Erratum: Quantum propagation of neutral atoms in a magnetic quadrupole guide

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The quantum number $\ell$ in the wave function $\psi(\rho, \phi)$ is in fact not an eigenvalue of the orbital angular momentum $L_z$ of the motion of the atom around the center of the guide, as we had mistakenly assumed without checking, but an eigenvalue of the operator $L_z - s_z$, where $s_z$ is the $z$ component of the total internal angular momentum of the atom that couples to the magnetic field. For a magnetic quadrupole field of the form of Eq. (1.1), $B = (4B_0/\sqrt{R}, -4B_0/\sqrt{R}, 0)$ the operators $L_z$ and $s_z$ do not individually commute with the Hamiltonian $H = p^2/(2m) + g \mu_B s \cdot B$,

$$[L_z, H] = ig \mu_B \hbar \left( \frac{4B_0}{R} (s_yx + s_xy), \right),$$

$$[s_z, H] = ig \mu_B \hbar \left( \frac{4B_0}{R} (s_yx + s_xy), \right),$$

but the difference $L_z - s_z$ commutes with $H$ and is thus a conserved quantity, $[L_z - s_z, H] = 0$. Calculation shows that the quantum number $\ell$ in the wave function $\psi(\rho, \phi)$ is the eigenvalue of $L_z - s_z$, e.g., for spin 1/2

$$\frac{1}{\sqrt{2}} \left[ F_+ (\rho) e^{i(\ell + 1/2) \phi} - F_- (\rho) e^{i(\ell - 1/2) \phi} \right] = \frac{\hbar}{2 \sqrt{2}} \left[ \frac{F_+ (\rho) e^{i(\ell + 1/2) \phi} - F_- (\rho) e^{i(\ell - 1/2) \phi}}{F_+ (\rho) e^{i(\ell + 1/2) \phi} - F_- (\rho) e^{i(\ell - 1/2) \phi}} \right],$$

and similarly for the spin 1 wave function in Eq. (4.5). Although $L_z$ and $s_z$ do not commute with $H$, they commute with $L_z - s_z$ and with each other, which means that any eigenstate of $L_z - s_z$ must be a linear combination of simultaneous eigenstates of $L_z$ and $s_z$. Since the eigenvalues of $L_z$ are always integer, it follows that the eigenvalue $\ell$ of $L_z - s_z$ must be half integer for half integer spin $s$ and integer for integer spin $s$. Only then does the wave function have the correct transformation properties under rotations. For spin 1/2 a rotation by $2\pi$ around the $z$ axis, for example, transforms the wave function of the above eigenstate into

$$\frac{1}{\sqrt{2}} F_+ (\rho) e^{i(\ell + 1/2) \phi} D_{mm}(0,0,2 \pi) \left| \begin{array}{c} 1 \\ 0 \end{array} \right\rangle + \frac{1}{\sqrt{2}} F_- (\rho) e^{i(\ell - 1/2) \phi} D_{mm}(0,0,2 \pi) \left| \begin{array}{c} 0 \\ 1 \end{array} \right\rangle.$$

Since the application of the spin rotation matrix $D_{mm}(0,0,2 \pi)$ causes the spinors to change sign, the $\phi$ dependent prefactors must not change sign under $\phi \rightarrow \phi + 2\pi$. Thus $\ell$ has to be half integer for spin 1/2, and not integer as was erroneously stated in Sec. III. It follows that $\ell$ cannot be zero, which, as explained by Eq. (5.3), has the consequence that there are no exact bound states in the case of spin 1/2 and makes Sec. IIIA redundant. Solving Eqs. (3.11) for $\ell = 1/2, 3/2, \ldots$ gives results that are qualitatively the same as those of Sec. III B. For $\ell = 1/2$ the energies and widths (Eq. 1.1) of the first three resonances are $(2.64, 0.34)$, $(4.25, 0.34)$, and $(5.62, 0.34)$, and for $\ell = 3/2$ they are $(3.53, 0.11)$, $(5.04, 0.15)$, and $(6.34, 0.18)$, in units of $(\hbar^2 G^2/2m)^{1/3}$. The similarity of the motion of spin 1/2 and of spin 1 atoms in the guide is underlined by the fact that Eqs. (3.7) for spin 1/2 and $\ell = 1/2$ are structurally the same as Eqs. (4.7a) and (4.7b) for spin 1 and $\ell = 0$ except for a replacement of $G$ by $2G$.

The energies and widths for $s = 1/2$ and the unphysical case of integer $\ell$ have been reproduced by a complex scaling calculation by Potvliege and Zehnlé [1]. These authors agree that $\ell$ should in fact be taken half integer [2]. The applicability of their method is independent of whether integer or half integer of $\ell$ are being considered.

Nothing changes for spin 1 in Sec. IV.

There are also two misprints. Contrary to what is stated in the text of Sec. II, we have used $G = 2g \mu_B B_0/R$ throughout the paper. The last paragraph of Sec. VII discusses the first excitation energy for an $\ell = 0$ atom of $^{87}$Rb, and not $\ell = 1$ as originally stated.

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