Modelling element abundances in semi-analytic models of galaxy formation

Robert M. Yates,1* Bruno Henriques,1 Peter A. Thomas,2 Guinevere Kauffmann,1 Jonas Johansson1 and Simon D. M. White1
1Max Planck Institut für Astrophysik, Karl-Schwarzschild-Str. 1, D-85741, Garching, Germany
2Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH, UK

ABSTRACT
We update the treatment of chemical evolution in the Munich semi-analytic model, L-GALAXIES. Our new implementation includes delayed enrichment from stellar winds, Type II supernovae (SNe-II) and Type Ia supernovae (SNe-Ia), as well as metallicity-dependent yields and a reformulation of the associated supernova feedback. Two different sets of SN-II yields and three different SN-Ia delay-time distributions (DTDs) are considered, and 11 heavy elements (including O, Mg and Fe) are self-consistently tracked. We compare the results of this new implementation with data on (a) local, star-forming galaxies, (b) Milky Way disc G dwarfs and (c) local, elliptical galaxies. We find that the $z=0$ gas-phase mass–metallicity relation is very well reproduced for all forms of DTD considered, as is the [Fe/H] distribution in the Milky Way disc. The [O/Fe] distribution in the Milky Way disc is best reproduced when using a DTD with $\leq50$ per cent of SNe-Ia exploding within $\sim400$ Myr. Positive slopes in the mass–[$\alpha$/Fe] relations of local ellipticals are also obtained when using a DTD with such a minor ‘prompt’ component. Alternatively, metal-rich winds that drive light $\alpha$ elements directly out into the circumgalactic medium also produce positive slopes for all forms of DTD and SN-II yields considered. Overall, we find that the best model for matching the wide range of observational data considered here should include a power-law SN-Ia DTD, SN-II yields that take account of prior mass-loss through stellar winds and some direct ejection of light $\alpha$ elements out of galaxies.

Key words: methods: analytical – Supernovae: general – Galaxy: abundances – galaxies: abundances – galaxies: evolution.

1 INTRODUCTION
Significant progress has been made in the field of galactic chemical evolution (GCE) since the first postulation of stellar nucleosynthesis by Arthur Eddington in the 1920s (Eddington 1920). The first techniques to determine element abundances in both gas (e.g. Aller 1942) and stars (e.g. Chamberlain & Aller 1951) were developed, and the theory of stellar nucleosynthesis was given a more formal footing by Burbidge et al. (1957). Later, more sophisticated studies of GCE were stimulated by the celebrated review by Beatrice Tinsley (Tinsley 1980). Now, it has been determined that a galaxy’s metallicity is related to its luminosity (e.g. Lequeux et al. 1979), age (e.g. Edvardsson et al. 1993, but see e.g. Friel 1995) and stellar mass (e.g. Tremonti et al. 2004), and that different types of stars contribute to GCE in different ways (e.g. Salaris & Cassisi 2005).

However, many questions relating to the cosmic abundances of heavy elements still remain. For example, it is still unclear what exact role different types of supernovae (SNe) and stellar winds play in the chemical enrichment of galaxies (e.g. McWilliam 1997), what the shape and universality of the stellar initial mass function (IMF) is (e.g. Bastian, Covey & Meyer 2010), how best to model the metal yields produced in stars (e.g. Romano et al. 2010) and what the progenitors and delay times of SNe-Ia are (e.g. Maoz & Mannucci 2012). These are important questions for us to address, as the chemical evolution of galaxies plays a key part in the evolution of galaxies in general; the presence of metals affects the cooling of gas (e.g. Sutherland & Dopita 1993), the formation of stars (e.g.Walch et al. 2011), stellar evolution (e.g. Salaris & Cassisi 2005) and the yields of newly synthesized metals (e.g. Woosley & Weaver 1995) which are released into the interstellar medium (ISM), circumgalactic medium (CGM) and even the intergalactic medium (IGM).

Aside from the ongoing observational studies into these questions, galaxy evolution models incorporating sophisticated GCE
modelling also provide an opportunity to further constrain the chemical evolution of galaxies. Many previous works have focused on reproducing the chemical signatures found in the solar neighbourhood (e.g. Tinsley 1980; Matteucci 1986; Matteucci & Greggio 1986; Thomas, Greggio & Bender 1998; François et al. 2004; Calura & Menci 2009; De Rossi et al. 2009; Calura et al. 2010, 2012; Romano et al. 2010; Pilkington et al. 2012; Tissera, White & Scannapieco 2012), chiefly in order to constrain the contributions from different types of SNe and stellar winds. Many others have focused on the chemical properties of local elliptical galaxies (e.g. Matteucci 1994; Thomas & Kauffmann 1999; Thomas, Greggio & Bender 1999; Pipino & Matteucci 2004, 2011; Nagashima et al. 2005; Calura & Menci 2009, 2011; Pipino et al. 2009a,b; Arrigoni et al. 2010a), chiefly to try to reconcile the observed positive slope in the relation between stellar mass ($M_\ast$) and $\alpha$ enhancement ([α/Fe]) with our theoretical understanding of metal production and galaxy formation.

The aim of this work is to address both of these issues, using a new implementation of detailed chemical enrichment in the Munich semi-analytic model of galaxy formation. We investigate if the chemical properties of Milky Way (MW) disc stars and local elliptical galaxies can be simultaneously obtained with a self-consistent model which assumes a Λ cold dark matter (ΛCDM) hierarchical merging scenario and varied star formation histories (SFHs). We also compare different SN-II yield sets and SN-Ia delay-time distributions (DTDs) to see which allow us to best match the observational data considered.

This paper is structured as follows: in Section 2, we give a general outline of the Munich semi-analytic model, L-GALAXIES. In Section 3, we describe the stellar yields, lifetimes and IMF used as inputs to our model. In Section 4, we describe the basic equations required to model GCE and discuss the SN-Ia DTD. In Section 5, we explain how this GCE model is implemented into the larger semi-analytic model and review the key physical processes governing the distribution of metals throughout galaxies. In Section 6, we discuss our model results for the chemical composition of (a) local, star-forming galaxies, (b) the G dwarfs of the MW disc and (c) the stellar components of local ellipticals, and compare these results to the latest observations. We conclude our work in Section 7.

2 THE SEMI-ANALYTIC MODEL

L-GALAXIES (Springel et al. 2001; De Lucia, Kauffmann & White 2004; Springel et al. 2005; Croton et al. 2006; De Lucia & Blaizot 2007; Guo et al. 2011, 2013; Henriques et al. 2013) is a semi-analytic model of galaxy evolution which extends the methods set out in White & Frenk (1991), Kauffmann, White & Guideroni (1993) and Kauffmann et al. (1999) so that the model can be run on sub-halo trees built from DM N-body simulations such as the Millennium (Springel et al. 2005). Galaxy evolution is governed by the transfer of mass among the various components of a galaxy (disc stars, bulge stars, halo stars, cold gas, hot gas, central black hole and ejecta reservoir), according to physical laws motivated by observations and simulations. L-GALAXIES is currently able to reproduce the large-scale clustering of galaxies, the Tully–Fisher relation and the optical colours, stellar mass function and gas-phase mass–metallicity relation observed in the local Universe. The model can also reproduce the abundance of galaxies as a function of stellar mass or luminosity out to $z = 3$. Analytical treatments of gas stripping and tidal disruption of satellites, as well as SN and AGN feedback are included (see Guo et al. 2011). The processes already included in L-GALAXIES that are of most relevance to this work are reviewed briefly in Section 5.3.

Prior to this work, L-GALAXIES included a simple GCE implementation. A fixed metal yield of 0.03Δ$M_\odot$ was assumed to be ejected into the ISM immediately after a star formation event, where Δ$M_\odot$ is the mass of stars formed at that time. A further 40 per cent of Δ$M_\odot$ was assumed to return immediately to the gas phase as H and He. Such an ‘instantaneous recycling approximation’ is often used in galaxy-formation models for its simplicity, but does not adequately describe the delayed enrichment of metals, particularly from long-lived low- and intermediate-mass stars and SNe-Ia. Previously, L-GALAXIES also did not consider individual chemical elements, but instead tracked only the total metal mass in each galaxy component. The tracking of individual elements allows us to compare with more detailed observational data on the chemical composition of the MW and other galaxies (see Section 6). For example, the ratio of $\alpha$ elements to iron is believed to be a good indicator of the star formation time-scale. A comparison of [α/Fe] between real galaxies and model galaxies with known SFHs will allow us to test this. Also, in future, tracking individual elements will provide a more realistic treatment of gas cooling, which depends not only on the total metallicity, but also on the relative abundance of different heavy elements, as well as the ultraviolet background radiation.

The model parameters we use in this paper are identical to those in Guo et al. (2011), with the exception of the ‘halo-velocity-dependent SN energy efficiency’, $\epsilon_v$, which we have increased in order to maintain the same total SN feedback energy that was used previously (see Section 5.3.3).

3 GCE INGREDIENTS

In order to model the chemical evolution of galaxies, we first need to know the total mass of heavy elements liberated from stars at any given time. To do this, we need to know (a) how many stars eject metals at that time and (b) how much of each element they eject. The former is given by the assumed stellar lifetimes, the IMF and the SFHs of galaxies. The latter is given by the stellar yields, obtained from stellar evolution models. The element ‘yield’ of a star is defined as the mass of that element that is synthesized and ejected (Tinsley 1980). If an element undergoes a net destruction during stellar nucleosynthesis (e.g. hydrogen), then its yield will be negative, whereas the mass of the element ejected will not.

The yields, as well as depending on the initial mass (and metallicity) of the star, also depend on the mode of ejection. We consider three modes in this work: stellar winds from low- and intermediate-mass stars during their thermally pulsing asymptotic giant branch phase (TP-AGB, or simply AGB phase), SNe-Ia from some intermediate-mass binary systems and the SN-II explosions of massive stars. Each of these three modes releases a different set of heavy elements at different times. Long-lived stars of mass $0.85 < M/\odot < 7$ release mainly H, C and N. SNe-Ia produce and eject mainly Fe and other iron-peak elements, whether they originate from single degenerate binaries (Whelan & Iben 1973), double degenerate binaries (Iben & Tutukov 1984; Webbink 1984) or otherwise (e.g. the binary progenitors of double-detonation, sub-Chandrasekhar-mass explosions; see Ruiter et al. 2011). Finally, short-lived stars of mass $\geq 7\odot$ explode as core-collapse SNe-II, ejecting chiefly $\alpha$ elements (e.g. O, Ne, Mg, Si, S and Ca).

We note here that we only consider 11 chemical elements in our GCE model, namely H, He, C, N, O, Ne, Mg, Si, S, Ca and Fe, as these elements are included in all of the yield sets we consider.
The following sub-sections outline in more detail these key ingredients for galactic chemical enrichment. The SFHs of galaxies are tracked self-consistently in our semi-analytic model and are discussed in Section 5.1.

3.1 The IMF

The IMF, \( \phi(M) \), is a probability density function, which tells us the fraction of stars in a 1 M_\odot simple stellar population (SSP) that are within a given mass range. It is obtained from the observable present-day mass function (PDMF) of field stars in the MW, or from PDMF indicators in extragalactic regions. In this work, we assume that the IMF is the same in all regions of space and does not evolve with time. There are, however, currently conflicting conclusions in the literature as to its universality (e.g. Elmegreen 2006; Weidner & Kroupa 2006; Bastian et al. 2010; van Dokkum & Conroy 2010; Fumagalli, Da Silva & Krumholz 2011; Gunnawardhana et al. 2011; Conroy & van Dokkum 2012b).

The IMF used in this work is taken from Chabrier (2003). This version is commonly used in chemical enrichment models, and is already utilized in L-GALAXIES via the stellar population synthesis (SPS) models of Bruzual & Charlot (2003) and Maraston (2005). Its use therefore provides both a good comparison to other works and self-consistency within the code. The Chabrier IMF is given analytically as

\[
\phi(M) = \begin{cases} 
A_\phi M^{-1} e^{-(\log M - \log M_\phi)^2/\sigma^2} & \text{if } M \leq M_\phi \\
B_\phi M^{-2.3} & \text{if } M > M_\phi
\end{cases}
\]

where \( M_\phi = 0.079 \) M_\odot and \( \sigma = 0.69 \). The values of the coefficients \( A_\phi \) and \( B_\phi \) are determined by requiring that (a) the overall function is continuous and (b) the IMF by mass is normalized to 1 M_\odot over the full mass range of stars considered;

\[
\int_{M_{\text{min}}}^{M_{\text{max}}} M \phi(M) \, dM = 1 \text{ M_\odot},
\]

where \( M_{\text{min}} = 0.1 \) M_\odot. When assuming \( M_{\text{max}} = 120 \) M_\odot, as we do in this work, the coefficients in equation (1) are \( A_\phi = 0.842 \) 984 and \( B_\phi = 0.235 \) 480.

Once normalized to the total mass of the SSP, equation (1) can be integrated over a certain mass range to tell us the number density \( n = N/V \) of stars in that mass range. Assuming that the IMF is the same everywhere, this is equivalent to the number of stars in a 1 M_\odot SSP in a given mass range. This integrated, normalized IMF has units of 1 M_\odot.

The Chabrier IMF predicts fewer stars of mass <1 M_\odot than the Salpeter (1955) IMF, and does so with a smoother transition than the multisegment power-law Kroupa (2001) IMF. At masses above 1 M_\odot, it has the same slope as the Kroupa IMF (an exponent of \(-2.3\) in linear mass units, rather than the \(-2.35\) used for the Salpeter IMF).

3.2 Stellar lifetimes

We adopt the metallicity-dependent lifetimes tabulated by Portinari et al. (1998, hereafter P98), kindly provided by Wiersma (Wiersma et al. 2009b). These lifetime tables, along with the yields tabulated by P98 for SN-II and the AGB yields considered here, were also used by the Overwhelmingly Large Simulations (OWLS) project for their cosmological, hydrodynamical simulations of galaxy evolution (Wiersma, Schaye & Britton 2009a; Wiersma et al. 2009b; Schaye et al. 2010). The P98 lifetime tables account for stars in the mass range \(0.6 \leq M/M_\odot \leq 120\) and five different initial metallicities, as predicted by Portinari, Chiosi & Bressan (1998).

3.3 AGB wind yields

We adopt the metallicity-dependent yield tables of Marigo (2001, hereafter M01) for low- and intermediate-mass stars, also kindly provided by Wiersma (Wiersma et al. 2009b). These stars eject their metals predominantly through stellar winds during their AGB phase, although total yields from the red giant branch and AGB phases combined are included in the M01 tables. For simplicity, we refer to these as ‘AGB wind’ yields hereafter. The SN-II yield tables of P98, which we also use, form a complete set with those of M01 for AGB winds. They are both based on the same Padova evolutionary tracks and do not require a large interpolation between them, as the AGB yields consider stars up to 5 M_\odot and the SN-II yields consider down to 7 M_\odot. In this work, we consider the ejecta from AGB winds to occur at the end of a star’s lifetime.

Fig. 2 shows the ejected mass of metals (top-left panel), total baryons (top-right panel) and individual elements (bottom two panels) from AGB stars as a function of initial mass. This is different from the yield, as it includes both the mass that passes through the stars unprocessed and any newly synthesized material. The element abundances of the Sun from Asplund et al. (2009) are used to scale the amplitudes of the curves in Fig. 2.

1 We note that it is also possible to link the M01 AGB yields to the P98 SN-II yields at 6 M_\odot, by including the P98 yields for electron-capture SNe (see P98, section 4.2). Doing this makes a negligible difference to the results discussed in this work.
Modelling element abundances

Figure 2. Mass released by AGB winds from the Marigo (2001) yield tables. Points indicate values from the yield tables. Solid lines indicate the interpolation used between these points. Dashed lines indicate extrapolations beyond the masses originally modelled. Top left: the mass of metals ejected as a function of mass, for three different initial metallicities. Top right: the total baryonic mass ejected as a function of mass, for three different initial metallicities. Bottom left: the mass of each element ejected as a function of mass, for stars of $Z_0 = 0.004$. Bottom right: same as bottom left, for stars of $Z_0 = 0.019$.

We note that no elements heavier than oxygen present in the wind have been synthesized or destroyed in the AGB stars, but have instead been formed in previous generations of stars and pass through the AGB stars unprocessed. We have extrapolated the AGB wind yields from 5 to 7 $M_\odot$, so that they meet with the SN-II yields used. The exact position of this interface within the region $5 < M/M_\odot < 8$ does not significantly affect our results.

3.4 SN-Ia yields

As with many other chemical enrichment models, we adopt the spherically symmetric ‘W7’ model for our SN-Ia explosive yields, originally tabulated by Nomoto, Thielemann & Yokoi (1984). We use a more recent iteration by Thielemann et al. (2003, hereafter T03). These tables provide the synthesized mass of 42 different element species. Unlike the AGB and SN-II yields, the SN-Ia yields used here are independent of the initial mass and metallicity of the progenitor system. The total mass ejected in an SN-Ia is assumed to be 1.23 $M_\odot$, the sum of the ejecta from the 11 elements considered in this work. As no H or He is ejected by SNe-Ia, this sum equals the mass of metals ejected. Fig. 3 shows the ejected mass of each element. Iron is the most abundant, while there are also non-negligible amounts of oxygen, silicon and nickel.

SN-Ia yields that depend on the initial mass and metallicity of the progenitors are now also available in the literature (e.g. Seitenzahl et al. 2013). We defer a study of the effect of such yields on our GCE model to future work.

Figure 3. The mass of each element ejected from SNe-Ia, according to the tabulation of Thielemann et al. (2003). Coloured circles represent elements that are considered in this work.

Rather than making assumptions about the type and lifetimes of the progenitor systems involved, we instead use observationally motivated DTDs to define the lifetimes of SN-Ia progenitors (see Section 4.1).
Figure 4. Mass released by SNe-II from the P98 yield tables. Points indicate values from the yield tables. Solid lines indicate the interpolation used between these points. Top left: the mass of metals ejected as a function of mass, for five different initial metallicities. Top right: the total baryonic mass ejected as a function of mass, for five different initial metallicities. Bottom left: the mass of each element ejected as a function of mass, for stars of $Z_0 = 0.004$. Dashed lines indicate the corrected C, Mg and Fe yields (see the text). Bottom right: same as bottom left, for stars of $Z_0 = 0.02$.

3.5 SN-II yields

Our preferred set of SN-II yields is tabulated by P98, and also kindly provided by Wiersma (Wiersma et al. 2009b). This set contains yields for initial masses ranging from 6 to 1000 $M_\odot$, and five initial metallicities from 0.0004 to 0.05. We only consider the existence of stars up to 120 $M_\odot$ here. Even then, the range provided by the P98 yields is significantly wider than, for example, the more commonly used yields of Woosley & Weaver (1995), which only go up to 40 $M_\odot$.\footnote{Unlike the tables of Woosley & Weaver (1995), both sets of SN-II yield tables considered in this work account for the decay of nickel into iron shortly after the SN. P98 do so by simply adding the $^{56}$Ni yield to that of $^{56}$Fe, and Chieffi & Limongi (2004) by only tabulating yields $10^8$ s after the explosion.} The P98 set also takes account of mass-loss through winds prior to the SN. The inclusion of prior mass-loss also affects the composition of the explosive yields, as we explain below.

Fig. 4 shows the ejected mass of metals (top-left panel), total baryons (top-right panel) and individual elements (bottom two panels) from SNe-II as a function of initial mass, as is done for AGB winds in Fig. 2. The dashed lines in Fig. 4 indicate corrections to the C, Mg and Fe yields that we include in our model, following the recommendation of Wiersma et al. (2009, see their section A3.2). These ad hoc corrections can be justified by uncertainties in the explosive yields tabulated by Woosley & Weaver (1995), on which the P98 SN-II yields are based. These corrections halve the yield of C and Fe and double the yield of Mg, relative to the originally tabulated values.

We note that the P98 yields show some sudden drops in the ejecta of certain elements. At low metallicities, the reduction in yield of the heaviest elements above $\sim 30 M_\odot$ is due to them being locked in the stellar remnant. Remnant masses increase significantly above $\sim 30 M_\odot$ at low metallicities due to low mass-loss efficiency prior to the SN. This effect is less severe for lighter elements, such as oxygen, as ‘pair creation’ SNe are believed to dominate over ‘core collapse’ SNe above $\sim 60 M_\odot$, allowing more of these elements to be ejected. At higher metallicities, more efficient mass-loss prior to the SN inhibits large remnant formation. Increased mass-loss at $Z_0 \geq 0.02$ from massive, Wolf–Rayet stars also causes the larger He and C yields in this metallicity range. The removal of these elements in the wind in turn suppresses the explosive $\alpha$ element yields. For more details, see section 5 of P98.

These specific features could have a significant impact on our results. We therefore also test our GCE implementation with an alternative set of SN-II yields, which do not take account of prior mass-loss, and therefore appear more stable as a function of initial mass and metallicity. This second set is taken from Chieffi & Limongi (2004, hereafter CL04). These account for stars of initial masses from 13 to 35 $M_\odot$, and so require both an extrapolation downwards to the upper mass limit for AGB winds (chosen here to be $7 M_\odot$) and upwards to a more reasonable maximum mass. We choose $M_{\text{max}} = 120 M_\odot$ when using the CL04 SN-II yields in order to match the maximum mass considered for the P98 SN-II...
yields, and because such massive stars are known to exist and could contribute to chemical enrichment in the real Universe. However, we caution that this represents a gross extrapolation into a regime well above that constrained by the original yield calculations. For this reason, we use the CL04 yields only as a comparison to those of P98, in order to discern what effect prior mass-loss might have on our overall results.

4 THE GCE EQUATION

In this section, we present the GCE equations required to calculate the mass ejection rate from stars. The implementation of these equations into our semi-analytic model is described in Section 5.

Following the prescriptions given by Tinsley (1980), the total rate of mass ejected by an SSP at time \( t \) is given by

\[
e_{\text{SSP}}(t) = \int_{M_{\text{min}}}^{M_{\text{max}}} (M - M_i) \psi(t - \tau_M) \phi(M) \, dM,
\]

where \( M_i \) is the initial mass of a star, \( \tau_M \) is its lifetime, \( \psi(t - \tau_M) \) is the star formation rate (SFR) when the star was born, \( M \) is the mass of the stellar remnant and \( \phi(M) \) is the normalized IMF by number, as given by equation (1). \( \psi(t - \tau_M) \cdot \phi(M) \) gives us the birthrate of stars of mass \( M \) at time \( t - \tau_M \). Multiplying this birthrate by \( (M - M_i) \), the mass ejected by one star of mass \( M \), then gives us the total mass ejection rate by stars of mass \( M \) at time \( t \). We can then integrate this quantity over a suitable range of masses (\( M_i \) to \( M_{\text{max}} \)) to obtain \( e_{\text{SSP}}(t) \).

The same equation can be written only considering the metals ejected by an SSP:

\[
e_{\text{Z}}(t) = \int_{M_{\text{min}}}^{M_{\text{max}}} M_{\text{Z}}(M, Z_0) \psi(t - \tau_M) \phi(M) \, dM,
\]

where \( M_{\text{Z}} = y_{\text{Z}}(M, Z_0) + (M - M_i) \) is the mass in metals returned to the gas phase by a star of mass \( M \) (as clarified by Maeder 1992, section 4.1). This is made up of the mass- and metallicity-dependent yield \( y_{\text{Z}} \), plus those metals present at the formation of the star that are later ejected unprocessed, \( Z_0 \cdot (M - M_i) \).

The same equation can be written again, when only considering individual chemical elements ejected by an SSP, replacing \( M_{\text{Z}} \) with \( M_{\text{Z}} = y_{\text{Z}}(M, Z_0) + (M_i/M)(M - M_i) \), the total mass of element \( i \) returned to the gas phase by a star of mass \( M \). However, for simplicity, we will proceed by describing the GCE equation in terms of the total metals ejected.

Equation (4) can be further split up into four sub-components, representing the three modes of ejection, AGB winds, SNe-Ia and SNe-II:

\[
e_{\text{Z}}(t) = \int_{M_{\text{min}}}^{M_{\text{max}}} M_{\text{Z}}^\text{GB}(M, Z_0) \psi(t - \tau_M) \phi(M) \, dM
\]

\[
+ \ A' \ k \ \int_{\tau_M}^{\infty} M_{\text{Z}}^\text{Ia} \psi(t - \tau) \ DTD(\tau) \, d\tau
\]

\[
+ (1 - A) \int_{7M_{\odot}}^{16M_{\odot}} M_{\text{Z}}^\text{II}(M, Z_0) \psi(t - \tau_M) \phi(M) \, dM
\]

\[
+ \int_{16M_{\odot}}^{M_{\text{max}}} M_{\text{Z}}^\text{II}(M, Z_0) \psi(t - \tau_M) \phi(M) \, dM.
\]

The first term in equation (5) represents the contribution to the ejected metals from AGB winds (approximating that the material is shed at the end of the stars’ lives), with the symbols representing the same quantities as in equation (4). As can be seen, the integral extends to masses above the minimum mass of SN-Ia-producing binary systems (~3 \( M_{\odot} \)). Therefore, we are explicitly accounting for the ejection of metals during the AGB phase of such stars, prior to the SN.

The second term represents the contribution from SNe-Ia, parametrized with an analytic DTD motivated by observed SN-Ia rates (see Section 4.1). Using a DTD means we do not have to make additional assumptions about the progenitor type of SNe-Ia, the binary mass function \( \phi(M_b) \), secondary mass fraction distribution \( f(M_z/M_b) \) or binary lifetimes in our modelling. These uncertain parameters become problematic when using the theoretical SN-Ia rate formalism of Greggio & Renzini (1983). The three SN-Ia DTDs that we consider in this work are described in Section 4.1.

The coefficient \( A' \) in the second term of equation (5) gives the fraction of objects from the whole IMF that are SN-Ia progenitors. This is subtly different from \( A \) in the third term, which is the fraction of objects only in the mass range 3–16 \( M_{\odot} \) that are SN-Ia progenitors. As clarified by Arrigoni et al. (2010a, section 3.3), these two coefficients are related by \( A' = A \cdot f_{3-16} \), where \( f_{3-16} \) is the fraction of all objects in the IMF that have mass between 3 and 16 \( M_{\odot} \). Our chosen value of \( A \) is 0.028 (i.e. 2.8% per cent of the stellar systems in the mass range 3–16 \( M_{\odot} \) are SN-Ia progenitors), as discussed in Section 5.4. The coefficient \( k \) is given by

\[
k = \int_{M_{\text{min}}}^{M_{\text{max}}} \phi(M) \, dM,
\]

and gives the number of stars in a 1 \( M_{\odot} \) SSP. For the Chabrier IMF used here, \( f_{3-16} = 0.0385 \) and \( k = 1.4772 \) when assuming \( M_{\text{min}} = 0.1 M_{\odot} \) and \( M_{\text{max}} = 120 M_{\odot} \).

The third term in equation (5) represents the ejection of metals, via SNe-II explosions, of all objects within the mass range 7.0 ≤ \( M/M_{\odot} \) ≤ 16.0 that do not produce SNe-Ia. Hence, the coefficient is \((1 - A)\).4

The fourth term represents the contribution to the ejection of metals from single, massive stars exploding as SNe-II.

We note here that equation (5) can also be rewritten so that all the modes of enrichment are expressed as time integrals, because the stellar lifetimes are a monotonic function of initial mass (e.g. P98, section 8.7).

4 Note that, because the distribution of SN-Ia-producing binaries is assumed to follow the distribution of all objects, the value of \( A \) is the same for any mass range within 3 < \( M/M_{\odot} \) < 16.

4.1 SN-Ia DTD

There have been many SN-Ia DTDs formulated in the literature. In this work, we consider three, shown in Fig. 5, and compare the results obtained from each.

The first is the power-law DTD with slope −1.12 proposed by Maoz, Mannucci & Brandt (2012), formed from a fit to the SN-Ia rate derived from 66 000 galaxies (comprising 132 detected SNe-Ia) from the Sloan Digital Sky Survey II (SDSS-II):

\[
\text{DTD}_{\text{PL}} = \frac{\alpha}{(\tau/\text{Gyr})^{1.12}},
\]

where \( \tau \) is the delay time since the birth of the SN-Ia-producing binary systems, and \( \alpha \) is the normalization constant, taken here to...
where $\tau < z< 0$. The redshift range $0.0$ is motivated by simultaneously fitting both lifetime separating the two components.

A line corresponds to the bi-modal DTD given by equation (9). All three DTDs correspond to the power-law DTD given by equation (7). The dotted line represents the bi-modal DTD proposed by Mannucci, Della Valle & Panagia (2006), based on observations of 56 SNe-Ia in the range $0.2 < z < 1.8$ from the Great Observatories Origins Deep Survey (GOODS) North and South fields. This form is given by

$$\text{DTD}_{NG} = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\left(\frac{\tau - \tau_c^2}{2\sigma^2}\right)},$$

where $\tau$ is again the delay time, $\tau_c = 1$ Gyr is the characteristic time (on which the Gaussian distribution is centred) and $\sigma = 0.2\tau_c$ Gyr is the characteristic width of the distribution.

The second is the narrow, Gaussian DTD proposed by Strolger et al. (2004), based on observations of 56 SNe-Ia in the range $0.2 < z < 1.8$ from the Great Observatories Origins Deep Survey (GOODS) North and South fields. This is given by

$$\text{DTD}_{NM} = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\left(\frac{\tau - \tau_c^2}{2\sigma^2}\right)}.$$

The third is the bi-modal DTD proposed by Mannucci, Della Valle & Panagia (2006), motivated by simultaneously fitting both the observed SN-Ia rate and the distribution of SNe-Ia with galaxy $B - K$ colour and radio flux, for a collection of samples of over the redshift range $0.0 < z < 1.6$. This DTD includes a ‘prompt’ component of SNe-Ia ($\sim 54\%$ of the total) that explode within $\sim 85$ Myr of the birth of the binary, followed by a broader, delayed distribution. The Mannucci et al. (2006) DTD has been expressed by Matteucci et al. (2006) as

$$\log(\text{DTD}_{BM}) = \begin{cases} 1.4 - 50(\log(\tau/\text{yr}) - 7.7)^2 & \text{if } \tau < \tau_0, \\ -0.8 - 0.9(\log(\tau/\text{yr}) - 8.7)^2 & \text{if } \tau > \tau_0, \end{cases}$$

where $\tau$ is the delay time and $\tau_0 = 0.0851$ Gyr is the characteristic lifetime separating the two components.

For all of these DTDs, the normalization requirement is

$$\int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \text{DTD}(\tau) \, d\tau = 1,$$

where $\tau_{\text{min}} = 5\, M_\odot$ and $\tau_{\text{max}} = 80\, M_\odot$ are the minimum and maximum assumed lifetimes of an SN-Ia-producing binary in the single-degenerate scenario (i.e. the lifetimes of the largest and smallest possible secondary stars), respectively. Strictly, their values depend on the stellar lifetime tables used, and therefore also on the metallicity of the stars. However, we choose to fix their values for equation (10) to those provided by the P98 lifetime tables for stars of $Z_0 = 0.02$, namely $\tau_{\text{min}} = 35$ Myr and $\tau_{\text{max}} = 21$ Gyr. Our chosen value of $\tau_{\text{min}} = 35$ Myr is in line with those commonly used in the literature, with assumed values ranging from $\sim 30$ Myr (e.g. Matteucci & Greggio 1986; Padovani & Matteucci 1993; Matteucci & Recchi 2001; Matteucci et al. 2009) to $\sim 40$ Myr (e.g. Greggio 2005). This choice also means that $\sim 48\%$ of SNe-Ia explode within 400 Myr when using the power-law DTD, which is close to the $\sim 50$ per cent predicted from observations of the SN-Ia rate by Brandt et al. (2010), and around the lower limit determined from SN remnants in the Small and Large Magellanic Clouds by Maoz & Badenes (2010).

## 5 IMPLEMENTATION

The GCE equation given by equation (5) has been implemented into our semi-analytic model so that the mass of chemical elements ejected is calculated at each simulation timestep. The key aspects of this implementation are outlined in the following sub-sections.

### 5.1 SFH, ZH and EH arrays

There are three galaxy-dependent values that are required for us to predict ejection rates from stars: the star formation history (SFH), the total gas-phase metallicity history (ZH) and the gas-phase element abundance history (EH). The SFH is required to identify $\psi(t - \tau_M)$, the ZH is required to identify $Z_0$ and the EH is required to calculate the unprocessed ejecta of each individual element within the semi-analytic model. We accommodate these histories into arrays in our code.

The L-GALAXIES time structure is made up of 63 snapshots (when run on the Millennium simulation), each containing 20 timesteps. Snapshots are arranged in intervals of $\log(1 + z)$ (see Boylan-Kolchin et al. 2009, equation 2), and 20 intermediate timesteps are chosen in the semi-analytic model to obtain a balance between reasonable computation time and reasonable time resolution. As there are nearly 26 million galaxies by $z = 0$ in the semi-analytic model, it would require a significant amount of memory for us to store the full histories of each galaxy at the resolution of one timestep. Therefore, we instead take a more dynamic approach; each galaxy has SFH, ZH and EH arrays of only 20 array-elements (hereafter, time ‘bins’). As time elapses in the simulation, the width of older bins (those storing data from higher redshifts) increases, while new bins are ‘activated’ with a default width of one timestep. Thus, the whole history of each galaxy can be stored with a time resolution that decreases with lookback time. High precision at recent times is especially important when calculating galaxy luminosities as a post-processing step, as young stars from recent star formation episodes tend to dominate the light. We have checked that changing the number of bins in the history arrays does not affect the chemical evolution in the model by testing our model with a range of history bin resolutions including full resolution (i.e. $63 \times 20$ bins per galaxy).

By $z = 0$, the older bins in such histories can be up to $\sim 3$ Gyr wide. This is acceptable when calculating the chemical enrichment within the code, as the bins are integrated over more finely at each timestep (see Section 5.2). However, when plotting enrichment relations using only the output $z = 0$ history bins, the lower resolution at high-z does not correctly represent the smooth chemical evolution actually
time structure of the history arrays is, by construction, the same for all galaxies at any given time. Therefore, we can know a priori the range of masses of stars that will explode in any given timestep.

We can re-write equation (4) as

\[ e_{Z}(t) = \psi(t - \tau) \int_{M_{L}}^{M_{U}} M_{Z}(M, Z_{0}) \cdot \phi(M) \, dM, \quad (11) \]

where \( \psi(t - \tau) \) can be put outside the integral if we assume, for each thin strip of the SFH integrated, that all the stars of mass \( M_{L} \leq M \leq M_{U} \) are born at the same time (i.e. \( \tau_{M_{L}} = \tau_{M_{U}} = \tau \)). We can then pre-calculate the integral in equation (11) numerically to obtain a value for each initial metallicity, in every history bin of every timestep that the semi-analytic model will run through, and store it in a three-dimensional look-up table. The true value of \( Z_{0} \) for a given galaxy is then used within the semi-analytic model to interpolate between these pre-calculated results at each timestep and \( \psi(t - \tau) \) is multiplied-in. The total mass in metals ejected is then given by \( e_{Z}(t) \cdot \Delta t \), where \( \Delta t \) is the width of the timestep. The same procedure is used to obtain the total mass ejected, and the total amount of each chemical element ejected at each timestep. Once the ejected masses are calculated, we transfer the material to either the galaxy’s ISM or CGM, as described in Section 5.3.3.

5.3 Infall, cooling and outflows

The chemical enrichment recipe outlined above is only part of the relevant physics needed to accurately model the chemical evolution of galaxies. The distribution of these metals between the various components of galaxies, and out into the IGM, as well as the infall and cooling of gas, are also important considerations when looking beyond a simple closed-box model. Treatments of these physical processes are already incorporated into L-GALAXIES, as described by Guo et al. (2011). A brief outline is also given below.

We note here that L-GALAXIES considers three classes of galaxy: those at the centre of a main DM halo, also known as a ‘friends-of-friends (FOF) group’ (type 0 galaxies), those at the centre of their own DM sub-halo but not of their associated FOF group (type 1 galaxies) and those galaxies that have lost their DM sub-halo through tidal disruption but have not yet merged with a central galaxy or been tidally disrupted themselves (type 2 galaxies). The prescriptions for the physical processes included in the model are then applied to galaxies according to their type. For example, infall of pristine gas is only allowed to occur for type 0 galaxies, whereas stripping of hot gas can only occur for type 1 galaxies (once they are within the virial radius of the central galaxy).

5.3.1 Infall

The mass of pristine gas (assumed to be 75 per cent hydrogen and 25 per cent helium) infalling on to the DM halo is simply determined by the difference between the assumed baryon fraction \( f_{b} \) and the actual baryon fraction \( M_{b} / M_{DM} \) in the DM halo. The assumed baryon fraction is reduced from the cosmic baryon fraction \( f_{b,cos} \) (assumed to be 0.17, as given by WMAP1) due to reionization, and is parametrized following Gnedin (2000) as

\[ f_{b}(z, M_{vir}) = f_{b,cos} \left[ 1 + (2/3 - 1) \left( \frac{M_{vir}}{M_{vir}(z)} \right) \right]^{-3/2}, \quad (12) \]

where \( M_{vir} \) is the mass within the radius at which the DM halo is 200 times denser than the critical density of the simulation (hereafter, the virial radius, \( R_{vir} \)) and \( M_{vir}(z) \) is the chosen characteristic mass of the virial radius of the main schematic.

Figure 6. Schematic illustrating how history arrays are ‘stitched together’ in post-processing to form higher resolution histories when plotting data. At every output snapshot (y-axis), a galaxy has a series of history bins (black boxes). The most recent bins from each output (in green) are extracted and used to form a higher resolution, non-overlapping history (shown in the bottom row). The other bins (in red) are discarded. So that there are no gaps in the reconstructed histories, a fraction of the mass from partially overlapping bins is also included (in orange). This means that many hundreds of bins and used to form a zoom-in of the bottom-right region of the main schematic.

5.2 Implementing the GCE equation

In order to model GCE, equation (4) needs to be implemented into L-GALAXIES as an algorithm, involving numerical integration and interpolation between values in a number of look-up tables. All non-model-dependent terms (i.e. everything except the SFHs, ZHs and EHs) are pre-calculated and stored in look-up tables, in order to speed-up the runtime of the code. This is possible because the occurring in our model. In these cases (for example, the [Fe/H]–[O/Fe] relation in Fig. 10), we construct higher resolution histories as a post-processing step, by ‘stitching together’ the highest resolution bins from the histories of all output snapshots, rather than just those from \( z = 0 \). This procedure is illustrated by the schematic in Fig. 6. In this way, a much smoother evolution can be plotted, which more accurately represents the chemical evolution occurring within the code. We note that when doing this for the disc components of galaxies, account needs to be taken of stars that move from the disc to the bulge through disc instabilities, by ensuring that the total mass formed in the stitched-together bins does not exceed the mass formed in the \( z = 0 \) history bins over the same time span.
halo mass, whose dependence on redshift has been calculated by Okamoto, Gao & Theuns (2008). In this formalism, $f_s$ tends towards $f_{s,\text{cool}}$ as $M_{\text{vir}}$ increases. Pre-enriched gas can also be re-accreted on to the DM haloes of central galaxies, in addition to this pristine infall (see Section 5.3.3).

### 5.3.2 Cooling

Following White & Frenk (1991), the cooling of gas from the CGM on to the disc is considered to fall into two regimes; at early times and in low-mass DM haloes, gas is able to cool rapidly in less than the free-fall time, with the cold-flow accretion on to the central galaxy modelled as

$$M_{\text{cool}} = \frac{M_{\text{acc}}}{t_{\text{dyn},h}}, \quad (13)$$

where $M_{\text{acc}}$ is the mass of gas accreted on to the DM halo, the dynamical time of the DM halo is $t_{\text{dyn},h} = R_{\text{vir}}/\sqrt{G M_{\text{vir}}}$ and $V_{\text{vir}} = (G M_{\text{vir}}/R_{\text{vir}})^{1/2}$. At late times and in massive DM haloes, the accretion shock radius is large, leading to the formation of a hot gas atmosphere. In this case, the accretion rate on to the central galaxy is reduced to

$$\dot{M}_{\text{cool}} = \frac{r_{\text{cool}}}{V_{\text{vir}}} \frac{M_{\text{hot}}}{t_{\text{dyn},h}}. \quad (14)$$

Here, the cooling radius $r_{\text{cool}}$ is set by the cooling function of Sutherland & Dopita (1993) and $M_{\text{hot}}$ is the mass of shocked gas in the hot gas reservoir (see Guo et al. 2011, section 3.2). This accreted gas is then able to form stars, following a simplified form of the Kennicutt–Schmidt law (see Guo et al. 2011, section 3.4).

### 5.3.3 SN feedback

SNe explosions can reheat cold gas and also eject it from the DM halo of galaxies. In previous versions of L-GALAXIES, the amount of energy released by SNe was assumed to be proportional to the mass of stars formed $\Delta M_*$ at that time. Now that we have discarded the instantaneous recycling approximation, it is more appropriate to relate this energy to the mass of material released by stars at that time. The total amount of energy produced by SN feedback is therefore parametrized as

$$E_{\text{SN}} = \epsilon_b \frac{1}{2} \epsilon_{\text{SN}}(t) \Delta t V_{\text{SN}}^2, \quad (15)$$

where $\epsilon_b$ is the halo-velocity-dependent SN energy efficiency, $\epsilon_{\text{SN}}(t)$ is the mass released by stars in one timestep (see equation 3) and $V_{\text{SN}}$ is the SN ejecta speed, assumed to be fixed at 630 km s$^{-1}$. This differs from the prescription used by Guo et al. (2011) in the use of $\epsilon_{\text{SN}}(t)$ rather than $\Delta M_*$. Due to this change, we have doubled the value of $\epsilon_b$, in order to have the same total SN feedback energy ($E_{\text{SN}}$) as previously used in the model. A thorough investigation into the precise values of model parameters required following our new GCE implementation is reserved for future work.

In our new, default GCE implementation, all stars dying in the stellar disc release material and energy into the ISM, whereas stars dying in the bulge and stellar halo release material and energy into the hot CGM. The energy dumped into the ISM by disc stars can then be used to reheat and possibly eject some (fully mixed) cold gas. Energy dumped into the CGM can also contribute to ejection. The amount of gas ejected from the DM halo into an external reservoir is given by

$$\Delta M_{\text{spec}} = \frac{E_{\text{SN}} - \frac{1}{2} \epsilon_{\text{DIS}}(t) \Delta t V_{\text{vir}}^2}{\frac{1}{2} V_{\text{vir}}^2}, \quad (16)$$

where $\epsilon_{\text{DIS}}(t)$ is the amount of gas that is reheated but does not escape the potential well. The ejected gas is then allowed to return to the DM halo over timescales that are proportional to $V_{\text{vir}}/t_{\text{dyn},h}$. This constitutes a second component of gas infall that has been pre-enriched by the galaxy.

We have also implemented an alternative feedback prescription which includes metal-rich winds. These winds dump some material released by disc SNe-II directly into the hot gas. This scheme is discussed in Section 6.3.3.

### 5.4 Default set-ups

There can be many free parameters involved when developing a chemical enrichment model. We have limited ourselves to only one new free parameter: the fraction, $A$, of objects in an SSP in the range $3 \leq M_*/M_\odot \leq 16$ that are SN-Ia progenitors. $A$ is specifically ‘tuned’ so that the peak of the [Fe/H] distribution for G dwarfs in our MW-type galaxy sample is around the solar value (see Section 6.2). An increase in $A$ corresponds to an increase in [Fe/H], and we find that the best value is $A \sim 0.028$ for all three of the DTDs we consider (see Section 4.1). A single value of $A$ was also found to be suitable for a range of different SN-Ia DTDs by Matteucci et al. (2009). All other results discussed in this work are obtained without further tuning. In the following, we label results using the bi-modal, power-law and narrow Gaussian DTDs with ‘BM’, ‘PL’ and ‘NG’, respectively.

We note that our preferred value of $A = 0.028$ is similar to that commonly found in the literature. For example, Greggio (2005) took a value of $A' = 0.001$ when also using a Chabrier IMF, which equates to a value of $A = 0.026$. Similarly, de Plaa et al. (2007) take a preferred value of 0.027 for a Kroupa IMF (assuming an SN-Ia progenitor mass range of 1.5–10 $M_\odot$). Arrigoni et al. (2010a) allow for a value between 0.015 and 0.05, preferring 0.03 when using a slightly top-heavy Chabrier IMF (i.e. a slope of 2.15 rather than 2.3 for $M > 1 M_\odot$, see equation 1). Other works, which have used IMFs with a smaller fraction of stars above 1 $M_\odot$ when using a slightly top-heavy Chabrier IMF (i.e. a slope of 2.15 rather than 2.3 for $M > 1 M_\odot$, see equation 1), have taken slightly higher values. For example, Matteucci & Recchi (2001) and François et al. (2004) prefer $A = 0.05$ when using a Scalo (1986) IMF. Calura & Menci (2009) and Matteucci et al. (2006) take values of $A' = 0.0020$ and 0.0025 for a Salpeter IMF, which corresponds to $A \sim 0.05$ and 0.06, respectively. And P98 find $A = 0.05–0.08$ when using a Salpeter IMF and an SN-Ia progenitor mass range of 3–12 $M_\odot$.

Our chosen value of $A \sim 0.028$ is also in line with expectations from observations of the SN-Ia rate, with the fraction of SN-Ia-producing stars in the range 3–8 $M_\odot$ believed to be between 0.03 and 0.1 (Maoz & Mannucci 2012). For the range 3–16 $M_\odot$, this equates to between $\sim 0.024$ and 0.081 (for a Chabrier IMF).

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5 We note again that the SN efficiency parameter $\epsilon_b$ has also been modified to ensure that the total SN feedback energy is unchanged (see Section 5.3.3). All other model parameters have been kept to the values used by Guo et al. (2011).
Results

In the following sub-sections, we compare results from our updated semi-analytic model to observational data for local star-forming galaxies (Section 6.1), MW disc stars (Section 6.2) and local elliptical galaxies (Section 6.3). In doing so, we are attempting both to assess the success of our GCE implementation and to further constrain which of the SN-II yield tables and SN-Ia DTDs described in Sections 3 and 4.1 perform best across the range of data considered. In what follows, ‘element enhancement’ refers to the ratio of element \(x\) to iron, \([x/\text{Fe}]\) and ‘element abundance’ refers to the ratio of element \(x\) to hydrogen, \([x/\text{H}]\). Throughout this work, we normalize our model values to the set of solar abundances used for the observations to which we compare. For clarity, we have selected a representative sample of \(\sim 480\,000\) \(z = 0\) galaxies and their progenitors for the plots in this section.

6.1 The mass–metallicity relations

One of the key diagnostics used to analyse the chemical evolution of galaxies is the relation between their stellar mass (\(M_*\)) and gas-phase metallicity \((\langle Z \rangle)\). The large statistical power of the SDSS allowed Tremonti et al. (2004) to determine the \(M_*–\langle Z \rangle\) relation for emission-line galaxies in the local Universe. They found a clear positive correlation below \(\sim 10^{10.5} M_\odot\) with a \(1\sigma\) scatter of only 0.1 dex. Above this mass, the relation was found to flatten. Here, we compare our \(z = 0\) model mass–metallicity relations for gas and stars with those observed. We also have the opportunity to directly compare L-GALAXIES results before and after the new GCE implementation – something we are not able to do when considering individual element ratios in later sub-sections.

Fig. 7 shows the \(M_*–\langle Z \rangle_{\text{cold}}\) relation for L-GALAXIES with the new GCE implementation and using the power-law DTD (points and black lines). 956 00 model galaxies were selected such that \(\log(M_*) \geq 8.6\) and \(-2.0 \leq \log(SFR) \leq 1.6\), in order to match the dynamic range of the SDSS data release 7 (SDSS-DR7) observations. We note here that both the gas-phase and stellar mass–metallicity relations are very similar for all three of the DTDs considered. This is because the SN-Ia DTD has little impact on the abundance of oxygen, which is the most abundant heavy element and is produced predominantly by SNe-II.

In Fig. 7, we also plot a fit to the same relation for L-GALAXIES prior to the new GCE implementation (red lines), and a fit to the observed \(M_*–\langle Z \rangle\) relation from the SDSS-DR7 (orange lines).\(^6\) We can see that there is very good agreement between the observations and our new model at \(z = 0\). Both the slope and amplitude of the new model relation are in better agreement with observations than those of the previous model. The increase in amplitude at lower mass is due to (a) our new GCE implementation (i.e. the input yields) allowing a different amount of metal into the ISM than the fixed 3 per cent yield assumed before, and (b) our new SN feedback scheme allowing more oxygen to stay in the ISM after it is released by stars, rather than being instantly ‘reheated’ into the CGM. This is because the energy input by a population of SNe is now distributed over time, rather than all dumped at once into the ISM straight after star formation, when a lot of oxygen is also released (see Section 5.3.3).

The scatter of our new model \(M_*–\langle Z \rangle\) relation is slightly larger than that seen in the SDSS. Studying the properties of outliers above and below the \(M_*–\langle Z \rangle\) relation can tell us a lot about the evolution of galaxies (e.g. Dellenbusch, Gallagher & Knezev 2007; Peebles, Pogge & Stanek 2008; Zahid et al. 2012a). We defer a detailed analysis of such galaxies in our model to later work.

We note here that the gas-phase metallicity is now defined as \(Z_{\text{cold}} = 12 + \log(N_0/N_\text{H})\) in our new model, in exactly the same way as in observations, where \(N_0\) and \(N_\text{H}\) are the number of atoms of oxygen and hydrogen, respectively. Previously, the approximation \(Z_{\text{cold}} = 9.0 + \log(M_{\text{cold}}/M_\odot)/0.02\) was used, where 9.0 was the assumed solar oxygen abundance and 0.02 the assumed solar metallicity. The difference in the value obtained when using these two methods is only small, with the new formulation estimating a metallicity \(\sim 0.04\) dex lower than the old formulation.

Fig. 8 shows the \(z = 0\) relation between the stellar mass and stellar metallicity \((Z_r)\) of our model galaxies (using a power-law DTD), after our new GCE implementation (points and black lines), and prior to it (red lines). In both cases, solar-normalized metallicities are calculated as \(Z_r = \log(M_*/M_{\odot})/0.02\), using the same solar metallicity of \(Z_\odot = 0.02\) assumed in the SPS models that obtained stellar metallicities in the SDSS-DR2 (Gallazzi, private communication).

Below \(M_* = 10^{10.5} M_\odot\), the new model \(M_*–Z_r\) relation is similar in shape to that of the previous model, but with an amplitude \(\sim 0.1\) dex higher. This is also higher than observed at low mass (although...
this is a region where observations are not well constrained). The mass-weighted $M_∗$–$Z_*$ relation of Panter et al. (2008) (green line) from the SDSS-DR3 probably provides the best comparison with our model, as we also consider mass-weighted metallicities. The Panter et al. (2008) relation also shows good correspondence with observations of Local Group dwarfs by Woo et al. (2008) (blue lines) and to a set of Local Group dwarf galaxies (blue lines) by Woo, Courteau & Dekel (2008).

Henriques & Thomas (2010) have shown that a more realistic treatment of stellar disruption, whereby satellite galaxies have their stellar component gradually stripped, can help steepen the slope of the $M_∗$–$Z_*$ relation in semi-analytic models. This could bring the low-mass end of our model relation into better agreement with observations. Including such a gradual disruption scheme into L-GALAXIES will be the focus of future work.

The model $M_∗$–$Z_{cold}$ and $M_∗$–$Z_*$ relations when using the CL04 SN-II yields have slightly shallower slopes and are ~0.1 dex higher than those assuming the P98 yields. They therefore have a higher amplitude than observed. This is because the CL04 yield set allows more oxygen to be produced and ejected from stars when extrapolated to $120 M_⊙$, particularly at low metallicity.

To conclude this section, we can say that our new GCE implementation improves the correspondence between our model and observations of gas-phase metallicities in local, star-forming galaxies. This was by no means a foregone conclusion, considering the significant changes to the chemical evolution modelling we have implemented. However, further improvement to the semi-analytic model is still required in order to better match the observed total stellar metallicities of galaxies at $z = 0$.

6.2 The MW disc

There is now a wealth of data available in the literature on the chemical composition of stars in the MW disc. These data allow us to put firm constraints on the success of our GCE implementation in reproducing realistic MW-type model galaxies. We construct a sample of ~5200 central (type 0) galaxies at $z = 0$ that are disc dominated (i.e. $M_{\text{bulge}}/(M_{\text{bulge}} + M_{\text{disc}}) < 0.5$), with DM halo masses in the range $11.5 \leq \log(M_{\text{vir}})/M_⊙ \leq 12.5$, and recent star formation rates of $1.0 \leq \text{SFR}/M_⊙\text{yr}^{-1} \leq 10.0$ over the redshift range $0.0 \leq z \leq 0.25$ (i.e. the last ~3 Gyr). Our results are not affected by small changes to these criteria. Three example SFHs from our MW-type model sample are shown in Fig. 9. The chemical evolution of the individual galaxy depicted in red is discussed in Section 6.2.2.

In order to compare with observations, we only consider G dwarfs ($0.8 \leq M/M_⊙ \leq 1.2$) still present in the stellar discs of our model MW-type galaxies at $z = 0$. When using the P98 stellar lifetimes (Section 3.2), not all G dwarfs live as long as the age of the MW disc. For example, stars of mass $1.2 M_⊙$ (the upper mass limit we assume for G dwarfs) live from 3.1 Gyr at $Z_0 = 0.0004$ to a maximum of 4.7 Gyr at $Z_0 = 0.02$ (see Fig. 1). These time-scales are clearly shorter than the typical ages of the oldest SSPs in our MW-type model discs (see Fig. 9). Therefore, we re-weight those SSPs for which some of the G dwarfs would no longer be present at $z = 0$ for the plots in this section. This correction removes a very small contribution from the oldest SSPs, reducing very slightly the number of low-[Fe/H], high-[O/Fe] stars. Although this is a more rigorous treatment, the main conclusions drawn from our MW-type sample also hold when assuming that all G dwarfs survive up to $z = 0$.

6.2.1 MW-type model galaxies

Fig. 10 shows the [Fe/H]–[O/Fe] relation for the G dwarfs in the stellar discs of our model MW-type galaxies, using the stitched-together histories described in Section 5.1, for the three DTDs we consider. Care needs to be taken when comparing Fig. 10 to observations. In observational studies of the MW disc, the chemical composition of individual stars of various ages are measured and plotted. In the case of our semi-analytic model, individual stars cannot be resolved, and so we must instead rely on the chemical composition of each population of stars, formed at each timestep during the evolution of a galaxy. Fig. 10 therefore shows the chemical composition of
Figure 10. The [Fe/H]–[O/Fe] relation for G dwarfs in the stellar discs of our MW-type model galaxy sample when using a bi-modal (top panel), power-law (middle panel) and narrow Gaussian (bottom panel) SN-Ia DTD. One galaxy contributes many hundreds of points to this relation (see Section 5.1). The grey-scale indicates the distribution of SSPs, weighted by the mass formed. Contours show the 68th, 95th, 99th and 99.9th percentiles. The chemical evolution of an individual MW-type model galaxy is overplotted on each of the panels in Fig. 10 (red tracks). This galaxy is discussed in detail in Section 6.2.2.

Each of the panels in Fig. 10 shows a clear decrease in [O/Fe] with increasing [Fe/H] towards the solar composition. There are, however, important differences in the distribution of SSPs for each of the three DTDs we consider. These differences can be seen more clearly in Fig. 11, where we compare the [Fe/H] and [O/Fe] distributions when using our three DTDs (black histograms) with those of 16 134 F and G dwarfs from the Geneva-Copenhagen Survey (GCS, orange histograms) (Nordström et al. 2004; Holmberg, Nordström & Andersen 2009) and 293 unique G dwarfs from the Sloan Extension for Galactic Understanding and Exploration (SEGUE, red histograms) survey (Yanny et al. 2009; Bovy, Rix & Hogg 2012a; Bovy et al. 2012b).

The difference in [Fe/H] distribution between the two observational samples is likely due to their different depths; the GCS probed strictly the solar neighbourhood (7.7 \(\leq \frac{R_{GC}}{\text{kpc}} \leq 8.31\) and 0.0 \(\leq \frac{|Z_{GC}|}{\text{kpc}} \leq 0.359\)), whereas SEGUE covered a wider range of galactic radii but also much higher galactic scaleheights (5 \(\leq \frac{R_{GC}}{\text{kpc}} \leq 12\) and 0.3 \(\leq \frac{|Z_{GC}|}{\text{kpc}} \leq 3.0\)).8 This means that the SEGUE sample includes a larger number of metal-poor, ‘thick-disc’ stars, and so has an [Fe/H] distribution spread to lower iron abundances. Our model, in turn, represents the average chemical composition of stars born at each timestep in the discs of MW-type galaxies, due to the full mixing of material in the various galactic components.

The model [Fe/H] distributions for all three of our set-ups are in reasonable agreement with the GCS data (partly by construction, as we have tuned \(A\) to obtain a peak of the [Fe/H] distribution around 0.0), although the NG set-up is skewed slightly more to higher iron abundances. However, there are significant differences in the model [O/Fe] distributions. For example, the high-[O/Fe] tail in our BM set-up is much less extended than seen in the \([\alpha/\text{Fe}]\) distribution from the SEGUE survey.9 This suggests that stars are being enriched with iron too quickly when using the bi-modal DTD – a conclusion also reached by Matteucci et al. (2009).

Interestingly, the extent of the high-[O/Fe] tail in the [O/Fe] distribution increases from left to right in Fig. 11. This is due to the different number of ‘prompt’ SNe-Ia assumed for each of the three DTDs. The smaller the prompt component, the larger the number of low-[Fe/H], high-[O/Fe] stars that can be formed before a significant amount of Fe gets into the star-forming gas. The bi-modal DTD allows \(\sim 54\) per cent of SNe-Ia to explode within 100 Myr of star formation (\(\sim 58\) per cent within 400 Myr), the power-law DTD allows \(\sim 23\) per cent within 100 Myr of star formation (\(\sim 48\) per cent.

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8 In Fig. 11, the stars with \(|Z_{GC}| < 0.3\) that are missing from the SEGUE survey are accounted for via the mass re-weighting of the [Fe/H] distribution described by Bovy et al. (2012a).

9 Note that Bovy et al. (2012a) and Bovy et al. (2012b) choose \(\alpha/\text{Fe}\) to be the average of \([\text{Mg/Fe}], [\text{Si/Fe}], [\text{Ca/Fe}]\) and \([\text{Ti/Fe}]\), with no oxygen lines included in the analysis. However, as oxygen is the most abundant \(\alpha\) element in galaxies, a comparison between their \(\alpha/\text{Fe}\) and our [O/Fe] is still valid here.
within 400 Myr) and the Gaussian has no prompt component at all. Only the Gaussian DTD has a high-[O/Fe] tail as extended as that seen for G dwarfs from SEGUE. However, we reiterate that the lack of any prompt component is in contradiction with recent observations (e.g. Maoz & Mannucci 2012). The smaller high-[O/Fe] tail produced when using the power-law DTD, although not as extended as seen in the SEGUE data, is still promising, especially when considering that (a) the SEGUE data contain a large number of α-enhanced, iron-poor stars at high galactic scaleheights, and (b) our model represents the chemical composition of MW-type stellar discs in a statistical sense, and also assumes full mixing of metals in the stellar disc. Including a treatment of the radial distribution of metals in galaxies, similar to that done by Fu et al. (2013), will be the focus of future work. We will also show in Section 6.3 that the power-law DTD also produces positive slopes in the $M_*^{-\alpha/\text{Fe}}$ relations of elliptical galaxies.

In Fig. 12, we show a finer binning of the model [Fe/H] and [O/Fe] distributions for the three SN-Ia DTDs considered (black histograms). Sub-distributions for three distinct age ranges (coloured histograms) are also plotted. All panels show nicely that older SSPs have lower [Fe/H] and higher [O/Fe] than younger SSPs, due to the delayed enrichment of the star-forming gas with iron from SNe-Ia. There is also no sign of an extended tail below [Fe/H] = −1.0 (the ‘G-dwarf problem’) that is common to closed-box models.

The broader plateau present in the [O/Fe] distribution for the PL set-up is due to the shape of the DTD; the power-law DTD assumes a smoother change in SN-Ia rate with time than the other two DTDs considered (see Fig. 5). This means that the ISM in a typical MW-type galaxy undergoes a fairly constant decrease in [O/Fe] of $\sim 0.025$ dex Gyr$^{-1}$ for the power-law DTD. In contrast, the bi-modal DTD causes a more gradual decrease in [O/Fe] of $\sim 0.016$ dex Gyr$^{-1}$, after significantly enriching the ISM with iron shortly after the start of star formation. In turn, the Gaussian DTD produces a steep decrease in [O/Fe] of $\sim 0.066$ dex Gyr$^{-1}$ from very high initial values until $\sim 1$ Gyr after the peak of star formation, with little change thereafter.

The CL04 SN-II yields produce qualitatively similar results to those discussed above, except that the [Fe/H] distribution is shifted to higher values and the [O/Fe] has a decreased high-[O/Fe] tail – in greater contradiction with observations. This is because, when...
extrapolated to $120 M_{\odot}$, the CL04 yields predict a higher production of O, Mg and Fe by SNe-II than the P98 yields, particularly at low metallicity.

6.2.2 An individual MW-type model galaxy

In this sub-section, we look more closely at the chemical evolution of an individual MW-type galaxy in our model. This galaxy’s SFH is plotted in red in Fig. 9, and its evolution in the [Fe/H]–[O/Fe] diagram is shown by a red track in each panel of Fig. 10. Points on the tracks in Fig. 10 denote the chemical composition at discreet times in the past, labelled by the lookback time in Gyr.

This galaxy nicely demonstrates the fairly smooth evolution that we would expect from an MW-type galaxy. However, it is not necessarily typical of our MW-type model sample as a whole. Some galaxies (such as that shown in blue in Fig. 9) undergo large infall and star formation events that can cause such a track to double-back on itself and otherwise deviate from a ‘smooth’ path (see also Calura & Menci 2009). However, our chosen galaxy provides a good example of the general chemical evolution undergone by MW-type galaxies in our model.

Fig. 13 shows the evolution from $z = 7$ to 0 of the mass, SFR, iron abundance, total metallicity and complete set of heavy element enhancements for this example MW-type galaxy, when using the power-law DTD. Four of the eight components of the galaxy (stellar disc, cold gas, hot gas and ejecta reservoir) are coloured according to the legend. Note that Fig. 13 shows the average chemical composition of a whole galaxy component at any given time.

Fig. 13 highlights the dependence of an element’s evolution on the mode of its release, namely SNe-II, SNe-Ia or AGB winds. Those elements that are predominantly produced in massive SNe-II (O, Ne and Mg) show a similar decline in their enhancement with cosmic time, as we would expect for a slowly declining SFR and a delayed enrichment of iron. These light $\alpha$ elements also show lower enhancements in the cold gas than in the stellar disc for this reason. The heavier $\alpha$ elements (Si, S and Ca) are produced mainly in lower mass SNe-II, and also have a greater contribution from SNe-Ia. They are therefore released into the ISM later than the lighter $\alpha$ elements, showing a gradual increase in enhancement with time (at a decreasing rate), and higher enhancements in the gas than in the stars while gas fractions are high. Nitrogen, an element with a dominant contribution from (delayed) AGB winds at low metallicity, shows a strong increase in [N/Fe] at the onset of AGB wind enrichment (at $z \sim 4$ for this galaxy), followed by a more gradual increase thereafter. Finally, the drop in [C/Fe] between $z \sim 3$ and 4 is due to a decrease in the C/Fe ratio in the ejecta of SNe-II at $Z_0 \sim 0.004$ compared to other metallicities. The increase in this ratio at higher metallicities, along with a significant contribution to C from AGB winds, causes the sharp rise in [C/Fe] shortly after. This is a specific property of the P98 SN-II yields. When using the CL04 SN-II yields, the [C/Fe] evolution follows that of the light $\alpha$ elements more closely.

To conclude this section, we can say that our new GCE implementation is able to reproduce the [O/Fe] distribution for G dwarfs in the MW disc if there is only a minor prompt component of SNe-Ia (i.e. $\leq 50$ per cent within $\sim 400$ Myr). Our NG set-up (narrow Gaussian DTD, delayed SNe-Ia only) and PL set-up (power-law DTD, $\leq 48$ per cent of SNe-Ia exploding within $\sim 400$ Myr) therefore reproduce the observed high-[O/Fe] tail best. However, the power-law DTD achieves this whilst assuming a more realistic fraction of prompt SNe-Ia. Our new GCE implementation also allows us to examine, in detail, the chemical evolution undergone by individual galaxies, which can help us explain the features seen in the sample as a whole.

Figure 13. The evolution, from redshift 7 to 0, of the mass (in $M_{\odot}$), SFR (in $M_{\odot}$ yr$^{-1}$), iron abundance, total metallicity and heavy element enhancements for four of the eight different galaxy components (see legend) in the example MW-type model galaxy shown in Fig. 10, using the power-law DTD.
6.3 Elliptical galaxies

The change in various element ratios as a function of velocity dispersion or \( M_\star \) in ellipticals can also provide insight into the chemical evolution of galaxies. It has been observed that \( \alpha \) enhancements increase with \( M_\star \) (e.g. Graves, Faber & Schiavon 2009; Thomas et al. 2010; Johansson, Thomas & Maraston 2012; Conroy, Graves & van Dokkum 2013). This has been mainly attributed to massive ellipticals undergoing the majority of their star formation at higher redshifts and over shorter time-scales. The stars in these galaxies are therefore likely to be deficient in iron, as they were formed before a significant number of SNe-Ia could enrich the star-forming gas. Less massive ellipticals, on the other hand, are believed to form a larger fraction of their stars later, from gas that has had time to be more enriched with iron. These galaxies should therefore have lower stellar \( \alpha \) enhancements.

Previous GCE models, working within a hierarchical merging scenario, have found it difficult to reproduce this trend between stellar mass and \( \alpha \) enhancement, without invoking either a variable or adapted IMF; morphologically dependent star formation efficiencies or additional prescriptions to increase star formation at high redshift (e.g. Thomas 1999; Thomas, Greggio & Bender 1999; Nagashima et al. 2005; Calura & Menci 2009, 2011; Pipino et al. 2009b; Arrigoni et al. 2010a; Arrigoni, Trager & Somerville 2010b).

We select \( z = 0 \) elliptical galaxies by bulge-to-mass ratio and \((g - r)\) colour, such that \( \mathcal{M}_{\text{bulge}}/(\mathcal{M}_{\text{bulge}} + \mathcal{M}_{\text{disc}}) \geq 0.7 \) and \((g - r) \geq 0.051 \log(M_\star) + 0.14\), to form a sample of \( \sim 8700 \) galaxies. These cuts are chosen to match the selection criteria used to obtain the sample of SDSS-DR7 ellipticals shown as green points in Fig. 18. The \((g - r)\) colour cut also nicely separates the red sequence from the blue cloud in our model at \( z = 0 \). Our model sample includes type 0, 1 and 2 galaxies (see Section 5.3).

Fig. 14 shows the SFHs of three example galaxies from our model elliptical sample. In this case, the sum of the SFRs from all progenitors at any given snapshot are plotted, rather than the SFRs from only the main progenitors. We can see that the lowest mass elliptical (blue) has a more extended SFH than the highest mass elliptical (red). This is the case in general for our model elliptical sample (see also De Lucia et al. 2006). We can also see that the two most massive ellipticals in Fig. 14 (red and green) had their star formation shutdown after a merger-induced starburst at \( \sim 5 \) and \( \sim 3 \) Gyr lookback time, respectively.

![Figure 14](http://mnras.oxfordjournals.org/) Three example SFHs from our model elliptical sample. The different colours correspond to different stellar masses at \( z = 0 \) (see legend). The points represent the sum of the SFRs from all progenitors at every output snapshot of the simulation. Low-mass ellipticals tend to have longer star formation time-scales than high-mass ellipticals in our model.

![Figure 15](http://mnras.oxfordjournals.org/) The \( M_\star \)-age relation for our model elliptical galaxies, coloured by galaxy type (see Section 5.3). Model ages are weighted by their \( r\)-band luminosity, and the mean relation and 1\( \sigma \) spread for the full sample (light blue lines) and the mass–age-selected sub-sample (dark blue lines) are also shown. A linear fit to the same relation for the SDSS-DR4 from JTM12 is given by the solid orange line, with the 1\( \sigma \) spread given by the dashed orange lines. Thick-rimmed points indicate model galaxies that lie within one standard deviation (\( \pm 0.222 \) dex) of the observed mass–age relation.

6.3.1 The mass–age relation

Before discussing element enhancements, we first show the \( M_\star \)-age relation for our model elliptical sample in Fig. 15. Also shown is a fit to the luminosity-weighted mass–age relation from the Johansson et al. (2012, hereafter JTM12) sample (solid orange line) and its 1\( \sigma \) scatter (dotted orange lines). The ages of model galaxies are \( r\)-band luminosity weighted in this plot, in order to make a fairer comparison with the observations. For the observed relation, we have used the stellar masses taken directly from the SDSS-DR7 catalogue.\(^\text{10}\) This is also the case for all subsequent plots showing data from the JTM12 sample.

It is known that semi-analytic models tend to produce too many old, red, dwarf galaxies by \( z = 0 \) compared to observations (e.g. Weinmann et al. 2006; Guo et al. 2011). This can be seen by comparing the model and observations in Fig. 15, which shows that the majority of ellipticals in the model have luminosity-weighted ages older than 10 Gyr. This is caused not just by the strong stripping of gas from satellites, but also by the strong SN feedback required to match the observed galaxy stellar mass function. Recent work by Henriques et al. (2013) has improved this problem to some extent, by allowing material ejected from model galaxies at high-\( z \) to be reaccreted over longer time-scales, allowing them to form more stars at low-\( z \), and therefore be younger and bluer at \( z = 0 \). However, this improvement is not implemented into the L-GALAXIES model presented here. Therefore, in the following sections, we will distinguish between our full elliptical model sample and a ‘mass–age-selected’ sub-sample, which includes only those model galaxies that lie within the 1\( \sigma \) scatter of the observed \( M_\star \)-age relation, noting that similar results are obtained when instead

\(^{10}\) Available at http://www.mpa-garching.mpg.de/SDSS/DR7.
selecting model galaxies that recover the mean observed relation (see Section 6.3.2). This sub-sample is not used in order to evade the evident issues still affecting the galaxy formation model, but rather as a means of testing the relation between mass, age and $\alpha$ enhancement in our new GCE implementation.

### 6.3.2 $[\alpha/Fe]$ relations

Fig. 16 shows the $M_*-[O/Fe]$ relation for the bulge and disc stars of our model ellipticals at $z = 0$, for the three SN-Ia DTDs we consider. Light blue contours represent our full elliptical sample. Dark blue, dashed, filled contours represent our mass–age-selected sub-sample. The observed relation from the JTM12 sample is given by the solid orange line. The slopes of the linear fits to these three relations in the range $10.0 \leq \log(M_*/M_\odot) \leq 12.0$ are given in the top-left corner of each panel. Model element ratios in this section have been normalized to the solar abundances measured by Grevesse, Noels & Sauval (1996), in accordance with the observations to which we compare.

We note here that estimates of element enhancements from SPS models, such as those used by JTM12, are found to be fairly good representations of the true global value, and are not as biased by small younger populations as age estimates can be (Serra & Trager 2007). It is therefore reasonable for us to compare our model mass-weighted element enhancements with these observations.

As with the extent of the high-$[O/Fe]$ tail in our MW-type sample (see Section 6.2.1), we can see from Fig. 16 that the strength of the slope in the model $M_*-[O/Fe]$ relation is inversely proportional to the fraction of prompt SNe-Ia assumed. For the BM set-up ($\sim 58$ per cent within 400 Myr), the model slope is much flatter than observed. For the PL set-up ($\sim 48$ per cent within 400 Myr), a positive slope is obtained, although shallower than observed. For the NG set-up (with no prompt component), a strong slope is obtained, with a larger scatter.

The increase in slope is because massive ellipticals have shorter star formation time-scales in the model, and so decreasing the fraction of prompt SNe-Ia has a bigger effect on their final iron abundance, increasing their final $\alpha$ enhancements more than for low-mass ellipticals. The increase in scatter at fixed mass is because older galaxies have shorter star formation time-scales in the model, and so undergo a greater increase in their $\alpha$ enhancements for the same reason. This correlation between mass, age and $\alpha$ enhancement can also be seen in the increasing difference between the slope for the full elliptical sample and the mass–age-selected sub-sample from the top to bottom panel, and also from the age–$[O/Fe]$ relation shown in Fig. 17 for the PL set-up. This result supports the canonical thinking that the slopes in $M_*-[\alpha/Fe]$ relations are driven by age.

### Figure 16

The $M_*-[O/Fe]$ relation for the bulge and disc components of our model elliptical sample, when using a bi-modal (top panel), power-law (middle panel) or narrow Gaussian (bottom panel) SN-Ia DTD. Light blue contours represent our full elliptical sample. Dark blue, dashed, filled contours represent our mass–age-selected sub-sample (see the text and Fig. 15). Contours represent the 68th and 95th percentiles. A linear fit to the observed relation from JTM12 is given by the orange line, with its $1\sigma$ spread (dotted orange lines). The slopes of these three relations are given in the top–left corner of each panel.

### Figure 17

The relation between mass-weighted age and oxygen enhancement for our model elliptical galaxies, when using the power-law DTD. Points show the full elliptical sample, with the grey-scale indicating the stellar mass. Dark blue, dashed contours show the mass–age-selected sub-sample. There is a clear positive correlation between age and $[O/Fe]$ in our model ellipticals, both in general and at fixed mass.
by differences in the star formation time-scale. If correct, then our model suggests there should only be a minor fraction of prompt SNe-Ia for any given SSP (i.e. ≲50 per cent within ∼400 Myr).

We note here that an even steeper slope is obtained for our mass–age-selected sub-sample when only selecting galaxies that recover the fit to the observed $M_\ast$–age relation (a slope of 0.043, rather than 0.032, for the PL set-up). This is because, at fixed mass, fewer old galaxies are present for such a selection.

Fig. 18 shows the enhancements of all the heavy elements that we track as a function of $M_\ast$ when using the power-law DTD. As in Fig. 16, light blue contours represent our full elliptical sample. Dark blue, dashed, filled contours represent our mass–age-selected sub-sample. Fits to observations of ellipticals drawn from the SDSS-DR4 (orange lines, JTM12), the SDSS-DR6 (red line, Graves et al. 2009) and the SDSS-DR7 (green points, see below) are also shown where possible. The PL set-up is shown here because it provides a positive slope for the $M_\ast$–[O/Fe] relation, while also assuming a more realistic fraction of ‘prompt’ SNe-Ia than the NG set-up.

Pleasingly, Fig. 18 shows that positive slopes are obtained for all the $\alpha$ elements when using our PL set-up (except for Mg, as explained below). This is again because of the correlation between mass, age and $\alpha$ enhancement in our model. The same is true for our NG set-up, but not for the BM set-up, which has a large fraction of prompt SNe-Ia. This is an important result, as it has been difficult previously for models to obtain positive slopes without invoking additional physics.

The reproduction of positive slopes is a better test for a GCE model than the reproduction of the observed amplitudes of particular element ratios. This is because the latter is mainly dependent on the input yields, whereas the former is also strongly dependent on mass-dependent galaxy evolution and metal distribution in the semi-analytic model.

It should be noted that the slopes (and amplitudes) of the different observational data shown in Fig. 18 differ substantially for some element enhancements. This is mainly due to the difference in the SPS modelling techniques used. Therefore, it is more important that...
our model produces positive slopes at all than reproduces the exact slopes of any particular observational data set.

The methodology of both JTM12 and Graves et al. (2009) is based on fitting observed and modelled Lick absorption line indices (e.g. Worthey 1994). JTM12 adopt the SPS models of Thomas, Maraston & Johansson (2011b) and 18 Lick indices, whereas Graves et al. (2009) adopt the models of Schiavon (2007) and use 7 Lick indices. The fits from JTM12 are based on a sample of visually classified, early-type galaxies in the redshift range $0.05 < z < 0.06$. Graves et al. (2009) selected red sequence galaxies, classified by the colour–magnitude diagram, in the redshift range $0.04 < z < 0.08$. Both samples exclude star-forming galaxies by applying cuts to certain emission-line strengths. Stellar masses are obtained from the Max Planck Institute for Astrophysics–John Hopkins University (MPA–JHU) catalogue for the JTM12 data, and from mass-to-light ratios obtained using the Bell et al. (2003) $(g-r) - M_\odot/L_\odot$ relation for the Graves et al. (2009) data. The additional observational data (green points), kindly provided by Conroy & Graves (private communication), are drawn from the SDSS-DR7, selecting galaxies in the redshift range $0.025 < z < 0.06$ with bulge-to-total light ratios $\geq 0.7$ and $(g-r) \geq 0.051 \log(M_\odot) + 0.14$. These galaxies are binned by $M_\odot$ and their stacked spectra are used to obtain element enhancements using the SPS models of Conroy & van Dokkum (2012a) and Conroy et al. (2013). For a detailed comparison of these different methods, see section 6 of Conroy et al. (2013).

Looking at the panels in Fig. 18 individually, we can see that the slopes of the model relations for [C/Fe] and [N/Fe] are shallower than observed. This is discussed further in Section 6.3.4. Although our PL set-up reproduces a slope of the $M_\odot$–[O/Fe] relation for the mass–age–selected sub-sample close to that obtained by JTM12 (as also shown in Fig. 16), the newly selected SDSS-DR7 data suggest a significantly steeper slope. Steeper slopes are obtained in the model when either using the Gaussian (delayed only) DTD, or when allowing direct ejection of light $\alpha$ elements out of galaxies via galactic winds, as explained in Section 6.3.3. An increase in $\tau_{\min}$ (the start time for SN-Ia explosions, see Section 4.1) also slightly increases the slope. For example, increasing $\tau_{\min}$ from 35 to 45 Myr increases the slope of the $M_\odot$–[O/Fe] relation by $\sim 0.004$ when using the power-law DTD.

Our PL set-up produces a flat $M_\odot$–[Mg/Fe] relation, with a slope slightly shallower than the well-constrained relation obtained from the SDSS-DR7 data. The SDSS-DR7 $M_\odot$–[Mg/Fe] relation, in turn, is flatter than the other observational data sets we compare to here. In our model, the slope of the $M_\odot$–[Mg/Fe] relation is flatter than the other light $\alpha$ elements because Mg is produced in greater amounts by low-metallicity SNe-II than high-metallicity SNe-II when using the P98 yields (compare the bottom two panels in Fig. 4). This is not the case for the CL04 SN-II yields, which produce a slope for [Mg/Fe] very similar to that of [O/Fe], due to the negligible difference in their metallicity dependence.

Strong, positive slopes for the heavier $\alpha$ elements (Si, S and Ca) are obtained for all three of the DTDs considered here when using the P98 SN-II yields. This is because these elements are produced only in small amounts by SSPs at low metallicity, as they are easily locked into the stellar remnants of the most massive, low-metallicity SNe-II (see Section 3.5). Low-mass elliptical galaxies, which never obtain high stellar metallicities, therefore never produce enough Si, S or Ca to obtain high enhancements. As the CL04 SN-II yields do not take account of the prior stellar winds’ effect on remnant composition, the slopes produced for the heavier $\alpha$ elements are very similar to those of the lighter $\alpha$ elements. This means that all slopes are equally sensitive to the choice of DTD when using the

\begin{equation}
\frac{\Delta \alpha}{\Delta \text{metallicity}} \propto \frac{1}{\text{total surface density}}
\end{equation}

A similar dependence on the gas surface density has also been used in the smoothed particle hydrodynamical simulations of Hopkins, Quataert & Murray (2012) and in the GALFORM semi-analytic model by Lagos, Lacey & Baugh (2013) (but see Newman et al. 2012). Interestingly, our preferred characteristic gas surface density of $\sim 10 M_\odot \text{pc}^{-2}$, below which all SN-II ejecta material is put into the wind, is very close to that below which the SFR surface density drops in local, spiral galaxies (e.g. Bigiel et al. 2008; Bigiel, Leroy & Walter 2011). This simple wind scheme lowers the stellar $\alpha$ enhancements of low-mass ellipticals more than high-mass ellipticals, because
relations is to assume a greater amount of N production in and further compound the problem, observations by JTM12 of the that these enhancements should also produce positive slopes. To they are at low metallicity in our model), yet observations suggest be expected if C and N are predominantly released in AGB winds (as than for the element. Our model produces slopes for C and N that are flatter case of C and N is more complicated than that of other heavy problems of GCE modelling.

6.3.4 Carbon and nitrogen

The case of C and N is more complicated than that of other heavy elements, not least because N is both a primary and secondary elements (see the top two panels in Fig. 18). This is to be expected if C and N are predominantly released in AGB winds (as they are at low metallicity in our model), yet observations suggest that these enhancements should also produce positive slopes. To further compound the problem, observations by JTM12 of the and relations show that these also have positive slope, due to the lower production of C and N by SNe-II that are more dependent on initial mass and metallicity), produce more realistic results in our GCE model.

To conclude this section, we reiterate that positive slopes in the relations of ellipticals, we do not claim that our simple wind model can solve all the problems of GCE modelling.

low-mass ellipticals tend to have lower density ISM, and so can dump their SN-II ejecta more efficiently into the CGM. This does not significantly affect the [Fe/H] distribution in the discs of MW-type model galaxies, although the number of low-[O/Fe] SSPs does increase slightly, as shown in Fig. 20. However, this simple wind scheme does cause a significant underenrichment of the ISM in low-mass star-forming galaxies by \( z = 0 \), which steepens the slope of the model \( M_* - Z_{\text{old}} \) relation away from that seen in observations. Therefore, although we show here that metal-rich winds are a way of strengthening positive slopes in the \( M_* - [\alpha/Fe] \) relations of ellipticals, we do not claim that our simple wind model can solve all the problems of GCE modelling.

Fig. 21 shows the \( M_* - [N/O] \) relation for our PL model (without additional galactic winds), along with the fit to observations from JTM12. This model relation is also flatter than observed, and its slope increases with the amount of prompt SNe-Ia in the DTD. We note that there is a scatter of high-mass model galaxies with [N/O] values more similar to those observed. However, these higher [N/O] galaxies tend to be younger for their mass in the model, whereas the opposite is true in the observational sample.

One way to increase the slopes in both the \( M_* - [N/Fe] \) and \( M_* - [N/O] \) relations is to assume a greater amount of N production in high-metallicity massive stars, as suggested by JTM12. Doing so implies a boost in secondary nitrogen production. Given the current uncertainty in the amount of secondary N production in stars, this could be a plausible solution, although this is far from certain. The red line in Fig. 21 is a fit to the full model elliptical sample when arbitrarily increasing the N released by SNe-II of metallicity \( \geq 0.02 \) by a factor of 1.5. A similar increase is also seen in the slope of the \( M_* - [N/Fe] \) relation. Although this is an ad hoc adjustment made to the stellar yields, it does indicate that such a change is capable of improving the values of both [N/Fe] and [N/O] in our model ellipticals.

When using the CL04 yields, the \( M_* - [C/Fe] \) and \( M_* - [N/Fe] \) relations are even flatter and the \( M_* - [N/O] \) relation has a negative slope, due to the lower production of C and N by SNe-II that these yield tables infer. This, again, is in contradiction with the observational data considered, suggesting that the P98 yields, which take account of prior stellar mass loss from massive stars (and so are more dependent on initial mass and metallicity), produce more realistic results in our GCE model.

To conclude this section, we reiterate that positive slopes in the \( M_* - [\alpha/Fe] \) relations of local elliptical galaxies can be obtained if an SN-Ia DTD with minor prompt component (i.e. \( \leq 50 \) per cent within \( \sim 400 \) Myr) is used, and/or galactic winds driven by SNe-II are allowed to directly eject metals out of galaxies.

Below \( \log(M_*) \sim 10.3 \, M_\odot \), slopes become shallower for our full elliptical sample. This is because the majority of low-mass ellipticals in the current model are significantly older than is observed. Recent improvements to the model that allow low-mass galaxies to be younger and bluer by \( z = 0 \) will likely remedy this problem. This is indicated by the results for our mass–age-selected sub-sample, but a re-analysis of the semi-analytic model results will also be required once these improvements are merged with the GCE model presented here.

7 CONCLUSIONS

We have implemented a new GCE model into the Munich semi-analytic model, L-GALAXIES, which accounts for the delayed enrichment of a series of heavy elements from SNe-II, SNe-Ia and AGB stars. We have also compared the results of this implementation with the chemical composition of local, star-forming galaxies,
the MW stellar disc and local, elliptical galaxies. Our conclusions are as follows.

(i) The gas-phase mass–metallicity relation for local, star-forming galaxies (when using the Bayesian, SDSS metallicities of Tremonti et al. 2004) is very well reproduced by our new model. However, we caution that both the slope and amplitude of the observed $M_\ast-Z_\odot$ relation depend strongly of the metallicity diagnostic chosen (e.g. Kewley & Ellison 2008). The stellar components of low-mass, star-forming galaxies tend to be more metal-rich in our model than observed.

(ii) The [Fe/H] distribution of G dwarfs in the MW disc is reasonably reproduced by our model, for all forms of SN-Ia DTD we consider. However, the high-[O/Fe] tail in the MW [O/Fe] distribution is best reproduced when using an SN-Ia DTD with a minor prompt component (i.e. $\lesssim 50$ per cent within $\sim 400$ kyr), such as a Gaussian DTD centred on $\sim 1$ Gyr, or a power-law DTD with slope $\sim -1.12$ and $\tau_{\min} \gtrsim 35$ Gyr.

(iii) Positive slopes in the $M_\ast-\alpha/\text{Fe}$ relations of ellipticals for local, elliptical galaxies are also obtained when assuming a minor prompt component (i.e. $\lesssim 50$ per cent within $\sim 400$ kyr). The strength of the slope is inversely proportional to the fraction of prompt SNe-Ia. These results are achieved when using the same implementation which produces our star-forming-galaxy and MW results.

(iv) The inclusion of metal-rich galactic winds, driven by SN-II explosions, strengthens the positive slopes in the $M_\ast-\alpha/\text{Fe}$ relations of ellipticals for all forms of SN-Ia DTD and SN-II yields considered. However, our simple, ISM-density-dependent wind scheme reduces the gas-phase metallicity of low-mass, star-forming galaxies, and so does not fully solve the problem.

(v) There is a clear correlation between mass, age and $\alpha$ enhancement in our model. This, along with the above findings, suggests that the chemical compositions of a diverse array of galaxies can be reconciled without requiring a variable IMF. Although an IMF that varied with SFR would likely produce similar results, it is instructive to see that this is not the only solution, given that the true behaviour of the stellar IMF is still uncertain.

(vi) The metallicity-dependent SN-II yields tabulated by P98, which take account of mass-loss from the progenitor star prior to the SN and its effect on the ejected yields, provide the best results for the range of galaxy types considered here. This supports the findings of the OWLS project (Schaye et al. 2010), which showed that the same set of yields also provided the best results in their galaxy formation simulations.

(vii) Overall, our results suggest, given the assumptions and limitations discussed, that the best model for matching the wide range of observational data considered here should include (a) a power-law SN-Ia DTD, (b) SN-II yields that take account of prior mass-loss through stellar winds and (c) some direct ejection of light $\alpha$ elements into the CGM.

We conclude by highlighting two unavoidable limitations of this work and GCE models in general. First, the stellar yields used as an input have a strong influence on the results, as shown here by comparing the SN-II yields of P98 and CL04, and also in other studies (e.g. François et al. 2004; Romano et al. 2010). Until the true yields ejected by stars of all masses and metallicities are better understood, the accuracy of GCE models will always be uncertain. Secondly, we only consider one free GCE parameter in this work, $\alpha$, the fraction of objects in a stellar SSP in the range $3 \lesssim M/M_\odot \lesssim 16$ that are SN-Ia progenitors. However, the values assumed for other GCE parameters (such as $\tau_{\min}, M_{\max}$ or the high-mass slope of the IMF) could also take on different values in reality. If so, our tuning of $A$ could also be correcting for these other uncertainties, and would not solely represent the efficiency of SN-Ia-progenitor formation. Further testing against additional observational data from Local Group dwarf galaxies, the intracluster medium of galaxy clusters and outliers from the mass–metallicity relation should help better constrain such uncertainties. These topics will be the focus of future work.

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