE} \), \( (3.7.19) \)
the change in welfare will depend on relative strengths of changes in welfare in Sector 1 and Sector 3.

The above results can be summarized in the following proposition:

**Proposition 4:** When the period of schooling and the wage rates are independent of schooling quality measured in terms of the teacher-student ratio, an exogenous increase (decrease) in the teacher-student ratio will decrease (increase) the total size of the educated labor force, employment in Sector 1 will fall, and the welfare level will rise (fall). The size of the educated labor force will rise (fall) if and only if the tax rate adjusts in such a way that the subsidy to the costs of education rises (declines) and raises (lowers) the minimum expected income of the educated worker and increases (decreases) in employment in the education sector offset the decline (rise) the Sector 1.

Thus, in this extended job ladder model with public financing of the education sector, an increase in the teacher-student ratio raises the expected wage, \( \bar{w} \), of an educated worker. This rise in \( \bar{w} \) expands the proportion of educated labor in the labor force due to the increasing number of students and teachers. If the increased cost of education is met by a higher subsidy from the government, lowering the private costs of education, additional job creation in the education sector brings more students. When the subsidy is met by higher tax revenue from Sector 1, employment in Sector 1 falls. Hence, part of the surplus educated labor gets absorbed in the education sector and the remaining portion of the surplus works in the subsistence sector.
Policy 2: Increase in the Teacher Wage Level ($\lambda_2$)

(a) An increase in $\lambda_2$, with $\lambda_1, t_s, \text{ and } \varepsilon$ remaining unchanged: This will produce similar effects on the size of the educated labor force and welfare of the economy. The results are given by:

$$\frac{dL^*_b}{d\lambda_2} = 0, \quad \frac{dL^*_b}{d\lambda_2} = \frac{\gamma \delta w_e - (1-\gamma)w_e}{(1-\gamma)(w+c) - \gamma w(\lambda_2-1)\varepsilon \delta} L^*_b \leq 0 \text{ since, } 0 \leq \delta \leq 1, 0 \leq \gamma \leq 1$$

(3.7.20)

$$\frac{dW^*}{d\lambda_2} = -w(1+\varepsilon)\delta \frac{dL^*_b}{d\lambda_2} = \frac{[(1-\gamma) - \gamma \delta][w \varepsilon w \delta (1+\varepsilon)L^*_b]}{(1-\gamma)(w+c) - \gamma w(\lambda_2-1)\varepsilon \delta} \geq 0,$$

(3.7.21)

since $(1-\gamma) \geq \gamma \delta$, given, $0 \leq \gamma \leq 1, 0 \leq \delta \leq 1$

An increase in the salary of teachers raises the expected average wage of an educated worker, which affects the size of the educated labor force. However, given any $\lambda_1$ and $t_s$, employment in Sector 1 does not change, and the private cost of education will rise. This will raise the opportunity cost of education. Since employment in Sector 1 remains unchanged, employment in the education sector is fixed by given $\varepsilon$ and $\delta$, the increased private cost of education will discourage labor to from getting educated when job opportunities are limited in the education sector, and wages are the same both for educated and uneducated workers in the subsistence sector. Only a fraction of the larger pool of educated workers gets employed as teachers and the remainder goes back to Sector 3, where educated labor has no extra productivity beyond the productivity of uneducated labor. Since the number of students withdrawn from Sector 3 falls due to higher costs of education, the welfare level in this framework will rise by the amount those students could have produced in Sector 3.

(b) When both $\lambda_1$ and $\lambda_2$ are increased: If wages in both Sectors 1 and 2 are increased, the effect on the size of the educated labor force and welfare will depend on $\xi$, the
elasticity of demand for labor in Sector 1 with respect to the wage rate. This is verified by the
following expression:

\[
\frac{dL^e}{d\lambda_1} = \left[(1-t_s)\eta + \left(1 + \frac{\gamma w}{N}\right)\right] \frac{L^e}{\lambda_1} \quad \text{Where,} \quad \eta = \frac{h^*}{h}\lambda w.
\]

\[N = \gamma w(\lambda_1 - 1) + t_s\lambda w((1-\gamma) / (\delta - \gamma))\]

This can also be expressed as

\[
\frac{dL^e}{d\lambda_1} = \left[\eta + \frac{\gamma \lambda_1 + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma))}{\gamma (\lambda_1 - 1) + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma)) (1-t_s)}\right] \frac{L^e}{\lambda_1}
\]

(3.7.22)

\[|\eta| \leq \frac{\gamma \lambda_1 + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma))}{\gamma (\lambda_1 - 1) + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma)) (1-t_s)}, \quad \frac{dL^e}{d\lambda_1} \geq 0, \quad \text{or,} \quad |\eta| \leq \nu, \quad \frac{dL^e}{d\lambda_1} \geq 0,
\]

where \(\nu = \frac{\gamma \lambda_1 + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma))}{\gamma (\lambda_1 - 1) + t_s \lambda_2 ((1-\gamma) / (\delta - \gamma)) (1-t_s)} \geq 1, \quad \text{for given} \quad 0 \leq \delta, \gamma, t_s \leq 1,

when \(t_s = 0\), i.e. education is not subsidized, \(\frac{dL^e}{d\lambda_1} = \left[\eta \frac{\lambda_1 - 1}{\lambda_1} + 1\right] \frac{L^e}{\lambda_1 - 1}\) the same as in equation 3.17 in original job ladder model in Section 3. Thus if \(|\eta| < \nu\), i.e., demand for labor in Sector 1 is not very elastic with respect to the wage rate, the size of the educated labor force will rise due to an increase in the expected wage of the educated worker. However, if demand for labor in Sector 1 is sufficiently elastic, the increase in \(\lambda_1\) will reduce the size of the educated labor force. The increase in \(\lambda_2\), on the other hand, will raise the private cost of education for any given \(t_s\), and hence the net effect will reduce the size of the educated labor force.

**Policy 3: Increase in the Subsidy**

If the education expenditure or subsidy is increased, which lowers the direct costs charged for education, similar results are again obtained, i.e., an increase in the size of the educated labor
force at the cost of a welfare loss to the economy. The following shows the effect of a higher rate of subsidy:

\[
c' = c - G_k / S = \frac{\lambda_1 w L^e_t}{S} - \frac{t_s \lambda_2 w L^e_t}{S} = c - \sigma, \text{ alternatively, in equilibrium: } S = \delta L^e_t
\]

and

\[
L^e_t = \lambda \delta L^e_t, c' = \lambda \delta \lambda_2 w e - \frac{t_s \lambda_1 w L^e_t}{\delta L^e_t}
\]

where \(\sigma\) is the rate of subsidy per student per schooling period. For any given \(\lambda_1, \lambda_2, \delta\) and \(e\), an increase in \(\sigma\) implies an increase in \(t_s\), since the increased subsidy is going to be financed by increased tax revenue from Sector 1. An increase in the subsidy will reduce the private cost of education and will attract more students to the education sector from the subsistence sector. However, with an unchanged \(\lambda_1\), the expected maximum income will be affected only if the increased tax reduces the size of employment in Sector 1, which employs only educated workers. Thus, a lowering of the opportunity cost of education of an educated worker will have an increasing effect on the size of the educated labor force, but lower employment opportunity in Sector 1 will have an effect on the expected wage of the educated worker. Hence the combined effect will depend on the size of the educated labor force, and welfare will depend on \(\eta_{L^e_t}^1\), the elasticity of demand for labor in Sector 1 with respect to the tax rate.

Thus if \(\eta_{L^e_t}^1 < \rho\), i.e., demand for labor in Sector 1 is not very elastic with respect to the tax rate, the size of the educated labor force will rise due to an increase in the subsidy to the cost of education. However, if demand for labor in Sector 1 is sufficiently elastic, the increase in \(t_s\) will reduce the size of the educated labor force. The impact on welfare will be positive or negative depending on the relative size of the educated labor force. The results are:

\[
\frac{dL^e_t}{dt_s} = \left[ \frac{\sigma(1-\gamma - \gamma \delta)}{(1-\gamma)(w + c) - \gamma w (\lambda_2 - 1) e \delta} - \lambda_1 w t_s \frac{h^1}{h} \right] L^e_t / t_s.
\]

(3.7.23)
This can also be expressed as:

\[
\eta_{L^E,t} = \frac{\sigma(1-\gamma - \gamma \delta)}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta} - \lambda_w \eta_{L^E,t} = \lambda_w \left( \rho - \eta_{L^E,t} \right) \quad (3.7.24)
\]

where, \( \rho = \frac{\sigma(1-\gamma - \gamma \delta)}{\lambda_w [(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta]} \)

and

\[
\frac{d W^{M*}}{d t_x} = [(1-t_x)\lambda_1 - 1]w \frac{d L^E_x}{d t_x} \frac{w \delta (1 + \varepsilon)}{1}\frac{d L^E_x}{d t_x}, \quad (3.7.25)
\]

when \( \eta_{L^E,t} = 0 \), i.e., perfectly inelastic with respect to \( t_x \)

\[
\frac{d L^E_x}{d t_x} = \frac{\lambda_w (1-\gamma - \gamma \delta) L^E_x}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta} \geq 0.
\]

### 3.7.2.1. Comparison of the Effects of Changes in \( \varepsilon, \lambda_2 \) and \( t_x \)

Comparing the expressions in equations 3.7.17, 3.7.17a, 3.7.20, and 3.7.22 obtains the following:

(a) Equation 3.17* has shown that for any given \( t_x \), if the increased cost of education due to an increase in the teacher-student ratio is shifted to the students for a non-negative rate of discount, then

\[
\frac{d L^E_x}{d \varepsilon} = \frac{\gamma w(\lambda_2 - 1) \delta - (1-\gamma) \lambda_2 w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta} \leq 0 \text{ since, } \gamma (\lambda_2 - 1) \delta - (1-\gamma) \lambda_2 \leq 0 \text{, for any given, } 0 < \gamma < 1, 0 < \delta < 1, \lambda_2 > 1, \gamma \delta (\lambda_2 - 1) \leq (1-\gamma) \lambda_2. \]

We also found (in equation 3.20*) that for an increase in \( \lambda_2 \), with \( \lambda_1, t_x \), and \( \varepsilon \) remaining unchanged,

\[
\frac{d L^E_x}{d \lambda_2} = \frac{\gamma \delta w \varepsilon (1-\gamma) w \varepsilon}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1) \varepsilon \delta} L^E_x \leq 0.
\]

Comparing these equations finds that if \( (\lambda_2 - 1) \geq \varepsilon \), then \( \left| \frac{d L^E_M}{d \varepsilon} \right| > \left| \frac{d L^E_M}{d \lambda_2} \right| \), i.e., for any given \( t_x \), the decline in the size of the educated labor force will be higher for an increase in the
teacher-student ratio, and hence a higher private cost of education, than in the case of an increase in the salary of the teachers, with $\lambda_1$, $t_s$, and $\varepsilon$ remaining unchanged:

$$\frac{dL_{EM}^*}{d\lambda_2} = \frac{\gamma \delta w e - (1-\gamma) w \varepsilon}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1) e \delta} L_{EM}^* \leq 0 \text{ since, } 0 \leq \delta \leq 1, 0 \leq \gamma \leq 1$$

$$|\frac{dL_{EM}^*}{d\varepsilon}| > |\frac{dL_{EM}^*}{d\lambda_2}| \text{ since, } |\frac{dL_{EM}^*}{d\lambda_2}| \leq 0, \text{ and } |\frac{dL_{EM}^*}{d\varepsilon}| \leq 0, \text{ for } (\lambda_2 - 1) \geq \varepsilon .$$

The size of the educated labor force will be reduced more for an increase in $\varepsilon$ than for an increase in $\lambda_2$. Thus, when part of the education cost is transferred to the students, the excess supply of the educated labor force can be reduced and there could be a greater efficiency gain, compared to the original job ladder model. When we compare the costs we also find that:

$$\frac{dL_{EM}^*}{d\lambda_2} \frac{dc}{d\lambda_2} = \frac{\gamma \delta - (1-\gamma)}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1) e \delta} L_{EM}^* \leq 0$$

$$\frac{dL_{EM}^*}{d\varepsilon} \frac{dc}{d\varepsilon} = \frac{\gamma \delta - (1-\gamma) - \frac{\gamma \delta}{\lambda_2}}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1) e \delta} L_{EM}^*$$

$$|\frac{dL_{EM}^*}{d\varepsilon} \frac{dc}{d\varepsilon}| < |\frac{dL_{EM}^*}{d\lambda_2} \frac{dc}{d\lambda_2}| .$$

Thus, the most cost-effective way to reduce the size of the excess supply of the educated labor force is to increase the teacher-student ratio, which will absorb part of the excess supply as teachers. However, due to increased cost of education, without changing the wage and size of the employment in Sector 1, higher opportunity cost will act as a disincentive to attract more students from the subsistence sector to the education sector.

(b) If employment in Sector 1 is relatively less elastic with respect to the wage rate in Sector 1 or the tax rate, then as a result of an increase in the wage rate in Sectors 1 and 2, and a subsidy to cost of education, $\frac{dL_{EM}^*}{d\lambda_1} \geq 0$, $\frac{dL_{EM}^*}{dt_s} \geq 0$, whereas $\frac{dL_{EM}^*}{d\lambda_2} \leq 0$. Thus, when employment in
Sector 1 is relatively inelastic with respect to the wage rate and tax rate, the increase in wages in Sector 1 and the increase in the subsidy to private costs of education will cause an increase in the size of the educated labor force, whereas, an increase in wages in the education sector will reduce the size of the educated labor force in the job ladder model with partial public financing of education sector.

### 3.7.2.2. Effects on the Marginal Product of Education

In this version of the job ladder model of education with public financing of the education sector, for the given schooling period \( t \) fixed for any educated worker, education cannot be varied at the margin. However, as in the original job ladder model, alternative methods—the flow approach and the stock approach—can be used to evaluate the contribution of education to an individual and society. Under the flow approach, the difference in wages between educated and uneducated workers is attributed as a private marginal product of education,

\[
\frac{w+(1-\gamma)c'}{\gamma} - w, \text{ or, } \frac{1-\gamma}{\gamma} (w+c') \text{ from equation 3.7.5,}
\]

where

\[
c' = \frac{c - G_R}{S} = \frac{\lambda_1 w L^E_{E1}}{S} - \frac{\lambda_1 w L^E_{E1}}{S} = c - \sigma
\]

is the private cost of education per student for each period. Under the stock approach, on the other hand, the difference in the sum of discounted lifetime earnings of an educated worker and that of an uneducated worker after schooling is attributed as the private marginal product of education. Given that the working and schooling periods are the same, and the same discount rates apply in all cases in the steady state, both the approaches yield identical results with respect to the difference between private and social returns to education. Now, the social marginal product of education can be easily derived from the social optimum model, where educated labor is chosen to maximize the welfare

\[
W = f(L^E_1) + w(1-(\delta)L^E) \text{ in such a way that the educated labor force is absorbed in Sector 1}
\]
and the education sector. The maximization process gives us the wage of the educated worker as

\[ f_1'(L^E_1) = \frac{w(1+\delta)}{1-\varepsilon\delta}, \]

where, \( L^E_1=(1-\varepsilon\delta)g\left(\frac{1+\delta}{1-\varepsilon\delta}\right) \). Thus, the social marginal product of education is the difference between wages of an educated and uneducated worker,

\[ \frac{w(1+\delta)}{(1-\varepsilon\delta)} - w \text{ or } \frac{w\delta(1+\varepsilon)}{(1-\varepsilon\delta)} . \]

Thus, the difference between the private and social marginal products is given by:

\[ R = \frac{w+(1-\gamma)c'}{\gamma} - \frac{w(1+\delta)}{(1-\varepsilon\delta)}, \quad (3.7.26) \]

or, rearranging terms, by:

\[ R = \left( \frac{w}{\gamma} + \frac{1-\gamma}{\gamma} c - \frac{1-\gamma}{\gamma} \sigma \right) - \left( w(1+\delta) + \delta \frac{w(1+\delta)}{(1-\varepsilon\delta)} \varepsilon \right) . \quad (3.7.26') \]

The term \( \varepsilon w(1+\delta)/(1-\varepsilon\delta) \), which represents the social cost of schooling per student year, is the product of the teacher-student ratio \( \varepsilon \) and the social wage of the teacher \( w(1+\delta)/(1-\varepsilon\delta) \).

Since for \( r \geq 0 \), \( \frac{1}{\gamma} \geq (1+\delta) \), as long as the cost of education, \( c'=(c-\sigma) \) is at least as much as it costs socially to educate the student, \( R<0 \). If due to a high subsidy \( \sigma \) by the government \( c' \) is smaller, then \( R < 0 \). The effects of a change in \( \varepsilon \) and a change in the subsidy per student due to a change in \( \varepsilon \) can be derived from the following:

\[ \frac{dR}{d\varepsilon} = \frac{1-\gamma}{\gamma} \frac{dc}{d\varepsilon} - \frac{1-\gamma}{\gamma} \frac{d\sigma}{d\varepsilon} - \frac{w\delta(1+\delta)}{(1-\varepsilon\delta)^2} = \frac{1-\gamma}{\gamma} \frac{dc}{d\varepsilon} - \frac{1-\gamma}{\gamma} \frac{d\sigma}{d\varepsilon} + \frac{dS_{MC}}{d\varepsilon} \], \quad (3.7.27)

where, \( \frac{dS_{MC}}{d\varepsilon} = -\frac{w\delta(1+\delta)}{(1-\varepsilon\delta)^2} \) = the change in social marginal cost of education.

Clearly \( a \) \( \frac{dR}{d\varepsilon} \geq 0 \) if \( \frac{dc}{d\varepsilon} \leq \frac{d\sigma}{d\varepsilon} - \frac{1-\gamma}{\gamma} \frac{dS_{MC}}{d\varepsilon}, \gamma \leq 1/2 \).
This leads to the following proposition:

**Proposition 5**: A rise in the teacher-student ratio raises the social marginal cost and hence affects the gap between the private and social marginal products of education. The change will depend on the relative changes in the subsidy per student, and the discount rate in the job ladder framework with public financing of the education sector. If the subsidy is adjusted in such a way that it equals the change in the social marginal cost of education, then the gap between the private and social marginal products of education remains unchanged.

### 3.8. The Job Ladder Model with Public Financing of Institutional Costs and Private Tuition Earnings by Teachers (Parallel Economy in Education)

This section makes a simple but important and major extension of the job ladder model to capture both public and private financing for the almost-compulsory “coaching cost” found predominantly in highly subsidized education at schools and colleges in developing countries. These schools and colleges are affiliated with central boards or universities in many countries such as India, Pakistan, Bangladesh, Malaysia, Indonesia, the Philippines, and Nepal. The boards and universities prescribe the curriculum and conduct all degree-level examinations. The teachers at schools and colleges are assigned to thoroughly teach the curriculum and courses prescribed by the boards and universities. However, in part due to overcrowded classes, lack of monitoring of the duties and responsibilities of teachers, political affiliation and activities of the teachers outside schools, associations with powerful unions, and other socio-economic factors,
the courses are not be properly taught in the classroom. Therefore, almost all the students pay the teachers for private tutoring outside the classes. Although education is officially free for students (or students pay a nominal subsidized fee at schools and colleges), they pay a high “private tuition cost” (roughly five to 10 times the monthly subsidized fee charged at schools) for private coaching by those teachers. For example, a report published by UNESCO (Bray, 2007) shows that the proportions of students receiving tutoring in Sri Lanka in 1990 were 62 percent for liberal arts students, 67 percent for commerce students, and 92 percent for science students. In Myanmar, a 1991 survey in the Yangon Division found that 91 percent of students received tutoring. In a more recent survey of 23 countries, Dang and Rogers (2008) found that about 25 to 90 percent of students received private tutoring at certain levels. This alarming rate has raised questions among many policymakers as to whether this imposes heavy costs on households (Psacharopoulos and Papakonstantinou, 2005), and exacerbates social inequalities. A simple extension incorporating these “private coaching earnings” shows that, despite substantial education subsidies by the governments, the size of the educated labor pool is smaller than intended.

This private tuition income is not taxed, and is therefore unaccounted for by the government. Overcrowded classes, limited class hours, little teacher monitoring or accountability, and the centralized examination system all seem to give teachers a certain degree of controlling power in the operation of this private coaching practice. The teaching positions are tenured almost from the beginning of employment (the customary probation period is just on paper), and there is virtually no accountability for teachers’ performance. This, together with other socio-political factors, is instrumental in sustaining the ever-growing parallel economy in the field of liberal arts education in much of the developing world.
To incorporate this aspect of the parallel economy in education, private tuition costs are included here in the cost function of the students. These payments are not paid to the institution but directly to the teachers by students. Though the institutional fee charged per each student is very low, and the government meets the expenses of the education sector from the tax revenue generated from Sector 1 (the modern sector), out-of-pocket costs for private tuition to pass the exams are paid by the students. Production functions in Sectors 1 and 3 (modern sector and subsistence sector), and education technology remaining unchanged, equations (3.8.1) to (3.8.3) will be of identical forms as in (3.7.1) and (3.7.3).

It is assumed that $G_R$ is the tax revenue generated from Sector 1 to meet part of the expenses of the education sector. The government revenue is given by:

$$G_R = t_s \lambda_1 w L_1^{E*}, \quad (3.8.4a)$$

where, $t_s$ is the tax rate levied on Sector 1.

The institutional cost of the education sector per student can be expressed as

$$c = \frac{\lambda_2 w L_2^{E*}}{S}, \quad (3.8.4b)$$

where, $S$ is the total number of students. Thus each student pays only $c^*$, a part of the institutional cost of education, and the remaining is met by the government tax revenue generated from Sector 1. In addition, the student now pays the private tuition costs to pass the exams. Thus:

$$c^* = c - \frac{t_s \lambda_1 w L_1^{E*}}{S} + k \lambda_2 w, \quad (3.8.4c)$$

which alternatively can be expressed as:

$$c^* S = c S - G_R + p_{tc} S = c^* = c - \sigma + p_{tc}, \quad (3.8.4d)$$

where, $\sigma$ is tax revenue spent per student and $p_{tc}$ is the private tuition cost paid by each student.
Therefore, equation 3.4 of the simple job ladder model that determines the equilibrium size of the educated labor force based on the principle of present value equalization will be now given by:

\[
\frac{1}{\gamma} \int_{0}^{T} e^{-r\tau} d\tau - e^{*} \int_{0}^{1} e^{-r\tau} d\tau = \frac{w}{\gamma} \int_{0}^{T} e^{-r\tau} d\tau
\]

or, \( \gamma \bar{w} = w + (1-\gamma)e^{*} \). \hspace{1cm} (3.8.4)

Other equations remaining the same, the average wage can now be written as:

\[
\bar{w} = \frac{(1-t_{1})\lambda_{1}wL_{E}^{*} + \lambda_{2}w(1+k/\varepsilon)e\delta L_{E}^{*}}{L_{E}^{*}} + \frac{(1-e\delta)L_{E}^{*}}{L_{E}^{*}}w. \hspace{1cm} (3.8.9)
\]

Substitution of \( \bar{w} \) in equation 3.8.4 gives \( L_{E}^{*} \), and substitution of the balanced budget condition for the education sector results in:

\[
L_{E}^{*} = \frac{\gamma w(\lambda_{1} - 1) + t_{1}\lambda_{1}w((1-\gamma)/\delta - \gamma)}{(1-\gamma)(w + c) - \gamma w(\lambda_{2} - 1)e\delta + k\lambda_{2}w(1-\gamma\delta)}L_{E}^{*}. \hspace{1cm} (3.8.10)
\]

This solution is meaningful if:

\[
\gamma w(\lambda_{1}(1-t_{1})(1-e\delta) + \lambda_{2}(1+k/\varepsilon)e\delta) \geq w + (1-\gamma)e^{*} \geq \gamma w(\lambda_{2}(1+k/\varepsilon)e\delta + (1-e\delta)). \hspace{1cm} (3.8.11)
\]

Here the additional private tuition cost paid by the student is a multiple \( k \) of the teacher’s wage. In the simplest case, \( k = 0 \), but more realistically \( k \) varies with the teacher-student ratio. Equation 3.8.11 implies that the opportunity cost of getting educated in this model lies between the expected minimum (when an educated worker gets a job only in the education sector and subsistence sector) and the expected maximum income of an educated worker (none of the educated workers are employed in the subsistence sector). As we have shown in section 3.2.1, in the job-ladder framework education is meaningful only if the size of the educated labor force net of teachers is at least as large as employment in Sector 1, and the educated labor force and the student body do not exhaust the total labor force. Thus, the opportunity cost of education (which
equals the sum of income forgone and the discounted sum of the charges for education) must be higher than the expected minimum income of an educated worker (which equals the discounted sum of income of an educated worker if that worker gets a job either in the education or subsistence sector). Also, the expected maximum income of an educated worker (which equals the discounted sum of income of an educated worker if that worker gets a job either in Sector 1 or the education sector) must be higher than the opportunity cost of education. The above equation becomes identical with the equation (3.13) of the original model, when \( t_s = 0 \) and \( k = 0 \).

The new welfare level is given by:

\[
W^{**} = f(L^{**}) + w(1 - L^{**} - (1 + \varepsilon)\delta L^{**}).
\]  
(3.8.12)

From equations 3.8.10 to 3.8.12, the following is obtained:

(a) for a non-negative discount rate \( r, \frac{1 - \gamma}{\gamma} \geq \delta \), we get \( L^{**} \leq L^{EM*} \)

(b) If \( k = 0, t_s = 0 \) then \( L^{**} = L^{**}^{EM} = L^{EM} \) as in the original job ladder model

(c) \( W^{**} \geq W^{**} \) The equality holds when \( r = 0 \). The new level of welfare is higher than the previous one.

This leads to the following proposition:

**Proposition 6:** When private tuition costs of students and additional earnings of teachers from private tuition are taken into account under the job ladder model of education with public subsidies to the education sector, the actual size of the educated labor force is smaller than that in either the pure job ladder model or the job ladder model with public financing of the education sector, mainly due to substantial private tuition costs incurred by students for their education. Since this private tuition cost raises the opportunity cost of education, it reduces the size of the educated labor force in the job ladder framework where the excess supply of educated
labor force ends up working in the subsistence sector without any additional productivity gain, and hence, the welfare loss is smaller than that in the pure job ladder model.

3.8.1. Comparative Static Results: Changes in Policy Parameters in the Job Ladder Model with Public Financing of Institutional Costs and Private Tuition Earnings by Teachers

The comparative static analysis of the sort made in the pure job ladder model leads to different results if \( k \) is invariant with those policy parameters. The results will change if \( k \) changes with them. For example, an increase in the teacher-student ratio obtains the following:

\[
\frac{dL^{E*}}{d\varepsilon} = \frac{\gamma \delta w(\lambda_2 - 1) - (1 - \gamma) \lambda_2 w - (1 - \gamma - \gamma \delta) \lambda_2 w \frac{dk}{d\varepsilon}}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1) \delta \delta + k \lambda_2 w(1 - \gamma - \gamma \delta)} L^{E*}.
\]

Thus,

- For any given \( t_x, k, \) and when \( (1 - \gamma) \geq \gamma \delta \) for \( r \geq 0 \), \( \frac{dL^{E*}}{d\varepsilon} \leq 0 \)
- For any given \( t_x, dk / d\varepsilon \geq 0, \) then \( (1 - \gamma) \geq \gamma \delta \) for \( r \geq 0, \), \( \frac{dL^{E*}}{d\varepsilon} \leq 0 \)
- For any given \( t_x, dk / d\varepsilon \leq 0, \) then \( (1 - \gamma) \geq \gamma \delta \) for \( r \geq 0, \), \( \frac{dL^{E*}}{d\varepsilon} \geq 0, \) if

\[
\gamma \delta (\lambda_2 - 1) - (1 - \gamma - \gamma \delta) \lambda_2 \frac{dk}{d\varepsilon} \geq (1 - \gamma) \lambda_2
\]

\[
\frac{dL^{E*}}{d\lambda_2} = \frac{\gamma \delta \varepsilon w - (1 - \gamma) \varepsilon w - \varepsilon k w (1 - \gamma - \gamma \delta)}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1) \delta \delta + k \lambda_2 w (1 - \gamma - \gamma \delta)} L^{E*}
\]

Thus for any given \( t_x \) and \( k, (1 - \gamma) \geq \gamma \delta, \) for \( r \geq 0, \), \( \frac{dL^{E*}}{d\lambda_2} \leq 0 \)

\[
\frac{dL^{E*}}{dk} = \frac{-(1 - \gamma - \gamma \delta) \lambda_2 w}{(1 - \gamma)(w + c) - \gamma w(\lambda_2 - 1) \delta \delta + k \lambda_2 w (1 - \gamma - \gamma \delta)} L^{E*}
\]

Thus for any given \( t_x \) and \( k, (1 - \gamma) \geq \gamma \delta, \) for \( r \geq 0, \), \( \frac{dL^{E*}}{dk} \leq 0 \)
The above results can be summarized in the following proposition:

**Proposition 7**: When private tuition costs of students and additional earnings of teachers from private tuition are taken into account, under the job ladder model of education with public subsidies to the education sector, the size of the educated labor force falls due to an increase in the teacher-student ratio, the salary of the teachers, and any increase in the private tuition costs to the students. This is due to private costs of education, given that the subsidy and wages in Sector 1 remain unchanged.

### 3.8.2. What if Government Ensures Quality Improvement in Schools, Charges Additional Fees, and Prohibits Private Tuition Practices of Teachers?

In many developing countries, most of the schools and colleges are public institutions and teachers are publicly appointed by strict administrative rules and monitoring. By making teachers accountable for the performance of the schools in the university board exams, the governments may try to ensure quality improvements by charging additional fees and prohibiting private tuition practices by public school teachers. This will allow students to pay an additional fee directly to schools when teachers are made accountable for the completion of courses there. Various states in India, such as West Bengal, have put regulations in place to ban private tuition by teachers who are employed in public schools.

Under this scenario, the student pays an additional fee $k \lambda_2 w$ and the government revenue to be spent on education rises to $(\frac{\lambda^2 w L^*E_1}{S} \cdot k \lambda_2 w)$. Hence, the individual cost of education for a student will be:

$$c^{**} = c + k \lambda_2 w - \frac{\lambda^2 w L^*E_1}{S} = c + k \lambda_2 w - \sigma,$$
where \( \sigma \) is tax revenue spent per student and \( k^* \lambda w \) is the additional fee paid by the student.

The schools presumably improve quality and prohibit private tuition by teachers.

Therefore, equation 3.4 of the simple job ladder model that determines the equilibrium size of the educated labor force based on the principle of present value equalization will now be given as:

\[
\gamma w = w + (1-\gamma)c**. \tag{3.8.2.4}
\]

Under the new system, the government absorbs the additional fees from students. Other equations remaining the same, the average wage can now be written as:

\[
\bar{w} = \frac{(1-t_s)\lambda wL\eta_{L\eta} + \lambda wL\eta + ((1-\varepsilon\delta)L\eta - L\eta_{L\eta})w}{L\eta}. \tag{3.8.2.9}
\]

Substitution of \( \bar{w} \) in equation (3.8.2.4) and substitution of a balanced budget condition for the education sector gives \( L\eta^{**} \) as:

\[
L\eta^{**} = \frac{[\gamma w(\lambda_1 - 1) + t_s\lambda w((1-\gamma)/\delta - \gamma)]L\eta}{[(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta + (1-\gamma)k^* \lambda w]}. \tag{3.8.2.10}
\]

From equations 3.8.9 to 3.8.10 and 3.8.2.9 to 3.8.210, the following can be derived:

(a) \( 0 \leq k^*/k \leq (1-\gamma - \gamma\delta)/(1-\gamma) \leq 1 \). In this case, students will pay less to the schools for additional fees than they would have paid to the teachers, and \( L\eta^{**} \geq L\eta^{**} \), i.e., the size of the educated labor force, is higher than under private tuition.

(b) If \( k^* = k \), then \( L\eta^{**} \leq L\eta^{**} \), i.e., the size of the educated labor force is lower than in the presence of private tuition, where additional earnings of teachers raise their average expected earnings, which attracts more students to get educated. The additional tuition fees paid to the teachers enter into the expected wage earnings of the educated worker.

(c) If \( k^* = k = 0 \), then the size of the educated labor force will remain the same as in the original job ladder model for \( t_s = 0 \), i.e., \( L\eta^{**} = L\eta^{**} = L\eta^{**} = L\eta^{**} = L\eta^{**} \).
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(d) \( c^{**} \leq c \), only if \( t_s \geq \frac{k^*}{\epsilon} \left[ \frac{\lambda_2 wL_{1}^{**}}{\lambda_1 wL_{1}^{**}} \right] \), i.e., the tax rate should be high enough so that government revenue would cover the part of wage bill for the teachers.

(e) If \( t_s \lambda_1 wL_{1}^{**} = k^* \lambda_2 wS = k^* \lambda_2 w\delta L_{1}^{**} \), the subsidy is just sufficient to cover the additional fees charged to the students, that the students end up paying the cost of hiring teachers in Sector 2, i.e., \( c^{**} = c \), and \( L_{1}^{**} = \frac{\gamma w ((\lambda_1 - 1) - t_s \Lambda_1) L_{1}^{**}}{w + (1-\gamma) c - \gamma \lambda_2 w \delta \delta - \gamma w (1-\epsilon \delta)} \), where the size of the educated labor force will be lower than in the original model with no public subsidy of education, i.e., \( L_{1}^{**} \leq L_{1}^{EM} \). This is due to the lower expected average income of the educated worker resulting from the tax on wages in Sector 1. The solution is meaningful only when \( t_s \geq \frac{\Lambda_1}{\Lambda_1 - 1} \), where \( 1 > \Lambda_1 > 0 \).

3.8.2.1. Comparative Static Results: Effects of Changes in Policy Parameters

The comparative static analysis of the sort made in the pure job ladder model leads to different results if \( k \) is invariant with those policy parameters. The results will change if \( k^* \) changes with them.

(a) For example, for an increase in the teacher-student ratio the following is obtained:

\[
\frac{dL_{s}^{**}}{de} = \frac{\gamma \delta w (\lambda_2 - 1) - (1-\gamma) \lambda_2 w (1 + \frac{dk^*}{d\epsilon})}{(1-\gamma)(w + c) - \gamma w (\lambda_2 - 1) \delta \delta + k^* \lambda_2 w (1-\gamma)} L_{1}^{**}.
\]

Thus,

- For any given \( t_s \), \( k^* \), and when \( 1 \geq \gamma \geq 0 \), for \( r \geq 0 \); and \( 1 \geq \delta \geq 0 \), \( \frac{dL_{s}^{EM}}{de} \leq 0 \).
Also, for $k = k^*$, \[ \frac{dL_{EM}^{**}}{d\varepsilon} \leq \frac{dL_{E}^{**}}{d\varepsilon} \leq \frac{dL_{EM}^{**}}{d\varepsilon} \]. Thus, for an exogenous increase in teacher-student ratio, the decline in the size of the educated labor force is highest when the students pay additional fees to their teachers directly for private tuition, compared to a situation where the government charges additional fees to remove private tuition and subsidize education.

- For any given $t_x$, \( dk^*/d\varepsilon \geq 0 \), then \( 0 \leq \gamma \leq 1 \) for \( r \geq 0 \), \( \frac{dL_{E}^{**}}{d\varepsilon} \leq 0 \)

- For any given $t_x$, \( dk^*/d\varepsilon \leq 0 \), then \( 1 \geq \gamma \geq 0 \) for \( r \geq 0 \), \( \frac{dL_{E}^{**}}{d\varepsilon} \geq 0 \), if

\[ \left| \frac{dk^*}{d\varepsilon} \right| \geq \frac{(1-\gamma)\lambda_2 - \gamma\delta(\lambda_2 - 1)}{(1-\gamma)\lambda_2} \]

\[ \frac{dL_{E}^{**}}{d\lambda_2} = \frac{\gamma\delta\varepsilon w - (1-\gamma)w\varepsilon - k^*w(1-\gamma)}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta + k^*\lambda_2 w(1-\gamma)} L_{E}^{**}. \]

Thus, for any given $t_x$, $k^*$, and when \( 1 \geq \gamma, \delta \geq 0 \), for \( r \geq 0 \), and \( \delta \leq (1-\gamma)/\gamma \)

\[ \frac{dL_{E}^{**}}{d\lambda_2} \leq 0, \text{ and when } k = k^* \quad \frac{dL_{EM}^{**}}{d\lambda_2} \leq \frac{dL_{E}^{**}}{d\lambda_2} \leq \frac{dL_{EM}^{**}}{d\lambda_2} \]

\[ \frac{dL_{E}^{**}}{dk^*} = \frac{-(1-\gamma)\lambda_2 w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta + k^*\lambda_2 w(1-\gamma)} L_{E}^{**}. \]

Also for any given $t_x$, when \( (1-\gamma) \geq \gamma\delta \) for \( r \geq 0 \), \( \frac{dL_{E}^{**}}{dk^*} \leq 0 \).

The above results can be summarized in the following proposition:

**Proposition 8:** For any exogenous increase in the teacher-student ratio, or an increase in the wages of teachers, the opportunity costs of education will rise, and hence the size of the educated labor force will fall. But the decline is higher in the presence of private tuition practices by teachers than in the scenario under which the additional fees are charged by the
schools to improve the quality of education, and education is partly subsidized by the government, which prohibits the private tuition practices.


3.9.1. Assumptions

This section develops an overlapping generations model of education that helps to further illustrate the issues of wage stickiness in the labor market for educated workers. Restuccia and Vandenbroucke (2011, 2013) have used the overlapping generations framework to study the educational attainment over much of the 20th century in the United States. Jones and Yang (2013) have used this framework to analyze the increasing costs of higher education in developed countries. The main contribution here is to incorporate the job ladder assumption into the overlapping generations models of education to analyze the impact of wage rigidities on the size of the educated labor force.

In this formulation, individuals can choose education based on ability and cost. Thus, an advantage of the overlapping generations model with the job ladder assumption is that demand for schooling is endogenous, and so it produces an endogenous teacher-to-student ratio in efficiency units. It is assumed that education decisions are based on labor market returns and the cost of tuition. In this model, a generation of mass one is born every period \( t = 1,2 \). A person is indexed by his or her innate ability, denoted by \( x \), with \( x \) in the interval \([x_1, x_2]\). The distribution of skills in each generation is described by the probability density function \( f(x) \), so that

\[
\int_{x_1}^{x_2} f(x)dx = 1,
\]

where \( x_1 \) is the lowest skill possible and \( x_2 \) is the highest skill (Figure 3.2). Every person lives for two periods. The first period corresponds to childhood and young adulthood (life until about age 20-25). The second period corresponds to mature (working)
adolescence (life from age 20-25 to about age 55-65). For simplicity, we abstract from retirement decisions.

**Figure 3.2. Distribution of Abilities and Education Threshold**

![Graph showing distribution of abilities and education threshold](image)

In the first period, children go to school, where the education they obtain, denoted by $s$, helps them accumulate skills useful for work during the second period. A student’s skills, $xh(s)$, depend on his or her ability $x$ and human capital, $h(s)$, acquired through investment in education, so that students with higher ability are more productive for a given level of schooling. Function $h(.)$ is the human capital production function, $h(0) = 1$, $h' > 0$, $h'' < 0$. Education is costly, requiring students’ own effort and time denoted by $d$, plus the time spent as guided by teachers denoted by $g$. Again, to keep the analysis simple, we abstract from school buildings, textbooks, and other material inputs. For every lecture hour, students must devote $\alpha$ hours reading textbooks, doing homework or lab work, going to the library, writing research papers, and otherwise engaging in student chores. $\alpha$ is assumed exogenous (equals about 3). The total time students devote to school-related activities is $s = (1 + \alpha)g$. The total cost of education from the student’s point of view is summarized by the cost function $C = C(s)$, with $C' > 0, C'' > 0$. That is, education costs are convex, rising in the total time and effort devoted by the student at an increasing rate. The gross interest rate corresponding to the 20-25 year period between when investments in education are made and when students and their families get their return is denoted by $1 + r = (1.03)^{25}$. 

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3.9.2. Job Ladder Assumption in the Three-Sector Framework

In this model, education provides opportunities for employment not available to uneducated workers, and schooling amounts to a lottery with probability \( \pi_1 \). Educated workers find employment in the formal sector (Sector 1), receiving a wage (per unit of human capital) of \( w_1 \).

With probability \( \pi_2 \), they find employment in Sector 2 (become teachers), receiving wage \( w_2 \); and with probability \( 1 - \pi_1 - \pi_2 \), the educated workers must go back to the subsistence sector with wage \( w_3 \). Let us assume that \( Ew \) is the expected wage associated with this lottery. Thus, the lifetime net income, denoted by \( A \), of an individual with ability \( x \) who chooses education level \( s \) is given by:

\[
A(x, s) = xh(s) \frac{Ew}{1 + r} - C(s).
\]  

(3.9.1)

Assuming risk neutrality, individuals (and their families) will choose their education effort, denoted by \( s^* \) (for schooling), to maximize lifetime net income. The value of \( s^* \) will equate the marginal discounted benefits of education with its marginal cost. The first order necessary condition for an interior solution \( s^* > 0 \) is implicitly given by:

\[
xh'(s^*) \frac{Ew}{1 + r} = C'(s^*).
\]  

(3.9.2)

The second order condition associated with maximization of equation 3.9.1 is also satisfied, given that \( C'' > 0 \) and \( h'' < 0 \). We are also interested, however, in situations in which the individual’s desired schooling choice is zero. This could be the case, in the model, for low enough ability levels \( (x) \).

A few observations are worth making about equation 3.9.2.
First, higher ability leads to more investment in education: \( ds*/dx > 0 \). This can be seen by differentiating equation 3.9.2:

\[
\frac{ds^*}{dx} = \frac{h'(s^*)E_w}{C^*(s^*) - xh''(s^*)} \frac{E_w}{1 + r} > 0.
\]  

(3.9.3)

The right-hand side of equation 3.9.3 is greater than 0.

Second, an increase in the expected wage, \( E_w \), or a decline in the real interest rate, \( 1+r \), each raises the effort going into schooling:

Since \( \frac{xh'(s^*)}{1 + r}E_w = C'(s^*) \Rightarrow for \ an \ \uparrow E_w \Rightarrow \uparrow C'(s^*) \)

Also, for \( an \ \downarrow (1 + r) \Rightarrow \uparrow C'(s^*) \)

Third, and most important for purposes here, privately desired education may be less than the administratively set minimum level for sufficiently low abilities. Assuming that the administratively set minimum level of schooling is zero, from equation 3.9.2, this threshold level \( \bar{x} \) such that \( s^* = 0 \) for all \( x < \bar{x} \) solves:

\[
\bar{x}h'(0) \frac{E_w}{1 + r} = C'(0).
\]  

(3.9.4)

Thus, this simple model endogenously generates the size of the unskilled labor force. All individuals with ability \( x \) in the interval \([x_1, \bar{x}]\) will be employed in the subsistence sector, each receiving the subsistence wage \( w_3 \).\(^{10}\)

\(^{10}\) This outcome—individuals choosing zero schooling—is likely to be inefficient if, as has been widely argued, there are positive externalities associated with receiving a minimum level of education. Even conservative libertarian economists agree on this (Friedman, 1962).
3.9.3. Flexible Market Equilibrium

In an equilibrium with flexible markets and prices, educated workers are absorbed in Sectors 1 and 2, and none of the educated workers are employed in Sector 3. So only unskilled workers are employed in the subsistence sector implying that:

\[ L_3 = \int_{x_h}^{x_f} x h(0) f(x) dx . \]

(3.9.5)

The size of the skilled labor force to be employed in the two modern sectors and corresponding wages can now be determined. Let \( L_1 \) be the level of employment in final goods production in the modern sector and \( L_2 \) be employment in the education sector. To simplify matters without a loss of generality, also assume that \( w_1 = w_2 \), so that skilled workers are indifferent between employment in Sectors 1 and 2.

In each period, employment and the common real wage in the two modern sectors (Sectors 1 and 2) are determined by the marginal productivity condition in Sector 1 and by the demand for education. At date \( t \), the supply of skilled adults is fixed by past education investments (in \( t-1 \)). The supply of skilled labor by mature adults (belonging to generation \( t-1 \)) who were educated at \( t-1 \) is

\[ L = L^s_1 + L^s_2 = \int_{x_1}^{x_2} x s^* (x) f(x) dx . \]

(3.9.6)

Employment in the education sector, \( L_2 \), at date \( t \), is determined by aggregate demand for education (by children and young adults) belonging to generation \( t \). The aggregate demand for education by the young can be obtained by simply summing up the demand for teacher inputs at each skill level that is \( g(x) = \frac{s^*(x)}{1 + \alpha} \), the overall ability level exceeding the threshold:

\[ g(x) = \frac{s^*(x)}{1 + \alpha} \]

11 We are assuming that each adult’s time endowment is one, which the person supplies inelastically.
Equating demand for education with supply and subtracting equation 3.9.7 from equation 3.9.6 yields the labor supply at date \( t \) in Sector 1 as a function of the wage and real interest rate (Figure 3.3).

To compute the momentary equilibrium (at date \( t \)) in the modern goods sector, we must also derive the labor demand curve for Sector 1. Denoting the production function in this sector as

\[
y_1 = A_i f(L_1^d),
\]

the labor demand is pinned down by the marginal productivity condition in the final goods sector:

\[
w^* = w_1 = w_2 = A_i f'(L_1^d).
\]

Finally, the equilibrium wage in the two sectors under flexible prices must satisfy the condition that labor demand equals labor supply (point E in Figure 3.3).

### 3.9.4. Fix-Price/Job Ladder Equilibrium

In reality, wages in the modern sectors are not always flexible. Unions are especially strong in the education sectors, and collective bargaining agreements result in wages being set via long-term contracts—a three-year contract duration is typical in many countries. Teacher unions are influential interest groups in many countries, donating significant sums of money and effort in
political campaigns. In most of the developing countries public sector wages are set by government agencies.

Assume therefore that the common wage in Sectors 1 and 2 is predetermined at $\bar{w}$ in period $t$, at a level that exceeds the market clearing wage $w^*$. Employment in the formal goods sector is then determined by aggregate demand (Figure 3.3):

$$\bar{w} = w_1 = w_2 = Af'(L_1^d).$$

(3.9.10)

Employment in the education sector will, as before, be set by aggregate demand for education (equation 3.9.7). Total demand for skilled workers, $L_1 + L_2$, will fall short of supply, $\bar{L}$, leading to $(\bar{L} - L_1 - L_2)$ skilled workers joining unskilled workers in producing final goods using subsistence techniques in Sector 3. In the fix-price equilibrium, the proportions of skilled workers employed in Sectors 1 and 2 must match the probabilities young adults perceive when they make their human capital investment decisions.

**Figure 3.3. Labor Market Outcomes in the Modern Sector: Flexible and Fixed-Price Cases**
3.9.5. A Comparison of Flexible Market Equilibrium and Fix-Price Equilibrium

This section examines the consequence of wage rigidities in the formal sector for the choice of education levels, the size of the traditional sector, and the distribution of worker income. In the flex-price equilibrium, all skilled workers are gainfully employed in the formal goods sector and as teachers (Sectors 1 and 2, respectively). With rigid wages in Sector 1, the presumption is that young people will have a stronger incentive to get educated. That is, one ordinarily expects that the rigid wage does not significantly raise the probability of not being able to find employment in the modern sectors (1 and 2) and $E\bar{w} > w^*$. In this normal case, demand for schooling will be higher in the fix-price equilibrium, but some skilled workers will be pushed to the traditional sector. In addition, a rigid wage in this case results in a lower threshold level of ability, so that the pool of skilled labor in the fix-price equilibrium is enlarged. This conclusion is similar to the results in the Bhagwati-Srinivasan job ladder model.

There is, however, a perverse case to consider in which $E\bar{w} < w^*$. That is, if the rigid wage creates a large enough probability of being pushed to the traditional sector, then young individuals will have incentives to get less education and the pool of educated skilled workers in the fix-price equilibrium will actually be smaller.

These two cases are considered in Proposition 9 later in this subsection.

The comparison between the two equilibrium solutions gives some interesting results. In order to compare these, assume that:

1. $w^* = w_1^* = w_2^*$ is the market clearing wage in the flexible market, where $\pi_1^*$ and $\pi_2^*$ are the respective probabilities of finding employment in Sectors 1 and 2. In flexible market equilibrium, educated labor gets fully absorbed in Sectors 1 and 2.
(2) $\tilde{w} = \tilde{w}_1 = \tilde{w}_2$ is the institutionally fixed or predetermined wage in Sectors 1 and 2 and $w_3$ is the wage in the subsistence sector under the fix-market or job ladder framework, where for an educated worker, $\tilde{\pi}_1$, $\tilde{\pi}_2$, and $(1 - \tilde{\pi}_1 - \tilde{\pi}_2)$ are the respective probabilities of finding employment in Sector 1, 2, and 3 (subsistence sector). In fix-market or job ladder equilibrium, the surplus educated labor goes back to the subsistence sector, earning the subsistence wage.

The expected wages for an educated worker under these flexible market and fix-price equilibrium conditions will then be given by

$$Ew^* = (\pi_1 + \pi_2)w^* \quad \text{and} \quad E\tilde{w} = (\tilde{\pi}_1 + \tilde{\pi}_2)\tilde{w} + (1 - \tilde{\pi}_1 - \tilde{\pi}_2)\tilde{w}_3$$

respectively.

A comparison of these two expected wages at the equilibrium condition gives us:

$$dEw = E\tilde{w} - Ew^* = (\tilde{\pi}_1 + \tilde{\pi}_2)\tilde{w} + (1 - \tilde{\pi}_1 - \tilde{\pi}_2)\tilde{w}_3 - (\pi_1^* + \pi_2^*)w^*$$

$$= (\tilde{\pi}_1 + \tilde{\pi}_2)(\tilde{w} - \tilde{w}_3) - (w^* - \tilde{w}_3).$$

Thus, the following scenarios result:

(1) $E\tilde{w} - Ew^* \geq 0 \quad \text{iff} \quad \frac{(\tilde{w} - \tilde{w}_3)}{(w^* - \tilde{w}_3)} \geq \frac{1}{(\tilde{\pi}_1 + \tilde{\pi}_2)} \quad \text{or} \quad \frac{(\tilde{w} - \tilde{w}_3)}{(w^* - \tilde{w}_3)} \geq \frac{\tilde{L}^e}{(\tilde{L}_1 + \tilde{L}_2)}$

(2) $E\tilde{w} - Ew^* \leq 0 \quad \text{iff} \quad \frac{(\tilde{w} - \tilde{w}_3)}{(w^* - \tilde{w}_3)} \leq \frac{1}{(\tilde{\pi}_1 + \tilde{\pi}_2)} \quad \text{or} \quad \frac{(\tilde{w} - \tilde{w}_3)}{(w^* - \tilde{w}_3)} \leq \frac{\tilde{L}^e}{(\tilde{L}_1 + \tilde{L}_2)}.$

These results can be summarized in the following proposition:

**Proposition 9:** If the institutionally fixed wage in the formal sector is sufficiently large, and/or the subsistence wage in the traditional sector is sufficiently low (all relative to the flexible equilibrium wage), such that the formal-subistence wage gap in fix-price equilibrium exceeds the wage gap of flexible equilibrium, then the ex ante expected wage of an educated worker is higher in the job ladder (fix-price) equilibrium. Hence the demand for education is higher in the fix-price scenario than in the flexible market equilibrium.
However, if the wage differentials are almost the same and the sum of the probabilities of getting employed in Sectors 1 and 2 is below 1, then $E\bar{w} \leq Ew^*$, and demand for education falls. This is shown in Figure 3.4, where in response to the higher expected wage, more people decide to get an education, and the minimum threshold level, $\bar{x}$, shifts to the left. However, if due to wage rigidity and inelastic demand of educated labor in the skilled sector expected wage falls, the demand for education declines, some workers may not opt for investing in education, as in line B, and the threshold level moves to the right. According to a recent study on tertiary education system and labor market for the Organization for Economic Cooperation and Development countries (Machin and McNally, 2007), equal wage differentials of .50 and 0.49, and a 0.68 percent employment rate of tertiary graduates may partly explain the shortage of tertiary educated workers in some of these countries.

**Figure 3.4. Labor Market Outcomes and Threshold Levels of Educated Labor**

![Figure 3.4. Labor Market Outcomes and Threshold Levels of Educated Labor](image-url)
3.9.6. A Numerical Example

This subsection examines the results using specific functional forms of the (1) human capital production function, (2) cost function of education, and (3) functional form of ability distribution. For the sake of the simplicity of the analysis, it is assumed that:

1. Human capital production function:
   \[ h(s) = \gamma_0 + \gamma_1 s - \frac{\gamma_2}{2} s^2, \]  
   where, \( h'(s) = \gamma_1 - \gamma_2 s, \quad h''(s) = -\gamma_2 \leq 0, 0 < \gamma_1, \quad \gamma_2 < 1 \)  

2. Cost function of education:
   \[ C(s) = c_0 + c_1 s + \frac{c_2}{2} s^2, \]  
   where \( c'(s) = c_1 + c_2 s, \quad c''(s) = c_2 \geq 0, 0 < c_1, \quad c_2 < 1 \)  

3. Substituting equations 9.11 and 9.12 into the first-order condition (equation 9.2) gives us:

   \[
   s^* = \frac{\frac{xy_1}{1+r} E_w - c_1}{\frac{xy_2}{1+r} E_w + c_2} \geq 0 \quad \text{iff} \quad \frac{xy_1}{1+r} E_w - c_1 \geq 0
   \]  

4. Evaluating equation 9.13 at the equilibrium level, we get the minimum threshold level of ability as

   \[ \bar{x} = \frac{c_1}{E_w \gamma_1} \]  

5. Equation 3.9.14 provides intuitive results as follows:

   - \( \frac{E_w}{1+r} \uparrow \Rightarrow \bar{x} \downarrow \), i.e., an increase in the present value of the expected wage increases the demand for education, the minimum threshold ability falls, and more people are encouraged to get an education
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- \( c_1 \uparrow \Rightarrow \bar{x} \uparrow \), i.e., an increase in the marginal cost of education in equilibrium reduces the demand for education, the minimum threshold ability rises, and less people are encouraged to get an education.

- \( \gamma_1 \uparrow \Rightarrow \bar{x} \downarrow \), i.e., an increase in the marginal product of education causes an increase in the demand for education, the minimum threshold ability declines, and more people are encouraged to get an education.

6. Assuming a uniform distribution function of ability \([0,1]\), we can derive

\[
\bar{L} = \int_{\bar{x}}^{1} \frac{(x \gamma_1 w^\gamma - c_1 x)}{x \gamma_2 w^\gamma + c_2} \, dx \quad \text{where,} \quad w_e = \frac{E w}{1 + r}
\]

\[
\bar{L} = \frac{2c_2(c_2 \gamma_1 + c_1 \gamma_2) \log((\gamma_2 w_e + c_1) / (\bar{x} \gamma_2 w_e + c_2))}{2 \gamma_2^3 w_e^2} - \frac{\gamma_2 w_e (2c_2 \gamma_1 + 2c_1 \gamma_2)(1 - \bar{x}) + \gamma_1 \gamma_2^2 w_e^2 (1 - \bar{x}^2) - c_1 (1 - \bar{x})}{2 \gamma_2^3 w_e^2}
\]  (3.9.15)

Also:

\[
L_2^d = \frac{1}{(1 + \alpha)(1 + r)} \int_{\bar{x}}^{1} \frac{(x \gamma_1 w_e - c_1)}{x \gamma_2 w_e + c_2} \, dx = \left[ \frac{\gamma_1 x}{\gamma_2 (1 + r)} - \frac{(c_1 \gamma_2 + c_1 \gamma_1) \ln(x \gamma_2 w_e + c_2)}{\gamma_2 E w} \right]_{\bar{x}}^{1}
\]  (3.9.16)

\[
L_2^d = \left[ \frac{\gamma_1 (1 - \bar{x})}{\gamma_2 (1 + r)} - \frac{(c_1 \gamma_2 + c_1 \gamma_1) \ln((\gamma_2 w_e + c_2) / (\gamma_2 \bar{x} w_e + c_2))}{\gamma_2 E w} \right]
\]

\[
L_3 = \int_{x_0}^{\bar{x}} x h(0) f(x) \, dx = \frac{\gamma_0}{2} \left[ \bar{x}^2 - x_0^2 \right].
\]  (3.9.17)

Thus, in addition to some interesting analytical results, using data on the educated labor force employed in skilled and unskilled sectors, average costs of education, some estimates of the discount rate, and estimates of wage differentials between skilled and unskilled workers, we can calibrate this model under various cost-specifications and ability density functions. We would like to do this calibration in our future research. We can also use this model to analyze
the impact of various policy instruments on the size of the educated labor force and minimum threshold levels that we have used in the Bhagawati-Srinivasan job ladder model.

3.10. Conclusion

It should be noted that no single theory of education can analyze the whole education system of any country. The job ladder specification explains certain aspects of highly subsidized higher education, especially liberal arts, in many developing countries in Asia and Africa. Using the job ladder framework, this chapter has analyzed the effects of selected education policies on the size of the educated labor pool and on economic welfare using the job ladder model of education, which is relevant to certain aspects of higher education in developing countries. The chapter has analyzed the flexible market and fix-market equilibria with wage rigidity, and compared the impact on the size of the educated labor force both in the simple job ladder and overlapping generations frameworks. The results show that, though in the original model the quality-enhancing policies increase the total size of the educated labor force, in extended versions of the model the results depend on the relative sizes of the cost-enhancing impact and the effect of an increase in the expected wage of an educated worker. In the overlapping generations formulation, an increase in present value of expected wages and/or an increase in the marginal product of education is found to increase the demand for education, the minimum threshold ability falls, and more people are induced to get an education.

The socio-economic benefits of higher education are discussed at greater length in World Bank and UNESCO Task Force (2000). Bloom, Canning, and Chan (2006), Devarajan, Monga, and Zongo (2012), and several other studies that show a strong correlation between higher education and GDP growth through the diffusion of human capital development and technology. However, the labor market distortions for those who are more highly educated, the gap between
private return and social returns, and resource constraints faced by governments with competing social goals raise questions about public financing of higher education in many developing countries. When resources are scarce and governments in many developing countries face other pressing demands, a general subsidy for higher education raises many valid policy concerns. A review of the survey of literature on financing of higher education can be found in Appendix 3.1.

Thus, many countries are introducing cost-sharing measures in higher education, by (1) introducing tuition fees, (2) employing student loan-grant-scholarship schemes, and (3) allowing the provision of higher education by quality private institutions. Obviously, in order to ease the political and social tensions that sometimes arise from implementing cost-sharing initiatives, reforms should be gradual, and a carefully designed set of options should be made available. However, as Pscharopoulos (1986, p. 40) notes, “beginning to reform the financing of education is better than continuing the existing situation [of public tax financing] in most countries.”

As a useful extension, it would be useful in a future study to analyze the impact of alternative higher education financing mechanisms under various labor market specifications (that is to say, minimum wage policy). If data are available, it would be worthwhile to undertake a cross-country comparison of the impact of labor market distortions on the size and earnings of educated workers in developing economies.

In recent reviews on public financing of higher education (Devarajan, Monga, and Zongo, 2012; Tilak, 2004; Sam, 2012, Montanini, 2013; World Bank 2010), several researchers pointed out the limited resources of governments, leading to a decline in expenditures per student and thereby affecting educational quality (Psacharopoulos, 1986; Gertel, 1991; Darrell and Dundar 2000). Some studies have also criticized the public funding strategy on the basis of discrimination against students from poorer backgrounds (Espinoza, 2008; Fahim and Sani, 2011; Abdessalem, 2011). Furthermore, in studies on rates of return to education, Psacharopoulos (1994) and Psacharopoulos and Patrinos (2004) showed that higher education has the lowest social returns, with private returns significantly exceeding social returns. In view of these criticisms, there has been a continued search for alternative policies for financing higher education.

Alternative Financing Strategies: An Appraisal

Among the alternative financing strategies, studies have mostly focused on privatization, tuition fees, financial assistance schemes, and internationalization. In almost all developing countries, there has been a significant increase in private sector participation in higher education, as evidenced in several studies, including Marimuthu (2008) and Bertolin and Leite (2008). Some studies have shown that in terms of efficiency, private institutions have been more efficient in using resources than their public counterparts (Al-Salamat et al., 2011). Some researchers, such as Barr (2004), argue that the quality of education improves with increased private sector participation due to increased competition. However, Bertolin and Leite (2008) contradict this viewpoint by providing evidence of poor performance in terms of qualitative indicators related to relevance and effectiveness. Teferra (2007) takes note of the emergence and growth of fraudulent operations in the sub-Saharan region. Franco (1991) points to the possibility that the
private sector might open undergraduate courses that require less investment and thereby better serve the profit motive. Johnstone et al. (2008) note the discrimination against poor students due to high tuition fees charged by the private institutions.

With respect to the need for sharing education costs, tuition fees are increasingly charged by public institutions. Barr (2004) suggests that tuition fees will increase efficiency and quality due to increased competition among universities to attract more students. Johnstone (2002) advocates for tuition fees because higher education is disproportionately partaken by students from high-income families and, hence, such fees are a more equitable way of financing the same. In a study of Kenya, Munene and Otieno (2007) show that despite a highly subsidized fee, education is still out of reach for children from poor families. On the issue of charging tuition fees, some countries have witnessed massive opposition, as described in the study by Eboh and Obasi (2002).

Because of the contradictory positions on tuition fees, a “dual track” system has evolved in many countries under which less-qualified students are enrolled on the basis of payment of full tuition fees. Johnstone (2002) notes the possibility that this system may lead to favoritism towards full-paying students, thereby limiting the seats for government-sponsored students.

Many developing countries have adopted various financial assistance schemes in order to reduce the burden of tuition fees on poor students. Pscharopoulos (1986) advocates loan schemes, as these will increase efficiency, with students enrolling for courses with the highest returns. Woodhall (2004) notes that this will improve participation rates among the economically poorer groups. Johnstone (2005), however, cautions that such loan schemes
should be need-based, rather than solely merit-based, as in the latter case the benefit will be mostly for middle-income students.

Another aspect of loan schemes studied by Johnstone and Marcucci (2007) is that loans cover only the tuition fees and are limited to students in public sectors only. However, the authors point out that the expenditure on housing and food are usually higher than tuition fees, and therefore, such loans contribute very little toward reducing the burden of cost of higher education. Also, students who could not survive the competition for admission to scarce public sector seats do not have easy access to loans for funding their private sector education fees.

Mingat and Tan (1986) point out that lower future income in African countries makes the cost of repaying loans higher than future salaries, and thus makes the loans difficult to bear. In order to make loans more affordable, income-contingent loans have emerged. Under these schemes, loan repayment is expressed as a percentage of future earnings (Barr, 2004; Chapman, 2005). Collecting on those loans, however, has been a challenging task in many developing countries, as pointed out by Chapman and Lounkaew (2010).

Along with the loan schemes, some scholarship schemes have also been introduced by governments in developing countries. However, such schemes have been criticized because of the inefficiency of the selection process and corrupt practices (Munene and Otieno, 2008; Abdessalem, 2011).

Another aspect of higher education that has received some attention from researchers is internationalization (Knight, 2007; Altbach, 2009). For example, branches of several prestigious institutes from developed countries have opened up in many developing countries in Asia. However, Teferra (2007) notes that such internationalization is not popular in sub-Saharan
African countries (with the exception of South Africa), which is explained by economies of scale and financial and infrastructure issues.

**Proposed Reforms**

The above review brings out that financing of higher education cannot be addressed by a single policy option. The discussion has shown that tuition fees are an effective means of cost sharing, as several researchers have suggested different ways of implementing the policy. Johnstone (2004a) argues that tuition fees should be complementary to public funding and not a substitute. Barr (2004) emphasizes allowing sufficient autonomy for the institutes to set different fee structures based on their operational costs. However, government regulation should be in place to avoid the charging of exorbitant fees. Pscharopoulos (1986) and Eboh and Obasi (2002) suggest that implementing tuition fees should be phased in and preceded by consultations with students and stakeholders, and at the same time be transparent.

Johnstone (2004b) recommends that tuition fees be implemented after financial assistance schemes are in place. Pscharopoulos (1986) points out that such loans should also be available for students in the private sector. Income-contingent loans have been found to be effective in countries with rich infrastructure and, hence, should also be promoted (Chapman, 2006; Barr 2004; Johnstone 2004b). However, in the case of relatively poorer countries, such loans have been characterized by high default rates (Chapman, 2005). Suggestions for private loan arrangements and small-scale loans have been made by Barr (2004) and Chapman and Nichols (2003), respectively. Those authors also note that micro-credit programs have met with greater success compared to government loans in achieving repayment rates. The schemes have been advocated as a more feasible and practical option for sharing the cost of education.
Researchers have also encouraged private participation in education and suggested several measures to reduce the negative impact that privatization might have on equality and quality (Pscharopoulos, 1986; McCowan, 2007). Measures may include public-private collaboration, and supportive measures like tax incentives for undertaking independent research and for enrollment of students from poor backgrounds. In order to combat fraudulent operations in higher education, accreditation institutions should be established and systems should be developed to regularly rank higher education institutions based on qualitative parameters of reaching and research (Altbach, 2009).

One may summarize the above studies by stating that no one policy can be adopted to finance higher education and, hence, a well-designed policy incorporating tuition fees, privatization, financial assistance schemes, and internationalization should be put in place to solve the issue of financing higher education.

Appendix 3.2A. A Second Numerical Example for Calibration Exercise

This subsection considers an alternative specification of the cost of education. The wages of teachers is undoubtedly a very important element of the cost of formal schooling that was not considered in the earlier example. Our example endogenizes the cost of schooling by considering wages together with material costs. Specifically, we assume the following specific functional forms for the (1) human capital production function, (2) cost function of education, and (3) functional form of ability distribution.
1. Human Capital Production Function: 
\[ h(s) = \gamma_0 + \gamma_1 s - \frac{\gamma_2}{2} s^2, \]  
(3.9.11*)
where \( h'(s) = \gamma_1 - \gamma_2 s, \) \( h''(s) = -\gamma_2 \leq 0, 0 < \gamma_1, \gamma_2 < 1 \)

2. Cost Function of Education: 
\[ C(s) = \left( p + \frac{w_2}{1 + \beta} \right) S, \]  
(3.9.12*)
where \( c'(s) = \left( p + \frac{w_2}{1 + \beta} \right), \) \( c''(s) = 0, \)
where \( p \) is the number of units of output (material costs) needed per unit of student school time and \( \frac{1}{1 + \beta} \) are the number of units of teacher time needed per unit of student time.

Note that the cost of schooling is related to the wages that are set for public school teachers. We focus on the case in which wages for teachers are set competitively and assume, for simplicity, that \( w_1 = w_2. \)

Toward the end, we make some remarks about how the model could be extended in a realistic and relevant direction in which the teachers’ professions are unionized.

3. Substituting equations 3.9.11* and 3.9.12* into the first-order condition (equation 3.9.2) gives us:
\[ s^* = \frac{\gamma_1}{\gamma_2} - \frac{p + \frac{w_2}{1 + \beta}}{\frac{x}{1 + r} \gamma_2 E_w} \geq 0 \text{ iff } \left| \frac{\gamma_1}{\gamma_2} - \frac{p + \frac{w_2}{1 + \beta}}{\frac{x}{1 + r} \gamma_2 E_w} \right| \geq 0 \]  
(3.9.13*)

4. Evaluating equation 9.13 at the equilibrium level, we get the minimum threshold level of ability as \( \bar{x} = \left( p + \frac{w_2}{1 + \beta} \right) \frac{\gamma_1}{1 + r E_w} \)  
(3.9.14)
5. Equation 3.9.14 provides intuitive results as follows:

- \( \frac{Ew}{1 + r} \uparrow \Rightarrow x \downarrow \), i.e., an increase in the present value of the expected wage increases the demand for education, the minimum threshold ability falls, and more people are encouraged to get an education.

- \( p \uparrow \Rightarrow x \uparrow \), i.e., an increase in the \( p \) (material costs needed per unit of student school time) in equilibrium reduces the demand for education, the minimum threshold ability rises, and less people are encouraged to get an education.

- \( w_2 \uparrow \Rightarrow x \uparrow \), i.e., an increase in \( w_2 \) teacher’s wages will raise the marginal cost of education, and hence reduces the demand for education, the minimum threshold ability rises, and less people are encouraged to get an education.

- \( \gamma_1 \uparrow \Rightarrow x \downarrow \), i.e., an increase in the marginal product of education causes an increase in the demand for education, the minimum threshold ability declines, and more people are encouraged to get an education.

6. Assuming a uniform distribution function of ability \([0,1]\), we can derive

\[
\bar{L} = \frac{1}{2} \frac{\gamma_1}{\gamma_2} (1 - \bar{x}^2) - \frac{p + \frac{w_2}{1 + r}}{\frac{\gamma_2 Ew}{1 + r}} (1 - \bar{x}) \quad (3.9.15*)
\]

\[
L_2^d = \frac{1}{(1 + \alpha)} \frac{\gamma_1}{\gamma_2} (1 - \bar{x}) + \frac{p + \frac{w_2}{1 + \beta}}{\frac{\gamma_2 Ew}{1 + r}} \ln \bar{x} \quad (3.9.16*)
\]
Assuming \( Y_i = f(L_i) = AL_i^{1-\alpha} \) we can find \( w_i = Af'(L_i^e) = A(1-\alpha)L_i^{\alpha} \), solving which we can derive

\[
L_i = \left( \frac{A(1-\alpha)}{w_i} \right)^{1/\alpha}
\]

(3.9.10*)

\[
\bar{L} = \int_\tau^1 \left( x^2 \gamma_1 w^\gamma - c_i x \right) dx / x \gamma_2 w^\gamma + c_2
\]

\[
\bar{L} = L_i^s + L_2^s = \int_\tau^r x s^*(x) f(x) dx = \bar{L} = L_i^s + L_2^s = \int_\tau^1 \left( x \gamma_1 \frac{p + w_2}{\gamma_2 Ew + 1 + \beta} \right) dx
\]

\[
\bar{L} = \frac{1}{2} \gamma_2 (1-x^2) - \frac{p + w_2}{1 + \beta} \gamma_2 Ew (1-\bar{x}), \text{ where } \bar{L} = \left[ \frac{x^2 \gamma_1}{2 \gamma_2} - \frac{p + w_2}{1 + \beta} \gamma_2 Ew / (1+r) \right]^{1/\alpha}
\]

\[
L_i^s = \int_\tau^1 s^*(x) f(x) dx = \left[ \frac{\gamma_1 x}{\gamma_2 (1+\alpha)} - \frac{p + w_2}{1 + \beta} \gamma_2 Ew / (1+r) \right]^{1/\alpha} = L_i^s = \frac{1}{(1+\alpha) \gamma_2} \gamma_1 (1-\bar{x}) + \frac{p + w_2}{1 + \beta} \gamma_2 Ew / (1+r)
\]

\[
L_3 = \int_\tau^\gamma xh(0) f(x) dx = \frac{\gamma_0}{2} \left[ \bar{x}^2 - \bar{x}_1^2 \right].
\]

(3.9.17)

Thus, in addition to some interesting analytical results, using data on the educated labor force in employed in skilled and unskilled sectors, average costs of education, some estimates of discount rate, and estimates of wage differentials between skilled and unskilled workers, we can calibrate this model under various cost Specifications and ability density functions. We would like to do this calibration in our future research. We can also use this model to analyze the impact of various policy instruments on the size of the educated labor force and minimum threshold levels that we have used in Bhagawati-Srinivasan job ladder model.

A simple and quick estimate shows that under fixed market equilibrium, if the marginal productivity of education rises, along with probabilities of getting jobs in skilled sector, demand
for education rises, and the minimum threshold level falls. Current probabilities used in this
exercise are estimates of employment rates in industry, services, and agricultural sectors. These
generate the value of $\pi$ at 34 percent of the labor force, which coincides with the percent of the
labor force (15+) without any education in 2010. In the future, the plan is to elaborate this
exercise using data for other low-income countries and also to examine the forms of the
production and cost functions of education that explain the best.

**Possible Extension: Teachers’ Unions**

We are extending the model to allow for the presence of teachers’ unions. When powerful
teachers unions are present, they can bargain with the government to raise the cost of schooling
$w_2$ above the competitive wage rate in the rest of the economy (comprising industrial and
services sectors). In the flexible prices equilibrium, skilled workers draw straws for the
government teaching positions and the losers are forced into the competitive private sector labor
market. A higher teacher’s wage will tend to reduce the demand for schooling and teachers.
This increases the supply of private skilled production workers and drives down $w_1$. What
happens to $E_w$ is not entirely clear. The teacher’s wage being higher makes $E_w$ increase.
However, the probability of receiving the teacher’s wage falls and the production worker’s wage

---

12 Other interactions between unions and governments are possible to model, including the provision of political
support from the unions to selected parties or candidates. These can take the form of campaign contributions à la
Grossman and Helpman (1994), or other forms.
also falls. It seems possible that $E_w$ would fall, which would further discourage schooling investments.

Unions can be modeled as caring about the wages received by their members and about their power, which depends on their size. Consider the following objective function for a teachers’ union:

$$U(w_2, L_2; w_1) = (w_2 - w_1)^2 L_2^\rho.$$ 

Unions can be employment- or wage-oriented depending on the relative sizes of $\lambda$ and $\rho$. Given the union’s objective, it engages in a Nash bargain with the government, assumed here to be interested in maximizing an objective function that depends on national welfare (proxied by national product, taken as the sum of the products of the three sectors) and also on political contributions from the teachers’ unions. The solution then depends on the bargaining power of the union and the government. To compute the remainder of the model, one takes the subsistence and teacher wages as given, along with the ability distribution. Then one needs to find $\bar{x}, \bar{L}, L_2, w_1$ (the other labor variables would follow directly). This will require a simultaneous solution of the analogs of (3.9.2), (3.9.15), (3.9.16) (depending on whether and how you alter things as suggested above) and a labor demand function from the manufacturing sector, i.e., a version of (9.10) based, say, on a Cobb-Douglas production function. Note also that $E_w$ is a function of the probability of getting a teaching job, which equals $L_2 / \bar{L}$. It may be possible to reduce these four equation systems by algebraically solving out for one or more of the endogenous variables in terms of the others.
Appendix 3.2B. Some Major Derivations

Derivation of 3.10

In the job-ladder model, expected Wage of the educated worker is given by

\[
\bar{w} = \frac{\lambda_1 w L^E_1}{L^E} + \frac{\lambda_2 w \varepsilon \delta L^E}{L^E} + \frac{((1 - \varepsilon \delta)L^E - L^E_1)w}{L^E}.
\]  \hspace{1cm} (3.9a)

Substitution of \( \bar{w} \) in equation (3.4) \( \bar{w} = w + (1 - \gamma)c \) gives us

\[
y\lambda_1 w L^E_1 + y\lambda_2 w \varepsilon \delta L^E + y((1 - \varepsilon \delta)L^E - L^E_1)w = [w + (1 - \gamma)c]L^E
\]

\[
[w + (1 - \gamma)c]L^E - \gamma\lambda_2 w \varepsilon \delta L^E - y((1 - \varepsilon \delta)L^E)w = yw(\lambda_1 - 1)L^E_1
\]

\[
L^E = \frac{yw(\lambda_1 - 1)L^E_1}{[w + (1 - \gamma)c] - \gamma\lambda_2 w \varepsilon \delta - y((1 - \varepsilon \delta)w} \hspace{1cm} (3.10)
\]

Derivation of 3.7.12

Derivation of equation 3.7.12, equilibrium value of \( L^E^* \):

Substitution of \( t, \lambda c \) and \( \delta \) gives us the following expression:

\[
y(1 - t)c \lambda_1 w L^E_1 + \frac{y\lambda_2 w \varepsilon \delta L^E}{L^E} + y\frac{((1 - \varepsilon \delta)L^E - L^E_1)w}{L^E} = w + (1 - \gamma)c \frac{t_\lambda w L^E}{L^E} \]

\[
y\frac{(1 - t)c \lambda_1 w L^E_1}{L^E} + \frac{y\lambda_2 w \varepsilon \delta L^E}{L^E} + y\frac{((1 - \varepsilon \delta)L^E - L^E_1)w}{L^E} = w + (1 - \gamma)c \frac{t_\lambda w L^E}{L^E} \]

\[
[w + (1 - \gamma)c]L^E - \gamma\lambda_2 w \varepsilon \delta L^E - y((1 - \varepsilon \delta)L^E)w = y(1 - t)c \lambda_1 w L^E_1 - \gamma L^E^*w + (1 - \gamma)t_\lambda \lambda_1 w L^E / \delta
\]

\[
L^E^* = \frac{y(1 - t)c \lambda_1 w - \gamma w + (1 - \gamma)t_\lambda \lambda_1 w / \delta}{[w + (1 - \gamma)c] - \gamma\lambda_2 w \varepsilon \delta - y((1 - \varepsilon \delta)w}
\]

\[
L^E^* = \frac{y(1 - t)c \lambda_1 w - \gamma w + (1 - \gamma)t_\lambda \lambda_1 w / \delta}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}
\]

\[
L^E^* = \frac{y\lambda_1 w - \gamma t_\lambda \lambda_1 w - \gamma + (1 - \gamma)t_\lambda \lambda_1 w / \delta}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}
\]

\[
L^E^* = \frac{yw(\lambda_1 - 1) + t_\lambda \lambda_1 w (1 - \gamma) / \delta - \gamma}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}
\]

\[
L^E^* = \frac{yw(\lambda_1 - 1) + t_\lambda \lambda_1 w (1 - \gamma) / \delta - \gamma}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}
\]
Derivation of 3.7.22: Comparative Statics Results for Job Ladder Model with Public Financing

\[ L^* = \frac{\gamma w(\lambda_1 - 1) + t_1 \lambda_1 w((1-\gamma) / \delta - \gamma)}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta + k \lambda_2 w(1-\gamma - \gamma\delta)} \]

\[ dL^* = \frac{\gamma w(\lambda_1 - 1) + t_1 \lambda_1 w((1-\gamma) / \delta - \gamma)}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} dL^* 
+ \frac{(\gamma w)d\lambda_1 + t_1 w((1-\gamma) / \delta - \gamma)d\lambda_2}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* 
+ \frac{[\gamma w\epsilon\delta(1-\gamma)w_\epsilon d\lambda_2] d\lambda_2}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \]

\[ dL^* = \frac{L^*}{L^*} dL^* + \frac{1}{D} \left[ \lambda_1 (\gamma w) + \lambda_1 t_1 w((1-\gamma) / \delta - \gamma) \right] L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\lambda_2 \gamma w \epsilon \delta - \lambda_2 (1-\gamma)w_\epsilon]}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\gamma w \epsilon \delta(1-\gamma)w_\epsilon d\lambda_2] d\lambda_2}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 \]

\[ dL^* = \frac{L^*}{L^*} dL^* + \frac{1}{D} \left[ \lambda_1 (\gamma w) + \lambda_1 t_1 w((1-\gamma) / \delta - \gamma) \right] L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\lambda_2 \gamma w \epsilon \delta - \lambda_2 (1-\gamma)w_\epsilon]}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\gamma w \epsilon \delta(1-\gamma)w_\epsilon d\lambda_2] d\lambda_2}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 \]

\[ dL^* = \frac{L^*}{L^*} dL^* + \frac{1}{D} \left[ \lambda_1 (\gamma w) + \lambda_1 t_1 w((1-\gamma) / \delta - \gamma) \right] L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\lambda_2 \gamma w \epsilon \delta - \lambda_2 (1-\gamma)w_\epsilon]}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 
+ \frac{[\gamma w \epsilon \delta(1-\gamma)w_\epsilon d\lambda_2] d\lambda_2}{(1-\gamma)(w + c) - \gamma w(\lambda_2 - 1)\epsilon\delta} L^* \frac{1}{\lambda_1} d\lambda_1 \]

\[ 1 = \eta_{\epsilon_1} + \eta_{\lambda_1} + \frac{\gamma \lambda_1 w L^*}{D L^*} - \eta_{\lambda_2} + \frac{[\lambda_2 \gamma w \epsilon \delta - \lambda_2 (1-\gamma)w_\epsilon]}{D} \eta_{\lambda_2} \]

In a similar way we derive \( L^* \) under extended job-ladder models.
\[
\gamma \left(1 - t_r \right) \lambda w L^E_x + \gamma \frac{\lambda w (1 + k / \epsilon) \epsilon \delta L^E_x}{L^E_x^{-1}} + \gamma \left( (1 - \epsilon \delta) L^E_x - L^E_x^{-1} \right) w = w + (1 - \gamma)(c - \frac{t_r \lambda w L^E_x}{L^E_x} + k \lambda w)
\]

\[
\gamma \left(1 - t_r \right) \lambda w L^E_x + \gamma \frac{\lambda w \epsilon \delta L^E_x + k \lambda \epsilon \delta L^E_x}{L^E_x^{-1}} + \gamma \left( (1 - \epsilon \delta) L^E_x - L^E_x^{-1} \right) w = w + (1 - \gamma)(c - \frac{t_r \lambda w L^E_x}{L^E_x} + k \lambda w)
\]

\[
[w + (1 - \gamma)c + (1 - \gamma)k \lambda w] L^E_x - \gamma \lambda w (1 - \epsilon \delta) L^E_x - \gamma w (1 - \epsilon \delta) L^E_x = \gamma (1 - t_r) \lambda w L^E_x - \gamma L^E_x w + (1 - \gamma)t_r \lambda w L^E_x / \delta
\]

\[
L^E_x = \frac{\gamma (1 - t_r) \lambda w - \gamma w + (1 - \gamma)t_r \lambda w / \delta}{(1 - \gamma)(w + c) - \gamma w (\lambda - 1) \epsilon \delta + k \lambda w (1 - \gamma - \gamma \delta)} L^E_x
\]

\[
L^E_x = \frac{\gamma (1 - t_r) \lambda w - \gamma w + (1 - \gamma)t_r \lambda w / \delta}{(1 - \gamma)(w + c) - \gamma w (\lambda - 1) \epsilon \delta + k \lambda w (1 - \gamma - \gamma \delta)} L^E_x
\]

\[
L^E_x = \frac{\gamma \lambda w - \gamma \lambda w - \gamma w + (1 - \gamma)t_r \lambda w / \delta}{(1 - \gamma)(w + c) - \gamma w (\lambda - 1) \epsilon \delta + k \lambda w (1 - \gamma - \gamma \delta)} L^E_x
\]

\[
L^E_x = \frac{\gamma w (\lambda - 1) + t_r \lambda w ((1 - \gamma) / \delta - \gamma)}{(1 - \gamma)(w + c) - \gamma w (\lambda - 1) \epsilon \delta + k \lambda w (1 - \gamma - \gamma \delta)} L^E_x
\]

\[
dL^E_x \over d\lambda_t = [\left(1 - t_r \right) \eta + \left(1 + \frac{\gamma w}{N} \right)] \frac{L^E_x}{\lambda_t} \text{ Where, } \eta = \frac{h}{h} \lambda_t w
\]

\[
dL^E_x \over d\lambda_t = [\left(1 - t_r \right) \eta + \left(1 + \frac{\gamma w}{N} \right)] \frac{L^E_x}{\lambda_t} \text{ Where, } \eta = \frac{h}{h} \lambda_t w
\]

\[
N = \gamma w (\lambda - 1) + t_r \lambda w ((1 - \gamma) / \delta - \gamma)
\]

or

\[
\frac{dL^E_x}{d\lambda_t} = \left[ \eta + \frac{\gamma \lambda_t + t_r \lambda_t ((1 - \gamma) / \delta - \gamma)}{\gamma (\lambda - 1) + t_r \lambda_t ((1 - \gamma) / \delta - \gamma)} \right] \frac{L^E_x}{\lambda_t}
\]

\[
|\eta| \leq \frac{\gamma \lambda_t + t_r \lambda_t ((1 - \gamma) / \delta - \gamma)}{\gamma (\lambda - 1) + t_r \lambda_t ((1 - \gamma) / \delta - \gamma)} \frac{dL^E_x}{d\lambda_t} \geq 0
\]

When \( t_r = 0 \), education is not subsidised,

\[
\frac{dL^E_x}{d\lambda_t} = \left[ \eta + \frac{\lambda_t - 1}{\lambda_t} \right] \frac{L^E_x}{\lambda_t}
\]

When \( t_r = 0 \), i.e. education is not subsidized, \( \frac{dL^E_x}{d\lambda_t} = \left[ \eta + \frac{\lambda_t - 1}{\lambda_t - 1} \right] \frac{L^E_x}{\lambda_t - 1} \), the same as equation 3.17 in the original job ladder model in Section 3 of this chapter.
Derivation of 3.8.2.10

\[
L^{E*} = \frac{[(1-\gamma)t,\lambda \omega / \delta + \gamma (1-t)\lambda \omega - \gamma \omega] L^{E}}{[w+ (1-\gamma)c + (1-\gamma)k \lambda \omega - \gamma \omega w(1-\varepsilon \delta)]}
\]

\[
L^{E*} = \frac{[(1-\gamma)t,\lambda \omega / \delta + \gamma (1-t)\lambda \omega - \gamma \omega] L^{E}}{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}
\]

\[
L^{E*} = \frac{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}
\]

when \( t,\lambda \omega L^{E*} = k \lambda \omega \delta L^{E*} \)

\[
L^{E*} = \frac{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w] L^{E}}{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}
\]

\[
L^{E*} = \frac{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}{[(1-\gamma)(w+c) - \gamma w(\lambda - 1)\varepsilon \delta + (1-\gamma)k \lambda \omega w]}
\]

Other Derivations: Effects on Income Share

\[
\frac{d\alpha^M}{d\varepsilon} = \frac{w\alpha^M dL^{EM}}{DL^{EM} d\varepsilon} > 0
\]

\[
\alpha^M = \left( \frac{wL^{EM}}{wL^{EM} + w(1-(1+\delta)L^{EM})} \right)
\]

\[
\frac{d\alpha^M}{d\varepsilon} = \frac{w dL^{EM}}{D d\varepsilon} + \frac{d\alpha^M}{d\varepsilon} - \alpha^M \left( \frac{w dL^{EM}}{D d\varepsilon} + \frac{L^{EM} d\alpha^M}{D d\varepsilon} \right) - \alpha^M \left( \frac{w(1-(1+\delta)) dL^{EM}}{D d\varepsilon} \right)
\]

\[
\frac{d\alpha^M}{d\varepsilon} = \frac{w dL^{EM}}{D d\varepsilon} - \alpha^M \left( \frac{w dL^{EM}}{D d\varepsilon} \right) + \alpha^M \left( \frac{w(1+\delta) dL^{EM}}{D d\varepsilon} \right)
\]
\[
\frac{d\alpha^M}{d\lambda_2} = \frac{w\alpha^M}{DL^{EM}} \frac{dL^{EM}}{d\lambda_2} > 0
\]

\[
\alpha^M = \left( \frac{\bar{w}L^{EM}}{\bar{w}L^{EM} + w(1 - (1 + \delta)L^{EM})} \right)
\]

\[
\frac{d\alpha^M}{d\lambda_2} = \frac{\bar{w}}{D} \frac{dL^{EM}}{d\lambda_2} + \frac{L^{EM}}{D} \frac{d\bar{w}}{d\lambda_2} - \alpha^M \left[ \frac{\bar{w}}{D} \frac{dL^{EM}}{d\lambda_2} + \frac{L^{EM}}{D} \frac{d\bar{w}}{d\lambda_2} \right] + \alpha^M \frac{w(1 + \delta)}{D} \frac{dL^{EM}}{d\lambda_2}
\]

\[
\frac{d\alpha^M}{d\delta} = \frac{\bar{w}}{D} \frac{dL^{EM}}{d\lambda_2} - \alpha^M \left[ \frac{\bar{w}}{D} \frac{dL^{EM}}{d\lambda_2} + \frac{L^{EM}}{D} \frac{d\bar{w}}{d\lambda_2} \right] + \alpha^M \frac{w(1 + \delta)}{D} \frac{dL^{EM}}{d\lambda_2}
\]

\[
\frac{d\alpha^M}{d\lambda_2} = \left[ \frac{(1 - \alpha^M)\bar{w} + \alpha^M w(1 + \delta)}{\bar{w}L^{EM} + w(1 - (1 + \delta)L^{EM})} \right] \frac{dL^{EM}}{d\lambda_2} = \frac{(1 - \alpha^M)\bar{w} + \alpha^M w(1 + \delta)}{DL^{EM}} \frac{dL^{EM}}{d\lambda_2}
\]
\[
\frac{d \bar{w}}{d \lambda_2} = \frac{w \lambda_2 - w(\lambda_1 - 1)L_1^E}{L^E \frac{dL^{EM}}{d \lambda_2}}
\]
\[
\frac{d \bar{w}}{d \lambda_2} = \frac{w \lambda_2 - w(\lambda_1 - 1)L_1^E}{L^E} \cdot \frac{\gamma w \lambda_2}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}
\]

Thus, \(\frac{d \bar{w}}{d \lambda_2} = \frac{w \lambda_2}{w \lambda_2} - w \lambda_2 = 0\)

Where, \(\frac{dL^{EM}}{d \lambda_2} = \frac{\gamma w \lambda_2}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}\)

And \(L^E = \frac{\gamma w(\lambda_1 - 1)L_1^E}{(1 - \gamma)(w + c) - \gamma w(\lambda_1 - 1)\varepsilon \delta}\)

Derivation of \(\frac{d \alpha^M}{d \varepsilon} = \frac{\alpha^M \cdot dL^{EM*}}{DL^{EM*}} \cdot d \varepsilon > 0:\)

\(\frac{d \alpha^M}{d \varepsilon} = \frac{\alpha^M \cdot dL^{EM*}}{DL^{EM*}} \cdot d \varepsilon < 0\)

\[\alpha^M = \left(\frac{\bar{w}L^{EM*}}{\bar{w}L^{EM*} + w(1 - (1 + \delta)L^{EM*})}\right)\]

\[\frac{d \alpha^M}{d \varepsilon} = \frac{\bar{w} \cdot dL^{EM*}}{D} \cdot d \varepsilon + \frac{L^{EM*}}{D} \cdot \frac{d \bar{w}}{d \varepsilon} - \alpha^M \left[ \frac{\bar{w} \cdot dL^{EM*}}{D} \cdot d \varepsilon + \frac{L^{EM*}}{D} \cdot \frac{d \bar{w}}{d \varepsilon} \right] - \alpha^M \cdot \frac{w(1 - (1 + \delta)) \cdot dL^{EM}}{D} \cdot d \varepsilon\]

\[\frac{d \alpha^M}{d \varepsilon} = \frac{\bar{w} \cdot dL^{EM}}{D} \cdot d \varepsilon - \alpha^M \left[ \frac{\bar{w} \cdot dL^{EM}}{D} \cdot d \varepsilon \right] + \alpha^M \cdot \frac{w(1 + \delta) \cdot dL^{EM}}{D} \cdot d \varepsilon\]

\[\frac{d \alpha^M}{d \varepsilon} = \frac{(1 - \alpha^M) \bar{w} + \alpha^M \cdot w(1 + \delta) \cdot dL^{EM}}{\bar{w}L^{EM} + w(1 - (1 + \delta)L^{EM})} \cdot d \varepsilon = \frac{(1 - \alpha^M) \bar{w} + \alpha^M \cdot w(1 + \delta) \cdot dL^{EM}}{D} \cdot d \varepsilon = \frac{\alpha^M \cdot dL^{EM}}{DL^{EM} \cdot d \varepsilon}\]

Also
\(\alpha^M \bar{w}L^{EM} + \alpha^M \cdot w(1 - (1 + \delta)L^{EM}) = \bar{w}L^{EM}\)

\(\alpha^M \cdot w = [(1 - \alpha^M) \bar{w} + \alpha^M \cdot w(1 + \delta)]L^{EM}\)

By definition
\[\bar{w} = \frac{\lambda_1 w_1 - 1/L^E_1 + \lambda_2 w_2 \cdot \delta L^E}{L^E} \cdot \frac{(1 - \varepsilon \delta)L^E - L^E_1)w}{L^E}\]

\(\bar{w}\)
\[
\frac{d\bar{W}}{d\varepsilon} = (\lambda_2 - 1)w\delta - \frac{w(\lambda_1(1-t_1) - 1)L^\varepsilon_1}{L^\varepsilon} \gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w \frac{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta}{(1-\gamma)(w+c)}
\]

\[
\frac{d\bar{W}}{d\varepsilon} = (\lambda_2 - 1)w\delta - \frac{w[\lambda_1(1-t_1) - 1][\gamma(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2]}{\gamma(\lambda_1 - 1) + t_1\lambda_1((1-\gamma)/\delta - \gamma)}
\]

\[
\frac{d\bar{W}}{d\varepsilon} = (\lambda_2 - 1)w\delta - \frac{w[\lambda_1(1-t_1) - 1][\gamma(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2]}{\gamma(\lambda_1 - 1) + t_1\lambda_1((1-\gamma)/\delta - \gamma)}
\]

When \( t_1 = 0 \), \( \frac{d\bar{W}}{d\varepsilon} = (\lambda_2 - 1)w\delta - \frac{w[\gamma(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2]}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \)

\[ t_1 = \frac{\gamma(\lambda_2 - 1)\lambda_2\varepsilon\delta}{\lambda_1(1-\gamma + \gamma\varepsilon\delta) - \gamma(\lambda_1 - 1)\lambda_2\varepsilon\delta} \]

thus, \( \frac{d\bar{W}}{d\varepsilon} = (\lambda_2 - 1)w\delta - w(\lambda_2 - 1) = 0 \)

where, \( \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \leq 0 \)

and \( L^\varepsilon = \frac{\gamma w(\lambda_2 - 1) + t_1\lambda_1((1-\gamma)/\delta - \gamma)}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \)

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} L^\varepsilon \]

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \]

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \]

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta - (1-\gamma)\lambda_2w}{(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta} \frac{dt_1}{d\varepsilon} \]

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{\gamma w(\lambda_2 - 1)\delta L^\varepsilon - \lambda_1 w L^\varepsilon}{\delta L^\varepsilon} \frac{dt_1}{d\varepsilon} \]

\[ \frac{dL^\varepsilon}{d\varepsilon} = \frac{[(1-\gamma)(w+c) - \gamma w(\lambda_2 - 1)\varepsilon\delta]}{\delta L^\varepsilon} \geq 0 \]

\[ \frac{(\lambda_2 - 1)w\delta L^\varepsilon}{\lambda_1 w L^\varepsilon} \geq \frac{dt_1}{d\varepsilon} \text{ i.e., } \frac{dt_1}{d\varepsilon} \leq \frac{\sigma}{(\lambda_2 - 1)w} \]
Derivation of 3.9.16 and 3.9.17

\[
\bar{I} = \left[ \frac{2c_2(c_2y_1 + c_1y_2 \log(y_2w^r + c_2) - y_2w^r(2c_2y_1 + 2c_1y_2 - xy_1y_2w^r) - c_1x)}{2y_2^3 (w^r)^2} \right]_x
\]

\[
\bar{I} = \left[ \frac{2c_2(c_2y_1 + c_1y_2 \log(y_2w^r + c_2) - y_2w^r(2c_2y_1 + 2c_1y_2 - xy_1y_2w^r) - c_1)}{2y_2^3 (w^r)^2} \right]_x
\]

\[
- \frac{2c_2(c_2y_1 + c_1y_2 \log(\bar{x}y_2w_c + c_2) - \bar{x}y_2w_c(2c_2y_1 + 2c_1y_2 - \bar{x}y_1y_2w_c) - c_1)}{2y_2^3 (w_c)^2}
\]

\[
\bar{I} = \frac{2c_2(c_2y_1 + c_1y_2 \log((\gamma_2w_c + c_2) / (\bar{x}y_2w_c + c_2)))}{2y_2^3 w_c^2}
\]

\[
- \frac{\gamma_2w_c(2c_2y_1 + 2c_1y_2)(1 - \bar{x}) + \gamma_2^2 w_c^2(1 - \bar{x}^2) - c_1(1 - \bar{x})}{2y_2^3 w_c^2}
\]

\[
L^d_2 = \frac{1}{(1 + r)^2} \int \frac{(y_1w_c - c_1)}{y_2w_c + c_2} dx \left[ \frac{\gamma_1x}{\gamma_2(1 + r)} \right] \frac{(c_1y_2 + y_2c_2) \ln(x(y_2w_c + c_2))}{\gamma_2Ew} \right]_x
\]

and,

\[
L^d_2 = \left[ \frac{\gamma_1(1 - \bar{x})}{\gamma_2(1 + r)} \right] \frac{(c_1y_2 + y_2c_2) \ln((\gamma_2w_c + c_2) / (\gamma_2\bar{x}w_c + c_2))}{\gamma_2Ew}
\]
Chapter 4

Total Factor Productivity in Economic Growth: Evidence from a Panel Data Model for Africa and Asia

Abstract
This chapter estimates the effects of openness, trade orientation, human capital, and other factors on total factor productivity (TFP) and output for a pooled cross-section, time-series sample of countries from Africa and Asia, as well as for the two regions separately. The models are estimated for the level and growth of both TFP and output by using panel fixed effects. The generalized method of moments is also applied to address endogeneity issues. Several variables related to political, financial, and economic risks are used as instruments, together with the lagged values of the dependent and endogenous explanatory variables. The data for this study span 40 years (1972–2011) and are grouped into five-year averages. Several sources were used to obtain the most updated data, including the newly released Penn World Table (Version 8.0). The chapter finds that inducing a greater outward orientation generally boosts TFP, per capita output, and growth. Greater accumulation of human capital has a consistently positive effect on output and TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than interactively with openness. Furthermore, inflation does not negatively affect growth, although inflation variability is found to adversely affect TFP and output in Africa.
4.1. Introduction

Total factor productivity (TFP) accounts for a large fraction of output growth in those parts of the world that have experienced modest to high growth. The accumulation of the basic factors of production such as capital per worker contributes significantly to output growth as well. Yet, while factors such as the rate of savings are commonly understood to affect capital accumulation, there is much less consensus about the exact factors that influence TFP. This has led to a burst of economic research over the last 20 years into the causes underlying TFP growth.

Availability of a comparable set of cross-country income data, particularly the Penn World Table, has facilitated such cross-country growth exercises. Most such datasets have lacked capital stock data in the past for many developing countries. Consistent series for variables that seem closely related to TFP in theory have also been difficult to obtain for long enough time periods, particularly for developing countries. With the availability of Version 8.0 the Penn World Table (Feenstra, Inklaar, and Timmer, 2013), a more consistent and accurate capital stock series can now be used, as has been done in this chapter. To understand the importance of this new release of the Penn World Table, Feenstra, Inklaar, Timmer (2011) recount recent history in a working paper preceding the release of Version 8.0:

Many researchers have been faced with the absence of established and harmonized data on capital input and have had to estimate series based on the available total investment data [and mainly based on the perpetual inventory method] (see, e.g., Caselli, 2005). The capital data will distinguish between different assets to account for compositional changes over time and differences across countries. (pp. 30-31)

This chapter starts with a discussion of the main empirical literature on TFP as a base case and makes three specific improvements. First, many papers have studied TFP for large panels of countries (e.g., 83 countries in Miller and Upadhyay, 2000) or for large economies such as India and China (Collins and Bosworth, 2008). Most of the large studies of TFP include
developing and developed countries and countries from all geographic regions in the same pool. This chapter focuses on developing countries alone based on the belief that pooling developing and developed countries in a single dataset is equivalent to making an erroneous assumption that all countries share the same technology. 13 Second, the chapter extends other studies by adding observations and updating the dataset up to and including 2011, which gives a relatively long period of 40 years of data, which are grouped into eight periods of five years each for all countries in the sample. Third, the chapter makes a comparative assessment of the findings through the use of alternative methods, in particular panel fixed effects and the generalized method of moments, to examine whether endogeneity problems really mar the fixed-effects results.

Since growth of TFP cannot be directly observed, economists employ alternative ways to estimate its contribution to the overall growth of an economy. One approach first estimates aggregate output as a function of the basic factors of production, namely labor ($L$) and physical capital ($K$). After the contribution of these factors has been determined, the resulting unexplained part of output is TFP, which can be studied further by examining its determinants. Several papers get significant results for TFP based on a panel data model of a large sample of countries that includes human capital and outward orientation. Some studies find a significant role for interaction between human capital, on the one hand, and openness and domestic price divergence from world prices, on the other, as these factors affect TFP.

This chapter uses data for 20 countries—10 each from Africa and Asia—but as mentioned earlier, extends the empirical model of others by including more observations per country and more variables. The chapter examines the differences in growth rates between developing and developed countries and countries from all geographic regions in the same pool.

13 Miller and Upadhyay (2002) also find that the assumption of a common technology for countries at all stages of the development ladder is not verified. As explained later in this chapter, the inclusion of country fixed effects does not fully address the cross-country or cross-region differences in technology.
countries in Africa with those in Asia by estimating the contribution of TFP to economic growth and relating the TFP differences to their underlying causes. A focus on Africa can also help in understanding why this region has failed to move on the path of convergence in per capita income with the rest of the world.

The effect of openness and trade liberalization on economic growth remains a subject of debate. A greater body of empirical evidence, however, supports a positive correlation between openness and growth. Trade forces domestic producers to compete with the rest of the world. This induces the adoption of more efficient technology, which causes TFP to grow faster. Larger exports can relax the foreign exchange constraint by permitting imports of key intermediate and capital goods in production. Sachs and Warner (1995), Harrison (1996), Edwards (1998), Miller and Upadhyay (2000), and Warner (2003) are some examples of empirical studies that show a strong trade liberalization-growth nexus. In a different vein, Rodriguez and Rodrik (2000) present a fairly comprehensive discussion of the topic and claim that no convincing empirical evidence exists to prove a positive trade-growth relationship. Warner (2003), however, makes strenuous efforts to refute the claims of Rodriguez and Rodrik and reasserts the existence of a close direct relationship between growth and outward orientation.

This chapter first calculates a measure of TFP from an aggregate production function in which output depends on physical capital, human capital, and labor.\(^4\) It then explores factors that influence the level and growth of TFP, especially openness, trade orientation, terms of trade, and external indebtedness, among external factors, as well as education and health-related variables (such as human capital, life expectancy, and child mortality), inflation, a demographic

\(^4\) Mankiw, Romer, and Weil (1992) and Islam (1995) also include human capital in the set of inputs. In many other studies including Miller and Upadhyay (2000), however, human capital does not prove to be a significant variable influencing output. In our new and updated data, we again explore whether human capital also directly affects output. We find human capital has a greater indirect effect on output, i.e., as a factor underlying the growth of total factor productivity.
factor (the dependency ratio), and governance indicators such as corruption, government, accountability and stability. Results of this setup are then compared with results from an alternative but popular framework—growth accounting—following Bernanke and Gurkaynak (2001) and Crafts (1999), taking the share of physical capital in output, $\alpha$, to be 30 percent for all countries in the sample.

The chapter estimates models separately for Africa, Asia, and all countries in the sample from these two continents. The presumption that all countries in the sample employ the same overall technology and hence can be studied in terms of a single regression equation can be unrealistic. It does not reconcile with the wide disparities in per capita income and growth rates observed across countries. It is likely that much of the income difference has to do with the differences in the countries’ ability to efficiently utilize the technology that is already available. Geographical and cultural differences can also pose a major constraint on the rate at which countries adopt modern technology.\footnote{Several studies now emphasize the importance of such differences in growth. Landes (1998) and Sachs (2000) find culture as playing a large role in causing income disparity between nations, while Sachs (2001, abstract) explores the role of geography and highlights “the difficulty of applying temperate-zone technological advances in the tropical setting.”} This chapter examines, among other things, whether countries in different geographic regions exhibit similar output elasticities with respect to capital and labor.

The chapter estimates the effects of human capital, openness, and trade orientation, among other factors, on the level and growth of TFP and also the level and growth of output for a pooled cross-section, time-series sample of countries from Africa and Asia, as well as for the two regions separately. The chapter first estimates TFP for the pooled and regional samples based on a Cobb-Douglas production function involving output per worker, capital per worker, and by including and excluding human capital. Another TFP series is constructed following the
growth accounting approach by assuming the share of capital in output equals 0.3. The chapter then searches for the possible determinants of TFP and output, with special emphasis on variables reflecting trade orientation and human capital.

The models in the chapter for the level and growth of both TFP and output are estimated by using panel fixed effects and by applying generalized method of moments to address possible problems with endogenous explanatory variables. Several feasible instruments are identified, including those related to political, economic, and financial riskiness of a country, together with the lagged values of the dependent and endogenous explanatory variables. This procedure generates a large number of moment conditions relative to the parameters being estimated. These overidentifying restrictions can be tested through the Hansen $J$-statistics. The results of such an exercise are found here to be reasonably satisfactory and the used instruments fairly valid.

The main results show that inducing a greater outward orientation generally boosts TFP. The chapter captures outward orientation through share of exports or total trade in GDP, terms of trade, and volatility in openness and terms of trade. Most of the middle-income and low-income countries instituted trade policy reforms during the sample period to make their economies more open and raise productivity. The significance of a positive influence of outward orientation on growth of TFP has important policy implications.

Among variables other than those that measure outward orientation, the chapter finds that greater accumulation of human capital has a positive effect on TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than interactively with openness. Furthermore, inflation, another domestic economic variable, does not have a
significant negative effect on TFP in the full sample or subsamples, although inflation variability is found to adversely affect TFP and output in Africa.

Finally, the social development indicator used here, life expectancy, positively and significantly affects TFP in Africa. Most poor countries are expected to improve their life expectancy over time and yet many countries in sub-Saharan Africa experienced an absolute decline during portions of the sample period. The chapter finds strong evidence to suggest that a rise in this indicator will raise TFP in Africa.

With regard to the data underlying the empirical model, the availability of Penn World Table version 8.0 released in 2013 has been highly useful. Capital stock data were reintroduced in this version after having been dropped for several releases in the past. The human capital variable was also introduced in this version after incorporating in the estimation the varying rates of return on different levels of educational attainment.

This chapter also uses data from the Risk Guide (PRS Group, 2013) for various economic and financial riskiness indicators of countries along with several governance indicators such as corruption, accountability, and stability of the government. Several other economic and social variables from the World Bank’s 2013 World Development Indicators were also included in the dataset. All of this raises confidence in the reliability of the results presented here.
4.2. Macroeconomic Scenario

This section briefly describes medium-to-long-term macroeconomic trends for Africa and Asia from 1972 to 2011. Figure 4.1 shows selected macroeconomic indicators for 20 countries, 10 of which are in Africa and 10 in Asia. Data for 40 years are divided into eight periods of five years each and averaged out for those five-year periods. The average growth of real GDP for 1972–2011 in Africa was 3 percent per year with sharp fluctuations from one five-year period to another. The first half of the 1980s (1.1 percent) and first half of 1990s (2.1 percent) witnessed much slower growth for Africa. Growth recovered largely during 2002–06 (4.3) and then rose further in 2007–11 (5.4 percent).

During the 40-year period (1972–2011), the export-GDP ratio for Africa grew from 23 percent to 33 percent, first declining in 1980s, then rising steadily from the 1990s onward, from the 1992–96 average of 23 percent to the 2007–11 average of 33 percent. The current account deficit declined from 9 percent of GDP in 1972-76 to 5 percent in 1992–96, but rose again to 9 percent during 2007–11. Human capital, measured as a weighted index of average educational attainment of the labor force and returns to education, as reported in the new Penn World Table 8.0, increased from 1.4 in 1972–76, to 2.0 in 2007–11.

The averages, however, conceal the fact that there are considerable variations among the countries in the region. Ghana, for instance, witnessed a sustained growth of average real GDP of around 5 percent during 2002–06, and 8 percent during 2007–11, whereas for Congo DR, there was a steady decline during 1987–2001, only rising to a positive growth rate during 2002–

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16 For our econometric study of economic growth and growth of total factor productivity, we use a balanced panel of 20 countries and eight five-year periods from 1972–76 to 2007–11. This is to ensure comparability of data across countries.

17 Figures 4.1 and 4.2 are based on macro indicator tables that were compiled from the World Bank’s World Development Indicators (2013), and the IMF’s World Economic Outlook (2013), among other sources. They are available from the World Bank and IMF websites, as well as from the author upon request.
11. For South Africa, the average was less than 1 percent (0.6) during 1988-92. Figure 4.2 reflects the movements of real GDP growth, exports, and external sector indicators in the Africa region from 1972–2011.

Selected macroeconomic indicators for countries in Asia are shown in Figure 4.3. In contrast to the trends in Africa, the average macroeconomic indicators for Asia show a steady improvement. Real GDP grew on average at a rate of 5.9 percent during the 1972–2011 period, compared to 3 percent for Africa. Asian growth performance remained impressive from the long-term perspective despite the fact that growth slowed from 1997–2001 to 3.7 percent, mainly due to the Asian crisis. Human capital, measured as a composite index of average educational attainment of the labor force and returns to education, increased from 1.72 to 2.42 years between 1972–76 and 2007–11.

For Asian economies, the average indicators show a steady improvement in human capital accumulation (as for Africa, but at a more elevated level), openness, and real GDP growth. Figure 4.3 shows the movements in real GDP growth rates, the export share of GDP, and domestic and external indicators in the Asia region during 1972–2011. Countries from both the regions have made progress in key indicators over time, although the Asian record looks more impressive on real GDP growth, human capital, and openness. Both regions also suffered from the Latin debt crisis of the 1980s and the Asian financial crisis of the late 1990s, when growth slowed down considerably.
Figure 4.1 Real GDP Growth, Openness, and Education: 1972-2011

**Openness, Growth, and Human Capital**

*All Countries*

- Growth (%)
- Education (index)
- Openness (% of GDP, right scale)

**Openness, Growth, and Human Capital**

*Africa*

- Growth (%)
- Education (index)
- Openness (% of GDP, right scale)

**Openness, Growth, and Human Capital**

*Asia*

- Growth (%)
- Education (index)
- Openness (% of GDP, right scale)

Source: International Monetary Fund, World Economic Outlook, own calculations.
Figure 4.2 Africa: Selected Macroeconomic Indicators: 1972-2011

Real GDP Growth, Inflation, and Trade-to-GDP Ratio\(^1\)
(In percent)

Investment, Fiscal Balance, and Broad Money
(In percent of GDP)

FDI, Current Account Balance, and External Debt
(In percent of GDP)

Source: International Monetary Fund, World Economic Outlook, own calculations.
1/ adjusted the series for hyperinflation for Congo, Zimbabwe, and Uganda.
Figure 4.3 Asia: Selected Macroeconomic Indicators: 1972-2011

Real GDP Growth, Inflation, and Trade-to-GDP Ratio
(In percent)

Investment, Fiscal Balance, and Broad Money
(In percent of GDP)

FDI, Current Account Balance, and External Debt
(In percent of GDP)

Source: International Monetary Fund, World Economic Outlook, own calculations.
1/ Adjusted the series for hyperinflation periods.
4.3. Estimates of the Production Function and Total Factor Productivity

This section explains how TFP is estimated in this chapter. First, fixed-effects regressions are used to determine output elasticity with respect to capital and labor, and the share of capital in output observed in the data. Growth of TFP comes out as log differences in the estimated series for log-TFP. Van Beveren (2012) explores methodological issues in the estimation of TFP for firms and industries and finds that the fixed-effects estimator practically removes the endogeneity bias. There are several other econometric problems that need to be addressed when dealing with firm-specific production functions, including selection of inputs and price and quantity of inputs and outputs. These micro issues, which must be dealt with as one moves from firms to industries (such as output versus employment shares of firms as weights in the calculation of industry-level TFP), need not be of concern here in this macro study of TFP and growth.

Second, even at the national level, the possible endogeneity issue arises with respect to several variables, including the capital stock in the production function. The generalized method of moments (GMM) is used here to address this problem. The basic Arellano-Bond GMM approach is extended through the assumption that the first differences of the instruments used are uncorrelated with the fixed effects. This GMM approach creates a system of two equations—one for differences and one for levels—and thus allows for introducing more instruments, thereby raising efficiency of the estimation significantly.

Third, following Bernanke and Gurkaynak (2001) and Crafts (1999), TFP growth is estimated on the assumption that the share of capital in output is 0.3 and this share is used in a growth accounting framework. This yields another TFP series (and its growth) that is further
studied here in terms of the possible factors that influence it, and depending on the results, seek to derive some implications for policy.

Researchers recently have cast doubts on the robustness of any relationships between economic growth and its explanatory factors. Robustness has been an issue since the influential paper by Levine and Renelt (1992) that advocated extreme bounds test to identify robust variables, followed by Sala-i-Martin (1997). Since many candidate variables emerged over time as affecting growth, some authors have favored Bayesian model averaging to finding robust variables (e.g., Fernandez, Ley, and Steel, 2001; Sala-i-Martin et al., 2004; and Danquah, Moral-Benito, and Ouattara, 2011). The method starts by assuming model uncertainty, i.e., which exact variables belong in the model is not clear. Hence it first assigns \textit{a priori} probabilities to alternative sets of candidate variables and then uses data to update them later in terms of posterior probabilities.

Ciccone and Jarocinski (2010), however, claim that this whole approach is dubious. Their analysis shows that continuous changes in income data and other statistics for countries around the world seem to make a given set of income determinants shift from being robust with one dataset to being fragile with another, and vice versa. Ciccone and Jarocinski (2010) show examples of changing strength of certain variables in affecting income with each updating of Penn World Table data, such as from version 6.0 to 6.1 and to 6.2. Their conclusion is that having agnostic priors about the importance of variables in the face of significant margins of measurement errors may not be sensible in the context of studies using Bayesian learning averages.

Rather than compare the results across different updates of income data, this chapter focuses on the most recent and fully and accurately updated version (8.0) of the Penn World
Tables. While recognizing that measurement problems still exist, and that methodological concerns remain, we have tried to provide a relatively balanced comparison of results from several econometric methods in this chapter.

4.3.1. Fixed-Effects Regressions

This section starts by estimating a simple Cobb-Douglas production function, which is commonly expressed as follows:

\[ Y = A K^{\alpha} L^{\beta}, \]  
\[ 0 < \alpha < 1 \text{ and } 0 < \beta < 1, \]  
(4.1)

where \( Y \) is real GDP, \( K \) is the total physical capital stock, \( L \) is the number of workers (labor force), and \( A \) is an index of TFP. (The exponents \( \alpha \) and \( \beta \) may or may not sum to 1 depending on whether or not the function displays constant returns to scale.) Dividing equation 4.1 by the labor force \( L \) yields output per worker and physical capital per worker. The resulting equation is:

\[ y = A k^{\alpha} L^{\alpha+\beta-1}, \]  
(4.2)

where \( y \) is real GDP per worker, and \( k \) is the per worker stock of physical capital. This production function exhibits increasing, constant, or decreasing returns to scale as \( \alpha + \beta \) and is greater than, equal to, or less than one, respectively.

Rewriting equation 4.2 in natural logarithms yields the following:
\[ \ln y = \ln A + \alpha \ln k + (\alpha + \beta - 1) \ln L. \]  

(4.3)

Thus, tests for constant returns to scale imply testing whether the coefficient of \( \ln L \) equals zero.

It has become popular to use human capital in growth regressions. However, a large set of models that include the role of human capital has not settled the question of whether it is best to treat it as a basic input in the production function or as a factor influencing TFP. Mankiw, Romer, and Weil (1992) and many others advocate the use of human capital growth in the output growth regression. Islam (1995) and several other researchers, particularly using panel regressions, find that human capital does not contribute significantly to explaining output in a direct way but it rather affects output through TFP.

The extension of equations 4.1–4.3 for human capital is straightforward, as shown in equations 4.4–4.6:

\[
Y = AK^\alpha L^\beta H^\gamma \quad (4.4)
\]

\[
y = Ak^\alpha h^\beta L^{\alpha + \beta + \gamma - 1} \quad (4.5)
\]

\[
\ln y = \ln A + \alpha \ln k + \gamma \ln h + (\alpha + \beta + \gamma - 1) \ln L \quad (4.6)
\]

Both specifications are used, and mixed results for human capital are found. Following the literature that tends to emphasize human capital having a primary role in TFP, and as explained in a subsequent section, human capital is also used here as a possibly important factor in TFP.
This chapter takes advantage of the new version (8.0) of the Penn World Table that includes data on human capital. Most of the empirical growth papers in the past have used proxies for human capital in terms of years of schooling among the adult population, or school enrollment rates. The new series, on the other hand, is based on the Mincerian rates of return estimated originally for all countries by Psacharopoulos (1994) and also in terms of the average years of schooling. The returns vary across levels of education. Thus, primary, secondary, and tertiary education each has its own return, with return on primary education exceeding that on secondary, and return on secondary exceeding that on tertiary education. The idea was also proposed by Hall and Jones (1999) and Caselli (2005), subsequently incorporated in the Barro-Lee education dataset, and now integrated in PWT8.0. The base of such measurement is still the average years of schooling in the adult population, but each level of education has a different Mincerian-type rate of return. Based on a cross-country Mincerian regression, Penn World Table 8.0 uses the following rates of return: for up to four years of education, the return is 0.134 per year, for the fifth through the eighth year of schooling, the return is 0.101 per year, and for the ninth year onward the rate is 0.068 per year. This gives a function, which translates years of schooling (s) into another number. For the average worker in a country, then, human capital equals the exponentiation of this number, as given by the function. This approach to measuring human capital seems more scientific than just the years of schooling that has been a popular method to date. All our results for human capital refer to this new concept of human capital.  

To estimate the production function, a panel data set is used for the period from 1972-2011 for a sample of 20 countries. There are 10 countries from sub-Saharan Africa and 10 countries from developing Asia. The countries are selected from well-defined regional country

\[18\] Please see Inklaar and Timmer (2013) for further details and illustrative examples.
groups that are used for economic analysis in the World Economic Outlook database of the International Monetary Fund (IMF). More data could have been included if it were desirable to use the sample size as the main criterion for country selection regardless of data sources or quality.

This chapter uses the most recent version (8.0) of the Penn World Table for all the variables for which the data were available from this source. For other variables, the chapter relies on the IMF’s highly comprehensive World Economic Outlook database, the World Bank’s World Development Indicators (2013), and the most recent International Country Risk Guide published by the PRS Group. The sample size can obviously be expanded in the future depending on data availability for earlier and later periods and for more countries from Africa and Asia.

The chapter uses panel data, constructed as five-year averages of variables for the following eight periods: 1972–76, 1977–81, 1982–86, 1987–91, 1992–96, 1997–2001, 2002–06, and 2007–11. Output is measured as real GDP per worker (2005 international prices) averaged over five-year blocks. Real GDP per capita, real GDP per worker, and population provide the data on the labor force, which are also averaged over five-year blocks. Data on physical capital stock per worker were based on the capital-output ratios assembled by the World Bank based on an earlier version of the Penn World Table.

Choice of the sample size was mainly constrained by data availability for all the major variables used (from PWT8.0, WDI, and PRS Group). With the availability of the Penn World Table version 8.0, a more consistent and accurate capital stock series can now be used. Capital stock data were reintroduced in this PWT8.0 version after having been dropped for several releases in the past. The human capital variable was also introduced in this version after incorporating in the estimation the varying rates of return on different levels of educational attainment. However, only 11 developing Asian countries were found to have data on all the major variables (income, employment, capital stock, human capital, etc.) for the entire 1972-2011 period without any gaps. Out of the 11, 10 countries were chosen for which policy variables and other WDI variables were available for the entire period. For Africa, a total of 16 countries had PWT variables for the entire 40-year period, yet the limited availability of PRS social variables and WDI variables constrained the choice to a smaller cross section of countries. Of course, selection of a shorter time span, such as if the sample period began in the 1980s instead of 1970s, might have increased the sample size as well. Under the circumstances, for our time period 1972-2011, 10 countries from each of the two regions provided a comparable sample for our growth analysis.
The data used in this model have 180 observations (20 countries and eight time periods). The estimated production function includes random errors to equations 4.3 and 4.6, which incorporate the effects of omitted variables. When using panel data, the omitted variables can be classified into three groups: country-varying time-invariant, time-varying country-invariant, and country-varying and time-varying variables. To find accurate estimates of the contribution of capital and labor to output, a panel model is used to control for institutional and other country-specific characteristics that may also have an effect on output. The estimation of production functions in the model thus follows the country fixed-effects approach. An alternative random-effects estimation, on the other hand, requires that the omitted variables be uncorrelated with the included right-hand-side variables, which would not be realistic in the context of the model used here.

The panel results of the production-function estimates are very similar between the fixed- and random-effects estimators. While random effects slightly outperform the fixed effects because of their efficiency, the theoretical assumption here is that the country-specific characteristics that are controlled for in the estimation are closely related to the included right-hand-side variables. Thus, particularly in the context of a growth model, it is reasonable to stay with the fixed-effects approach. It was determined by using the Hausman test that the coefficients are very similar, and hence the TFP estimates are not very different between the two methods.

---

20 For more details about panel estimation, see Hsiao (1986) and Greene (2007).
Finally, time-specific dummy variables are also considered in order to see if there is any similarity of effects across countries in any given period. The inclusion of time dummies would modify the estimated equation as follows:\(^{21}\)

\[
\ln y = \ln A + \alpha \ln k + (\alpha + \beta - 1) \ln L + \sum_{i=1}^{9} \theta_i \text{time}_i + \varepsilon
\]  

(4.7)

where \(\text{time}_i\) \((i = 1, \ldots, 8)\) represents the time dummy variables, whereas the variables for each country measure deviations from their country means over time. The country-specific fixed effects \((cintj)\) are then calculated as given below:

\[
cfix_j = \bar{\ln y}_j - \hat{\alpha} \bar{\ln k}_j - (\hat{\alpha} + \hat{\beta} - 1) \bar{\ln L}_j
\]

(4.8)

or,

\[
cfix_j = \bar{\ln y}_j - \hat{\alpha} \bar{\ln k}_j - \hat{\gamma} \bar{\ln h}_j - (\hat{\alpha} + \hat{\beta} + \hat{\gamma} - 1) \bar{\ln L}_j
\]

(4.9)

where a bar over a variable indicates the mean of that variable, a caret over a parameter indicates the estimate of that parameter, \(\delta = (\alpha + \beta - 1)\), or \(\delta = (\alpha + \beta + \gamma - 1)\), and \(j = \{1,2,3,\ldots,20\}\) is the index across countries. The time-specific fixed effects in this setup, with the direct inclusion of the time dummies, would appear directly as the respective coefficients of these dummy variables.

Table 4.1 reports the estimates of equation 4.6. Column 2 gives the estimates for all countries. The coefficient \(\ln k\) is the elasticity of output with respect to the physical capital stock and equals 0.45. The production-function specification for “all” countries assumes that countries in Africa and Asia follow the same technology, except to the extent that technological differences can be captured by the country-specific effects (i.e., fixed effects in the model here).

---

\(^{21}\) The estimation of an aggregate production function confronts the researcher with numerous problems. One major concern is the possible endogeneity of physical and human capital, since these factors are accumulated over time. Benhabib and Spiegel (1994) examine this issue and conclude that the coefficients of physical and human capital probably overestimate their effects, while the coefficient of labor probably understates its effect. These potential biases should be kept in mind when interpreting the findings here.
Technology can also affect the quality of physical and human capital and if so the elasticity of output with respect to these factors can vary across countries or regions. A cursory look around the two continents, however, indicates that the representative technologies in the two regions may be different. A more recent argument also emphasizes the role of geography in growth and development.\(^{22}\) Columns three and four in Table 4.1 report the production-function estimates for Africa and Asia separately. They show that the output elasticity with respect to physical capital—which under marginal product pricing of factors measures the share of capital in output—equals 0.25 for Africa and 0.44 for Asia.

### Table 4.1. Panel Fixed-Effects Estimates of the Production Function with Human Capital

*(Dependent Variable: Logs of PPP-adjusted Output per Worker (log \(y_t\))*

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>4.8551***</td>
<td>6.3083***</td>
<td>4.2507***</td>
</tr>
<tr>
<td></td>
<td>(0.285)</td>
<td>(0.588)</td>
<td>(0.518)</td>
</tr>
<tr>
<td>log (k_t)</td>
<td>0.4504***</td>
<td>0.2510***</td>
<td>0.4441***</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.056)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>log (L_t)</td>
<td>-0.3034***</td>
<td>-0.4078***</td>
<td>0.1046</td>
</tr>
<tr>
<td></td>
<td>(0.092)</td>
<td>(0.135)</td>
<td>(0.251)</td>
</tr>
<tr>
<td>log (h_t)</td>
<td>0.5206*</td>
<td>0.6060*</td>
<td>-0.0939</td>
</tr>
<tr>
<td></td>
<td>(0.286)</td>
<td>(0.331)</td>
<td>(0.423)</td>
</tr>
<tr>
<td>(\bar{R}^2)</td>
<td>0.939</td>
<td>0.927</td>
<td>0.940</td>
</tr>
<tr>
<td>S.E.E.</td>
<td>0.210</td>
<td>0.247</td>
<td>0.149</td>
</tr>
</tbody>
</table>

Notes: Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

Thus, the elasticities of output with respect to capital stock are found to be different between the two regions. In response to a 1 percent change in capital stock per person, output

\(^{22}\) See Sachs (2001) and the large literature subsequently spawned by that study.
changes by 0.25 percent in Africa and 0.44 percent in Asia. This makes it hard to conclude that output is subject to greater diminishing returns to capital in Asia than in Africa. It was expected that the larger capital accumulation in Asia would have led to a slower rate of output response in Asia than in Africa, but the findings contradict this hypothesis and indicate that a rather opposite conclusion is warranted. If these results are confirmed by more studies, they would probably support the hypothesis that greater capital accumulation would be associated with faster output growth, particularly when physical capital is accompanied by growing human capital as well.

The contribution of human capital to output is, however, not very efficiently measured according to the results here. For the pooled sample, and for Africa, the coefficient of human capital comes out significant at the 10 percent level, while it is not at all significant for Asia. While the effects are suspect, for the entire sample (0.52) and for Africa (0.61), the elasticity of output with respect to human capital is even higher than with respect to physical capital. It should be noted that the way human capital is measured in Penn World Table 8.0 reflects different rates of return for different levels of schooling. Primary schooling has the highest rate of return, followed by secondary and then by tertiary schooling. This measure captures the notion of human capital better than does the secondary enrollments used in older studies such as Mankiw, Romer, and Weil (1992), Islam (1995), and many others. And yet, because it excludes health status of the population and the effect of on-the-job training, the new measure cannot fully capture the broad concept of human capital.

Table 4.2 shows the results of the production-function estimates without human capital. Strikingly, the share of (physical) capital stays almost identical to its share in the human-capital-inclusive specification. For all countries, the capital’s coefficient is 0.47, and for Africa and Asia it is 0.25 and 0.44, respectively. One of the goals of this study is to explore the impact of human capital on output either directly or through TFP. Because of poor statistical significance of
human capital’s direct effect on output, we will rely more on its indirect (TFP) effect, as discussed in the next section.

**Table 4.2. Panel Fixed-Effects Estimates of the Production Function without Human Capital**

*(Dependent Variable: Logs of PPP-adjusted Output per Worker (log yt))*

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>4.5335***</td>
<td>6.2744***</td>
<td>4.3398***</td>
</tr>
<tr>
<td></td>
<td>(0.201)</td>
<td>(0.568)</td>
<td>(0.212)</td>
</tr>
<tr>
<td><strong>log k</strong></td>
<td>0.4675***</td>
<td>0.2514***</td>
<td>0.4414***</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.057)</td>
<td>(0.045)</td>
</tr>
<tr>
<td><strong>log L</strong></td>
<td>-0.1279***</td>
<td>0.2110***</td>
<td>0.0702</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.080)</td>
<td>(0.113)</td>
</tr>
<tr>
<td><strong>$R^2$</strong></td>
<td>0.939</td>
<td>0.928</td>
<td>0.941</td>
</tr>
<tr>
<td><strong>S.E.E.</strong></td>
<td>0.213</td>
<td>0.246</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>N = 160</td>
<td>N = 80</td>
<td>N = 80</td>
</tr>
<tr>
<td>No. of countries</td>
<td>= 20</td>
<td>= 10</td>
<td>= 10</td>
</tr>
</tbody>
</table>

Notes: Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

Why is there a huge difference in the size of the effect of physical capital on output between Africa and Asia? Neoclassical theory generally predicts that countries with greater capital accumulation are likely to have declining marginal productivity of capital. Africa’s accumulation has been much slower over the last 40 years than that in Asia. However, the neoclassical prediction is only true holding other factors constant, particularly supplementary inputs such as human capital and institutional factors. The lower output elasticity of capital for Africa must thus be explained in terms of changes in these other factors. Starting at a lower base, human capital has, for instance, expanded rapidly in Africa since the 1970s, when the data used here begin. A greater role for explaining the differential impact of physical capital accumulation,
therefore, lies in other factors that are investigated in Section 4.4. Since it cannot be assumed that countries will share the same technology across geographical regions, the reasons for productivity movements over time are studied for each region as well as for the entire sample of countries in the dataset. Even if one accepts that technological differences are partly captured by the country fixed effects, which include many other unobserved but important differences across countries, technology can also affect the quality of physical and human capital and, if so, the elasticity of output with respect to these factors can vary across countries or regions.

One of the two main series of estimated TFP results from the production-function estimates reported in Table 4.2. Another series is also estimated, under the assumption that \( \alpha \), the share of physical capital in output per worker, equals 0.3, a standard assumption in the literature.\(^{23}\) This allows a comparison of the results from regressions of TFP where TFP is generated from the parametric (production-function-based) approach and a non-parametric (also called the growth-accounting) approach with an identical \( \alpha \) for all countries. A second important point is that the parametric TFP series includes not only the residual of output after subtracting the share-weighted growth of capital stock, but also includes country-specific “fixed” effects. Growth-accounting-based TFP reflects no adjustment of this kind. Since it cannot be assumed that countries will share the same technology across geographical regions, the reasons for productivity movements over time are studied for each region as well as for the entire sample countries in the dataset.

\(^{23}\) See Bernanke and Gurkaynak (2001) and Crafts (1999), among others.
4.3.2. Growth Accounting

The growth-accounting approach to estimating TFP assumes that the share of capital is set by past estimates of the output going to the owners of capital. Studies on developing and developed countries usually make different estimates of the capital income share to calculate TFP. For instance, while Collins and Bosworth (1996) find a 0.3–0.4 share to be reasonable for all countries pooled together, Kim and Lau (1994) find a 0.4–0.5 share to be appropriate for the East Asian “tigers” and a share of around 0.3 appropriate for the developed world. On the other hand, in Harrison’s (1996) estimates the capital income share varies between 0.41 and 0.63 depending on a country’s openness. Bernanke and Gurkaynak (2001) and Crafts (1999) also use 0.3 as the share of capital in income. We follow this literature by assuming a 30 percent share of capital for all countries in our data. TFP growth per worker then equals output growth per worker minus the share-weighted growth of capital per worker. According to this popular and simple accounting method, growth of TFP on average was 0.22 percent for all countries. TFP growth contributed 23 percent to income growth during the sample period. For Africa and Asia, the figures are starkly different. Asia marched on with a 38 percent contribution of TFP, whereas for Africa the contribution of TFP is negative. While Asia saw annual 2.3 percent growth of output per worker for the entire period from 1972–76 to 2007–11, output per worker actually fell in Africa at an annual average rate of 0.30 percent. TFP growth in Africa was even more negative at 0.43 percent. The simple correlation between the average growth rates of output and TFP is 0.88. The overall worst performer in the growth of TFP (output) is Congo (Congo), followed by Bangladesh (Zambia) and then Zambia (Kenya). The best performers are all concentrated in Asia. The best in TFP growth are China, Thailand, and India, whereas the best in output growth are China, Vietnam, and Thailand. Note that Korea, Taiwan, Singapore, and Hong Kong do not belong in the sample because this study is limited to low- to middle-income countries from Asia.
and Africa. The income-TFP relationship would change somewhat if the focus were on the last two decades, when some countries in Africa picked up both income and TFP growth, which restored their growth rates in the positive territory.

It is interesting to note that the Spearman rank correlation between regression-based TFP growth and growth-accounting-based TFP growth is 0.93, which strongly rejects the null of independence of the two series (with a p-value of 0.000). This seems to indicate that studies of TFP based on the production-function approach and a growth accounting framework could yield similar policy implications. Yet the levels of TFP according to the two methods are very different, with growth accounting TFP series on average standing 3.3 times the parametric TFP series, even though their growth rates are similar. The sections which follow examine TFP estimated both ways.

4.4. Determinants of Total Factor Productivity

This section studies the role of economic and other factors that influence changes in TFP. It uses three different approaches to study the level or growth of TFP. The first looks at estimates from the panel fixed effects. The second and third methods are based on panel GMM estimations. The third approach uses system GMM procedure on the TFP series estimated from the growth accounting approach. In contrast, the second approach examines the determinants of TFP indirectly through output growth regressions (rather than those for TFP growth) per worker, though these regressions also use the system GMM framework. In that framework any factor other than a direct input in production could be said to affect output through TFP.
4.4.1. Fixed-Effects Estimates

4.4.1.1. The Framework

To study the level and growth of TFP, this chapter considers domestic variables such as stock of human capital, inflation, life expectancy, and child mortality, as well as external variables, such as openness, terms of trade, and external debt for their influence in TFP.

The estimating equation takes the following form:

$$\ln \text{tfp} = a_1 + a_2 \ln \text{hucap} + a_3 \ln \text{open} + a_4 \ln (1+\pi) + a_5 \ln \text{trmstrd} + a_6 \ln \sigma_{	ext{open}}$$
$$+ a_7 \ln \sigma_{n} + a_8 \ln \sigma_{\text{tot}} + a_9 \text{lifexp} + a_{10} \text{childmort} + a_{11} \text{debtgdp}$$
$$+ a_{12} \text{corruptn} + a_{13} \text{gvtstabil} + \varepsilon$$

(4.10)

where economic variables include $\text{hucap}$, the stock of human capital in the economy; $\text{open}$, the degree of economic openness, $\pi$, the rate of inflation (where the number 1 is added to address the problem arising from logarithms of negative inflation), $\text{trmstrd}$, the terms of trade as measured by the export-to-import price index, and $\sigma_i$ equals the standard deviation of $i$ (= $\text{open}$, 1+$\pi$, and $\text{trmstrd}$) over the five-year subperiods. The equation also includes some socio-political and other indicators of an economy such as life expectancy ($\text{lifexp}$), child mortality ($\text{childmort}$), corruption ($\text{corruptn}$), and government stability ($\text{gvtstabil}$).

Variable definitions: The measurement of several variables in the dataset deserves mention. The output, labor force, and capital stock that underlie the estimates of TFP come from the Penn World Table 8.0. The same is true of human capital, for which the calculation is based on Barro and Lee (2013) data on education attainment and on Psacharopoulos (1994) rate of return data on levels of schooling (primary, secondary, and tertiary). The human capital variable
thus follows the Hall and Jones (1999) weighted average of years of schooling, weights being the estimates of Mincerian returns to educations for various years of schooling.

Openness is measured by either the ratio of total trade to GDP or the ratio of exports to GDP available from Penn World Table 8.0 and the IMF’s World Economic Outlook (WEO) database. Inflation is based on either the consumer price index (World Bank, World Development Indicators 2013) or the GDP deflator (IMF/WEO). The export and import price indices, whose ratio defines the terms of trade, are derived from the IMF’s International Financial Statistics 2013. The lifexp, childmort, and dpndncy are, respectively, life expectancy at birth, the mortality rate for children under five, and the ratio of the population under 15 and over 65 years old to the rest of the population, all extracted from the World Development Indicators.

Finally, selected political variables from the International Country Risk Guide are used. The variable govtstabil measures a government’s ability to stay in office depending on factors such as the type of governance, cohesion of the government and governing parties, approach toward an election, and command of the legislature. Similarly corruptn measures an index of perceived corruption in the government, and demoact indicates not just whether elections are free and fair but how accountable the government is toward its people, comporisk is a composite political, financial, and economic risk rating ranging from very high risk (high negative values) to very low risk (low negative values), i.e., the higher the value the lower the risk.

Furthermore, extlconflict is used to indicate risk to the incumbent government and to inward investment, ranging from trade restrictions and embargoes to geopolitical disputes, armed threats, border incursions, foreign-supported insurgency, and full-scale warfare. Finally, the measure of ethnictension indicates the degree of tension attributable to racial, national, or language divisions, with lower ratings showing higher risk.
The last four variables—demoact, comporisk, extlconflict, and ethnictension—are used as instruments for some of the endogenous variables in the GMM estimations of the economic model used here. The list of variables and definitions are given in Appendix 4.1.

While the relationship between human capital and output was not found to be highly significant in the last section, human capital could be related significantly with TFP, an issue explored in this section. In a comprehensive theoretical examination of the relationship between human capital and technology, Galor (2005) employs a model that predicts a positive relation between technological progress and schooling in equilibrium. Tamura (2006) shows that TFP growth is positively related to human capital accumulation, which in turn is helped by declining young adult mortality. This section also considers under-five child mortality as having an indirect effect on TFP. A similar consideration can lead one to examine a chain of relationships from child mortality reduction to higher life expectancy, a decrease in fertility, human capital accumulation, and TFP growth.

Among the more recent studies in favor of including life expectancy in a growth model, Acemoglu and Johnson (2007) study the effect of life expectancy in a cross-section analysis for two years: 1940, when drugs for many diseases that affected a large segment of population, such as malaria and cholera, had not been discovered; and 1980, when those drugs had become available even to people from very poor countries. Acemoglu and Johnson (2007) find a negative effect of a rise in life expectancy on income. Control of many diseases led to higher population growth through increased life expectancy and, in line with the neoclassical growth theory, exerted a negative impact on per capita income. More recently, Cervellati and Sunde (2011a, b) have shown using the demographic transitional theory that the relationship between life expectancy and growth should be modified to highlight two distinct phenomena in data. In the
“pre” regime of Acemoglu and Johnson (2007), i.e., 1940, the effect of life expectancy on growth is consistently negative, whereas for countries in the “post” regime it is consistently positive, as the mortality drop caused a sufficient drop later in fertility. The sample used here begins in 1972-76, a period when the disease-preventing drugs had become widely available in most countries, though not necessarily to entire populations in some developing countries. This calls for a relatively monotonic relationship between life expectancy and development, which is assumed in this chapter. To address discrepancies in the mortality rates across developing countries, however, we also control for the under-5 mortality in the model, as noted above.

External factors are claimed in the literature to have a substantial impact on TFP and income growth. The ways this can occur include, among other factors, improving terms of trade, growth in the global economy, better business conditions that attract foreign direct investment (FDI), and access to foreign capital at low interest rates. Delpachitra and Van Dai (2012) find that trade significantly affects TFP growth but that no such relationship of TFP exists with either human capital or FDI. This contrasts with the results in Driffield and Jones (2013), who show that FDI acts as an important factor in increasing productivity and output in developing economies. An attempt is made in this chapter to study the relationships between human capital, trade and FDI in the 20 countries in the sample.

Among other factors, macroeconomic stability, as measured by inflation and inflation volatility, could be important in influencing TFP and output. Governance indicators such as stability of governments and external debt levels within reasonable bounds can point to regime stability as well.

Equation 4.10 is estimated using a fixed-effects method where each variable in each country is measured as a deviation from its mean over time. Tables 4.3a and 4.3b report the results of the estimates of equation 4.10 where the series of TFP per worker is calculated after
the production-function regression as shown in Table 4.2. On the other hand, Table 4.4 shows the results of the TFP regression where TFP is the “residual” of output per worker estimated according to the growth accounting framework where the share of physical capital equals 0.3. Three main results, one for the pooled sample and two for the separate continents of Africa and Asia, are shown in each of those tables. Different specifications of the model have been estimated, including interaction terms between human capital, openness and FDI, interaction between other factors, and the time dummies.

The results for the growth of TFP on the growth of the same set of variables will be discussed later in the chapter. Finally, we repeat our exercise for output per worker and output growth per worker to check if a direct estimate of output growth produces more sensible results. All these results appear in Tables 4.3 through 4.8.

### 4.4.1.2. Fixed-Effects Total Factor Productivity Results for All Countries

The results for \( \log TFP \) appear in Tables 4.3a and 4.3b. Table 4.3a shows a common specification for the full sample (designated as “All”), Africa, and Asia, while Table 4.3b makes a slight variation in the model for different regions. In all sets of results, the main focus remains on human capital, though attention is also given to one or more of the external factors and socio-political variables. The results show that human capital has a positive relationship with TFP and for the full sample has a coefficient of 0.10, which is significant at the 10 percent level, and for Africa and Asia is significant at 5 or 1 percent. A 10 percent increase in human capital, measured in this chapter as return-weighted schooling attainment, is likely to raise TFP by 1 percent, holding other factors constant. For example, between 1999 (median year for the 1997–2001 period) and 2009, Senegal achieved a 10 percent increase in human capital from 1.73 to 1.90 per
person, close to the average human capital of 2.0 in the overall data. Holding other factors constant, Senegal’s TFP is therefore likely to have grown by 1 percent during the same decade because of human capital accumulation.

Table 4.3a. Panel Fixed-Effects Estimates of Total Factor Productivity
(Dependent Variable: Log TFP per Worker (log tfpt), Production-Function Approach)

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>4.2137***</td>
<td>5.7183***</td>
<td>3.9928***</td>
</tr>
<tr>
<td></td>
<td>(0.135)</td>
<td>(0.133)</td>
<td>(0.178)</td>
</tr>
<tr>
<td><strong>hucap</strong></td>
<td>0.1007*</td>
<td>0.2228***</td>
<td>0.1575**</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.102)</td>
<td>(0.047)</td>
</tr>
<tr>
<td><strong>infltn</strong></td>
<td>-0.4134</td>
<td>-3.13e-05</td>
<td>-4.00e-06</td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td>(3.25e-05)</td>
<td>(8.89e-06)</td>
</tr>
<tr>
<td><strong>inflvolat</strong></td>
<td>-1.864*</td>
<td>-2.2015**</td>
<td>-0.9306</td>
</tr>
<tr>
<td></td>
<td>(0.839)</td>
<td>(0.968)</td>
<td>(1.019)</td>
</tr>
<tr>
<td><strong>openness</strong></td>
<td>0.0015***</td>
<td>0.0029***</td>
<td>-0.00024</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0012)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td><strong>termstrd</strong></td>
<td>0.0012***</td>
<td>0.0017</td>
<td>0.0023***</td>
</tr>
<tr>
<td></td>
<td>(0.00035)</td>
<td>(0.0011)</td>
<td>(0.0007)</td>
</tr>
<tr>
<td><strong>lifexpect</strong></td>
<td>0.0073***</td>
<td>0.0890**</td>
<td>0.00098</td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(0.004)</td>
<td>(0.005)</td>
</tr>
<tr>
<td><strong>corrupt</strong></td>
<td>-0.0478**</td>
<td>-0.0572***</td>
<td>-0.0538*</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.008)</td>
<td>(0.032)</td>
</tr>
<tr>
<td><strong>debtgdp</strong></td>
<td>-0.0015**</td>
<td>-0.0014***</td>
<td>0.00087</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0004)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td><strong>gовstabil</strong></td>
<td>-0.0345***</td>
<td>-0.0430***</td>
<td>-0.0216**</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.010)</td>
</tr>
<tr>
<td><strong>( \bar{R}^2 )</strong></td>
<td>0.906</td>
<td>0.940</td>
<td>0.939</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>N = 160</td>
<td>N = 80</td>
<td>N = 80</td>
</tr>
<tr>
<td><strong>No. of countries</strong></td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
## Table 4.3b. Panel Fixed-Effects Estimates of Total Factor Productivity
*(Dependent Variable: Log TFP per Worker (log tfpt); Production-Function Approach)*

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>4.2594***</td>
<td>4.8171***</td>
<td>4.0770***</td>
</tr>
<tr>
<td></td>
<td>(0.235)</td>
<td>(0.753)</td>
<td>(0.156)</td>
</tr>
<tr>
<td><strong>hucap</strong></td>
<td>0.0988*</td>
<td>0.6002***</td>
<td>0.2188***</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.225)</td>
<td>(0.070)</td>
</tr>
<tr>
<td><strong>infltn</strong></td>
<td>-0.4324</td>
<td>-0.4255</td>
<td>-0.1369</td>
</tr>
<tr>
<td></td>
<td>(0.322)</td>
<td>(0.523)</td>
<td>(0.186)</td>
</tr>
<tr>
<td><strong>inflvolat</strong></td>
<td>-1.9040*</td>
<td>-3.1404*</td>
<td>-1.1695</td>
</tr>
<tr>
<td></td>
<td>(1.067)</td>
<td>(1.721)</td>
<td>(0.944)</td>
</tr>
<tr>
<td><strong>openness</strong></td>
<td>0.0058</td>
<td>0.0065***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td><strong>openvolat</strong></td>
<td></td>
<td>-0.0040</td>
<td>-0.0012</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.012)</td>
<td>(0.005)</td>
</tr>
<tr>
<td><strong>termstrd</strong></td>
<td>0.0014***</td>
<td>0.00015</td>
<td>0.0019**</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.002)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td><strong>totvolat</strong></td>
<td>-0.0032**</td>
<td></td>
<td>0.0029</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td></td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>lifexpect</strong></td>
<td>0.0177***</td>
<td>0.0134**</td>
<td>-0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.0063)</td>
<td>(0.004)</td>
</tr>
<tr>
<td><strong>childmort</strong></td>
<td>-1.45e-07***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.28e-08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>corruptn</strong></td>
<td>-0.0509**</td>
<td>-0.1125***</td>
<td>-0.0487*</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.026)</td>
<td>(0.029)</td>
</tr>
<tr>
<td><strong>debtgdp</strong></td>
<td>-0.0014**</td>
<td>-0.0017**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.00068)</td>
<td></td>
</tr>
<tr>
<td><strong>govstabil</strong></td>
<td>-0.0306**</td>
<td>-0.0035</td>
<td>-0.0261**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.011)</td>
</tr>
<tr>
<td><strong>fdigdp</strong></td>
<td>0.0200*</td>
<td></td>
<td>0.0200*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.011)</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.885</td>
<td>0.932</td>
<td>0.922</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

While the effect of human capital on TFP in Africa is generally much bigger, its coefficient varies considerably between specifications. In Table 4.3a, the coefficient of this factor is 0.22, but it rises to 0.60 when terms of trade volatility is introduced into the model, as in Table 4.3b. Although not to the extent as for the full sample, human capital’s effect in Asia is

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also not the same across specifications, rising only from 0.16 in Table 4.3a to 0.22 in Table 4.3b. From results in these two tables, it can be inferred that TFP is closely and positively associated with human capital accumulation.

This conclusion is also supported by the numbers in Table 4.4, where the TFP series is based on the growth accounting exercise undertaken in this chapter. At various significance levels between 1 and 10 percent, human capital again provides a significant impact on TFP. The coefficient of human capital in Table 4.4 is 0.31 for the full sample, which is greater than for the same sample in the case where TFP is based on production-function estimates (0.10). While the two TFP series are different, as they are calculated using different methods, the correlation between them is high, as alluded to earlier.

Another set of covariates examined in this chapter is external factors in growth. The variables used are openness, terms of trade, and external debt. Together they are associated significantly with TFP, although individually their impact on TFP is mixed. Openness has a positive impact on TFP in all countries and in Africa, but not in Asia, for which the coefficient is either insignificant or numerically marginal. For all, it is found that a 7 percent increase in openness (from the mean value of 33 percent rising to 35.3 percent of GDP) is likely to lead to a 1 percent increase in TFP. According to the growth-accounting-based TFP exercise, on the other hand, the required rise in openness is 3.5 percent.

While all countries have opened up in trade in recent years, Asian economies, especially in East and Southeast Asia, have had a high degree of openness for several decades now. Hence, compared with Africa, for which primary exports have been dominant, further increases in TFP resulting from a rise in openness may be limited for the Asian economies. For Africa, the
coefficient is significant and large, which indicates that this region will benefit more in TFP from further increases in openness.

**Table 4.4. Panel Fixed-Effects Estimates of Total Factor Productivity  
(Dependent Variable: log tfp; Growth-Accounting-based TFP, \( \alpha = 0.3 \))**

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>4.5041***</td>
<td>3.9960***</td>
<td>5.3037***</td>
</tr>
<tr>
<td></td>
<td>(0.627)</td>
<td>(0.698)</td>
<td>(0.308)</td>
</tr>
<tr>
<td><strong>hucap</strong></td>
<td>0.3083*</td>
<td>0.5275**</td>
<td>0.1745***</td>
</tr>
<tr>
<td></td>
<td>(0.186)</td>
<td>(0.206)</td>
<td>(0.058)</td>
</tr>
<tr>
<td><strong>Infln</strong></td>
<td>-0.0081</td>
<td>-0.5130</td>
<td>0.2021</td>
</tr>
<tr>
<td></td>
<td>(0.390)</td>
<td>(0.556)</td>
<td>(0.262)</td>
</tr>
<tr>
<td><strong>inflvolat</strong></td>
<td>1.4042</td>
<td>-2.8481*</td>
<td>1.1118</td>
</tr>
<tr>
<td></td>
<td>(1.188)</td>
<td>(1.653)</td>
<td>(0.863)</td>
</tr>
<tr>
<td><strong>openness</strong></td>
<td>0.0028***</td>
<td>0.0058***</td>
<td>1.68e-03**</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.001)</td>
<td>(6.89e-04)</td>
</tr>
<tr>
<td><strong>openvolat</strong></td>
<td>0.0015</td>
<td>-0.0049</td>
<td>-0.0066</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.011)</td>
<td>(0.006)</td>
</tr>
<tr>
<td><strong>termstrd</strong></td>
<td>0.0016***</td>
<td>0.0015</td>
<td>1.63e-03**</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0015)</td>
<td>(8.45e-04)</td>
</tr>
<tr>
<td><strong>totvolat</strong></td>
<td>-0.0016</td>
<td>-0.0023**</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.0029)</td>
</tr>
<tr>
<td><strong>lifexpect</strong></td>
<td>0.0144***</td>
<td>1.11e-02*</td>
<td>0.0041</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(6.37e-03)</td>
<td>(0.006)</td>
</tr>
<tr>
<td><strong>childmort</strong></td>
<td>-2.53e-07***</td>
<td>6.04e-07</td>
<td>-2.29e-07***</td>
</tr>
<tr>
<td></td>
<td>(7.99e-08)</td>
<td>(6.01e-07)</td>
<td>(5.74e-08)</td>
</tr>
<tr>
<td><strong>corruptn</strong></td>
<td>-0.0609***</td>
<td>0.1087***</td>
<td>-0.0329</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.026)</td>
<td>(0.030)</td>
</tr>
<tr>
<td><strong>debtdgdp</strong></td>
<td>-0.0021**</td>
<td>-1.26e-03</td>
<td>0.00098</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(9.97e-04)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td><strong>govstabil</strong></td>
<td>-0.0213*</td>
<td>0.0031</td>
<td>-0.0180*</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.021)</td>
<td>(0.011)</td>
</tr>
<tr>
<td><strong>( R^2 )</strong></td>
<td>0.938</td>
<td>0.954</td>
<td>0.943</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated using the growth accounting approach from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

As for Asia, the terms of trade seems to be more important than openness. Its effect is significant for Asia and for the full sample, but not for Africa. The terms of trade for primary goods, in which Africa remains more specialized, fluctuates a lot more than the terms of trade for...
manufactures. This volatility in the terms of trade has been fairly damaging for Africa, which is also indicated in the results by a significant negative sign for its coefficient (Tables 4.3b and 4.4). Thus, the effect of terms of trade volatility seems to mirror differences in trade structure between the two regions.

External debt is another factor whose impact on TFP parallels the effect of openness. While external indebtedness generally seems to reduce TFP for the full sample, its impact is not significant in Asia. With a mix of countries—some of which (e.g., the Philippines) have not been able to manage their debt well in the past and others (e.g., Vietnam) that have made an efficient use of their foreign capital (Pritchett, 2003)—the result is not significant for Asia. For Africa, debt overhang has been severe for much of the sample period. Although Africa has been a beneficiary of international debt relief provided several times in the last few decades, debt forgiveness has mostly followed severe growth problems created by, among other factors, debt overhang itself.

Among socio-political factors in TFP, three variables in Tables 4.3a, 4.3b, and 4.4 present a mixed story. Life expectancy has the expected positive and significant impact in the full sample and in Africa, but its significance is not evident in Asia. The same is true of corruption, which has a negative impact in the full sample and in Africa. On the other hand, with respect to government stability, an unexpected negative sign is found for the full sample and for Asia. The stability variable is comprised of such factors as the type of governance, cohesion of the government and political parties, and command over the legislature, which are all expected to indicate the government’s ability to carry out its policies and programs. In general, more countries in Asia have had relatively stable regimes as well as higher TFP than in Africa, yet the
results indicate a negative impact of this variable on Asian TFP. As will be shown later, we do find expected results for this variable when the GMM estimation procedure is used. Again, as noted for Asia, we do not find even life expectancy and corruption yielding conventional results.

Finally, inflation does not associate with TFP level according to any of the three tables of results discussed in this section. However, volatility of inflation, which theoretically is more relevant for output and TFP, posts the expected negative sign and significance for its coefficient in the whole sample and in Africa. It is not significant in Asia.

4.4.1.3. Fixed-Effects Estimates of Growth of Total Factor Productivity

This section reports results for the growth of TFP. A movement from the level to the change reduces the sample size by one for each country, or from 160 to 140 overall. The results are presented in three tables paralleling the results for the level of TFP—Tables 4.5a, 4.5b, and 4.6. Starting off with human capital, the last subsection found that human capital makes a positive and significant impact on TFP. With growth, however, the favorable impact is limited to Asia (and the full sample) and does not extend to Africa. The results indicate that a typical country in Asia that achieves a 1 percentage point faster accumulation of human capital is likely to gain at least 0.13 percentage point and up to 0.43 percentage point in its TFP growth. The former rate applies to the TFP that follows from the production function, whereas the latter applies to TFP that comes from growth accounting. While even the smaller of these two numbers (0.13) is substantial, the range of such coefficients seems a bit too large considering the findings reported earlier. In the TFP-level specification, as discussed in the last subsection, the estimated coefficients for human capital varied within a narrow range, between 0.16 and 0.22. This issue
will be taken up again in connection with the results from the GMM estimation discussed in the next section.

Table 4.5a. Panel Fixed-Effects Estimates of Total Factor Productivity Growth
(Dependent Variable: dlog tfp; Production-Function-based TFP)

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.7411***</td>
<td>-0.4781</td>
<td>-0.2441</td>
</tr>
<tr>
<td></td>
<td>(0.169)</td>
<td>(0.333)</td>
<td>(0.421)</td>
</tr>
<tr>
<td>hu_cap</td>
<td>0.1599**</td>
<td>0.0093</td>
<td>0.01059</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.100)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>he*fdi</td>
<td>0.0062*</td>
<td>0.0056**</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>(0.0036)</td>
<td>(0.026)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Infln</td>
<td>-0.6171***</td>
<td>-0.7789***</td>
<td>-0.5853**</td>
</tr>
<tr>
<td></td>
<td>(0.159)</td>
<td>(0.263)</td>
<td>(0.250)</td>
</tr>
<tr>
<td>openness</td>
<td>0.00922**</td>
<td>0.0047***</td>
<td>0.00050</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.014)</td>
<td>(0.0062)</td>
</tr>
<tr>
<td>openvolat</td>
<td>-0.0028</td>
<td>-0.0060</td>
<td>-0.0049</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>corruptn</td>
<td>-0.0115</td>
<td>-0.0209</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.013)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>-0.0012***</td>
<td>-0.0013***</td>
<td>-0.0043***</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.003)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>childmort</td>
<td>-8.93e-08****</td>
<td>-5.45e-07*</td>
<td>-1.06e-07***</td>
</tr>
<tr>
<td></td>
<td>2.50e-08</td>
<td>3.10e-07</td>
<td>2.06e-08</td>
</tr>
<tr>
<td>dpndncy</td>
<td>0.0073***</td>
<td>0.0057***</td>
<td>0.0041</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.002)</td>
<td>(0.0026)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.531</td>
<td>0.517</td>
<td>0.548</td>
</tr>
<tr>
<td>N = 140</td>
<td>N = 70</td>
<td>N = 70</td>
<td></td>
</tr>
<tr>
<td>No. of countries = 20</td>
<td>No. of countries = 10</td>
<td>No. of countries = 10</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated using the growth accounting approach from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
Regarding the factors related to the external sector of the economy—openness, terms of trade, and external debt—the impact of openness mirrors the impact under the level TFP models. Thus, greater openness generally benefits TFP growth in both the production-function and growth-accounting-based TFP series. Openness, however, exerts a significant effect consistently in all three samples if one limits to the growth accounting TFP. For the other TFP series, the results are consistent for Africa regardless of variations in model specification, whereas for Asia
the impact of openness is model dependent, analogous to what was found for the level TFP. On the other hand, the effect of a faster rise in the external-debt-to-GDP ratio is to clearly reduce TFP growth. This result is invariant to the choice of model or region. With respect to the terms of trade, TFP changes are not significantly affected, for the most part.

Table 4.5b. Panel Fixed-Effects Estimates of Total Factor Productivity Growth
(Dependent Variable: $d\log tfp$; Production-Function-based $TFP$)

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.9459***</td>
<td>-0.6523**</td>
<td>-1.0953**</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.320)</td>
<td>(0.448)</td>
</tr>
<tr>
<td>huca$\dagger$</td>
<td>0.2543***</td>
<td>0.0434</td>
<td>0.4290***</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.125)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>infl$\dagger$</td>
<td>-0.000015**</td>
<td>-0.00005**</td>
<td>-0.000013</td>
</tr>
<tr>
<td></td>
<td>(0.000007)</td>
<td>(0.000019)</td>
<td>(0.00001)</td>
</tr>
<tr>
<td>inflvolat$\dagger$</td>
<td>0.1788</td>
<td>0.2441</td>
<td>-1.7955</td>
</tr>
<tr>
<td></td>
<td>(0.389)</td>
<td>(0.710)</td>
<td>(1.275)</td>
</tr>
<tr>
<td>openness$\dagger$</td>
<td>0.00047</td>
<td>0.0027***</td>
<td>0.0007*</td>
</tr>
<tr>
<td></td>
<td>(0.00038)</td>
<td>(0.00053)</td>
<td>(0.00039)</td>
</tr>
<tr>
<td>openvolat$\dagger$</td>
<td>0.1886</td>
<td>-1.1388</td>
<td>-0.4910</td>
</tr>
<tr>
<td></td>
<td>(0.739)</td>
<td>(0.732)</td>
<td>(0.804)</td>
</tr>
<tr>
<td>termstrd$\dagger$</td>
<td>-0.0002*</td>
<td>-</td>
<td>0.0018***</td>
</tr>
<tr>
<td></td>
<td>(0.00012)</td>
<td></td>
<td>(0.00048)</td>
</tr>
<tr>
<td>totvol$\dagger$</td>
<td>-</td>
<td>-0.00030**</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00013)</td>
<td></td>
</tr>
<tr>
<td>corruptn$\dagger$</td>
<td>-0.0024</td>
<td>-0.0323</td>
<td>0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0066)</td>
<td>(0.023)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>debtgdp$\dagger$</td>
<td>-0.00098***</td>
<td>-0.0005**</td>
<td>-0.0033***</td>
</tr>
<tr>
<td></td>
<td>(0.00021)</td>
<td>(0.00028)</td>
<td>(0.00074)</td>
</tr>
<tr>
<td>govstabil$\dagger$</td>
<td>-0.0080</td>
<td>-0.0018</td>
<td>-0.0320***</td>
</tr>
<tr>
<td></td>
<td>(0.0058)</td>
<td>(0.0088)</td>
<td>(0.0095)</td>
</tr>
<tr>
<td>he*fdi$\dagger$</td>
<td>0.0059**</td>
<td>0.0078**</td>
<td>0.0078*</td>
</tr>
<tr>
<td></td>
<td>(0.0030)</td>
<td>(0.0033)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>dpndncy$\dagger$</td>
<td>0.0076***</td>
<td>0.0074***</td>
<td>0.0052*</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0022)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.3569</td>
<td>0.5517</td>
<td>0.6204</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated using the growth accounting approach from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
Table 4.6. Panel Fixed-Effects Estimates of Total Factor Productivity Growth
(Dependent Variable: dlog tfp; Growth-Accounting-based TFP)

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-0.7231***</td>
<td>-0.8634**</td>
<td>-0.0499</td>
</tr>
<tr>
<td></td>
<td>(0.165)</td>
<td>(0.364)</td>
<td>(0.318)</td>
</tr>
<tr>
<td>hucap</td>
<td>0.1572**</td>
<td>0.1143</td>
<td>0.1293*</td>
</tr>
<tr>
<td></td>
<td>(0.069)</td>
<td>(0.075)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>he*fdi</td>
<td>0.0077**</td>
<td>0.0059**</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td>(0.0033)</td>
<td>(0.0028)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>infln</td>
<td>-0.4989***</td>
<td>-5.06e-05***</td>
<td>-0.5265**</td>
</tr>
<tr>
<td></td>
<td>(0.164)</td>
<td>(1.67e-05)</td>
<td>(0.212)</td>
</tr>
<tr>
<td>openness</td>
<td>0.00082*</td>
<td>0.0025**</td>
<td>0.00082**</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0012)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>openvolat</td>
<td>0.00089</td>
<td>-0.0049**</td>
<td>-0.0049</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.0047)</td>
</tr>
<tr>
<td>corrupt</td>
<td>0.0018</td>
<td>-0.0293</td>
<td>-0.0091</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.024)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>-0.0014***</td>
<td>-0.00069*</td>
<td>-0.0034***</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.00037)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td>childmort</td>
<td>-1.51e-07***</td>
<td>-2.22e-07</td>
<td>-1.88e-07***</td>
</tr>
<tr>
<td></td>
<td>(3.15e-08)</td>
<td>(1.68e-07)</td>
<td>(3.34e-08)</td>
</tr>
<tr>
<td>dpndncy</td>
<td>7.06e-03***</td>
<td>0.0071**</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>(1.35e-03)</td>
<td>(0.0034)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.534</td>
<td>0.493</td>
<td>0.718</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated using the growth accounting approach from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

It is interesting to point out that we have attempted to uncover any interactive effects of human capital with external variables. A significant interaction of openness with human capital is not found, which is at variance with Miller and Upadhya (2000), among other studies, who report significant results for that interaction in the larger dataset for 83 countries. They calculate a minimum threshold for openness that a country must have before human capital begins to influence TFP. In the results for the 20-country sample presented here, however, not only do we...
not find any significance for human capital-openness interaction but even the openness term loses its significance when the interaction term is brought in. (These results are not shown in Table 4.6.) It is possible that the difference in results is primarily due to the focus here on developing countries and the use of much-updated data.

One exception to all this is the interaction between human capital and FDI. While human capital data are different in the models presented here, even in the new data used some support is found for the argument initially made by Borensztein et al. (1998) that FDI affects growth only after human capital rises to pass a certain threshold level. We also find that the interaction between these two variables is significant in the model for the growth of TFP. No such support is found, however, in the single-stage output growth regressions discussed in the next section.

Finally, regarding inflation, compared to a rise in inflation having no impact on the level TFP, we now find accelerating inflation hurting TFP growth regardless of which TFP series or sample is used. Changes in life expectancy and government stability are not associated with TFP growth and those effects are not reported in the tables. For its part, falling child mortality provides a relatively trivial though significant positive effect on TFP growth.

4.4.1.4. Fixed-Effects Estimates of Level and Growth of Output per Worker

This section discusses the single-stage growth regressions where TFP is just a part of output. Investment share of GDP and possibly human capital represent the basic factors of production, whereas other factors can explain TFP. The level and growth of output per worker is discussed as being related to the level and changes in the covariates on the right-hand side of the regressions (Table 4.7)
### Table 4.7. Panel Fixed-Effects Estimates of Output per Worker

(Dependent Variable: Logarithms of Output per Worker (log(Y/L))@

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>5.6203***</td>
<td>6.7557***</td>
<td>6.2498***</td>
</tr>
<tr>
<td></td>
<td>(0.845)</td>
<td>(0.655)</td>
<td>(0.168)</td>
</tr>
<tr>
<td><strong>hucap</strong></td>
<td>0.6076**</td>
<td>0.4661**</td>
<td>0.5981***</td>
</tr>
<tr>
<td></td>
<td>(0.264)</td>
<td>(0.222)</td>
<td>(0.063)</td>
</tr>
<tr>
<td><strong>invgdp</strong></td>
<td>0.0103***</td>
<td>0.0138***</td>
<td>0.0115***</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.003)</td>
<td>(0.0029)</td>
</tr>
<tr>
<td><strong>infltn</strong></td>
<td>0.00000</td>
<td>-0.00005***</td>
<td>0.000007</td>
</tr>
<tr>
<td></td>
<td>(0.00002)</td>
<td>(0.000)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td><strong>inflvolat</strong></td>
<td>0.1996</td>
<td>-3.0782***</td>
<td>3.9026***</td>
</tr>
<tr>
<td></td>
<td>(1.524)</td>
<td>(1.058)</td>
<td>(1.135)</td>
</tr>
<tr>
<td><strong>openness</strong></td>
<td>0.0031***</td>
<td>0.0016</td>
<td>0.0015***</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.001)</td>
<td>(0.00046)</td>
</tr>
<tr>
<td><strong>termstrd</strong></td>
<td>0.0009***</td>
<td>0.0003*</td>
<td>0.0019***</td>
</tr>
<tr>
<td></td>
<td>(0.00009)</td>
<td>(0.000)</td>
<td>(0.00076)</td>
</tr>
<tr>
<td><strong>totvol</strong></td>
<td>-0.0026***</td>
<td>-0.0033***</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>(0.00093)</td>
<td>(0.0011)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td><strong>lifexpect</strong></td>
<td>0.0250***</td>
<td>0.0056</td>
<td>0.0109***</td>
</tr>
<tr>
<td></td>
<td>(0.0070)</td>
<td>(0.004)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td><strong>corrupt</strong></td>
<td>-0.0618***</td>
<td>-0.1200***</td>
<td>-0.0501***</td>
</tr>
<tr>
<td></td>
<td>(0.0179)</td>
<td>(0.013)</td>
<td>(0.0182)</td>
</tr>
<tr>
<td><strong>debtgdp</strong></td>
<td>-0.0031***</td>
<td>-0.0017**</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td>(0.00074)</td>
<td>(0.001)</td>
<td>(0.00096)</td>
</tr>
<tr>
<td><strong>dpndncy</strong></td>
<td>0.0052</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.952</td>
<td>0.973</td>
<td>0.969</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>N = 159</td>
<td>N = 79</td>
<td>N = 80</td>
</tr>
<tr>
<td><strong>No. of countries</strong></td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

*Notes: Total factor productivity series is based on production-function estimates. Data used are for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.*

Starting with the results for the log-level of per worker output, both physical capital investment and human capital are closely associated with output. The respective coefficients are significant at the 1 and 5 percent levels. The coefficient for investment share of GDP equals 0.0103 for the entire sample, whereas its mean share in the data is 0.199, or about 20 percent.

To understand the implications of these numbers, take the case of Ghana, which during the last sample period of 2007-11 stood near the median per worker income, with about $5,000
in 2005 purchasing power parity (PPP) dollar terms. Raising investment by 10 percent, given
GDP, will increase the investment share to about 0.220 (or 22 percent), which might take some
time to accomplish. When this is achieved, however, the per capita output will have risen by
0.0103 \times 100 \times 10, or 10.3 percent, thus increasing Ghana’s per worker income from $5,000 to
$5,515. A rise in the investment share of 10 instead of 2 percentage points would require raising
investment by 50 percent at a given level of GDP. Rapidly developing economies have
succeeded in achieving such an increase in investment in a short order. The results do not seem
unrealistic if the right conditions for investment prevail in the economy.

The human capital result tells a similar story. Its coefficient for all countries is 0.61 and
the mean and median levels of human capital are both 2.0 in the data presented here according to
the newly constructed human capital measure in Penn World Table 8.0. A 10 percent increase in
human capital is therefore likely to be associated with a 6 percent increase in per capita income.
In the case of Bangladesh, for example, for which the human capital median of 2.03 during
2007-11 is close to the overall median, such a rise in human capital will increase per capita
output from $4,316 to $4,575, which does not appear to be quite as large. Moreover, there is
another problem with such a calculation for the effect of human capital. As policy and other
factors raise the years of schooling, the return to human capital falls, which will cause output to
respond in a diminishing returns fashion. This nonlinear relation of human capital to schooling in
the new Penn World Table needs to be borne in mind while considering policies related to
education.\footnote{Attempts to test diminishing returns to human capital through the introduction of the square of human capital yielded statistically insignificant results for all countries and for Africa. Regressions for Asia came closer, but still failed, to show a significantly negative coefficient for the square term. Because of imprecise estimation, the human capital square term was dropped from all subsequent specifications.}
Among the external variables, openness, terms of trade, and external debt all have coefficients with the expected signs: positive, positive, and negative, respectively. The size of these effects are, however, very small. For instance, if the terms of trade, which is indexed to 100 for the base year, were to become 20 percent more favorable, per capita income would likely go up trivially by about 0.02 percent. Similar effects are obtained for the effects of openness (mean openness or trade/GDP = 67 percent) and external debt to GDP (mean = 63 percent). The terms of trade volatility also shows the expected negative result, and its effect is also small. Thus, for instance, if volatility falls by one standard deviation, which admittedly is large (62 percent of the mean volatility), output is affected positively by one-sixth of 1 percent.

Other variables in Table 4.7 are life expectancy, indicative of health conditions, which is significantly positive, and corruption, indicative of administrative weakness, which is significantly negative, as expected. Their coefficients do not translate much into output effects, however.

Regressions for Africa and Asia are generally in line with the results for all countries presented above. In particular, human capital and real physical investment are again highly significant statistically and have a substantial impact on output. External factors present a mixed picture, with openness significant only for Asia and external indebtedness and volatility in terms trade only for Africa. The terms of trade, however, affects output significantly in both the regions. While the effect of corruption is significant in both regions, life expectancy is significant only in Asia.

The last sets of results are for the growth of output per worker (Table 4.8). Growth of human capital and investment are positively associated with growth of output. Of the external factors, openness and terms of trade variables are not significant, whereas foreign indebtedness
has a significant negative impact on output growth. This is true of the full sample and Africa, but such is not the case for Asia. Inflation provides a weakly (at the 10 percent level) negative effect on output. Changes in life expectancy and government stability are not related to output growth, except in Africa, where government stability shows a positive effect.

**Table 4.8. Panel Fixed-Effects Estimates of Output Growth per Worker**

*(Dependent Variable: dlog(Y/L)*

<table>
<thead>
<tr>
<th></th>
<th>All Countries</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant</strong></td>
<td>-0.9461***</td>
<td>-0.5764</td>
<td>-1.5798**</td>
</tr>
<tr>
<td></td>
<td>(0.342)</td>
<td>(0.693)</td>
<td>(0.620)</td>
</tr>
<tr>
<td><strong>hucap</strong></td>
<td>0.2252**</td>
<td>0.0037</td>
<td>0.4001***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.202)</td>
<td>(0.139)</td>
</tr>
<tr>
<td><strong>invgdp</strong></td>
<td>0.0114**</td>
<td>0.0094</td>
<td>0.0146***</td>
</tr>
<tr>
<td></td>
<td>(0.0046)</td>
<td>(0.0069)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td><strong>infln</strong></td>
<td>-0.4930**</td>
<td>-0.6447*</td>
<td>-0.4076*</td>
</tr>
<tr>
<td></td>
<td>(0.237)</td>
<td>(0.371)</td>
<td>(0.206)</td>
</tr>
<tr>
<td><strong>openvolat</strong></td>
<td>0.00201</td>
<td>0.0074***</td>
<td>-0.0029</td>
</tr>
<tr>
<td></td>
<td>(0.0051)</td>
<td>(0.0025)</td>
<td>(0.0073)</td>
</tr>
<tr>
<td><strong>termstrd</strong></td>
<td>-0.00018</td>
<td>-0.00023</td>
<td>0.0009**</td>
</tr>
<tr>
<td></td>
<td>(0.00013)</td>
<td>(0.00017)</td>
<td>(0.00046)</td>
</tr>
<tr>
<td><strong>lifexpect</strong></td>
<td>-0.0016</td>
<td>-0.0004</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.0041)</td>
<td>(0.0057)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td><strong>debtgdp</strong></td>
<td>-0.0016***</td>
<td>-0.0019***</td>
<td>-0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0004)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td><strong>govstabil</strong></td>
<td>0.0088</td>
<td>0.0397***</td>
<td>-0.0232***</td>
</tr>
<tr>
<td></td>
<td>(0.0077)</td>
<td>(0.013)</td>
<td>(0.0051)</td>
</tr>
<tr>
<td><strong>dpndncy</strong></td>
<td>0.0062**</td>
<td>0.0034</td>
<td>0.0077**</td>
</tr>
<tr>
<td></td>
<td>(0.0025)</td>
<td>(0.0047)</td>
<td>(0.0038)</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.492</td>
<td>0.498</td>
<td>0.553</td>
</tr>
<tr>
<td></td>
<td>N = 140</td>
<td>N = 70</td>
<td>N = 70</td>
</tr>
<tr>
<td></td>
<td>No. of countries = 20</td>
<td>No. of countries = 10</td>
<td>No. of countries = 10</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated using growth accounting approach from the production-function estimates. Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Heteroskedasticity-robust standard errors appear in parentheses. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

Note that the tables of results report the numbers for the same models for different sample groups—the full sample (“All”), Africa, and Asia—to the extent that the models are not hampered by problems of multicollinearity or when some variables do not seem to add much
value when those controls are not of central interest in this study of human capital and growth. Only in the latter cases have certain controls been dropped from the regressions.

4.4.2. Regressions Using the Generalized Method of Moments

The discussion in Section 4.4.1 was limited to the estimation of panel data models with fixed effects, without much consideration of the possibility that some variables on the right-hand side may be endogenous. If so, they would be correlated with the error term and thereby suffer from bias in their estimated coefficients. Many macroeconomic models face such a possibility. A difficulty in handling such a problem in macro models arises from the fact that most of the economic variables are likely to be endogenous. Literature has increasingly attempted to introduce instruments for some of the problem correlates of growth. But this has subsequently led to further criticism that instruments may not be valid or strong. Addressing the skepticism created by failure to identify deeper causes of growth, Bazzi and Clemens (2013) urge researchers who apply the GMM to make more efforts to delve into the method with complementary methods that can evaluate the instrument’s strength. As these authors show, GMM can address endogeneity issues in some important ways, yet it does not invalidate the results derived using other techniques.

This section attempts to supplement the analysis presented in this chapter of growth correlates with the use of GMM. GMM is applied as an additional method to estimate the model and use the results to provide a comparative assessment of these techniques for the purposes at hand. The GMM comes in handy when it is hard to establish the exogeneity of instruments, since ones does not clearly know the data-generating process. The simple Arellano-Bond GMM (or difference GMM, for the first difference transformation) removes the fixed effect at the cost of introducing correlation between the differenced lagged values of the dependent variable and the
differenced error term. But the system GMM allows estimation of the model without the requirement that instruments be “excellent,” which implies that the instruments could be drawn from within the existing dataset. The simple instrumental variables regression instead does not use all the available information in the data, making the ‘system GMM’ that exploits this information more efficient.

The exclusion restriction that the empirical growth models usually place on some of the instruments may be inappropriate because judging a given variable to be exogenous in the context of economic growth may contain an element of arbitrariness. The reason is that so many factors, not all included in the original model before the use of instruments, can have a direct or indirect bearing on income and growth. Yet, in a paper that is focused on determining the effect of human capital and other economic variables, there is some rationale for taking factors that reflect political decision-making and that are inherently built in our chosen instruments, as discussed below, as relative “outsiders.” In addition, following empirical literature, we take one or more periods of lags of dependent and other variables together with changes in these variables as instruments. This procedure can lead to the use of many more instruments than the number of estimated parameters in the model. In turn, this necessitates a test of overidentifying restrictions that can be performed as done below by using the Hansen $J$-statistic.

The model here is essentially the same as that analyzed in the preceding section except that it pays more attention to addressing the possible endogeneity issue. A large volume of literature has accumulated over the last couple of decades on the ways in which such variables as human capital, inflation, and openness can depend in part on per capita income and the growth rate of an economy—a case of reverse causality. For example, greater income permits households to invest in their children’s education. Similarly, expansionary monetary and fiscal
policy that tends to raise output in normal times can also lead to inflation when idle resources cannot be brought to use as fast as additional demand created by the policy. Further, basic macroeconomics teaches that imports that depend on income and exports from a small economy can also rise in response to supply-induced growth of output as well as to a surge in foreign demand for output at home.

In the models in this chapter for the level and growth of TFP, and the level and growth of output, risk indicators of the political and economic situation of a country are used as instruments for what are seemingly endogenous variables. One instrument used is economic risk-taking which, according to the 2013 International Country Risk Guide, serves as a way to evaluate the strengths and weaknesses of economies (PRS Group, 2013). If strengths outweigh weaknesses, the economic risk will be low and vice versa. Risk components are assessed based on real GDP growth, inflation, government budget balance, and current account balance. The variable has the highest possible value of 50 for the lowest and 0 for the highest economic risk, although a value near 15 seems to be around the bottom of the economic riskiness data for an economy. Since low values show a high risk, a number like 15 might damage an economy’s process of human capital accumulation, give rise to inflation, and hinder integration into the world economy.

A measure of financial riskiness of a country provides another instrument by serving as a way to evaluate the ability of a country to manage financially by financing its official, commercial, and trade debt obligations (PRS Group, 2013). This measure integrates foreign debt into GDP, foreign debt service into exports, current account balance into exports, international liquidity as months of import cover, and exchange rate stability. It also uses a government
accountability index that includes whether elections are free and fair and whether the
government actually responds to people’s concerns.

Among noneconomic instruments, ethnic tension, which has afflicted many developing
countries, is another avenue through which the acquisition of human capital and trade can be
disrupted. Racial, ethnic, and linguistic divisions can make opposing groups of people intolerant
of one another, potentially leading to chaos and disruption in the economy.

The lagged values of human capital, inflation, and openness are also used here as
instruments, together with other factors already in the model such as corruption, government
stability, and the dependency ratio. All this leads to the use of many more moment conditions
than the parameters being estimated, which causes the model to be overidentified. While we
understand the limitation of the Hansen $J$-statistic for overidentifying restrictions, in most cases
it shows, as reported below, that the null hypothesis of the validity of the instruments used here
cannot be rejected.

The GMM /instrumental variables model can be briefly written as follows:

$$ y_i = X_i \beta + \varepsilon_i $$

$$ E[Z_i \varepsilon_i] = 0 $$

where there are $K$ variables in $X_i$ and $Q$ instrumental variables in $Z_i$, and where $Q \geq K$. The
GMM model starts from the assumption that some elements in $X_i$ are correlated with the error
term but their instruments $Z_j$ are not:

$$ \text{cov}[\varepsilon_i, \varepsilon_j | X, Z] = \sigma^2 \omega_i, $$
which can be extended for panel data with an additional country subscript.

The results of the GMM estimation are given in Tables 4.9 through 4.14. Table 4.9 shows the results for logTFP based on the production-function approach and Table 4.10 shows the results for logTFP based on the growth accounting framework. Both tables report numbers for dynamic GMM estimates where logTFP_{t-1} appears on the right-hand side. The lagged dependent variable allows for lagged adjustment to past values of the explanatory variables. A country that has invested significantly in human capital and removed many restrictions on trade is likely to enjoy greater productivity in a way that further raises productivity in the future. The coefficients, which are significant at the 1 percent level, are relatively large, indicating that a country with high productivity in the last five-year period will, ceteris paribus, experience high productivity in the current period as well. The coefficients for the full sample, Africa, and Asia are 0.62, 0.49, and a remarkably high 1.19, respectively. It appears that high productivity in the past gives an accelerated inertial push to current productivity in Asia.

Before discussing the results for other variables, it should be noted that the $p$-values for the Hansen $J$-statistic in Table 4.9 are too high (0.44, 0.18, 0.14) for the null of valid instruments to be rejected. Indeed, the same conclusion holds for all the GMM result tables for all three country groups. For the purposes of the discussion here, therefore, the coefficients of the variables can be accepted for being not too unreliable.

Comparing the GMM results in Table 4.9 with those in Table 4.3a (basic fixed-effects model for logTFP) finds that the GMM coefficients for human capital are larger than those under
fixed effects. It should be remembered, however, that model specifications in the two cases are slightly different. The important finding from the GMM exercise in any case is that most of the variables that are found to affect the dependent variable statistically significantly under fixed effects also influence TFP significantly under the GMM estimation. This is not limited to human capital as just discussed, but is verified for openness of a country (for the full sample and for Africa), the terms of trade (full sample and Asia), external indebtedness (full sample), inflation (Africa), and government stability (Asia).

Using the TFP series based on growth accounting (Table 4.10), human capital and openness again have a significant impact on TFP. On the other hand, there are some differences between GMM and the fixed-effects results. For instance, life expectancy now is no longer significant, in contrast to its significant impact for the full sample and Africa.

The results for the growth of TFP are available in Table 4.11 (production-function-based TFP growth) and Table 4.12 (growth accounting). Using identical specifications shows that human capital is highly significant and the elasticities of TFP with respect to this variable are fairly comparable across methods that were used to derive the TFP series. These GMM elasticities are 0.95 (production function) and 0.84 for the two approaches. The main lesson from all of the estimations of the effects of human capital is that TFP responds positively to the stock of human capital, and its growth also responds positively in most countries to faster accumulation of human capital.
### Table 4.9. Panel General Method of Moments Estimates of Total Factor Productivity

*(Dependent Variable: log tfp; TFP from the Production Function)*

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.9542</td>
<td>1.8286***</td>
<td>-1.0027</td>
</tr>
<tr>
<td></td>
<td>(1.204)</td>
<td>(0.614)</td>
<td>(0.882)</td>
</tr>
<tr>
<td>logtfp-1</td>
<td>0.6161***</td>
<td>0.4585***</td>
<td>1.1893***</td>
</tr>
<tr>
<td></td>
<td>(0.191)</td>
<td>(0.076)</td>
<td>(0.178)</td>
</tr>
<tr>
<td>hucap</td>
<td>0.1838**</td>
<td>0.6035***</td>
<td>0.3580**</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.194)</td>
<td>(0.160)</td>
</tr>
<tr>
<td>infln</td>
<td>0.00008</td>
<td>0.1819</td>
<td>-0.3262</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.894)</td>
<td>(0.561)</td>
</tr>
<tr>
<td>infvolat</td>
<td>-1.6243**</td>
<td>-3.8019**</td>
<td>-0.6629</td>
</tr>
<tr>
<td></td>
<td>(0.851)</td>
<td>(2.040)</td>
<td>(2.908)</td>
</tr>
<tr>
<td>open</td>
<td>0.0018***</td>
<td>0.0091***</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.0032)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>openvolat</td>
<td>-0.0060</td>
<td>-0.0215*</td>
<td>-0.0083</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.013)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>life</td>
<td>0.0026</td>
<td>0.0063</td>
<td>-0.0088</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0096)</td>
<td>(0.0085)</td>
</tr>
<tr>
<td>termstrd</td>
<td>0.0014*</td>
<td>0.00001</td>
<td>0.0025***</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0002)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>totvolat</td>
<td>-0.0008</td>
<td>-0.0016</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.0010)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>corrupt</td>
<td>-0.0088</td>
<td>-0.0433</td>
<td>0.0313</td>
</tr>
<tr>
<td></td>
<td>(0.0092)</td>
<td>(0.029)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>-0.0014***</td>
<td>-0.0009</td>
<td>-0.0032*</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0006)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>govtstabil</td>
<td>-0.0051</td>
<td>0.0223</td>
<td>-0.0222**</td>
</tr>
<tr>
<td></td>
<td>(0.0079)</td>
<td>(0.022)</td>
<td>(0.0083)</td>
</tr>
<tr>
<td>J-stat</td>
<td>3.743</td>
<td>4.9085</td>
<td>6.9803</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.243)</td>
<td>(0.179)</td>
<td>(0.137)</td>
</tr>
</tbody>
</table>

*Notes:* Total factor productivity series are calculated from the production-function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
### Table 4.10. Panel General Method of Moments Estimates of Total Factor Productivity

*(Dependent Variable: log tfp; TFP from Growth Accounting)*

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
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<td>(0.749)</td>
<td>(0.814)</td>
<td>(1.562)</td>
</tr>
<tr>
<td>logtfp$_{-1}$</td>
<td>0.9458***</td>
<td>0.4143**</td>
<td>0.6888**</td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.185)</td>
<td>(0.262)</td>
</tr>
<tr>
<td>hucap</td>
<td>0.5747**</td>
<td>0.7539**</td>
<td>0.2040*</td>
</tr>
<tr>
<td></td>
<td>(0.233)</td>
<td>(0.378)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>infl</td>
<td>-0.2576</td>
<td>0.1241</td>
<td>0.1489</td>
</tr>
<tr>
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<td>(0.808)</td>
<td>(0.770)</td>
<td>(0.525)</td>
</tr>
<tr>
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<td>-5.4295***</td>
<td>-3.7974**</td>
<td>-0.5350</td>
</tr>
<tr>
<td></td>
<td>(1.785)</td>
<td>(1.763)</td>
<td>(1.653)</td>
</tr>
<tr>
<td>open</td>
<td>0.0019</td>
<td>0.0109***</td>
<td>0.0017*</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0027)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>openvolat</td>
<td>0.0027</td>
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<td>-0.0069</td>
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<td>(0.0057)</td>
<td>(0.021)</td>
<td>(0.0072)</td>
</tr>
<tr>
<td>termstrd</td>
<td>-0.0018</td>
<td>0.00008</td>
<td>0.0020*</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0001)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>totvolat</td>
<td>-0.0018</td>
<td>-0.0020***</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0013)</td>
<td>(0.0007)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>-0.0010**</td>
<td>-0.0012</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0008)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>life</td>
<td>-</td>
<td>0.0096</td>
<td>-0.0127</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0089)</td>
<td>(0.0081)</td>
</tr>
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<td>-0.0636**</td>
<td>0.0132</td>
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<td>(0.0083)</td>
<td>(0.031)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>childmort</td>
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<td>-0.0000</td>
<td>-0.0000***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>govtstabil</td>
<td>0.0324</td>
<td>0.0054</td>
<td>-0.0233***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.020)</td>
<td>(0.0076)</td>
</tr>
<tr>
<td>dpndncy</td>
<td>0.0038**</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J-stat (p-value)</td>
<td>4.1414</td>
<td>4.5924</td>
<td>7.3899</td>
</tr>
<tr>
<td></td>
<td>(0.529)</td>
<td>(0.332)</td>
<td>(0.117)</td>
</tr>
</tbody>
</table>

*Notes:* Total factor productivity series are calculated from the production function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at: *** 1 percent, ** 5 percent, * 10 percent levels.
Table 4.11 Panel General Method of Moments Estimates of Total Factor Productivity Growth  
(Independent Variable: dlog tfp; TFP based on the Production Function)

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
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<td>-1.5295***</td>
<td>-1.2308**</td>
</tr>
<tr>
<td></td>
<td>(0.818)</td>
<td>(0.495)</td>
<td>(0.476)</td>
</tr>
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<td>hucap</td>
<td>0.9465***</td>
<td>0.3374**</td>
<td>0.4406***</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.138)</td>
<td>(0.151)</td>
</tr>
<tr>
<td>infln</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>inflvolat</td>
<td>-</td>
<td>-0.6408</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.303)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>open</td>
<td>0.0030***</td>
<td>0.0037***</td>
<td>0.0016**</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0011)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>openvolat</td>
<td>-7.9689*</td>
<td>-0.3248</td>
<td>1.1802</td>
</tr>
<tr>
<td></td>
<td>(4.851)</td>
<td>(3.148)</td>
<td>(2.691)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>0.0016</td>
<td>-0.0008*</td>
<td>-0.0026**</td>
</tr>
<tr>
<td></td>
<td>(0.0009)</td>
<td>(0.0004)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>corruptn</td>
<td>-0.0431</td>
<td>-0.0089</td>
<td>0.0109</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>he*fdi</td>
<td>0.0276**</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dpndncy</td>
<td>0.0068**</td>
<td>0.0100*</td>
<td>0.0060*</td>
</tr>
<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.0039)</td>
<td>(0.0031)</td>
</tr>
<tr>
<td>childmort</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>-0.0000***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>J-stat (p-value)</td>
<td>1.94 (0.963)</td>
<td>7.6468 (0.469)</td>
<td>12.0216 (0.150)</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated from the production-function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
### Table 4.12. Panel General Method of Moments Estimates of Total Factor Productivity Growth

(Independent Variable: $d\log tfp$; TFP based on Growth Accounting)

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-2.5049***</td>
<td>-2.1632***</td>
<td>-1.8428*</td>
</tr>
<tr>
<td></td>
<td>(0.725)</td>
<td>(0.796)</td>
<td>(0.932)</td>
</tr>
<tr>
<td>hucap</td>
<td>0.8400***</td>
<td>0.5426***</td>
<td>0.6569***</td>
</tr>
<tr>
<td></td>
<td>(0.294)</td>
<td>(0.200)</td>
<td>(0.2366)</td>
</tr>
<tr>
<td>infln</td>
<td>0.6943</td>
<td>-0.0000**</td>
<td>0.2676</td>
</tr>
<tr>
<td></td>
<td>(0.943)</td>
<td>(0.0000)</td>
<td>(0.897)</td>
</tr>
<tr>
<td>infvolat</td>
<td>-</td>
<td>-1.6690</td>
<td>-2.3786</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.378)</td>
<td>(2.368)</td>
</tr>
<tr>
<td>open</td>
<td>0.4330</td>
<td>0.0050**</td>
<td>0.0016*</td>
</tr>
<tr>
<td></td>
<td>(0.588)</td>
<td>(0.0024)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>openvolat</td>
<td>-1.3649*</td>
<td>4.0978</td>
<td>4.5114</td>
</tr>
<tr>
<td></td>
<td>(0.808)</td>
<td>(3.615)</td>
<td>(3.358)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>0.0003</td>
<td>-0.0012</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0008)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>corruptn</td>
<td>-0.0007</td>
<td>-0.0036</td>
<td>0.0233</td>
</tr>
<tr>
<td></td>
<td>(0.0095)</td>
<td>(0.024)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>childmort</td>
<td>-0.0000**</td>
<td>-0.0000</td>
<td>-0.0000***</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>dpndncy</td>
<td>0.0097***</td>
<td>0.0109**</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(0.0041)</td>
<td>(0.0076)</td>
</tr>
<tr>
<td>he*fdi</td>
<td>0.0345***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0084)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>J-stat (p-value)</td>
<td>2.4094</td>
<td>7.3415</td>
<td>6.7206</td>
</tr>
<tr>
<td></td>
<td>(0.7900)</td>
<td>(0.5002)</td>
<td>(0.5670)</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated from the production function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.

Tables 4.13 and 4.14 show the GMM results for the level and growth of the output per worker of which TFP is a part. In addition to the variables in other tables, these tables also have the investment share of GDP representing the effect of changes in physical capital. Once again, the focus is on the changes in physical and human capital and external factors in growth. The investment-GDP ratio is significantly positive and its coefficients in Table 4.13 are similar to those found under fixed effects (Table 4.7) for all countries and Africa, but somewhat smaller for
Asia. Repeating the exercise from the previous section about investment affecting output, we find that a 10 percent increase in investment share will now lead to a 11.9 percent rise in per capita output, (such as from PPP $5,000 to $5,600). Thus, the GMM estimates are more favorable for output than under fixed effects, where the output increase was slightly lower at 10.3 percent.

**Table 4.13. Panel General Method of Moments Estimates of Output per Worker**

*(Dependent Variable: log Y/L)*

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>2.1595</td>
<td>-0.3833</td>
<td>-0.5587</td>
</tr>
<tr>
<td>log(Y/L)_1</td>
<td>0.5146**</td>
<td>0.6334**</td>
<td>0.9401***</td>
</tr>
<tr>
<td></td>
<td>(0.2305)</td>
<td>(0.241)</td>
<td>(0.248)</td>
</tr>
<tr>
<td>hu cap</td>
<td>0.5515*</td>
<td>0.5925**</td>
<td>0.3168</td>
</tr>
<tr>
<td></td>
<td>(0.304)</td>
<td>(0.290)</td>
<td>(0.296)</td>
</tr>
<tr>
<td>invgdp</td>
<td>0.0119*</td>
<td>0.0153***</td>
<td>0.0052*</td>
</tr>
<tr>
<td></td>
<td>(0.0072)</td>
<td>(0.0056)</td>
<td>(0.0035)</td>
</tr>
<tr>
<td>infln</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>-0.0000</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>infvolat</td>
<td>-4.4690**</td>
<td>-2.4689</td>
<td>-0.4856</td>
</tr>
<tr>
<td></td>
<td>(2.321)</td>
<td>(2.447)</td>
<td>(0.350)</td>
</tr>
<tr>
<td>open</td>
<td>0.0027**</td>
<td>0.0084***</td>
<td>0.0075*</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0023)</td>
<td>(0.0039)</td>
</tr>
<tr>
<td>openvolat</td>
<td>-</td>
<td>-1.8016</td>
<td>-2.8683*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.167)</td>
<td>(1.733)</td>
</tr>
<tr>
<td>termstrd</td>
<td>0.0003</td>
<td>0.0109*</td>
<td>-0.0055</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0066)</td>
<td>(0.0065)</td>
</tr>
<tr>
<td>totvolat</td>
<td>-0.0099***</td>
<td>-0.0000</td>
<td>-0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0033)</td>
<td>(0.0014)</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>debtgdp</td>
<td>-0.0010</td>
<td>-0.0012</td>
<td>-0.0019***</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0008)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>life</td>
<td>0.0093</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0074)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>corruptn</td>
<td>0.0263*</td>
<td>-0.0955*</td>
<td>-0.0258</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.047)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>dpndncy</td>
<td>-</td>
<td>0.0097***</td>
<td>0.0131*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0032)</td>
<td>(0.0079)</td>
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<td>J-stat</td>
<td>5.0620</td>
<td>9.6115</td>
<td>4.7089</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.4083)</td>
<td>(0.0870)</td>
<td>(0.4524)</td>
</tr>
</tbody>
</table>

*Notes: Total factor productivity series are calculated from the production-function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.*
The GMM estimate for the human capital elasticity is also similar for all countries (0.55 instead of the earlier 0.61), whereas the coefficients for Africa and Asia are substantially different—much larger for Africa (0.59 against 0.47) and only about half as much for Asia (0.32 against 0.60).

Table 4.14. Panel General Method of Moments Estimates of Output Growth per Worker
(\textit{Dependent Variable: dlog Y/L})

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Africa</th>
<th>Asia</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{constant}</td>
<td>-1.9685***</td>
<td>-2.3992***</td>
<td>-2.0115**</td>
</tr>
<tr>
<td></td>
<td>(0.426)</td>
<td>(0.623)</td>
<td>(0.956)</td>
</tr>
<tr>
<td>\textit{hucap}</td>
<td>0.4286***</td>
<td>0.6429*</td>
<td>0.6777***</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.354)</td>
<td>(0.245)</td>
</tr>
<tr>
<td>\textit{invgcdp}</td>
<td>0.0137**</td>
<td>0.0277**</td>
<td>0.0078*</td>
</tr>
<tr>
<td></td>
<td>(0.0058)</td>
<td>(0.014)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>\textit{infln}</td>
<td>-0.0000</td>
<td>-0.0001**</td>
<td>-0.0000</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>\textit{infvolat}</td>
<td>-1.5747**</td>
<td>-2.5474***</td>
<td>-1.5353</td>
</tr>
<tr>
<td></td>
<td>(0.741)</td>
<td>(0.807)</td>
<td>(1.288)</td>
</tr>
<tr>
<td>\textit{open}</td>
<td>0.0025***</td>
<td>0.0119***</td>
<td>0.0020**</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0022)</td>
<td>(0.0007)</td>
</tr>
<tr>
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<td>0.0026</td>
<td>-0.0040</td>
<td>-0.0018</td>
</tr>
<tr>
<td></td>
<td>(0.0042)</td>
<td>(0.012)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>\textit{debtgdp}</td>
<td>-0.0006**</td>
<td>-0.0013</td>
<td>0.00001</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0009)</td>
<td>(0.0014)</td>
</tr>
<tr>
<td>\textit{corruptn}</td>
<td>-0.0177</td>
<td>-0.0935*</td>
<td>0.0095</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.053)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>\textit{dpndnc}</td>
<td>0.0129***</td>
<td>0.0079**</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
<td>(0.0035)</td>
<td>(0.0059)</td>
</tr>
<tr>
<td>\textit{childmort}</td>
<td>-0.0000***</td>
<td>-0.0000</td>
<td>-0.0000**</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>\textit{J-stat} (p-value)</td>
<td>1.7715</td>
<td>5.4466</td>
<td>3.4526</td>
</tr>
<tr>
<td></td>
<td>(0.621)</td>
<td>(0.142)</td>
<td>(0.327)</td>
</tr>
</tbody>
</table>

Notes: Total factor productivity series are calculated from the production-function estimates (columns 1 and 3) and growth accounting (2). Results are based on data for 40 years, from 1972 to 2011, assembled into eight periods of five years each. Coefficients are significant at the *** 1 percent, ** 5 percent, and * 10 percent levels.
In the GMM model for output growth (Table 4.14), both physical and human capital again have a substantial and significant impact on output growth. Among external factors in growth, openness is highly significant in affecting both the level and growth of output. Volatility of inflation matters for output in the full sample of countries and for output growth in the full sample and in African economies.

**Summary of Results**

Looking at all of the estimates presented in this chapter, it seems that the GMM estimation may be superior because of its fundamental difference from the panel fixed effects. The GMM methodology tries to address the endogeneity issue in a reasonable way, in contrast to the fixed-effects estimates, without the use of instruments. However, in terms of the size and significance of the estimated coefficients for the key variables, both approaches have produced broadly similar results. Investment in human capital seems to be a sure way to increase both TFP and overall output. Sam (2012) reviews many studies on education financing schemes, including providing subsidies to the needy before instituting a policy of charging tuition fees, giving autonomy to educational institutions in the setting of tuition fees, and involving the private sector more in the provision of educational services. These are more micro issues that are beyond the scope of this chapter. The finding in this chapter of a TFP-human capital relationship, however, contrasts with Delpachitra and van Dai (2012), who find no relationship between human capital and TFP growth in the economies of Southeast Asian nations. Our results do match their result with respect to trade and openness, however. In our work, while the two regions of Asia and Africa perform slightly (and in some cases substantially) differently in terms of the magnitude of effects, the increases in physical and human capital, and the more favorable
international conditions, improve macroeconomic situation generally in all developing economies.

It should be noted that in the column for Asia on Table 4.9, a coefficient of the lagged dependent variable larger than unity is most likely indicating an unstable difference equation. While Asian economies grew much faster than those in Africa during the sample period in terms of income and productivity, such economic behavior should not generally lead to a relatively explosive change in TFP. It is worth mentioning that similar results have been reported by various other studies. To solve this problem, Arellano and Bond (1991) suggest using a first-differenced estimator in GMM and using lagged levels of the dependent variable and other endogenous explanatory variables as instruments for the first-differenced equation. Using this approach (as used by Levine and Warusawitharana, 2014; Ziesemer, 2012; Fukase, 2010), when we take the first difference in the GMM specification for Asia, we get a smaller coefficient for the lagged dependent TFP variable (0.63) in contrast to 1.19 reported in Table 4.9. Also, we find that the resulting coefficients of human capital, openness, and inflation are significant with expected signs. However, as Arellano and Bover (1995) and Blundell and Bond (1998) have documented, the difference GMM estimator can have biased and imprecise finite sample properties when the dependent variable is highly persistent (which seems not to be the case in our data). They recommend the use of system GMM estimator that includes both the lags and differences of the dependent variable as instruments in such a situation.

Another point to remember is that the large coefficient of the lagged dependent variable may also indicate that there has been no discernible pattern of conditional convergence of TFP in
Asia during the period of our study (consistent with Di Liberto et al., 2011; Aiyar and Feyrer, 2002; and Edwards, 1998).

On the other hand, for the GDP per worker model (Table 4.13), when we include time fixed effects, the estimated parameter for one period lag changes to 0.70 for Asia as compared to 0.94 that was obtained without the time fixed effects. The resulting coefficients for human capital, openness, and volatility of the terms of trade also have expected signs and are statistically significant.

4.5. Conclusion

This chapter has estimated the effects of openness, trade orientation, and human capital, among other factors, on total factor productivity and output for a pooled cross-section, time-series sample of countries from Africa and Asia, and for the two regions separately. The chapter first estimated two different series of TFP for the pooled and regional samples based on a Cobb-Douglas production function involving output per worker, capital per worker, and the labor force, with and without human capital. Another TFP series was constructed following the growth accounting approach by assuming the share of capital in output equals 0.3. Then, we searched for the possible determinants of TFP and output, with special emphasis on variables reflecting trade orientation and human capital.

The models were estimated for the level and growth of both TFP and output by using panel fixed effects, and they also applied GMM to address endogeneity issues. Several variables related to political, financial, and economic risks were used as instruments together with the lagged values of the dependent and endogenous explanatory variables. This generated a large
The main results show that inducing a greater outward orientation generally boosts TFP. Outward orientation was proxied in terms of share of exports or total trade in GDP, terms of trade, and volatility in the export-GDP ratio. Most of the middle-income and low-income countries instituted trade policy reforms during the sample period to make their economies more open and raise productivity. The significance of a positive influence of outward orientation on TFP growth has important policy implications. The results seem compatible with Berg and Krueger's (2003) assertion that “openness has important positive spillovers on other aspects of reform, so the correlation of trade with other pro-reform policies speaks to the advantages of making openness a primary part of the reform package.”

Among variables other than those that measure outward orientation, the chapter has found that greater accumulation of human capital has a positive effect on TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than interactively with openness. Furthermore, inflation, another domestic economic variable, does not have a significant negative effect on TFP in the full sample or subsamples, although inflation variability is found to adversely affect TFP and output in Africa.

Finally, the social development indicator, life expectancy, positively and significantly affects TFP in Africa. Most poor countries are expected to improve their life expectancy over time, and yet many countries in sub-Saharan Africa experienced an absolute decline during portions of the sample period. We find strong evidence to suggest that a rise in this indicator will raise TFP in Africa.
While the data compilation for this chapter has been a result of much painstaking research into various sources, the availability of Penn World Table Version 8.0 released in 2013 has been most welcome. Capital stock data were reintroduced in this version after having been dropped from several versions in the past. The human capital variable was also introduced in this version. It has been estimated after incorporating different rates of return on various levels of educational attainment. The chapter has also used data from the 2013 International Country Risk Guide for various economic and financial riskiness indicators of countries along with several governance indicators such as corruption, accountability, and stability of the government. Several other economic and social variables from the World Bank’s 2013 World Development Indicators were also included in the dataset. All of this raises confidence in the reliability of the estimates.

Despite efforts to come up with a good model estimated with the most updated data available, there is significant scope for further improvement in terms of a more comprehensive dataset and more estimation methods against which to check the results. Other measures of human capital, measures of policy-based (ex ante) openness, and alternative measures of socio-political variables could potentially be added. As a useful extension, it seems appropriate to combine several different measures of openness used in the literature to estimate a single indicator of outward orientation that would reflect the nature of the protective regime of an economy.\(^{25}\) The policy implication in favor of greater openness and faster human capital accumulation, therefore, needs further empirical verification. With respect to production-function-based TFP estimates, it is possible to go beyond the first approximation of Cobb-

\(^{25}\) Warner (2003) adopts this approach to dispute the claim made by Rodriguez and Rodrik (2000) that the relation between trade policy and economic growth is not robust.
Douglas. What function is best for a given country or a group of countries needs further research.\(^\text{26}\)

Also worthy of mention is the analysis by Ciccone and Jarocinski (2010), who argue that continuous changes in income data and other statistics for countries around the world have led to a significant change in the robustness of a given set of factors affecting income. Among different versions of the Penn World Table, for instance, what looked like a robust relationship between factors when data from Version 6.0 were used turned out to be a very fragile relationship under Version 6.1 and a mixed one under 6.2. Ciccone and Jarocinski (2010) show examples of the changing strength of certain variables in affecting income with each updating of the Penn World Table data.

With regard to estimation methods, several other possibilities exist as well. First, notwithstanding the TFP dynamics implied by coefficients from the GMM estimation for the levels of TFP and GDP, a more careful look at the lag structure of the dependent variables would be desirable in the GMM specification for Asia. It would also be useful to perform a comparative evaluation of results for Asia with those for Africa and the entire sample according to the new specifications. Finally, with respect to convergence issues, it would be interesting to examine whether the 1997-99 Asian crisis and subsequent slowdown in the middle-income countries has repositioned Asian economic growth toward a convergence path in contrast with a diverging trend witnessed before the crisis. Secondly, the use of a maximum likelihood method, the systems approach through 3SLS, the frontier production function to calculate TFP, Bayesian learning averages, and extreme bounds analyses by first distinguishing core and other factors in growth could be attempted to check the robustness of results. Each of these methods has its own

\(^{26}\) A related paper (Miller and Upadhyay, 2003) explores this question in some depth.
strengths and weaknesses, so none are clearly superior to the study here. However, if the results from the current research could be verified further through other techniques, the reliability of these results would clearly increase.
### Appendix 4.1. Variable Definitions and Descriptions

<table>
<thead>
<tr>
<th>Number</th>
<th>Variable Name</th>
<th>Data Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y/L</td>
<td>Penn World Table 8.0</td>
<td>Output per worker (2005 PPP dollars).</td>
</tr>
<tr>
<td>2</td>
<td>LABOR</td>
<td>Penn World Table 8.0</td>
<td>Labor force (in millions).</td>
</tr>
<tr>
<td>3</td>
<td>CAPL</td>
<td>Penn World Table 8.0</td>
<td>Capital per worker (in GDP units). Capital stocks are estimated based on accumulating, and applying depreciation to, past investments using the perpetual inventory method. Then $CAPL$ is obtained by dividing this variable by labor input.</td>
</tr>
<tr>
<td>4</td>
<td>HC</td>
<td>Penn World Table 8.0</td>
<td>Human capital per worker. Average years of schooling and assumed rate of return.</td>
</tr>
<tr>
<td>5</td>
<td>INFLATION</td>
<td>Penn World Table 8.0</td>
<td>Rate of consumer price index inflation.</td>
</tr>
<tr>
<td>6</td>
<td>INFLAVOL</td>
<td>Calculated from Penn World Table 8.0</td>
<td>Standard deviation of inflation.</td>
</tr>
<tr>
<td>7</td>
<td>INVGDP</td>
<td>Penn World Table 8.0</td>
<td>Fixed investment as a percent of GDP.</td>
</tr>
<tr>
<td>8</td>
<td>M2GDP</td>
<td>IMF, World Economic Outlook</td>
<td>Broad-money-to-GDP ratio.</td>
</tr>
<tr>
<td>9</td>
<td>OPEN</td>
<td>Penn World Table 8.0</td>
<td>Total trade as a percent of GDP.</td>
</tr>
<tr>
<td>10</td>
<td>OPENVOL</td>
<td>Calculated from Penn World Table 8.0</td>
<td>Standard deviation of $OPEN$.</td>
</tr>
<tr>
<td>11</td>
<td>TOT</td>
<td>Penn World Table 8.0</td>
<td>Export price to import price (percent)</td>
</tr>
<tr>
<td>12</td>
<td>TOTVOL</td>
<td>Calculated from Penn World Table 8.0</td>
<td>Standard deviation of terms of trade.</td>
</tr>
</tbody>
</table>
### Appendix 4.1. Variable Definitions and Descriptions (continued)

<table>
<thead>
<tr>
<th>Number</th>
<th>Variable Name</th>
<th>Data Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>DEBTGDP</td>
<td>IMF, World Economic Outlook</td>
<td>External debt to GDP (percent).</td>
</tr>
<tr>
<td>14</td>
<td>FBGDP</td>
<td>IMF, World Economic Outlook</td>
<td>Fiscal balance to GDP (percent).</td>
</tr>
<tr>
<td>15</td>
<td>XGDP</td>
<td>Penn World Table 8.0</td>
<td>Exports as a percentage of GDP.</td>
</tr>
<tr>
<td>16</td>
<td>GOVTSTABIL</td>
<td>International Country Risk Guide</td>
<td>Index of government stability (larger more stable). A measure of the government's ability to stay in office and carry out its declared program(s), depending upon such factors as type of governance, cohesion of the government and governing parties, approach toward election, and command of the legislature.</td>
</tr>
<tr>
<td>17</td>
<td>LIFE</td>
<td>World Bank, World Development Indicators</td>
<td>Life expectancy at birth (years).</td>
</tr>
<tr>
<td>18</td>
<td>CHILDMORT</td>
<td>World Bank, World Development Indicators</td>
<td>Under 5 mortality rate per thousand population.</td>
</tr>
<tr>
<td>19</td>
<td>DEPNDNCY</td>
<td>World Bank, World Development Indicators</td>
<td>Dependency ratio (15&lt;age&lt;65/others).</td>
</tr>
<tr>
<td>20</td>
<td>CORRUPTION</td>
<td>International Country Risk Guide</td>
<td>Index of corruption (larger more corrupt). A measure of corruption within the political system that is a threat to foreign investment by distorting the economic and financial environment, reducing the efficiency of government and business by enabling people to assume positions of power through patronage rather than ability, and introducing inherent instability into the political process.</td>
</tr>
</tbody>
</table>
### Appendix 4.1. Variable Definitions and Descriptions (continued)

<table>
<thead>
<tr>
<th>Number</th>
<th>Variable Name</th>
<th>Data Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>DEMOACT</td>
<td>International Country Risk Guide</td>
<td>Democratic accountability index. A measure of not just whether there are free and fair elections, but how responsive government is to its people. The less responsive it is, the more likely it will fall. Even democratically elected governments can delude themselves into thinking they know what is best for the people, regardless of clear indications to the contrary from the people.</td>
</tr>
<tr>
<td>23</td>
<td>ECONRISK TAKING</td>
<td>International Country Risk Guide</td>
<td>Index of economic risk taking Ability. A means of assessing a country's current economic strengths and weaknesses. To ensure comparability between countries, risk components are based on accepted ratios between the measured data within the national economic/financial structure, and then the ratios are compared, not the data. Risk points are assessed for each of the component factors of GDP per head of population, real annual GDP growth, annual inflation rate, budget balance as a percentage of GDP, and current account balance as a percentage of GDP. Risk ratings range from a high of 50 (least risk) to a low of 0 (highest risk), though lowest de facto ratings are generally near 15.</td>
</tr>
<tr>
<td>24</td>
<td>ETHNIC TENSION</td>
<td>International Country Risk Guide</td>
<td>Index of Ethnic Tension (larger more tension). A measure of the degree of tension attributable to racial, national, or language divisions. Lower ratings (higher risk) are given to countries where tensions are high because opposing groups are intolerant and unwilling to compromise.</td>
</tr>
</tbody>
</table>
Appendix 4.1. Variable Definitions and Descriptions (continued)

<table>
<thead>
<tr>
<th>Number</th>
<th>Variable Name</th>
<th>Data Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>EXTLCONFLICT</td>
<td>International Country Risk Guide</td>
<td>Conflict with neighboring countries. A measure of the risk to the incumbent government and to inward investment, ranging from trade restrictions and embargoes through geopolitical disputes, armed threats, border incursions, foreign-supported insurgency and full-scale warfare.</td>
</tr>
<tr>
<td>26</td>
<td>FINRISKTAKING</td>
<td>International Country Risk Guide</td>
<td>Index of financial risk taking ability. A means of assessing a country’s ability to pay its way by financing its official, commercial and trade debt obligations. To ensure comparability between countries, risk components are based on accepted ratios between the measured data within the national economic/financial structure, and then the ratios are compared, not the data. Risk points are assessed for each of the component factors of foreign debt as a percentage of GDP, foreign debt service as a percentage of exports of goods and services ($XGS$), current account as a percentage of $XGS$, net liquidity as months of import cover, and exchange rate stability. Risk ratings range from a high of 50 (least risk) to a low of 0 (highest risk), though lowest de facto ratings are generally near 20.</td>
</tr>
</tbody>
</table>

Chapter 5

Summary and Conclusions

Human capital expansion and labor migration from villages to cities are aspects of the labor market structure in poor countries that are continuously influenced by public policies. In many developing countries, however, either public policies are ineffective in producing economic growth or they have unintended adverse consequences. The existence of significant unemployment levels among educated workers indicates low marginal productivity of resources devoted to education in a typical developing country. Yet the expansion of education continues to be a major objective of social policy. Human capital, especially that acquired in liberal arts education, has expanded in response to large public subsidies in much of the developing world. Human resource policy in these countries is mainly directed toward increasing the supply of educated labor, which has raised unemployment. Such a labor market imbalance therefore requires new approaches in government policies and interventions.

In addition, government attempts to establish or enhance modern sectors of the economy have led to large public investments in physical and social infrastructure in cities, such as roads and transportation. However, this belief in the potential for productivity expansion in modern sectors appears to have generated an overenthusiastic response, as governments allocate disproportionately large fractions of their budgets to infrastructure in and around cities (as opposed to villages). This “urban bias” has created a wide gap in public services between rural and urban areas in most countries.
The purpose of the Chapters 2 and 3 in this dissertation has been to analyze the effects of public policies on rural-urban migration and human capital expansion, while Chapter 4 has examined the role of human capital and outward orientation in the growth of total factor productivity and real per capita income in developing countries of Africa and Asia. These chapters have analyzed the outcomes of human resource and other government policies, and offered suggestions on how the policies might be improved.

Chapter 2 extended a two-sector general equilibrium model of rural-urban migration to include government spending. Provision of public goods acts as a productivity-enhancing input in private production, which results in external economies of scale. This approach is generalized by introducing unbalanced allocation of public expenditure in rural and urban sectors due to political economy considerations, different sectoral output elasticities with respect to government input, and taxation on urban output. The chapter studied the effects of an increase in public spending and taxation on sectoral outputs, factor prices, urban unemployment, and welfare. Of particular concern here is to study the effect of an unbalanced allocation of government spending between rural and urban areas.

Chapter 2 presented two dual-economy models of migration between rural and urban sectors of a developing economy with an explicit government budget constraint. In addition to capital and labor, government expenditure serves as an input in the production process that brings forth external economies. The expenditure must, however, be financed through a tax on urban output. The chapter rules out rural taxation because of real-world evidence that such taxation does not directly contribute much to government revenue, and in order to keep the model used here sufficiently simple.
The first model is the neoclassical general-equilibrium model of two sectors with full employment of labor in both. The second model (presented in Section 2.6) retains external economies of scale in public expenditure but augments the original model to consider unemployment in the urban sector. Migration from overpopulated villages to cities occurs as a response to a higher urban wage and stops when the rural wage equals the expected urban wage that includes the effect of urban unemployment.

Each of the two models analyzes two important cases. They examine what happens to output, employment, and urban unemployment when the government varies the fraction of total expenditure going to the two sectors while still satisfying the budget constraint. A basic assumption of the model is that the urban sector uses more capital per unit of labor than does the rural sector; that is, the urban output is relatively capital-intensive.

In the full-employment version of the model, the effects of an increase in the share of government budgetary allocation to the rural sector represent an interesting case study. This is so because of the evidence that public expenditure suffers in much of the developing world from an urban bias, while an enormous potential to raise productivity in rural agriculture exists but goes untapped. The chapter finds that a higher expenditure share going to the rural sector leads to three distinctly possible results. First, assuming that public goods contribute more to the productivity of capital and labor in rural than in urban areas, rural output increases and so does national output when the urban sector is not very large. Second, if employment and output in the urban sector are large enough, the reduction of the tax base that follows from a decrease in the urban share of public expenditure and hence urban output can lead to a decrease in overall national output as well. Third, the last result may not hold (and rural and total output can still
rise) if government services contribute sufficiently more to the change in rural output than to urban output.

Second, the chapter analyzes the effects of an increase in the tax on urban output. An urban tax increase, holding expenditure shares constant, will reduce urban output but will raise both rural and urban expenditures. Thus, rural output invariably increases unless government services yield a marginal productivity of zero. This also leads to a reduction in out-migration from rural to urban areas, and can cause reverse migration from urban to rural areas. The effect on urban output is, however, ambiguous. Under the condition that scale economies of greater public expenditure on urban output dominate the contractionary effect of the tax increase, urban manufacturing will expand. However, the chapter also finds several mixes of parameters that yield a loss of urban output, a reduction in the tax base, and an overall decline in total output and welfare in the country. The result on the direction of migration similarly depends on specific values of the parameters in the model.

The second model used is the Harris-Todaro model augmented in terms of public expenditures. The first point to note here is that the stability condition (the Khan-Neary condition) that shows the stability of the model in employment-adjusted terms in the conventional Harris-Todaro version is now more stringent in this augmented version.

One main result of this second model is that an increased allocation of public spending to the rural sector can still increase the proportion of the urban population employed. This results under the condition that the negative returns-to-scale effect on urban output due to the urban expenditure cut is more than offset by increased labor productivity in urban employment when the level of urban employment falls. The positive employment effect in the urban sector arises from the positive output effect in the rural sector when rural spending is greater. This leads to a
rise in the rural wage and induces reverse migration from the urban sector. If this rising urban productivity more than offsets the output effect of the public expenditure decline, the urban employment rate increases and the unemployment rate falls.

The welfare effects in the unemployment model are subject to a combined influence of a set of elasticities and go different ways depending on the relative influence of composite parameters of the model.

Finally, the results of the model indicate several directions for further research, two of which look especially promising. First, since poor countries typically have a large rural sector, governments are inclined to impose a tax on this sector as well in various ways. Second, the implications of government expenditure can be drastically different and much more positive on national output and welfare if there is a significant inflow of foreign aid or concessional loans. While its qualitative results are explored and discussed in Chapter 2, it would be highly interesting to determine their effects more concretely as well.

The models can be extended to include the dynamic versions following Mas-Colell and Razin (1973). The growth aspects of the models can be studied in a dynamic setting. A simple calibration exercise using the data on rural-urban migration and output and income shares from India, China, Ghana, and many other South Asian and sub-Saharan economies where rural-urban migration is an important aspect of the labor market will help to get estimates on the costs of dualism. Also following Bhagwati and Srinivasan (1974) and Krichel and Levine (1999), it would be worthwhile to calibrate the model for policy rankings and welfare implications in a dual-economy framework. Another interesting study for Africa and Asia would be to analyze the determinants of migration, incorporating human capital, housing stock, health care, road
provision, and other public services into the Harris-Todaro framework, following an empirical study by Ghatak, Mulgern, and Watson (2008).

Chapter 3 then studied the effects of selected education policies on the size of the educated labor pool and on economic welfare using the “job ladder” model of education, which is relevant to liberal arts education in developing countries. The policies considered were increasing the teacher-student ratio, raising the relative wage of teachers, and providing a larger direct subsidy to education. In one of the important findings, an increase in the teacher-student ratio raises the marginal social cost of education and reduces the gap between the private and social marginal products of education.

Second, a reduction in the costs of schooling, which is equivalent to a policy of higher education subsidy, lowers the opportunity cost of education and increases the difference between private and marginal social costs of education. In each of these cases, the result is an increase in the total size of the educated labor force, but a reduction in the level of economic welfare. Also, an extension of the model to include private tutoring practices shows that despite an increased government subsidy, the size of the educated labor force is smaller than intended.

Empirical findings by both professional educators and education economists suggest that the advocacy of simple expansion of education without quality improvement is misguided. In many developing countries such as India, however, labor markets for the educated are still characterized by wage stickiness, fairness-in-hiring rules, and the lack of a clear correlation between productivity and education quality. Thus, under the present conditions that exist in South Asian countries, education quality reforms will fail to integrate higher education, production, and welfare in a desirable way. For higher education, some recent country studies suggest that in addition to providing government resources, public-private partnerships help
design innovative programs not only for improving the quality of education, but also making it accessible to many in developing countries (Patrinos, Felipe, and Gauqueta, 2009). A good extension of the model in Chapter 3 would be to analyze the impact of higher education on rural-urban migration, rural-urban wage inequality, and total factor productivity (TFP), depending on the availability of data from sub-Saharan Africa, as well as developing Asia. Another interesting study would be to explore the link between education and other human capital variables such as fertility and life expectancy.

Chapter 4 also examines the role of human capital but now in the context of overall economic growth and growth of TFP. This is empirical research aimed at identifying the importance of several domestic and external economic factors, as well as human capital, in long-term growth. Empirical evidence suggests that poor but growing nations can rely on accumulation of capital per worker for significant periods of time while they are on the path to steady-state growth. Once they get closer to their production frontier, however, much of the growth in per capita income must depend on technological progress, or “TFP growth,” which summarizes changes in a large number of economic and noneconomic factors in addition to direct improvements in production technology.

TFP accounts for a large fraction of output growth in those parts of the world that have experienced modest to high growth. The accumulation of the basic factors of production such as capital per worker contributes significantly to output growth as well. Yet, while factors such as the rate of savings are commonly understood to affect capital accumulation, there is much less consensus about the exact factors that influence TFP. This has led to a burst of economic research over the last 20 years into causes underlying TFP growth.
Availability of a comparable set of cross-country income data, particularly the Penn World Table, has facilitated such cross-country growth exercises. Yet most such datasets have lacked capital stock data in the past for many developing countries. Consistent series for variables that seem closely related to TFP in theory have also been difficult to obtain for long enough time periods, particularly for developing countries. With the availability of Version 8.0 of the Penn World Table, a more consistent and accurate capital stock series can now be used, as is done in Chapter 4.

The chapter starts with a discussion of the main empirical literature on TFP as a base case and makes three specific improvements. First, many papers have studied TFP for large panels of countries by including in the same pool developing and developed countries and countries from all geographic regions. Chapter 4 focuses on developing countries alone because pooling developing and developed countries in a single dataset is equivalent to making a relatively unrealistic assumption that all countries share the same technology. Second, the model estimation strategy extends other studies by adding observations and updating the dataset up to and including 2011, which gives a relatively long period of data for 40 years (which are grouped into eight periods of five years each for all countries in the sample). Third, a comparative assessment is also made of findings through the use of alternative methods, in particular the panel fixed effects and the generalized method of moments, to examine whether endogeneity problems really mar the fixed-effects results.

Chapter 4 uses alternative ways to estimate the contribution of TFP to the overall growth of an economy. One approach first estimates aggregate output as a function of the basic factors of production, namely labor (L) and physical capital (K). Another method, called growth accounting, uses fixed shares of capital and labor in output. After the contribution of these
factors has been determined, the resulting unexplained part of output is TFP, which can be studied further by examining its determinants. The chapter uses data for 20 countries—10 each from Africa and Asia—and for 40 years for all the variables. The chapter examines the differences in growth rates between countries in Africa with those in Asia by estimating the contribution of TFP to economic growth and relating the TFP differences to their underlying causes. A focus on Africa also helps in understanding why this region has failed to move on the path of convergence in per capita income with the rest of the world.

After calculating a measure of TFP from an aggregate production function, the chapter explores factors that influence the level and growth of TFP, especially openness, trade orientation, terms of trade, and external indebtedness among external factors, as well as others such as education and health-related variables (namely, human capital, life expectancy, and child mortality); inflation; a demographic factor (the dependency ratio); and governance indicators such as corruption, government accountability, and stability. Results of this setup are then compared with those from an alternative but popular framework—growth accounting—following Bernanke and Gurkaynak (2001) and Crafts (1999), where the share of physical capital in output, $\alpha$, is assumed, as in much of the related literature, to be 30 percent for all countries in the sample.

Another approach to the study of TFP is to directly estimate the per capita income growth by including basic inputs in production as well as other factors that influence growth. These other factors are assumed to affect income growth through TFP growth. Chapter 4 estimates the effects of human capital, openness, and trade orientation, among other factors, on the level and growth of TFP and also the level and growth of output for a pooled cross-section, time-series sample of countries from Africa and Asia, and for the two regions separately.
The models for the level and growth of both TFP and output are estimated by using panel fixed effects, and by applying the generalized method of moments to address possible problems with endogenous explanatory variables. Several feasible instruments are identified to address endogeneity. These include variables related to political, economic, and financial riskiness of a country together with the lagged values of the dependent variable as well as endogenous explanatory variables. This procedure generates a large number of moment conditions relative to the parameters being estimated. The resulting overidentifying restrictions can be tested through the Hansen $J$-statistics. The chapter finds the results of such an exercise to be reasonably satisfactory and the instruments used fairly valid.

The main results show that inducing a greater outward orientation generally boosts TFP. Most of the middle-income and low-income countries instituted trade policy reforms during the sample period to make their economies more open and to raise productivity. The significance of a positive influence of outward orientation on growth of TFP has important policy implications.

Among variables other than those that measure outward orientation, greater accumulation of human capital is found to have a positive effect on TFP growth in both Africa and Asia. Its positive influence comes rather independently of trade variables than through interaction with openness. Furthermore, inflation, another domestic economic variable, does not have a significant negative effect on TFP in the full sample or subsamples. although inflation variability is found to adversely affect TFP and output in Africa.

Finally, life expectancy, the social development indicator in the model, positively and significantly affects TFP in Africa. Most poor countries are expected to improve their life expectancy over time and yet many countries in sub-Saharan Africa experienced an absolute
decline in life expectancy during portions of the sample period. There is strong evidence to suggest that a rise in this indicator will raise TFP in Africa.

While the data compilation for Chapter 4 has been a result of much painstaking search into various sources, there is significant scope for further improvement of data quality and the updating of statistics. Further, even meticulous research into growth factors hardly seems to offer conclusive evidence about how best to accelerate economic growth. As Deaton (2010) says about the difficulty in identifying specific policies, including those that relate to the external sector, that could help poor countries to achieve faster growth: “[E]mpiricists and theorists seem further apart now than at any period in the last quarter century. Yet reintegration [to the world economy] is hardly an option because without it there is no chance of long-term scientific progress” (as quoted by Lin, 2012, p. 31).

Chapter 4 uses Cobb-Douglas as the first approximation for the actual production functions for countries in the sample. What function is best for a given country or a group of countries needs further research.

As a useful extension, it would be appropriate to combine several different measures of openness used in the literature to estimate a single indicator of outward orientation that would reflect the nature of the protective regime of an economy. Another valuable exercise would be to derive the TFP growth estimates using nonparametric estimation methods (Pagan and Ullah, 1999) and compare the results with those derived from conventional TFP estimates. Also, it is worth exploring whether there is a clear nexus between human capital accumulation and public spending on education and health as they contribute to the growth of TFP, since it is possible, as explored in an earlier chapter, for parts of human capital to remain unutilized.
Understanding various effects of policy interventions will provide useful policy insights for improving the effectiveness of government expenditures in enhancing economic growth. From a long-term perspective, targeting government expenditure merely to reduce poverty is not sufficient. Public expenditure must also stimulate economic growth in order to generate resources needed for future expenditures to support growth and provide a self-sustained and lasting solution to the problem of the high incidence of poverty. Public expenditure allocation among rural and urban areas should aim at enhancing growth and reducing regional disparities.


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