Asymptotic safety and Kaluza-Klein gravitons at the LHC

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We study Drell-Yan production at the LHC in low-scale quantum gravity models with extra dimensions. Asymptotic safety implies that the ultraviolet behavior of gravity is dictated by a fixed point. We show how the energy dependence of Newton’s coupling regularizes the gravitational amplitude using a renormalization group improvement. We study LHC predictions and find that Kaluza-Klein graviton signals are well above standard model backgrounds. This leaves a significant sensitivity to the energy scale $\Lambda_T$ where the gravitational couplings crossover from classical to fixed point scaling.

I. INTRODUCTION

By allowing the fundamental Planck scale to be as low as the electroweak scale, models with extra dimensions have raised enormous interest in the past decade. This way, the notorious hierarchy problem of the standard model (SM) is elegantly circumvented [1,2]. In these scenarios, SM particles propagate on the four-dimensional brane, whereas gravity lives in the higher-dimensional bulk. If realized in nature, high-energy particle collisions at the LHC become sensitive to Planck scale physics and offer the exciting possibility to experimentally test the quantization of gravity.

A description of graviton mediated scattering processes at energies close to the Planck scale requires a fundamental quantum theory for gravity. Unfortunately, perturbative quantization of four- and higher-dimensional gravity seems not applicable, and a fully satisfying theory is currently not at hand. In recent years, field-theoretical studies indicate that higher-dimensional gravity becomes asymptotically safe [3,4], meaning that the ultraviolet behavior of gravity is dictated by a nonperturbative fixed point under the renormalization group (RG) [5]. In the asymptotic safety scenario for gravity, the energy dependence of the gravitational couplings is characterized by an energy scale $\Lambda_T$ where the gravitational couplings cross over from classical behavior at low energies to fixed point scaling at high energies [6–9]. The nontrivial UV fixed point of gravity can be seen as the gravitational analogue of the perturbative UV fixed point of Yang-Mills theory. In gravity, the onset of fixed point scaling manifests itself through a weakening of Newton’s coupling. Experimental signatures for asymptotic safety within low-scale quantum gravity are discussed in [3,4,10–12].

A generic prediction of large extra dimensions is a tower of massive Kaluza-Klein (KK) gravitons. At colliders, Kaluza-Klein gravitons appear both as real final states [13] and virtual intermediate states [14,15]. Real gravitons leave the detector unnoticed and appear as missing energy. Virtual gravitons mediate Drell-Yan processes [16], and lead to deviations in standard model reference processes such as $pp \rightarrow \ell^+\ell^-$. Missing energy signals and the study of benchmark spectra are standard search tools for physics beyond the SM.

Previously, gravitational Drell-Yan production was studied within asymptotically safe gravity [10,11], string theory [17], and effective theory [18,19] (see [20] for a review). The relevant amplitude involves a sum over all intermediate Kaluza-Klein gravitons. Within effective theory, this sum is strongly UV divergent in two and more extra dimensions and requires a UV cutoff of the order of the fundamental Planck scale. This scale cannot be precisely determined by low-energy considerations alone. Within asymptotically safe gravity, high-energy interactions become dynamically suppressed [4], rendering the single graviton amplitude finite [10]. The amplitude no longer requires an artificial UV cutoff and is solely determined through the dynamical scale $\Lambda_T$. Form-factor approximations of the fixed point which treat the Kaluza-Klein gravitons perturbatively also require a UV cutoff, which ideally should be fixed by quantum gravity itself [11].

In this paper, we analyze gravitational Drell-Yan production at the LHC, extending the study of [10] towards Planckian center-of-mass energies. The UV finiteness of the amplitude in principal offers access to the crossover scale $\Lambda_T$. We implement explicit RG equations for the running of Newton’s coupling and also study the RG scheme dependence of amplitudes. We show how to relate the RG scale with the Kaluza-Klein mass and kinematic variables of the scattering process. We evaluate the amplitude both numerically and analytically, using analytical continuation in energy or principal value integration,
respectively. We also explain how the form-factor approximation and effective theory fit within our picture in specific limits. Interestingly, we find that the Kaluza-Klein gravitons act as messengers of the higher-dimensional fixed point which tend to suppress the amplitude on the brane. While the energy dependence of parton-level amplitudes is folded with strongly decaying parton distribution functions, we still find significant sensitivity to the scale \( \Lambda_F \) over SM backgrounds.

We organize this paper as follows: we start with a brief review of the single graviton amplitude and its significant UV sensitivity within effective theory (Sec. II). The basics of the asymptotic safety scenario for quantum gravity are discussed (Sec. III), including explicit RG equations for the running of Newton’s coupling. We then implement quantum gravity (Sec. III), including explicit RG equations for the running of gravitons by its RG-improved counterpart (Sec. IV). We evaluate various schemes to relate the RG scale with the UV cutoff scale introduced to regularize the amplitude would behave as

\[ S(s) = - \frac{S_{n-1}}{M_*^2} \left( \frac{1}{M_*^2} \right)^{n-2} C, \]  

but the numerical coefficient \( C \) is divergent. A physical meaning can be given to the amplitudes for \( n > 2 \) by analytical continuation of the result for \( 0 < n < 2 \), corresponding to dimensional regularization (DR). In this case one finds \( C_{\text{DR}} = \pi^{n/2} \Gamma(1 - \frac{n}{2}) \). For odd dimensions \( C_{\text{DR}} \) is finite, but remains divergent for even dimensions. The regularized version of the effective operator takes the form

\[ S = - \frac{S_{n-1}}{M_*^2} \left( \frac{\Lambda}{M_*} \right)^{n-2} C \]

and \( F(x) \) denotes the explicit regularization for the integral \( (x = m/\Lambda) \). Hence, the coefficient \( C \) now parametrizes the effective operator in terms of the (unknown) UV completion of the theory. There are several ways to implement a UV cutoff \([19,22–26]\). For a sharp cutoff the UV modes are suppressed by hand above \( m = \Lambda \) and \( F_{\text{sharp}}(x) = \theta(1 - x) \). This leads to

\[ C_{\text{sharp}} = \frac{1}{n - 2}. \]  

Note that the strong UV divergence of the single graviton amplitude (3) is now parametrized by the strong sensitivity of (5) on the UV cutoff scale \( \Lambda \). Note, additionally, that the logarithmic divergence at \( n = 2 \) becomes a simple pole in the coefficient (7).
On dimensional grounds and to lowest order in $s \ll M_s$, the effective operator representing $s$-channel graviton exchange can also be written as

$$S = -\frac{4\pi}{M_s^4}.$$  \hfill (8)

This parametrization using the scale $M_{\text{eff}}$ is commonly applied in the literature. In general, the mass scale $M_{\text{eff}}$ cannot be determined from low-energy considerations alone, but requires input from the full quantum dynamics at the Planck scale. The regularization (6) also fixes the mass scale $M_{\text{eff}}$ in (8) as

$$\left(\frac{M_s}{M_{\text{eff}}}\right)^4 = \frac{S_{n-1}}{4\pi} \left(\frac{\Lambda}{M_s}\right)^{n-2} C.$$  \hfill (9)

Below, we will denote the effective theory cutoff on the KK masses as $\Lambda_{\text{eff}}$. The energy integration over (3) also requires a cutoff in effective theory, which we denote as $\Lambda_s$. For the remainder of this paper we study how quantum gravity regularizes the amplitude (3) dynamically, provided that the high-energy behavior of gravity is governed by a renormalization group fixed point.

### III. Quantum Gravity

In this section, we discuss the asymptotic safety scenario for gravity and adopt it for the purposes of the present paper. It is well known that the standard perturbative quantization of gravity faces problems. In this section we discuss S. Weinberg’s scenario known as asymptotic safety [5] (see [6–9] for reviews), where the high-energy behavior of gravity is nonperturbative and dictated by an interacting fixed point under the renormalization group. We also introduce some of our notation and discuss the main implications as well as explicit renormalization group equations for gravity.

#### A. Asymptotically safe gravity

As in any nontrivial quantum field theory, once quantum fluctuations of the propagating field—the metric field—are taken into account, the corresponding (Newton’s) coupling $G_N$ becomes a “running” coupling as a function of the renormalization group scale $\mu$. To set the stage, we will assume a general $d$-dimensional gravitational action including matter couplings. We are interested in the coupling of standard model particles living on a four-dimensional brane with $d$-dimensional gravity. The fundamental Planck scale $M_s$ is of the order of a few TeV. Within effective theory, quantum gravitational corrections can reliably be computed for characteristic energies below the fundamental Planck scale $E^2 \ll M_s^2$, provided that the theory is equipped with a UV cutoff. We write the renormalized (running) gravitational coupling as

$$G(\mu) = G_N Z^{-1}(\mu).$$  \hfill (10)

where $\mu$ denotes the RG scale. We have also introduced the wave-function renormalization factor $Z(\mu)$ accounting for the scale (or energy) dependence of Newton’s coupling. The energy dependence of (10) is given by the Callan-Symanzik equation for the running of Newton’s coupling [6]

$$\frac{dG(\mu)}{d\ln \mu} = \eta G(\mu).$$  \hfill (11)

Here $\eta$ denotes the graviton anomalous dimension $\eta(\mu) = -\frac{d\ln Z(\mu)}{d\ln \mu}$. Note that the anomalous dimension is a function of all couplings in the theory including the gravitational couplings, the gravity-matter couplings, and the cosmological constant. Once $\eta = \eta(\mu)$ is known explicitly as a function of the RG scale $\mu$ we have

$$Z^{-1}(\mu) = Z^{-1}(\mu_0) \exp\left(\int_{\mu_0}^\mu dk \eta(k)\right).$$  \hfill (12)

At low energies $\mu \ll M_s$, the quantum corrections are weak, and can be computed perturbatively provided

$$G(\mu) = G(\mu_0) \mu^{d-2} \ll 1.$$

To leading order, the anomalous dimension obeys $|\eta| \ll 1$ in the perturbative regime. However, once energies approach the Planck scale, quantum corrections become relevant and the graviton anomalous dimension becomes large $|\eta| \sim O(1)$. In this regime the sensitivity to the high-energy structure of gravity is strong, and an effective theory description is no longer applicable.

Within S. Weinberg’s scenario of asymptotically safe gravity, the domain of validity of (10) is extended to energies at and above the Planck energy. This regime is central for collider signatures of low-scale quantum gravity investigated in this paper. Within the asymptotic safety scenario it is stipulated that the metric field remains the main carrier of the gravitational force even at high energies. This is achieved provided that the relevant gravitational couplings display a nontrivial UV fixed point. The fixed point tames the virulent UV divergences of standard perturbation theory and implies a weakening of gravitational interactions at short distances, sometimes referred to as “antiscreening.” Implications of the fixed point become visible once we introduce dimensionless variables as appropriate for fixed point studies. We define the dimensionless gravitational coupling as $g(\mu) = G(\mu) \mu^{d-2}$. The corresponding Callan-Symanzik equation reads

$$\frac{dg}{d\ln \mu} = (d - 2 + \eta)g(\mu).$$  \hfill (13)

From (13) we conclude that the gravitational coupling displays two types of fixed points, a free theory fixed point at $g_* = 0$ and an interacting fixed point with

$$\eta_* = 2 - d.$$  \hfill (14)

The free theory fixed point is the so-called Gaussian one. In its vicinity the anomalous dimension is small and hence the
running coupling \( G(\mu) = G_N \) is not running at all. The Gaussian fixed point governs the regime of classical general relativity. The second fixed point—an implicit one—requires that the anomalous dimension \( \eta \) counterbalances the canonical dimension of Newton’s coupling. It is interesting to compare this behavior with standard perturbation theory in Newton’s coupling. The effective expansion parameter is the dimensionless quantity \( G_N E^{d-2} \), where \( E \) denotes the energy scale. While this coupling is small for low energies, it grows with increasing energy, ultimately causing a breakdown of standard perturbation theory. In contrast, the dimensionless coupling at high energies remains bounded within asymptotically safe gravity and achieves a fixed point. When expressed in terms of the renormalized Newton’s coupling, the fixed point scaling leads to the behavior \( G(\mu) = g_s / \mu^{d-2} \) for large \( \mu \gg M_\ast \). Hence gravity is antiscreening and weakens in the onset of fixed point scaling.

Next we discuss implications of the RG running at the level of the graviton propagator [6–9]. Neglecting tensor indices, the \( d \)-dimensional graviton propagator \( \Delta(p^2) \) behaves as \( \sim 1/p^2 \) in the perturbative regime where \( p^2 \ll M_\ast^2 \). RG corrections are obtained via the wave-function renormalization factor, leading to \( \Delta(p^2) \rightarrow 1/(Z(\mu)p^2) \). Evaluating the RG at the scale set by the graviton momentum \( \mu^2 = p^2 \) implies that the anomalous dimension modifies the propagator \( \Delta(p^2) \rightarrow (p^2)^{-1+\eta/2} \). Clearly, for small anomalous dimension \( |\eta| \ll 1 \), the propagator remains unchanged. In turn, the propagator behaves as \( \Delta(p^2) \sim (p^2)^{-d/2} \) in the fixed point regime where the anomalous dimension grows large. Hence, the renormalization group improvement of the propagator due to the dressing with virtual gravitons leads to an enhanced suppression for large external momenta [3,4]. Below, we exploit the onset of fixed point scaling in the regime where center-of-mass energies approach the fundamental Planck scale.

### B. Functional renormalization

Explicit RG equations for gravity—such as (11) and (13), and similar for other couplings—are studied in [3,4,27] using the functional renormalization group [28,29]. This technique is based on a Wilsonian cutoff for the propagating modes. Diffeomorphism symmetry is controlled with the help of the background field method [27,30]. In order to illustrate the main result, and also for later use (see Secs. IV and V), we discuss the RG equations for the gravitational coupling in the absence of a cosmological constant and matter couplings. Following [3], the RG equation for \( g \) is given by (13), where the anomalous dimension is

\[
\eta = (2 - d) \frac{2(d + 2) g}{1 - 2(d - 2) g}.
\]

We have rescaled \( g \) as \( g \rightarrow g / c_\eta \) with \( c_\eta = (4\pi)^{d/2-1} \times \Gamma(\frac{d}{2} + 2) \) to simplify the expressions. Note that (15) is nonperturbative in the running coupling \( g \). The RG equations (13) and (15) are integrated to give the explicit solution

\[
\frac{\mu}{\mu_0} = \left( \frac{g(\mu)}{g_0} \right)^{1/\theta_0} \left( \frac{g_s - g(\mu)}{g_s - g_0} \right)^{-1/\theta_{nG}}.
\]

The parameter \( \theta_G \) and \( \theta_{nG} \) are the scaling exponents at the Gaussian \( (G) \) fixed point \( g_s = 0 \) and the non-Gaussian \( (nG) \) fixed point \( g_s \neq 0 \), with \( g_0 = g(\mu = \mu_0) \). The former is given by the canonical mass dimension of Newton’s coupling in \( d \) dimensions, whereas the latter is determined from the UV dynamics of the theory [3] with

\[
\theta_G = d - 2, \quad \theta_{nG} = 2d - 2 \frac{d}{d + 2}.
\]

As a function of the RG scale \( \mu \), (13) and (15) displays a crossover from classical scaling to fixed point scaling. The scaling indices \( \theta \) parametrize how rapidly each gravitational coupling and its anomalous dimension crossover from their classical to fixed point values on a logarithmic energy scale. Note that both indices grow with dimension. In particular, the larger \( \theta \), the faster the crossover regime in units of the RG “time” \( \ln \mu \). Furthermore, \( \theta_{nG} \) is larger than \( \theta_G \) and the ratio \( \theta_{nG} / \theta_G = 2d/(d + 2) \) ranges between \([4/3, 1/2]\) for \( d \) between \([4, \infty]\). It follows that the crossover region between IR and UV scaling narrows with increasing dimension [3,10].

### C. Running Newton’s coupling

For the applications below it is useful to have explicit expressions for the functions \( g(\mu), G(\mu), Z(\mu), \) and \( \eta(\mu) \) at hand. We introduce three different approximations capturing the essential features. Since the transition from perturbative to fixed point scaling becomes very narrow with increasing dimension, the scale-dependence is well-approximated by an instantaneous jump in the anomalous dimension at the energy scale \( \mu = \Lambda_T^{(0)} \). This is the quenched approximation, solely characterized by a transition scale of the order of the fundamental Planck scale \( M_\ast \), where

\[
Z^{-1}(\mu) = \begin{cases} 
1 & \text{for } \mu < \Lambda_T^{(0)} \\
(\Lambda_T^{(0)})^{d-2} / \mu^{d-2} & \text{for } \mu \geq \Lambda_T^{(0)} 
\end{cases}
\]

\[
\eta(\mu) = \begin{cases} 
0 & \text{for } \mu < \Lambda_T^{(0)} \\
2 - d & \text{for } \mu \geq \Lambda_T^{(0)} 
\end{cases}
\]

In the quenched approximation, continuity in \( G(\mu) \) follows if the fixed point value \( g_s \) is related to Newton’s coupling at low energies and the transition scale via \( g_s = G_N(\Lambda_T^{(0)})^{d-2} \). We will identify the crossover scale \( \Lambda_T \) with the parameter \( \Lambda_T^{(0)} \) in (18).

In order to achieve explicit continuous expressions for \( \eta(\mu) \) we resolve (16) for \( g(\mu) \) to find \( Z(\mu) \) and \( \eta(\mu) \) by
using mild additional approximations for the ratio $\theta_G/\theta_{nG}$, see (17). For the additional approximation $\theta_G/\theta_{nG} = 1$, we find an explicit expression for the running coupling (10) from (16) with

$$Z(\mu) = 1 + G_N \mu^{d-2}/g_*,$$

$$\eta(\mu) = (2 - d) \left( \frac{1}{1 - \frac{G_N}{2g_* \mu^{2-d}}} \right).$$

We have taken the limit $g_0 \ll g_*$ and $\mu_0 \ll \Lambda_T$ while keeping the gravitational coupling $G_N = g_0/\mu_0^{d-2}$ fixed to its $d$-dimensional value. We have also used $g_* = G_N(\Lambda_T^{(1)})^{d-2}$. Here, the anomalous dimension is linear in the dimensionless coupling $g$ and reads $\eta = (2 - d)g/g_*$. Provided that $(2 - d)/g_*$ is identified with the one-loop coefficient from perturbation theory in the presence of a UV cutoff, (19) becomes equivalent to a one-loop approximation. We refer to (19) as the linear approximation.

A second, nonperturbative, expression for the running coupling is achieved in the approximation $\theta_{nG}/\theta_G = 2$, meaning that the approach towards the UV fixed point is twice as fast as the approach towards the Gaussian fixed point. This approximation becomes exact with increasing $d$, see (17). The anomalous dimension and the wavefunction renormalization factor $Z$ are given by

$$Z^{-1}(\mu) = \sqrt{1 + \left( \frac{G_N}{2g_* \mu^{2-d}} \right)^2} - \frac{G_N}{2g_* \mu^{2-d}},$$

$$\eta(\mu) = (2 - d) \frac{Z^2(\mu) - 1}{Z^2(\mu) + 1}.$$

Since $\eta = (2 - d)g/(2g_* - g)$ in terms of the dimensionless coupling $g$, (20) is nonperturbative. The fixed point value is defined as $g_* = G_N(\Lambda_T^{(2)})^{d-2}$. We refer to (20) as

the quadratic approximation. Note that the crossover width in (20) is smaller than the one in the linear approximation (19). More generally, for any dimension, the crossover width of the solution (16) is bounded by the width of the linear crossover (19) and the width of the quadratic approximation (20), see Fig. 1 and 2.

D. Relevant scales

The scales relevant in the above setup are the $d$-dimensional Planck mass $M_s$, obtained from the $d$-dimensional Newton coupling via $M_s^{d-2} = 1/G_N$, and the scale $\Lambda_T$ where the gravitational coupling crossover from classical to fixed point scaling. The scale $M_s$ is set by the infrared behavior of the $d$-dimensional theory, $G_N = G(\mu = 0)$. In turn, the scale $\Lambda_T$ is set by the RG dynamics of the gravitational sector in the ultraviolet limit. Therefore we view $\Lambda_T$ as the dynamical Planck scale. We expect that $\Lambda_T$ is of the order of $M_s$, provided that the RG effects induced up to the scale $\mu = \Lambda_T$ are moderate. Because of the finite width of the crossover, the scales $\Lambda_T^{(0)}$, $\Lambda_T^{(1)}$, and $\Lambda_T^{(2)}$ appearing in the approximated RG equations (18)–(20) can be numerically different from the physical crossover scale $\Lambda_T$. The quantitative relation we work out in Sec. IV.

In scenarios where many particles couple to gravity, the externally accelerated RG running can lead to a large separation between $\Lambda_T$ and $M_s$. We will not be concerned with these scenarios in this paper. Therefore, in the approximations (18)–(20) the fixed point value $g_*$ parameterizes the uncertainty in the determination of the dynamical Planck scale $\Lambda_T$. We also note that an analogous pattern is observed when RG improving the physics of black holes [12]. The RG running leads to a dynamical mass scale $M_s$. 

FIG. 1 (color online). Crossover of the gravitational coupling $G(\mu)/G_N$ from classical (dashed line) to fixed point scaling (full lines) in the Einstein Hilbert theory with $n = 3$ extra dimensions: classical behavior, linear crossover (19), quadratic crossover (20), and the quenched approximation (18) with $\Lambda_T = \Lambda_T^{(0)} = \Lambda_T^{(1)} = \Lambda_T^{(2)}$ (see text).

FIG. 2 (color online). Crossover of the graviton anomalous dimension from classical (dashed) to fixed point scaling (full lines) in the Einstein Hilbert theory with $n = 3$ extra dimensions: classical behavior, linear crossover (19), quadratic crossover (20), and the quenched approximation (18), with $\Lambda_T = \Lambda_T^{(0)} = \Lambda_T^{(1)} = \Lambda_T^{(2)}$ (see text).
IV. VIRTUAL GRAVITON EXCHANGE

In this section, we explain how quantum gravity effects are implemented via a renormalization group improvement of the single graviton amplitude.

A. Scale identification

The tree-level single graviton amplitude in extra dimensions includes an integral over perturbative KK propagators. Quantum gravitational corrections are included by replacing perturbative propagators with quantum corrected running propagators, and perturbative vertex functions with running vertex functions. Our key assumption is that quantum gravity corrections are primarily accounted for by replacing the perturbative propagator by the RG-improved one. This step requires that the RG scale $\mu$ is identified with the scale relevant for the physical process under investigation

$$\mu = \mu(\sqrt{s}, m),$$

in general, depending on some combination of the center-of-mass energy $\sqrt{s}$ and the KK mass $m$. A discussion of a matching which involves several scales is given in [12] in the context of black hole production. The RG-improved amplitude takes the form

$$S(s) = -\frac{S_{n-1}}{M_T^2} \left( \frac{\Lambda_T}{M_T} \right)^{n-2} C(s; \Lambda_T),$$

where

$$C(s; \Lambda_T) = \int_0^\infty dx \frac{x^{n-1}}{-s/\Lambda_T^2 + x^2} Z^{-1}(\mu = \mu(\sqrt{s}, m, \Lambda_T)),$$

and $x = m/\Lambda_T$. The energy dependence of (23) is encoded in the wave-function factor $Z(\mu)$, itself defined via the running of the gravitational coupling (10) and (11). Note also that $Z(\mu)$ depends on $\Lambda_T$ through the fixed point and the $d$-dimensional Planck scale $M_*$, see Sec. III C.

Now we compare different approximations for the RG-improved couplings at a given crossover scale $\Lambda_T$. The definition of $\Lambda_T$ is fixed through the low-energy amplitude $S(s = 0)$. In particular, this implies that the crossover scale $\Lambda_T$ coincides with the scale $\Lambda_T^{(0)}$ set in the quenched approximation. The definition of the corresponding scale $\Lambda_T^{RG}$ in terms of $\Lambda_T$ for a smooth crossover of Newton’s coupling is then obtained from

$$(\Lambda_T)^{n-2} C_{\text{quench}}(s = 0; \Lambda_T) = (\Lambda_T^{RG})^{n-2} C_{\text{RG}}(s = 0; \Lambda_T^{RG}).$$

This uniquely fixes the scale $\Lambda_T^{RG}$ in terms of $\Lambda_T$ for any explicit wave-function factor $Z_{RG}^{-1}$. Quantitatively, the scales $\Lambda_T$ and $\Lambda_T^{RG}$ will at best differ by a few percent (see Fig. 3).

B. Kaluza-Klein gravitons

First, we analyze the effective operator for single graviton exchange (22) and (23) in the limit of small center-of-mass energy. This approximation is motivated at the LHC by the sharply falling parton densities. Within effective theory, the integration over the KK gravitons is cut off and we obtain the well-known result (7). Next, we introduce the renormalization group corrections predicted by asymptotically safe gravity. In the limit of small $\sqrt{s} \ll M_*$, the matching (21) becomes

$$\mu = m,$$

so that the RG scale is matched to the KK mass. For the coefficient (23), we find

$$C = \int_0^\infty dx x^{n-3} Z^{-1}(x\Lambda_T).$$

Here, $x = m/\Lambda_T$ measures the KK mass in units of $\Lambda_T$. Note the similarity between (26) and the regularized expression (6) from effective theory. Within the renormalization group, the scale dependence of the wave-function factor $Z^{-1}$ takes over the role of the regulator function $F(x)$ in (6). The coefficient (26) is computed for a given renormalization group trajectory $G(\mu) = G_NZ^{-1}(\mu)$. Analytical results for (26) are obtained in the approximations introduced in Sec. III C. In the quenched approximation (18), we find [10]

$$C_{(0)} = \frac{1}{n - 2} + \frac{1}{4} = C_{\text{EFT}} + C_{\text{UV}}.$$
We emphasize that the first term $C_{\text{EFT}}$ is identical to the contribution within effective theory (7), provided the effective theory cutoff coincides with the energy scale where gravity becomes nonperturbative. The second term $C_{\text{UV}}$ originates solely from the KK gravitons with mass $m > \Lambda_T$ (x > 1). For low numbers of extra dimensions these terms are similar in magnitude. However, with increasing $n$ the contributions from the trans-Planckian domain dominate over the sub-Planckian regime. In the linear approximation (19), we have

$$C_{(1)} = \frac{1}{4} \Gamma\left(\frac{n-2}{n+2}\right) \Gamma\left(\frac{n+6}{n+2}\right) = \frac{1}{4} + \frac{2}{3} \frac{\pi^2}{n} + \mathcal{O}\left(\frac{1}{n^2}\right).$$

(28)

We see that (28) reaches the same limit as (27). For the quadratic approximation (20), we obtain

$$C_{(2)} = -\frac{1}{(n+2) \Gamma(2 - \frac{n}{n+2})} = \frac{1}{4} + \frac{1}{2n} + \mathcal{O}\left(\frac{1}{n^2}\right).$$

(29)

The large-$n$ limit implies that $C_{(0)} \simeq C_{(2)} \simeq C_{(1)}$, which furthermore holds for all $n > 2$. Using $\Lambda_T$ within the quenched approximation as the reference point, we employ the results (27)–(29) to identify the relative normalization of scales using (24). We obtain $\Lambda_T / \Lambda_T^{(i)} = (C_{(0)} / C_{(i)})^{1/(n-2)}$. This normalization implies a small shift of the order of a few percent or less, see Fig. 3.

Next we discuss the full $n$ dependence of the amplitude (22) depending explicitly on $n$ via $S_{n-1}$, and implicitly through the RG factor $C$ and scale $\Lambda_T$ (see Fig. 4). The phase space factor $S_{n-1}$ grows until $n = 8$, and decays for larger $n$. The coefficients (27)–(29) decrease until $n = 10$ and stay at their asymptotic value thereafter. Interestingly, their product is $n$ independent for $n = 3, \ldots 8$ so that the main $n$ dependence of (22) originates from the prefactor $(\Lambda_T / M_*)^{n-2}$. Therefore, the main dependence is on the value of $\Lambda_T$ in $d = 4 + n$ dimensions.

In Fig. 5, we compare the production amplitude (23) at small $s \ll \Lambda_T^2$, parametrized by coefficient $C$ from effective theory and the renormalization group. Within effective theory, the IR dynamics of KK gravitons are retained but their UV dynamics are suppressed due to a cutoff at $m = \Lambda_T$. The amplitude becomes very small for large $n$.

Within asymptotically safe gravity, the UV dynamics of KK modes is controlled by a fixed point for the gravitational couplings. Quantitatively, the UV contributions to the amplitude dominate with increasing $n$. The amplitude remains bounded and approaches a finite universal limit for large $n$ as a consequence of the underlying fixed point. This can be understood from (26) as follows. For small $x < 1$, the factor $Z(x\Lambda_T) = 1$ is bounded. Because of the volume factor $\sim x^{-3}$ in the KK integration, the sub-Planckian modes with $x < 1$ will no longer contribute to (26) in the limit of many extra dimensions. In the trans-Planckian regime $x > 1$ on the other hand, fixed point scaling implies $\mu \partial_\mu Z = (n + 2) Z$, see (10), (11), and (15). Then, under the integral we can trade the RG scaling $\mu \partial_\mu$ with $x \partial_x$ scaling. In the limit $1/n \to 0$ the scaling regime sets in at $x \approx 1$ and the integrand becomes a total derivative $C = \frac{1}{4} \int_0^1 dx \partial_x(x^{-4})$, whence $C = \frac{1}{4}$ independently of the details of the RG trajectory $Z(x\Lambda_T)$. The same conclusion is reached by recalling that the volume of an $n$-dimensional sphere is concentrated in the vicinity of its boundary for large $n$. We also note that the coefficient approaches the limit $C = \frac{1}{d-n}$ in a setup where particles are confined to a $(d-n)$-dimensional brane.

![FIG. 4. The $n$ dependence of the amplitude $|S| M_*^2/(4\pi)$ defined in (22). Thin grey lines cover $\Lambda_T / M_* = 0.5$ to 1.5 in steps of 0.05 (from bottom to top). Close to $\Lambda_T / M_* = 1$ (thick black line), the overall $n$ dependence of the amplitude is very weak.](image)

![FIG. 5. Comparison of fixed point gravity with effective theory predictions for the amplitude (23) in the limit of small center-of-mass energies $s \ll \Lambda_T^2$ for various numbers of extra dimensions. The dashed line indicates the effective theory result, with a sharp cutoff on the KK mass at $m = \Lambda_T$. The full line represents the fixed point gravity result with a crossover scale $\Lambda_T$. Their difference is mainly due to the UV dynamics of the KK gravitons, and becomes more pronounced with increasing $n$.](image)
C. Planckian energies

Next, we are interested in the behavior of amplitudes at Planckian center-of-mass energies. Expressions for the RG-improved single graviton amplitude are obtained by identifying the renormalization group scale $\mu$ with the $d$-dimensional Euclidean momentum scale,

$$\mu^2 = s + m^2. \quad (30)$$

This identification is most natural from a $d$-dimensional perspective as the effective single graviton amplitude is sensitive to the $d$-dimensional momenta running through the propagator. We evaluate (30) together with an analytically continuation to the Euclidean signature, and obtain finite results for the single graviton amplitude. Using the definition for $Z(\mu)$ via (10) and (11), we find

$$C(s) = \int_0^\infty dx \frac{x^{n-1}}{y^2 + x^2} Z^{-1}\left(\sqrt{y^2 + x^2} \Lambda_T\right) \quad \text{with} \quad y^2 = s/\Lambda_T^2. \quad (31)$$

We emphasize that the $s$ dependence of (31) originates from both the single graviton exchange and the RG dressing of the propagator. In Fig. 6, we compute (31) in the quenched approximation for various extra dimensions. For small $s/\Lambda_T^2 \ll 1$ the variation compared to $C(s = 0)$ is small, though slightly stronger for increasing $n$. With increasing number of extra dimensions, the crossover from perturbative scaling to high-energy scaling sets in at lower energy scales $s < \Lambda_T^2$. This new pattern is largely insensitive to the details of the RG running as can be seen from Fig. 7, where we establish the RG scheme (in-)dependence by comparing the quenched, linear, and quadratic approximation. Moreover, while a small difference at $s = \Lambda_T^2$ remains visible for $n = 4$, the results with increasing $n$ in all three approximations are barely distinguishable at all values of $s$. The pre-Planckian suppression also indicates that the RG-improved amplitudes are compatible with perturbative unitarity. This pre-Planckian suppression in the effective operator is also noteworthy because the underlying $d = (4 + n)$-dimensional running coupling, with increasing $n$, displays an increasingly sharp crossover at the scale $\Lambda_T$. Hence, the extra-dimensional integration of KK modes implies that the transition region between classical and fixed point scaling becomes wider, and shifted towards scales below the fundamental transition scale $\Lambda_T$. This result also implies that the domain of applicability for standard effective theory becomes smaller with increasing $n$.

D. Universality

For asymptotically large $s/\Lambda_T^2 \gg 1$, the amplitude (31) is solved analytically and independent of the renormalization group trajectory $Z^{-1}(\mu)$. Since $Z$ is deep in the scaling regime for large $\sqrt{s}$ we find that $\lim_{\mu \to \infty} \mu^{n/2} Z^{-1}(\mu)$ becomes a $\mu$-independent constant, also using (12) and (15). Therefore, the leading behavior reads

$$C(s) = c_n \left(\frac{\Lambda_T^2}{s}\right)^2 + \text{subleading}, \quad (32)$$

with the coefficient

$$c_n = \lim_{s \to \infty} \int_{\Lambda_T/\sqrt{s}}^\infty dz \frac{z^{n-1}}{(1 + z^2)^{n/2 + 2}} = \frac{2}{n(n + 2)}, \quad (33)$$

where we have substituted $z = m/\sqrt{s}$. The leading large-$s$ decay (32) is universal, independent of the number of extra dimensions, and fully dominated by the gravitational fixed point. All reference to the renormalization group trajectory have dropped out. On the other hand, nonuniversal information is encoded in the prefactor of the leading term in (32). This includes the dependence on the number of extra dimensions.
dimensions $n$ in (33), the dependence on the choice of the RG scheme implicit to the matching condition (30), and the $n$ dependence of the crossover scale $\Lambda_T$ in (32).

This result can also be understood on dimensional grounds. The tree-level amplitude $S(s)$ in (22) has canonical mass dimension $[S(s)]=-4$. From the four-dimensional (brane) perspective it can be read as $\propto G_{\text{eff}}(s)/s$, where $G_{\text{eff}}(s)$ is the effective energy-dependent Newton’s coupling in four dimensions. In the scaling regime $\sqrt{s}/\Lambda_T \gg 1$, $\sqrt{s}$ is the only remaining energy scale. Therefore, $S(s) \propto 1/s^2$, and the number of extra dimensions is not differentiated by the scaling. This behavior equally implies $G_{\text{eff}}(s) \propto 1/s$, which is the expected pattern for an RG fixed point in four dimensions. Hence, after the KK modes are integrated out, the amplitude $S(s)$ behaves as if an effective four-dimensional graviton is exchanged whose high-energy behavior is governed by a four-dimensional fixed point with a crossover scale $\Lambda_T$ parametrically smaller than the four-dimensional Planck scale $M_{\text{Pl}}$. In this case, the effective four-dimensional fixed point is inherited from the underlying higher-dimensional fixed point.

**E. Form-factor approximation**

From a four-dimensional (brane) perspective, the relevant kinematical scale is the exchanged momentum $\sqrt{s}$, suggesting the 4-dimensional scale identification

$$\mu^2 = s.$$  

(34)

For the coefficient (31), this matching implies that the KK gravitons are treated perturbatively and their UV dynamics are cut off, similar to effective theory. Furthermore, the $s$ dependence originates solely from the running gravitational coupling, leading to the form-factor approximation

$$C(s) = Z^{-1}(\sqrt{s})C_{\text{KK}}.$$  

(35)

with $Z(\mu)$ given by the renormalization group equations for gravity. Here, $C_{\text{KK}}$ denotes a regularized version of the KK integral (6).

The matching (34) with (35) has been adopted by Hewett and Rizzo [11], together with the approximation (19) for Newton’s coupling and an independent UV cutoff for the KK integral $\Lambda_{kk}$. This is conceptually different from our approach (see also [10]), which does not require an additional UV cutoff. The Hewett-Rizzo parameter $t$ [11] is related to $g_s$ in (19) and the scale $\Lambda_T$ as $t = (g_s)^{(n+2)/(n+1)} = \Lambda_T/M_{\text{Pl}}$. Note that for $t$ below $= 2$, it has been argued that scattering within asymptotically safe gravity is compatible with perturbative unitarity [11].

The additional UV cutoff on the KK modes in (35) should be set by the dynamical Planck scale. In this case, $C_{\text{KK}}$ will be a similar order of magnitude as the coefficients $C$ obtained from the RG at $s = 0$. In this case, we conclude that the form-factor approximation (35) can be obtained from a matching (21) provided the factorization $Z(\mu(\sqrt{s}, m)) \approx Z_s(\sqrt{s})Z_m(m)$ holds true in the relevant kinematical regime. We also note that the $s$ dependence in (35) originates solely from the wavefunction factor of the $d$-dimensional gravitational coupling. Therefore, the large-$s$ decay of the amplitude is given by the large-$\mu$ decay of the gravitational coupling,

$$C(s) \approx \left(\frac{\Lambda_T^2}{s}\right)^{n/2+1}.$$  

(36)

With increasing dimension, the suppression becomes increasingly strong. In Fig. 8, we discuss the impact of the KK modes within asymptotically safe gravity for various $n$. The grey curves include the KK dynamics following (31) and (32), whereas the black curves neglect their dynamics for KK masses above $\Lambda_T$, see (35) and (36). Both sets of curves are based on the linear approximation (19) for the RG running of the gravitational coupling, normalized to the same strong-gravity scale $\Lambda_T$. For (35), the energy dependence of the amplitude becomes visible only very close to $\Lambda_T$, which is followed by a rapid crossover and an $n$-dependent large-$s$ decay. Furthermore, the large-$s$ behavior shows no universality if the KK dynamics are omitted. In contrast to this, fully integrating out the KK sector leads to a suppression of the amplitude already below $\Lambda_T^2$, and to universal scaling in the trans-Planckian regime. This summarizes the differences between our approach and the matching (35). We conclude that the dynamics of the KK modes is relevant already at sub-Planckian energies. The form-factor approximation (35) together with (34) is applicable for sufficiently small $s \ll M_{\text{Pl}}^2$, provided that the additional UV cutoff $\Lambda_{kk}$ is set to the scale $\Lambda_T$.

**FIG. 8.** Impact of KK dynamics on the single graviton amplitude within asymptotically safe gravity. All curves are based on Newton’s coupling in the linear approximation with transition scale $\Lambda_T$. Gray (black) lines denote the amplitudes with (31) [without (35)] the UV dynamics of KK gravitons; $n = 4, 8,$ and 12 (from right to left). The inclusion of KK dynamics leads to a universal suppression of the amplitude.
**F. Pole region and virtuality**

In this section, we evaluate the KK integral in the pole region, where $s = m^2$ and the graviton is deemed on-shell. On-shell KK gravitons can also decay into lighter particles and therefore have a nonzero decay width. We explore the implications of the graviton width in more detail in Sec. IV G below. Here, we will adopt the principal value prescription. In addition, we compare the matching (30) with the alternative choice

$$\mu^2 = |s - m^2|, \quad (37)$$

where the RG scale is identified with the virtuality of a KK graviton. Note that the main difference between the choice $\mu^2 = s + m^2$ from (30) and (37) relates to the pole region where $s = m^2$. Around the pole, (37) implies $\mu = 0$, meaning that this region is only sensitive to classical physics $G(\mu = 0)$, even for trans-Planckian energies $s$, $m^2 \gg M_*^2$. In contrast, the matching (30) leads to $\mu \approx m$ at the pole, which is sensitive to RG corrections.

For illustration, the physics content of the matching (37) is depicted in Fig. 9 for a specific choice of $M_*$ and $\Lambda_T$. Single graviton exchange is reliable for $\sqrt{s}$ below the fundamental Planck scale. For KK modes starting with masses of the order $\sqrt{s + \Lambda_T^2}$, RG corrections must be taken into account. This is denoted as the high mass region (HM) in Fig. 9. A high-energy region (HE) requiring RG effects may also exist, provided that the crossover scale $\Lambda_T$ is below the fundamental scale of gravity $M_*$. As the collider energy approaches $M_*$, we expect multigraviton processes or the production of mini-black holes to become important. Finally, we define the pole region (P), where $s = m^2$ and the graviton is on-shell.

We quantify the pole region using the RG running in the linear approximation (19), together with (37). The KK kernel falls off sufficiently quickly as a function of $m$ and we can adopt the prescription

$$Z^{-1} \frac{m^{n-1}}{s - m^2 + i\epsilon} = Z^{-1} P \frac{m^{n-1}}{s - m^2} + i\pi \frac{m^{n-1}}{\sqrt{s}} \delta(\sqrt{s} - m). \quad (38)$$

We also use the fact that $Z^{-1} = 1$ at $\mu = 0$ for all approximations. The second term in (38) is responsible for the on-shell production of KK gravitons. The principal value integration we can perform analytically for all $n = 2 + 4i$ and integer $i$. For $n = 2$ we obtain

$$C(s) = P \int ds \frac{x}{s + x^2} Z^{-1}(\mu^2 = |s - x^2|) = \frac{1}{4} \ln \left(1 + \frac{1}{s^2} \right) \quad (39)$$

with $x = m/\Lambda_T$ and $s \rightarrow s/\Lambda_T^2$. We note that the amplitude is well-defined for all $s$, and the result agrees with the one obtained by analytical continuation. The amplitude decays asymptotically as $\propto 1/s^2$. For comparison, effective theory gives $C_{\text{eff}}(s) = \frac{1}{2} \ln(1/s - 1)$ valid for $s/\Lambda_T^2 < 1$ [21], which has the same logarithmic singularity as (38) for ultrasoft $s/\Lambda_T^2 \ll 1$. Next we consider $n = 6$, where

$$C(s) = \frac{\pi s}{2\sqrt{2}} + \frac{s^2}{8} \ln \left(1 + \frac{1}{s^2} \right) - \frac{s}{4\sqrt{2}} \ln(A(s) + s^2) - \frac{A(s)}{4} \arctan A(s) + (s \rightarrow -s). \quad (40)$$

and $A(s) = 1 - \sqrt{2s}$, $x = m/\Lambda_T$, while $s$ is again expressed in units of $\Lambda_T^2$. The result agrees with (31) after analytical continuation $s \rightarrow -s$. For large negative $s$, we find that the amplitude (40) decays as $s^{-2}$. Again, this matches the coefficient from (32) in this limit. For large positive $s$, (40) grows linearly with $s$, approaching $\pi s/\sqrt{2} + O(1/s^2)$. We stress that the linearly growing term in $s$ originates from the density of states at the pole region for $s, m^2 > \Lambda_T^2$. As discussed above, this behavior is an effect of the virtuality matching (37), which treats the modes at the pole with $s, m^2 \gg \Lambda_T^2$ as classical. In fact, the growing term is absent for the Euclidean matching (30), which induces a suppression of the density of states for large KK masses. In comparison with effective theory, we note that the amplitude from fixed point gravity has a similar energy dependence for $|s|/M_*^2 \ll 1$. For $s \rightarrow M_*^2$, effective theory is no longer applicable and the energy...
dependence differs between the approaches. Effective theory predicts a weaker decay for large \( s \) after analytical continuation as it is not sensitive to the fixed point behavior of the KK gravitons.

To conclude, we obtain a good approximation for the amplitude (23) within a principal value integration via the Euclidean matching (30). The amplitude decays \( \propto s^{-2} \) for large \( s \) and quantitatively becomes equivalent to analytical continuation in \( s \), (31). The significance of the \( s^{-2} \) scaling is that a necessary condition for perturbative unitarity is that the cross section be bounded by \( 1/s \). Examining the full \( s \)-channel amplitude we see that \( C(s) \sim s^{-2} \) produces exactly this behavior at the level of \( \sigma \). Hence, the amplitude decays asymptotically as required by perturbative unitarity for all physically motivated matchings. For \( \Lambda_T/M_* \) up to the order of a few, the \( s \)-channel amplitude is unitary. In turn, for large \( \Lambda_T/M_* \gg 1 \), our results fall back on those from standard effective theory where perturbative unitarity is violated for center-of-mass energies approaching the fundamental Planck scale. Hence, the high-energy fixed point improves \( s \)-channel unitarity of the single graviton amplitude.

G. Graviton width

In order to justify the virtuality matching, we discuss the narrow-width assumption for KK gravitons. This aspect is independent of the UV sector, and also applies within standard effective theory. Using a Breit-Wigner propagator in the KK sum means we ignore interference between KK states, as the standard resummation ignores mixing between different KK states. In terms of the KK mass splitting this requires

\[
\delta_{KK} \gg \Gamma ,
\]

for \( m \sim \sqrt{s} \) where \( \delta_{KK} \) is the spacing between consecutive KK modes. It is simple to compute the lowest order graviton decay width to standard model particles, as it appears in (41). This decay is suppressed by \( 1/M_{Pl}^2 \) since it occurs only on the brane. A calculation based on the Feynman rules given in Ref. [21] yields

\[
\Gamma(G \rightarrow SM) = \frac{293}{960\pi} \frac{m^3}{M_{Pl}^2}.
\]

We may also consider on-shell decays of a single massive graviton to two other (less) massive gravitons. This contribution would produce a much larger decay width, suppressed only by factors of \( 1/M_* \). However, extra-dimensional momentum conservation persists as KK number conservation in the compactified theory (also the basis for dark matter in UED models [31]). The occupation vectors of all gravitons participating in an interaction must sum to zero at any vertex. When combined with energy-momentum conservation \( |\vec{n}_1|^2 > |\vec{n}_2|^2 + |\vec{n}_3|^2 \), the on-shell decay amplitude vanishes for all graviton self-interactions.

In fact, the next contribution to the decay width will come from higher orders in the expanded action based on power counting. We emphasize again the null contribution due to self-interactions. The second diagram includes a sum over the intermediate graviton as the KK number is not conserved due to the matter coupling. Indeed, below \( M_* \) the graviton width is tiny, but at this scale the resummation quickly breaks down and the width in general exceeds the spacing. Therefore, we have little reason to extend our classical results in the pole region above the scale \( M_* \). We emphasize that the gravitons contributing in this case probe IR physics. A rephrasing of this physical picture is that multigraviton exchange such as the second diagram in (43) becomes relevant at the scale \( M_* \). It is still possible to use perturbation theory in the pole region above \( \Lambda_T \) as long as we have a hierarchy \( \Lambda_T < M_* \).

The KK mass spacing is often estimated using \( \delta_{KK} = 1/R [32] \). We improve this estimate to assure that the narrow-width condition in (41) does not break down at scales significantly below \( M_* \). To that end, we consider a state \( \vec{n}_1 \) where \( \vec{n}_1^2 = N \) is some integer. In general, there will exist another state \( \vec{n}_{i-1} \) such that \( \vec{n}_{i-1}^2 = N - 1 \). If there is no state satisfying this relation then the spacing
will be larger than our estimate, leaving intact (41). The integer \( N \) fixes \( m = \sqrt{N}/R \) so that for two nondegenerate KK modes the spacing is at least

\[
\delta_{KK} \geq \frac{\sqrt{N} - \sqrt{N - 1}}{R} = \left( \frac{2\pi R M_{\text{Pl}}^n}{(2\pi R M_{\text{Pl}}^n + 2)} \right)^{1/(2n)} \times \left[ \sqrt{m} - \sqrt{m - \left( \frac{(2\pi R M_{\text{Pl}}^n)^n}{M_{\text{Pl}}^n} \right)^{1/n}} \right].
\]  

(44)

We note that (44) is sensitive to the equality of the radii of the extra dimensions. For the case of unequal radii, the lower bound may be set by choosing the largest radius. In Fig. 10 we see that this lower bound for the spacing is much larger than the lowest order graviton width for any number of extra dimensions \( n \geq 2 \) and for all energy scales relevant to LHC physics. For example, for \( n = 2 \) and \( M_\ast = 5 \text{ TeV} \) we expect the lines to cross only around \( m \sim 1000 \text{ TeV} \), far above the scale at which the leading order approximation width breaks down. For the LHC observables computed in the following, (44) confirms the hierarchy between the tiny KK width and the level spacing.

V. VIRTUAL GRAVITONS AT THE LHC

The LHC has several on-going searches for extra-dimensions (for a recent example see [33]). These include real graviton emission, black-hole production, and virtual graviton exchange. Real emission searches are generally well defined, and have little sensitivity to Planck scale physics at sub-Planckian collider energies. Similarly, black-hole production only becomes relevant at center-of-mass energies above the Planck scale. An important issue in understanding black hole production is the question of a remnant, i.e., a minimum black hole mass which is predicted by fixed point gravity [12] as well as by string theory. Virtual graviton exchange probes the quantum gravity effects in the ultraviolet range of the Kaluza-Klein spectrum in a well-defined field-theory environment. This makes this production process the prime candidate to test different descriptions of quantum gravity in low-scale gravity models. Hence, we apply our findings from the previous sections to gravitational Drell-Yan production at the LHC.

A. Kinematics and cuts

The generic prediction for virtual graviton exchange at the LHC is an enhancement of rates at increasing invariant masses. This does not include a mass peak or any kind of side-bin analysis, which means we must test the standard-model-only hypothesis as well as the Standard-Model-plus-graviton hypothesis on the kinematic distributions of the Drell-Yan process [19,21]. This signature may be further refined through the use of angular correlation techniques [34]. Our reference channel is \( pp \rightarrow \mu^+ \mu^- \) [35] at \( \sqrt{s} = 14 \text{ TeV} \) but other channels will show similar behavior. For the di-muon final-state one advantage from the theory point of view is that we need only consider \( s \)-channel diagrams.

Based on the discussion in Sec. IV F, we evaluate the relevant amplitudes using analytic continuation, which sets a lower bound. Moreover, since the SM background can originate only from a \( q\bar{q} \) initial state we have an automatic signal-over-background advantage due to probing the gluons in the initial state. Therefore, the collider energy plays an obvious role affecting the discovery reach in two ways. First, for valence quarks in the initial states it determines the upper limit of the Kaluza-Klein tower we can probe. Second, because of the rapid drop of the gluon parton density in the proton the lower end of the Kaluza-Klein tower will be enhanced over the background significantly.
once the collider energy increases. However, unlike for black hole production the luminosity is the main limiting factor because of the relatively small and perturbative gravitational coupling. An optimized strategy for the discovery of virtual gravitons at the LHC given an energy and a luminosity is beyond the scope of this paper. Our theoretical considerations, on the other hand, will strongly impact such a study. As model parameter we consider 2, 3, and 6 extra dimensions and fix the fundamental Planck scale to $M_s = 5$ TeV. The experimental cuts which we apply throughout the section to take into account detector features are

$$ p_{T,\mu} > 50 \, \text{GeV} \quad |\eta_\mu| < 2.5. \quad (45) $$

The detection efficiency for muons we assume to be 100%, including triggering, and their excellent energy resolution is perfect compared with the distributions we are going to look at. In addition, whenever we compare signal and background rates we require $m_{\mu\mu} > M_s/3$ to enhance the graviton signal over the standard model backgrounds. Combined with the cuts in (45) this suppresses the highly IR peaked photon and $Z$ backgrounds which generically fall off with the partonic center-of-mass energy fast. As said above, the idea of this section is not to present a comprehensive LHC analysis, but to test our results for the behavior of the graviton KK integral at the level of LHC signals.

### B. LHC signatures of the fixed point

We first compare LHC predictions from fixed point gravity with those from effective theory. Additionally we will evaluate the sensitivity of fixed point gravity signatures to the crossover scheme, see Sect. III C and IVA. The relevant amplitude is given by (23). We compare the total cross-section from fixed point gravity using $Z^{-1}$ from the quenched approximation (18) with the predictions from effective theory with UV cut-offs at $m = \Lambda_T$ and at $\Lambda_s = M_s$, which we implement as $Z^{-1}_{\text{eff}} = \theta(\Lambda_T - m)\theta(M_s - \sqrt{s})$. The result is displayed in Fig. 11 for $M_s = 5$ TeV and $\Lambda_T = M_s$. Dashed lines denote the result from effective theory, full lines stand for fixed point gravity. The difference between effective theory and fixed point gravity (solid versus dashed curves) is negligible at low scales $s \ll M_s^2$, provided the UV cutoff in effective theory is identified with the crossover scale, $\Lambda = \Lambda_T$. With increasing energy $\sqrt{s} = M_s$, the difference becomes more pronounced, indicating the limit of validity for an effective theory description. It turns out that with increasing $n$ the signal becomes more sensitive to the UV modes. This is exactly what is expected based on the discussion in Sec. IV B.

In Table I we compare the production cross-sections in various approximations for $M_s = \Lambda_T = 5$ TeV. For the UV contributions arising from KK modes, comparing fixed point gravity (a), (b) with effective theory (c), we find an increase in the total rate. On the other hand, including the UV events in $\sqrt{s}$ by comparing (a) with (b) we find only a

<table>
<thead>
<tr>
<th>$\sigma$ [fb]</th>
<th>$n = 3$</th>
<th>$n = 6$</th>
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<tr>
<td>$M_s$ [TeV]</td>
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<tr>
<td>(a) 0.317</td>
<td>0.15</td>
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<tr>
<td>(b) 0.320</td>
<td>0.15</td>
<td>0.394</td>
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<tr>
<td>(c) 0.215</td>
<td>0.09</td>
<td>0.209</td>
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<tr>
<td>(d) 0.182</td>
<td>0.09</td>
<td>0.190</td>
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FIG. 12 (color online). Signal distribution for di-muon production via virtual graviton exchange at the LHC with $M_s = 5$ TeV and $n = 3$ (6) extra dimensions show left (right). Comparison of several RG schemes and transition scales $\Lambda_T$: quenched, linear, and quadratic approximation with $\Lambda_T/M_s = 1.2$ (dotted lines), $\Lambda_T/M_s = 1$ (full lines), and $\Lambda_T/M_s = 0.8$ (dashed lines). With increasing $\Lambda_T/M_s$, the di-muon production rate increases. The scheme dependence is moderate.
modest increase. Evidently, the $1/s^2$ behavior for large energies makes these contributions subleading. The form-factor approximation (d) comes out below the effective theory estimate (c), introducing a modest extra reduction of overall rates with energy at below-Planckian energies. Provided the crossover scale takes the larger value $M_s = 8$ TeV, we find that the LHC is only very weakly sensitive to the UV KK modes, leaving no difference between (a) and (b), and (c) and (d), respectively. Still, the difference between (a), (b) and (c), (d) persists.

Next we compare the results from fixed point gravity using the quenched (18), linear (19) and quadratic (20) forms of the RG running, at fixed transition scale $\Lambda_T$. The variation of results under changes in the RG running allow an estimate for the “RG scheme dependence” in the present setup. The weak scheme dependence of the $s$-dependent amplitudes was already demonstrated in Sec. IV C, see also Fig. 7. We would like to verify this at the level of LHC cross-sections. In Fig. 12 we establish scheme independence for $n = 3$ and $n = 6$

![Graphs showing normalized and un-normalized differential cross sections](image_url)

**FIG. 13** (color online). Normalized differential cross sections $(1/\sigma)d\sigma/dm_{\mu\mu}$ (left column) and un-normalized ones $d\sigma/dm_{\mu\mu}$ (right column) for gravitational di-muon production at the LHC for $n = 2$, 3, and 6 (top, middle, bottom row) with $M_s = 5$ TeV and crossover scale $\Lambda_T = 5$ TeV. Fixed point gravity (FP) with full $s$ dependence (full lines); fixed point gravity in the form-factor approximation with UV cutoff at $m = \Lambda_T$ (full lines); effective theory (EFT) with UV cutoff at $\Lambda_T$ (dotted lines); effective theory in the $s = 0$ approximation (low $s$) and UV cutoff at $\Lambda_T$ (dotted lines).
extra-dimensions, respectively, for several values of the transition scale \( \Lambda_T \). The variation is minute at the low end of the spectrum but increases at higher energies as expected. In all cases the uncertainty due to the scheme is smaller than uncertainties coming from QCD effects or from parton densities, conservatively estimated at \( \sim 15\% \) here. A second point illustrated in Fig. 12 is that the total rates depend strongly on the transition scale \( \Lambda_T \). This is especially true for higher \( n \), as has been pointed out in Sec. IV B, see Fig. 4. The reason for this is that the suppression of amplitudes due to the gravitational fixed point sets in at about \( \Lambda_T \), leading to larger rates for larger \( \Lambda_T \). However, the \( s \) dependence in the signal is mostly independent of \( \Lambda_T \). In the next section we study the normalized distributions in order to obtain a handle on the signals. 

C. Phenomenology

In this section, we discuss signatures of asymptotically safe gravity and large extra dimensions at the LHC. If not stated otherwise, we set \( M_* = \Lambda_T = 5 \text{ TeV} \) and examine distributions with respect to \( m_{\mu\mu} \), the invariant mass of the produced di-muon pair. We also compare our results, and, in particular, the \( m_{\mu\mu} \) dependence, with those from effective field theory \([18,19,25]\), the form-factor approximation \([11]\), and the standard model background.

In Fig. 13, we compute normalized and un-normalized differential cross sections for \( M_* = 5 \text{ TeV} \) and \( \Lambda_T = M_* \) for \( n = 2, 3, 6 \) extra dimensions. Again, we contrast fixed point gravity results with those from effective theory. Comparing the full \( \sqrt{s} \) dependence from (a) asymptotically safe gravity with (b) effective theory (full versus dashed lines), we note that they essentially agree for \( n = 2 \). The reason for this is that the perturbative amplitude is only logarithmically divergent. For \( n = 3 \) and 6 the decay of the differential cross section with invariant mass becomes more pronounced within fixed point gravity. Here, the difference is due to the UV divergence of the perturbative amplitude. Note, in particular, that with increasing number of extra dimensions \( n \), the difference between (a) and (b) grows. Turning to the low-\( s \) approaches, we note that the differential cross section decays more slowly within the form-factor approximation (c) (full black lines) for all \( n \). This comes about because a dynamical suppression with energy sets in only close to \( \sqrt{s} = \Lambda_T \), unlike (a). Also, the form-factor approximation (c) deviates from the effective theory approximation (with \( s = 0 \)) (d) only close to \( \Lambda_T \). Overall, we see that fixed point gravity falls off most quickly with increasing energy \( \sqrt{s} \). Technically, this results from the powers of \( s \) in the denominator of the RG-improved KK sum, not present in the lowest dimensional operator from effective theory.

In Fig. 14 we present a comparison of our data including the SM background. For larger values of \( m_{\mu\mu} \sim M_* \) signal to background \( S/B \) becomes of order 1. Note that the cross section for \( n = 2, 3 \) and \( n = 6 \) are very close to each other. This near degeneracy is a consequence of our choice \( \Lambda_T = M_* \), which is lifted as soon as \( \Lambda_T \neq M_* \), see Fig. 4. In view of perturbative unitarity, our calculations in (32) and the behavior in Fig. 13 indicate a clear improvement over effective theory. Phenomenologically, for the LHC we now know that the onset of fixed point scaling should be distinguishable form, e.g., effective field-theory descriptions. Turning this argument around, the \textit{ad hoc} assumption of an effective field theory is not well suited to analyze LHC data which should be expected to strongly depend on ultraviolet effects. This is even more obvious for string theory signatures \([17]\) with their peak structure dictated by the Veneziano amplitude. But also the two existing descriptions of quantum gravity at the LHC, fixed point gravity and string theory could not look more different in their predictions for the \( m_{\mu\mu} \) distribution. Figure 14 provides a realistic signature for asymptotic safety. Moreover, the differential cross section as a function of \( m_{\mu\mu} \) can be used to distinguish our effect uniquely and study in some detail the transition from the classical to the quantum gravity regime.

VI. DISCUSSION

We studied gravitational di-lepton production at the LHC, provided gravity displays a nontrivial fixed point at high energies \([3,4]\). Previously, it was shown that the RG-improved single graviton amplitude becomes finite \([10]\). Furthermore, with increasing \( n \) the UV Kaluza-Klein gravitons dominate over the IR Kaluza-Klein gravitons, leading to an increased production cross section over estimates from effective theory in the kinematical regime with \( s \ll M_*^2 \).
Here, we have extended this study towards intermediate center-of-mass energies. We have worked out how the RG scale links with the kinematics and the Kaluza-Klein mass and compared several scale identifications such as $(25), (30), (34),$ and $(37).$ We have stressed the importance of incorporating the KK mass, as done in $(25), (30),$ and $(37),$ and evaluated the amplitude via analytic continuation in $s$. We found a suppression of the amplitude at Planckian energies (Fig. 6) compared to the low-energy amplitude. The extra-dimensional Kaluza-Klein gravitons act as messengers of the UV fixed point in the higher-dimensional theory, whose existence leads to a dynamical suppression of the single graviton amplitude on the brane. This phenomenon is dictated by the fixed point, and largely independent of the number of extra dimensions. Furthermore, with increasing $n$, the dynamical suppression sets in at lower scales. This pattern is neither visible within effective theory nor within form-factor approximations as these operate a cutoff which screens the fixed point dynamics of the KK gravitons. Provided the dynamics of the KK modes are taken into account, differential cross sections show a decrease with invariant mass, compatible with multigraviton scattering [36]. For recent developments along these lines within string quantum gravity, see [37,38].

We also compared our results with those from effective theory, where amplitudes depend on an unknown ultraviolet cutoff scale $\Lambda_{kk}$. This scale is fixed once the fundamental theory for gravity is known. Formally, our results reduce to those from effective theory in the limit where the transition scale $\Lambda_T$ becomes asymptotically large. In this case, the running of Newton’s coupling is switched off. Also, the UV cutoff of the effective theory can be expressed through the fundamental scale $\Lambda_T$ and the energy. This map shows that the domain of validity for effective theory varies parametrically with $n$, because the amplitude becomes sensitive to the UV completion already for center-of-mass energies below the fundamental Planck scale.

From an experimental point of view, it will be most important to determine the scale $\Lambda_T$. We saw that the single graviton amplitude is sensitive to $\Lambda_T$, which sets the overall normalization of production cross sections. Interestingly, the amplitude is largely insensitive to the details of the crossover. Slight variations in the result are below other experimental and theoretical uncertainties. Therefore, we expect that the qualitative change of the amplitude due to the fundamental crossover scale $\Lambda_T$ are stable predictions of our scenario. The details of the UV fixed point, such as the momentum dependence of the amplitude, could in principal be studied by careful examination of the distributions with respect to the di-muon invariant mass.

VII. CONCLUSION

Extra-dimensional models with a fundamental Planck scale as low as the electro weak scale offer an excellent opportunity to test the quantization of gravity at colliders. If gravity is asymptotically safe, gravitational interactions become weaker towards high energies. The onset of asymptotic safety is characterized by the energy scale $\Lambda_T$ which marks the transition from Newtonian scaling at low energies to fixed point scaling at high energies. We find that graviton-induced Drell-Yan spectra are well above standard model backgrounds with a significant sensitivity for measuring $\Lambda_T$ and thus conclude that Drell-Yan production has exciting prospects in probing quantum gravity at the LHC.

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Note added.—After finalizing this paper a study of LHC bounds on virtual gravitons appeared which has some overlap with part of the discussions presented above [39].


